Aalto University
School of Science

Decision Programming

for Multi-Stage Optimization under Uncertainty

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(with Juho Andelmin and Fabricio Oliveira)

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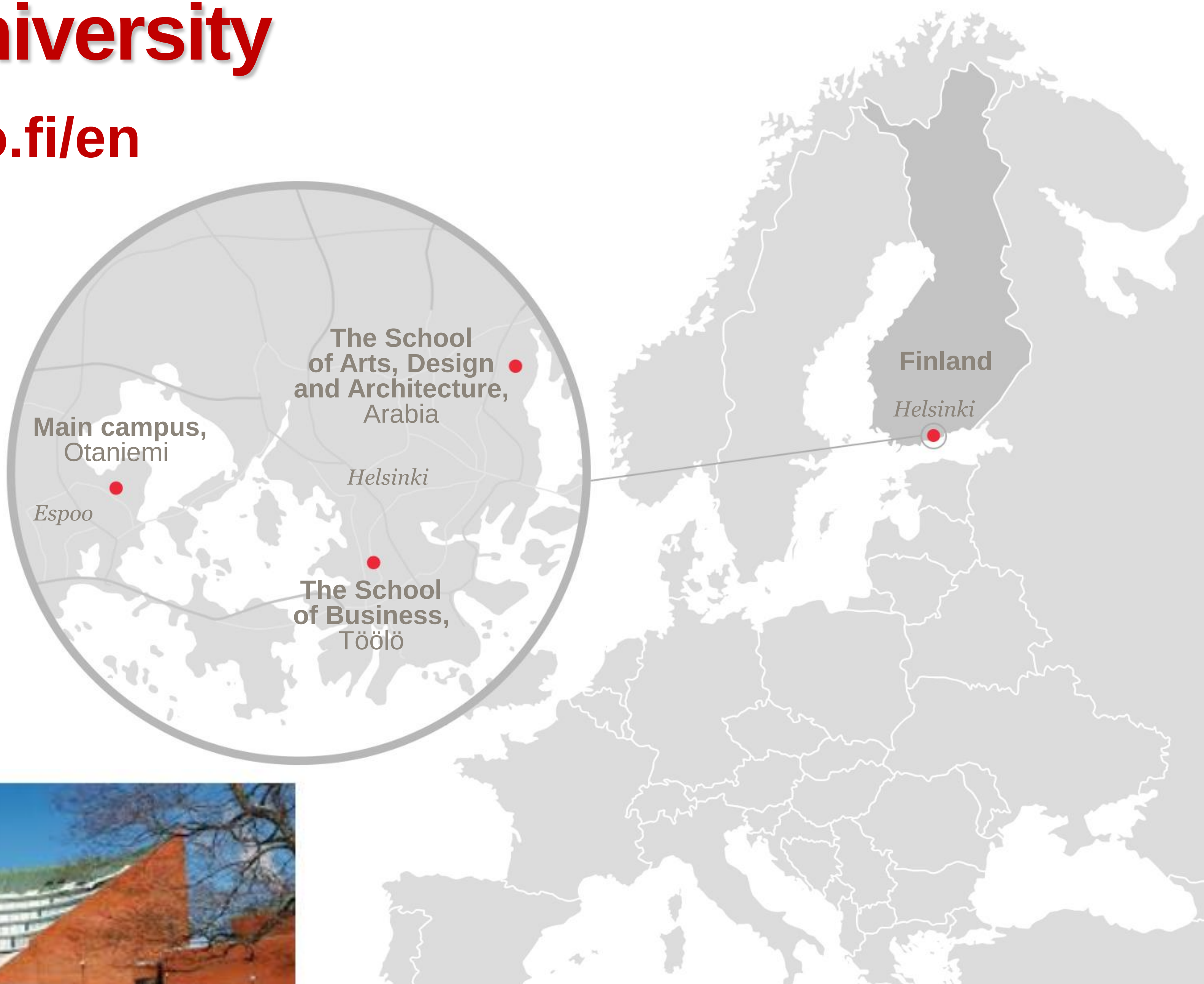


School *emphasis*

School of Engineering

- Arctic technology
- Mechanics and material technology
- Multidisciplinary energy technologies
- Sustainably built environment
- Systems design and production

School of Business

- Microeconomics
- Behavioral finance and financial markets
- Management systems and decision-making
- Strategic management in the global context
- Customer behavior
- New business creation: entrepreneurship, new business models and the service economy

School of Chemical Technology

- Process technology
- Industrial biotechnology
- Biomaterials science
- Metals and minerals recovery processes
- Active and functional materials

School of Science

- Computational and mathematical sciences
- Condensed-matter and materials physics
- Energy sciences
- Computer sciences
- Neuroscience and -technology
- Creating and transforming technology-based business

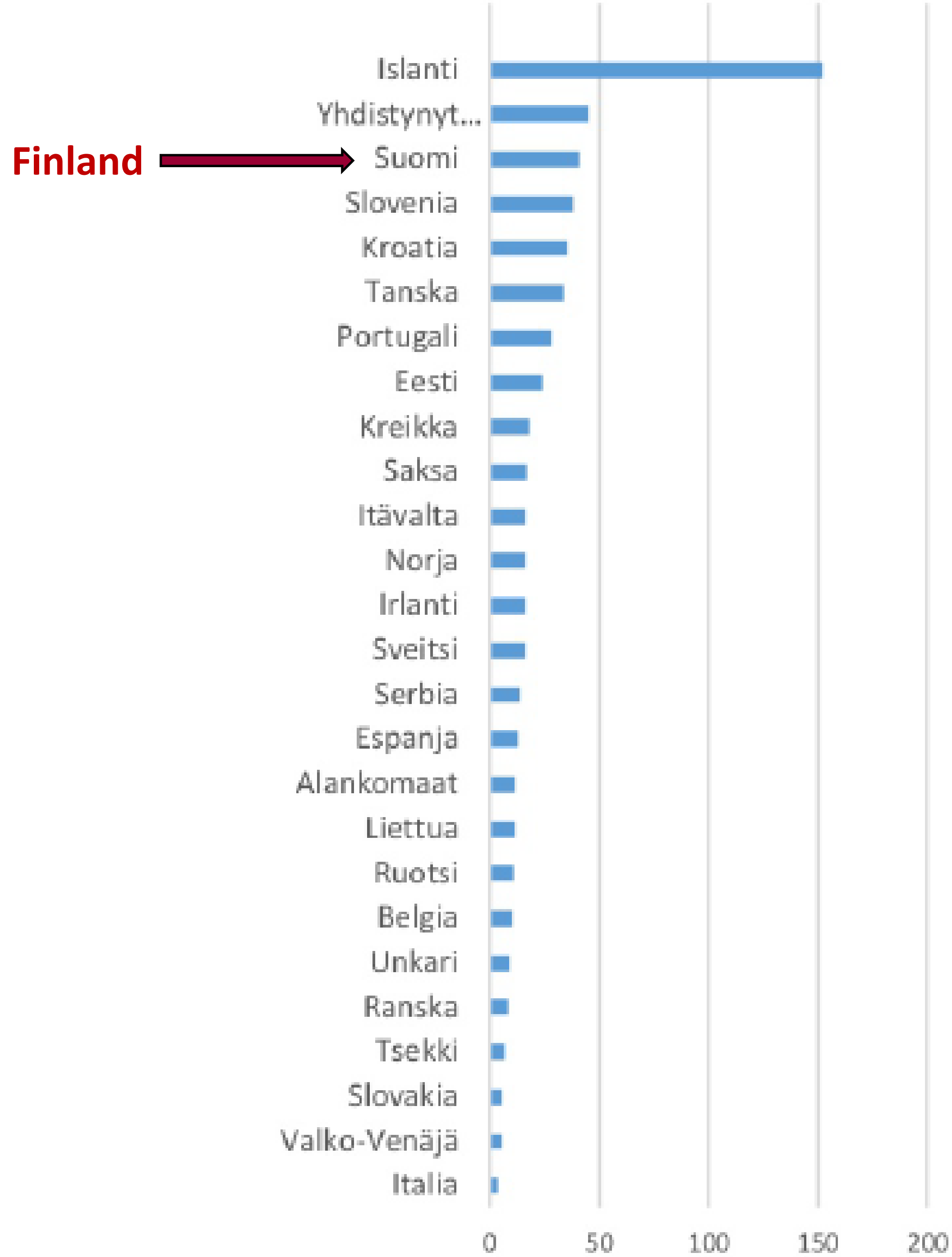
School of Electrical Engineering

- Information and communication technology ICT
- Micro- and nanotechnology
- Energy and environment
- Health and wellbeing

School of Arts, Design and Architecture

- Environmental design
- Meanings and expressions, storytelling
- Artistic research practice
- Culture of sharing: new ways of planning, producing and distribution
- Digital society

European operational research societies per capita



Professors at the Systems Analysis Laboratory

Harri Ehtamo
- Optimization
- Game theory

Fabricio Oliveira (9/2017 ➔)
- Stochastic optimization
- Supply chain mgmt

Ahti Salo
- Risk and decision analysis
- Foresight and innovation

Kai Virtanen
- Operations research in defence

Raimo Hämäläinen (emeritus 8/2016 ➔)
- Environmental decision making
- Systems intelligence

Antti Punkka (12/2017 ➔)
- Decision analysis
- Resource management

Risto Lahdelma (9/2017 ➔, 50% double affiliation)
- Linear programming
- Energy modelling

Ahti Salo

Professor
Doctor of Technology, Director of the Laboratory

Professor Salo has worked extensively on the development of decision analytic methods and their uses in resource allocation, innovation management, risk management, technology foresight, and efficiency analysis. He has published widely in leading international journals (including Management Science and Operations Research) and received awards for his research from the Decision Analysis Society of the Institute for Operations Research and the Management Sciences (INFORMS). He serves on the Editorial Boards of several refereed journals.

Professor Salo has directed a broad range of basic and applied research projects funded by leading industrial firms, industrial federations, and funding agencies. He has been visiting professor at the London Business School, Université Paris-Dauphine and the University of Vienna. He has been the President of the Finnish Operations Research Society (FORS) for two biennial terms. In 2010-11, he served as the European and Middle East representative of the International Activities Committee of INFORMS. In 2010-16, he was jury member of the EDDA Doctoral Dissertation Award of the Association of European Operational Research Societies (EURO), and chaired this jury in 2016. He has been on the Board of the Association of Parliament Members and Researchers (Tutkas) since 1999.

Ahti Salo

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http://sal.aalto.fi/ahti

Ongoing research projects

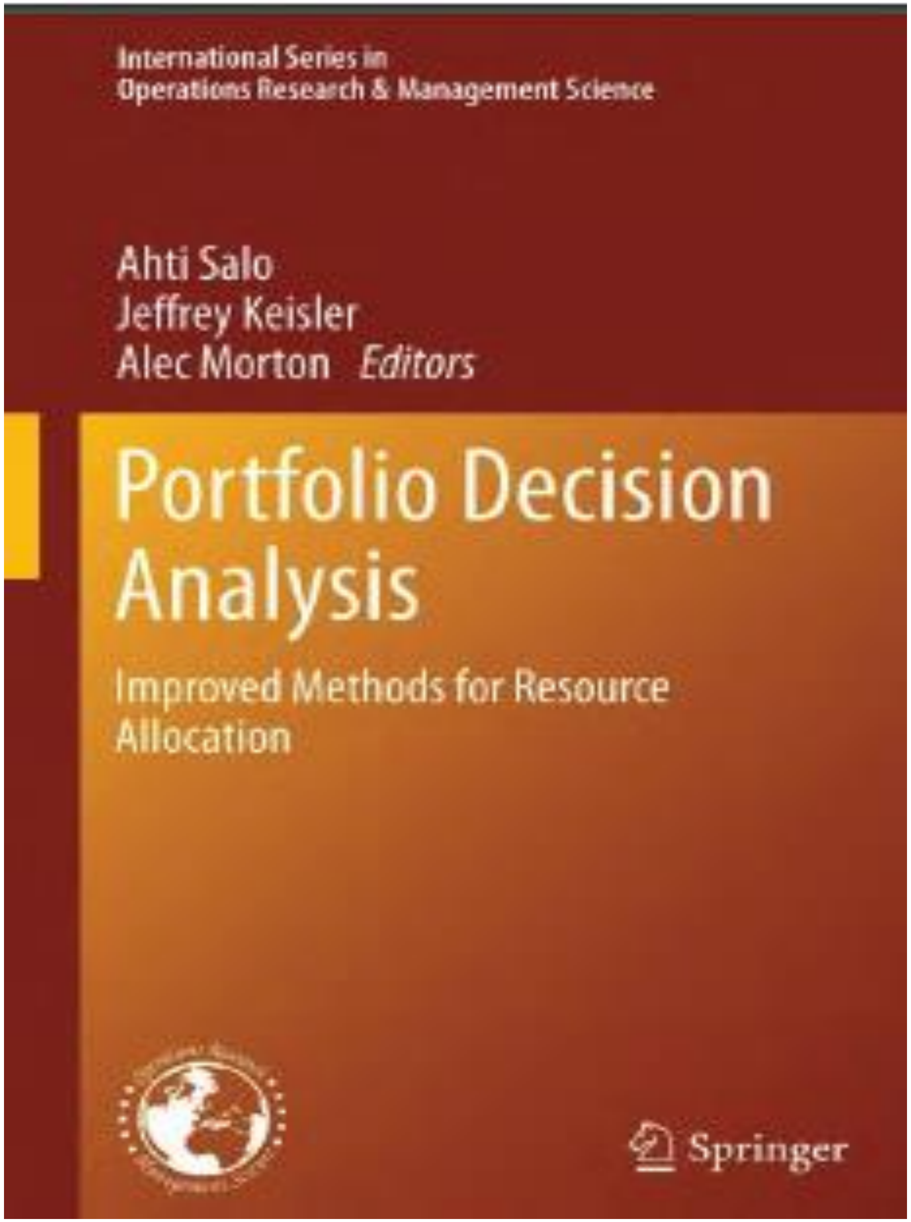
- Platform Value Now
  (Strategic Research Council of the Academy of Finland)

- Adversarial risk analysis in the assessment of weapons systems,
  (Scientific Advisory Board for Defense)

- Probabilistic risk assessment method development and applications
- Systematization of methodologies for safety justification
  (Finnish Research Programmes on Nuclear Power Plant Safety and Nuclear Waste)

- Earlier ones
  - First technology assessment study for the Finnish Parliament
  - Evaluation of national RTD programmes in electronics and telecommunication
  - National foresight study ‘FinnSight 2015’


PLATFORM VALUE Now
Signals Viewpoints Science Program Contact Conference 2017
The Platform Value Now project, funded by Finland's Strategic Research Council, focuses on understanding the fast emerging platform ecosystems, their value creation dynamics and requirements of the supportive institutional environment.
www.platformvaluenow.org



"By Portfolio Decision Analysis (PDA) we mean a body of theory, methods, and practice

which seeks to help decision makers make informed multiple selections from a discrete set of alternatives

through mathematical modeling that accounts for relevant constraints, preferences, and uncertainties."

Winner of the 2013 Publication Award of the Decision Analysis Society of the Institute for Operations Research and the Management Sciences (INFORMS)

Influence diagrams

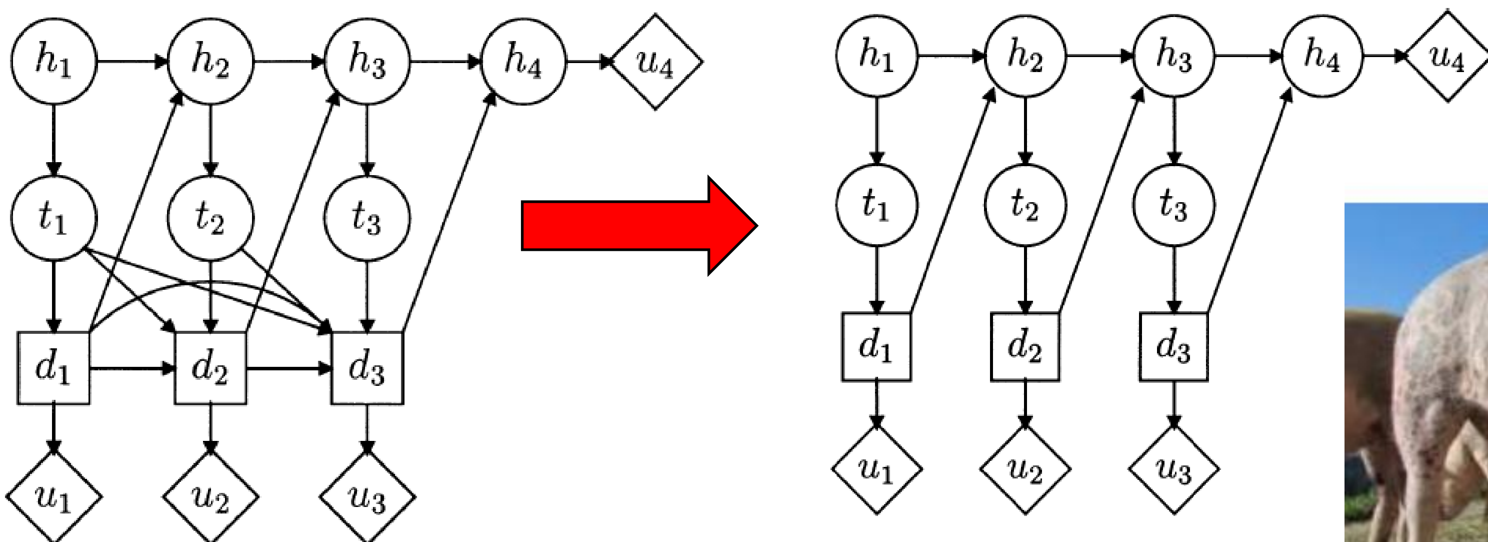
- Graphical representations of
  1. chance, decision and value nodes
  2. arcs which indicate dependencies between these

- Common solution approaches (Howard & Matheson, 2005; Bielza et al 2010)
  1. Form the decision tree and it solve with dynamic programming
  2. Eliminate nodes (after arc reversals, if needed)

- Assumptions and limitations
  - Earlier decisions must be recalled ('no forgetting')
  - Risk constraints cannot be easily handled
  - Problems of portfolio decision analysis become unwieldy

Limited information influence diagrams (LIMID)

- Pigs are grown for four months and then sold
- Diagnosing t_i and treating pigs d_j through injections (Lauritzen and Nilsson, 2001)



Pig farm problem

- Pigs sold after 4 months,
  diseased ones for 300 DKK, healthy for 1000 DKK
- A pig has the disease after the first month with 10 % probability
- Montly tests: diseased pigs indicated with 80% probability
  and healthy 90% probability
- Based on tests, pigs can be injected at for 100 DKK
- If injected, a healthy pig develops the disease in the following month with 10% probability; and without injection, with 20%
- If injected, a diseased pig remains diseased in the following month with 50% probability%; without injection, with 90% probability

Janne Gustafsson

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Past Awards

2005

DAS Student Paper Award: Winner(s)

Winning material: Contingent Portfolio Programming for the Management of Risky Projects

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Contingent Portfolio Programming for the Management of Risky Projects

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Methods for selecting a research and development (R&D) project portfolio have attracted considerable interest among practitioners and academics. This notwithstanding, the industrial uptake of these methods has remained limited, partly because of the difficulties of capturing relevant concerns in R&D portfolio management. Motivated by these difficulties, we develop contingent portfolio programming (CPP), which extends earlier approaches in that it (i) uses states of nature to capture exogenous uncertainties, (ii) models resources through dynamic state variables, and (iii) provides guidance for the selection of an optimal project portfolio that is compatible with the decision maker's risk attitude. Although CPP is presented here in the context of R&D project portfolios, it is applicable to a variety of investment problems where the dynamics and interactions of investment opportunities must be accounted for.

Subject classifications: research and development: project selection; decision analysis: theory; programming: linear, applications.
Area of review: Decision Analysis.
History: Received November 2002; revision received November 2003; accepted July 2004.

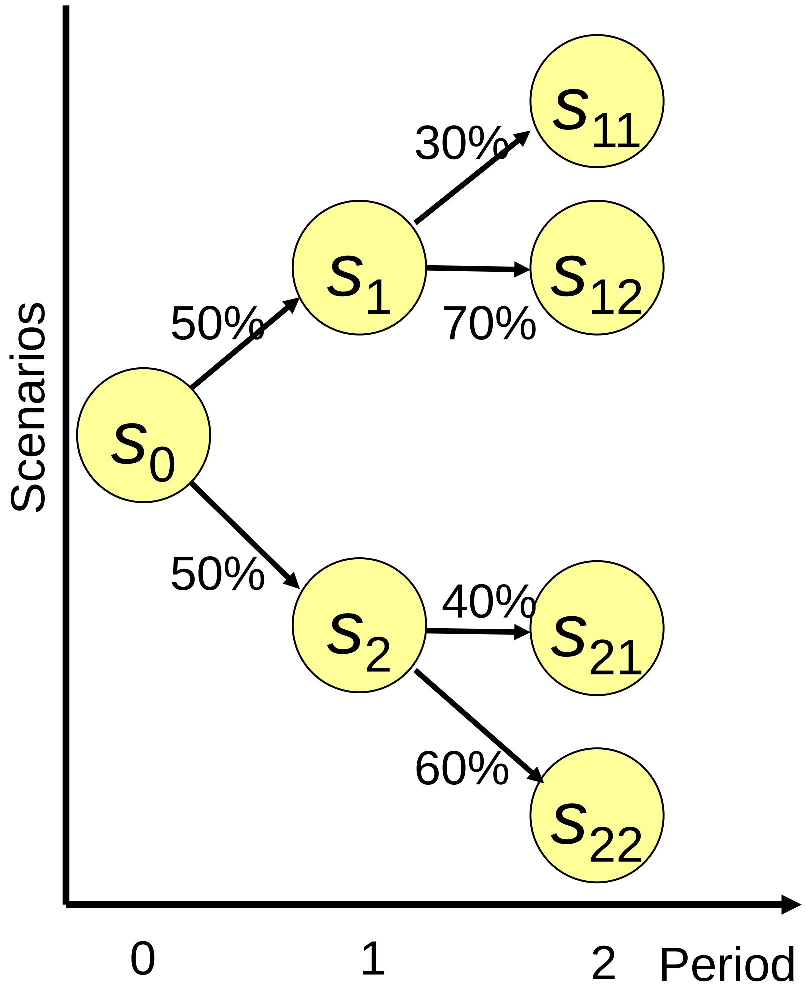
Characteristics of PDA in R&D project selection

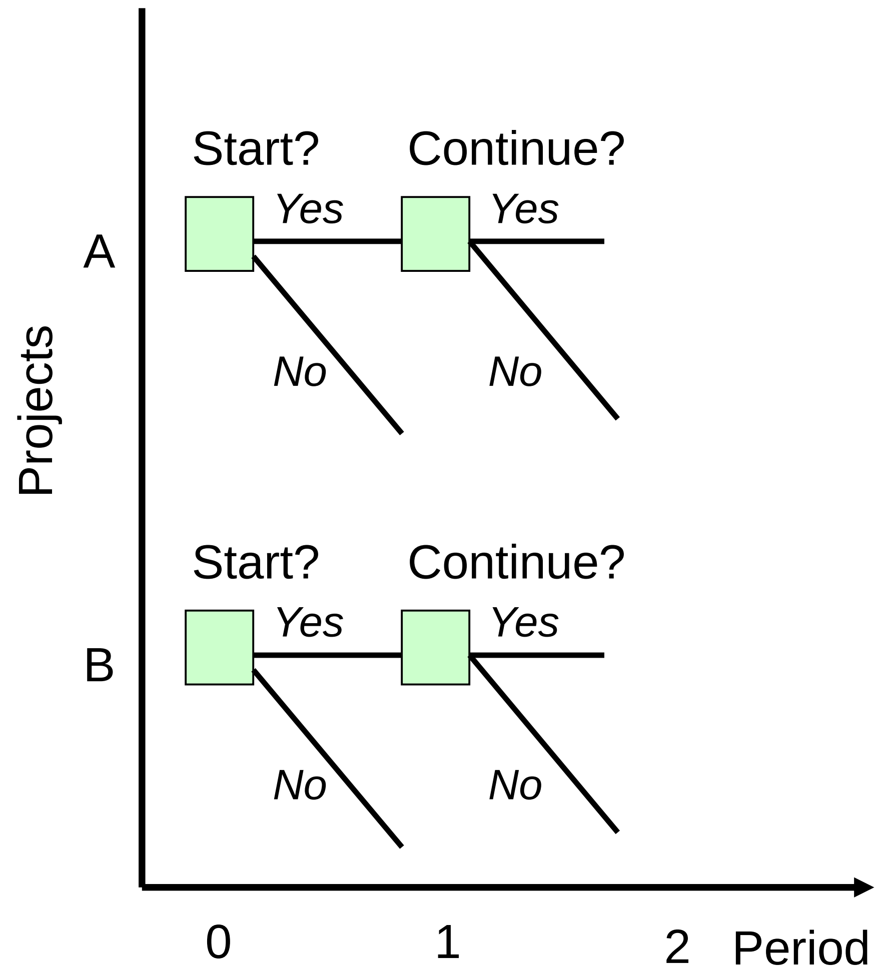
- Projects
  - Can be started in different ways (not only ‘go/no-go’ decisions)
  - Can offer opportunities for follow-up investments
  - Can involve interdependencies (synergies, cannibalization)
- Uncertainties
  - **Exogenous** – do not depend on project decisions (e.g., total market size)
  - **Endogenous** – are influenced by project decisions (e.g., time-to-market)

An example of Contingent Portfolio Programming (CPP)

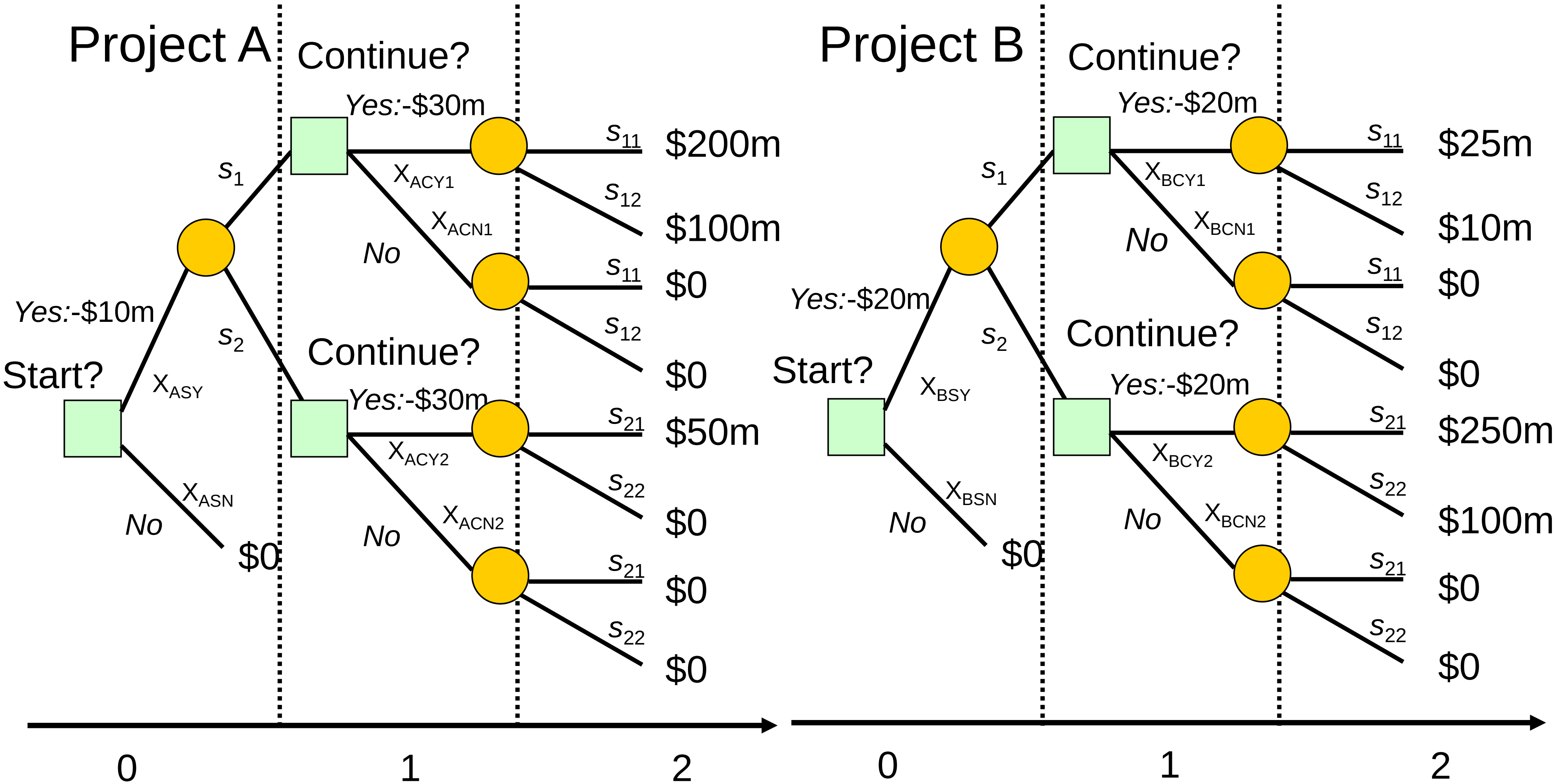
- A portfolio of two projects: A and B
  - One or both can be started at $t=0$
  - Continued investments possible at $t=1$
  - If completed, projects yield cash flows at $t=2$
- Cash flows from projects are contingent on scenarios
- Money as the only resource
  - Initial budget b = \$100m
  - Leftover budget invested at the risk free interest rate r = 8%

Scenario tree and decisions

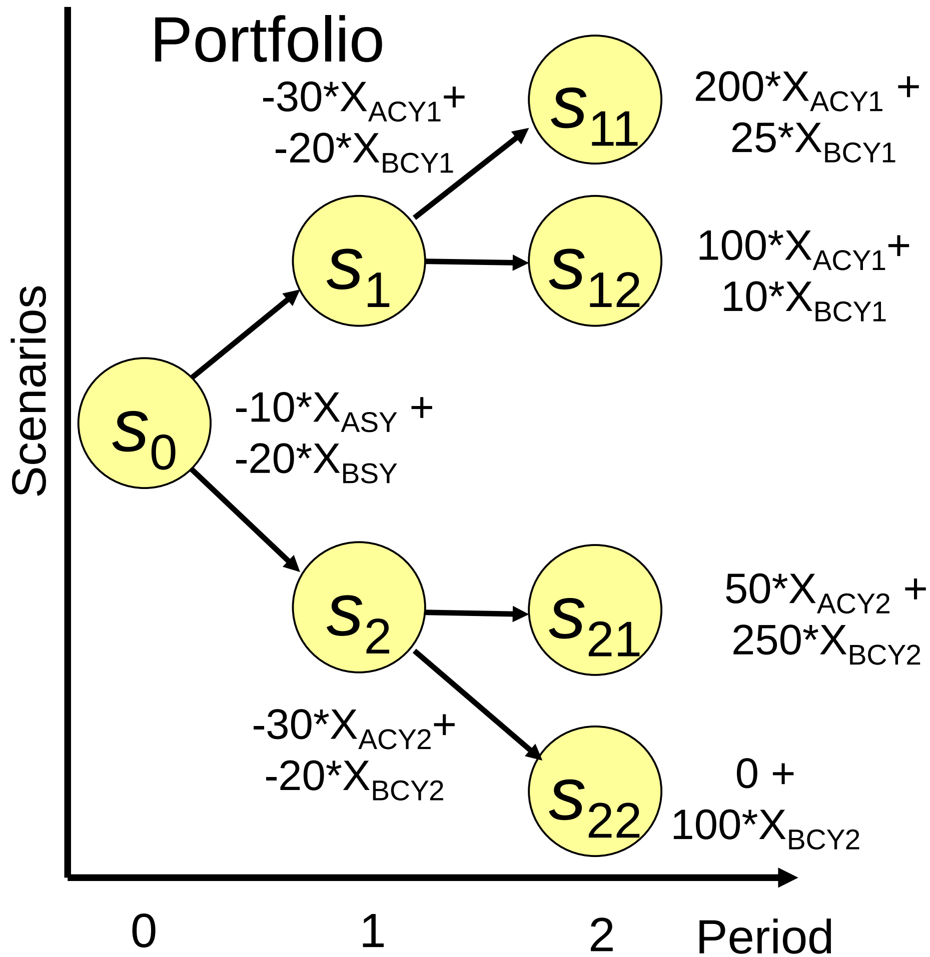




Parallel decision trees



Cash flows for the portfolio of two projects



- Maximize NPV certainty equivalent
  - Approximated by the linear model

    $E[NPV] - k \cdot absdev[NPV]$
  - *absdev* = mean absolute deviation (model remains linear)
- Subject to
  - Consistency constraints
  - Resource constraints
  - Deviation constraints
- Note: Project cash flows are negatively correlated

Constraints

<u>Consistency constraints:</u>

$X_{ASY} + X_{ASN} = 1$

$X_{ACY1} + X_{ACN1} = X_{ASY}$

$X_{ACY2} + X_{ACN2} = X_{ASY}$

$X_{BSY} + X_{BSN} = 1$

$X_{BCY1} + X_{BCN1} = X_{BSY}$

$X_{BCY2} + X_{BCN2} = X_{BSY}$

<u>Resource constraints:</u>

$-10 \cdot X_{ASY} - 20 \cdot X_{BSY} + 100 - RS_{s0} = 0$

$-30 \cdot X_{ACY1} - 20 \cdot X_{BCY1} + 1.08 \cdot RS_{s0} - RS_{s1} = 0$

$-30 \cdot X_{ACY2} - 20 \cdot X_{BCY2} + 1.08 \cdot RS_{s0} - RS_{s2} = 0$

$200 \cdot X_{ACY1} + 25 \cdot X_{BCY1} + 1.08 \cdot RS_{s1} - RS_{s11} = 0$

$100 \cdot X_{ACY1} + 10 \cdot X_{BCY1} + 1.08 \cdot RS_{s1} - RS_{s12} = 0$

$50 \cdot X_{ACY2} + 250 \cdot X_{BCY2} + 1.08 \cdot RS_{s2} - RS_{s21} = 0$

$100 \cdot X_{BCY2} + 1.08 \cdot RS_{s2} - RS_{s22} = 0$

Resource surplus variables *RS* indicate how much resources there are after each scenario

Deviation constraints

$$NPV_{sp}^{r}(\mathbf{X}) - ENPV^{r}(\mathbf{X}) - \Delta NPV_{sp}^{r+} + \Delta NPV_{sp}^{r-} = 0$$

<u>First deviation constraint:</u>

$$\underbrace{-10 \cdot X_{ASY} - 20 \cdot X_{BSY} + 1/1.08 \cdot (-30 \cdot X_{ACY1} - 20 \cdot X_{BCY1}) + 1/1.08^2 \cdot (200 \cdot X_{ACY1} + 25 \cdot X_{BCY1})}_{NPV_{s11}(\mathbf{X})}$$

$$\underbrace{\begin{aligned} &- [-10 \cdot X_{ASY} - 20 \cdot X_{BSY} \\ &+ 50\% \cdot 1/1.08 \cdot (-30 \cdot X_{ACY1} - 20 \cdot X_{BCY1}) \\ &+ 50\% \cdot 1/1.08 \cdot (-30 \cdot X_{ACY2} - 20 \cdot X_{BCY2}) \\ &+ 50\% \cdot 30\% \cdot 1/1.08^2 \cdot (200 \cdot X_{ACY1} + 25 \cdot X_{BCY1}) \\ &+ 50\% \cdot 70\% \cdot 1/1.08^2 \cdot (100 \cdot X_{ACY1} + 10 \cdot X_{BCY1}) \\ &+ 50\% \cdot 40\% \cdot 1/1.08^2 \cdot (50 \cdot X_{ACY2} + 250 \cdot X_{BCY2}) \\ &+ 50\% \cdot 60\% \cdot 1/1.08^2 \cdot (100 \cdot X_{BCY2})] \end{aligned}}_{ENPV(\mathbf{X})}$$

$$- \Delta NPV_{sp11}^{+} + \Delta NPV_{sp11}^{-} = 0$$

Deviation variables

ΔNPV's indicate by how much the present value in each scenario path deviates from the expected NPV

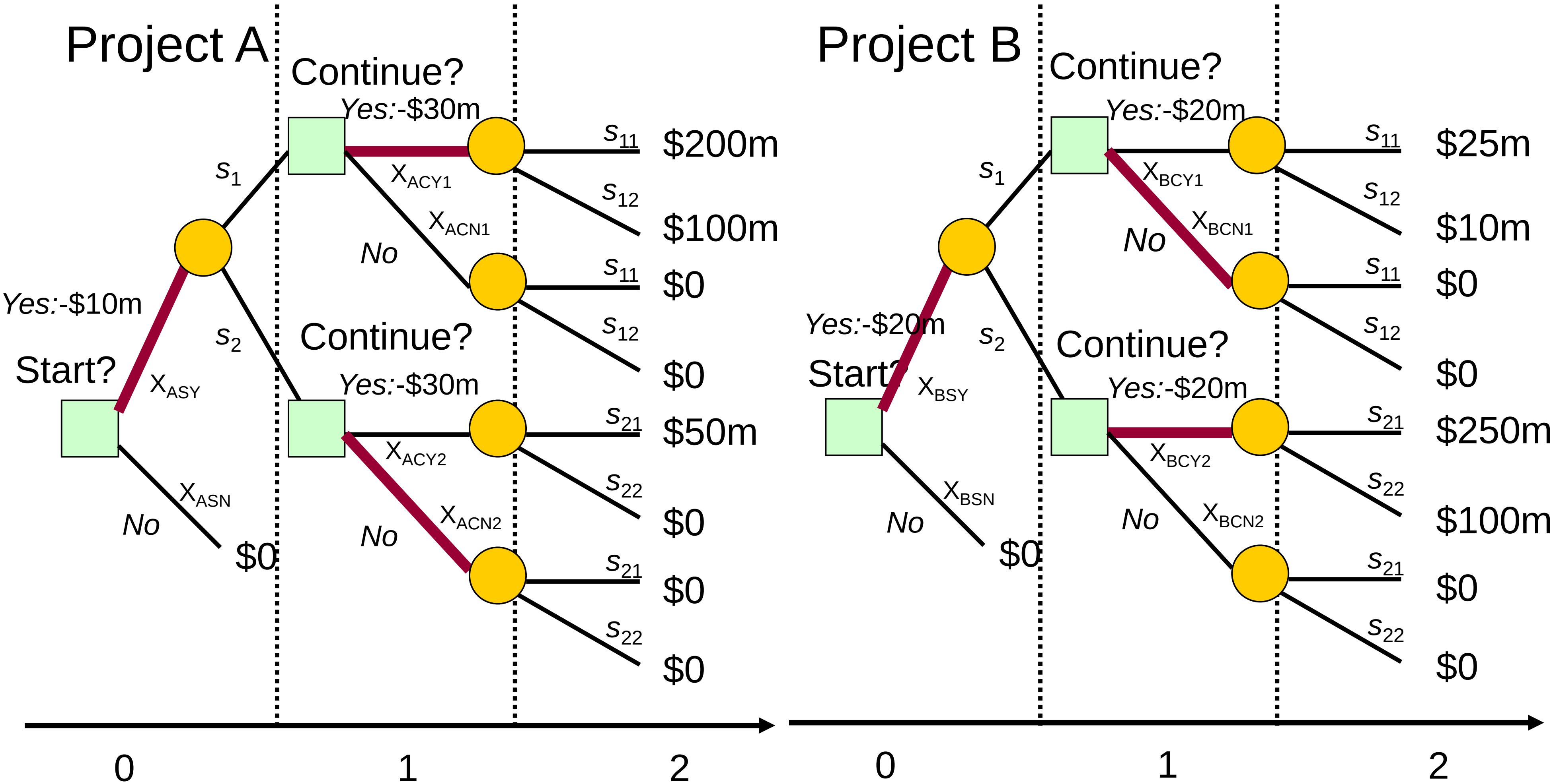
Objective function

$$\text{Maximize}\;\; \underbrace{\begin{cases} -10\cdot X_{ASY} - 20\cdot X_{BSY} + \\ 50\% \cdot 1/1.08 \cdot (-30\cdot X_{ACY1} - 20\cdot X_{BCY1}) + \\ 50\% \cdot 1/1.08 \cdot (-30\cdot X_{ACY2} - 20\cdot X_{BCY2}) + \\ 50\%\cdot 30\% \cdot 1/1.08^2 \cdot (200\cdot X_{ACY1} + 25\cdot X_{BCY1}) + \\ 50\%\cdot 70\% \cdot 1/1.08^2 \cdot (100\cdot X_{ACY1} + 10\cdot X_{BCY1}) + \\ 50\%\cdot 40\% \cdot 1/1.08^2 \cdot (50\cdot X_{ACY2} + 250\cdot X_{BCY2}) + \\ 50\%\cdot 60\% \cdot 1/1.08^2 \cdot (100\cdot X_{BCY2}) \end{cases}}_{ENPV}$$

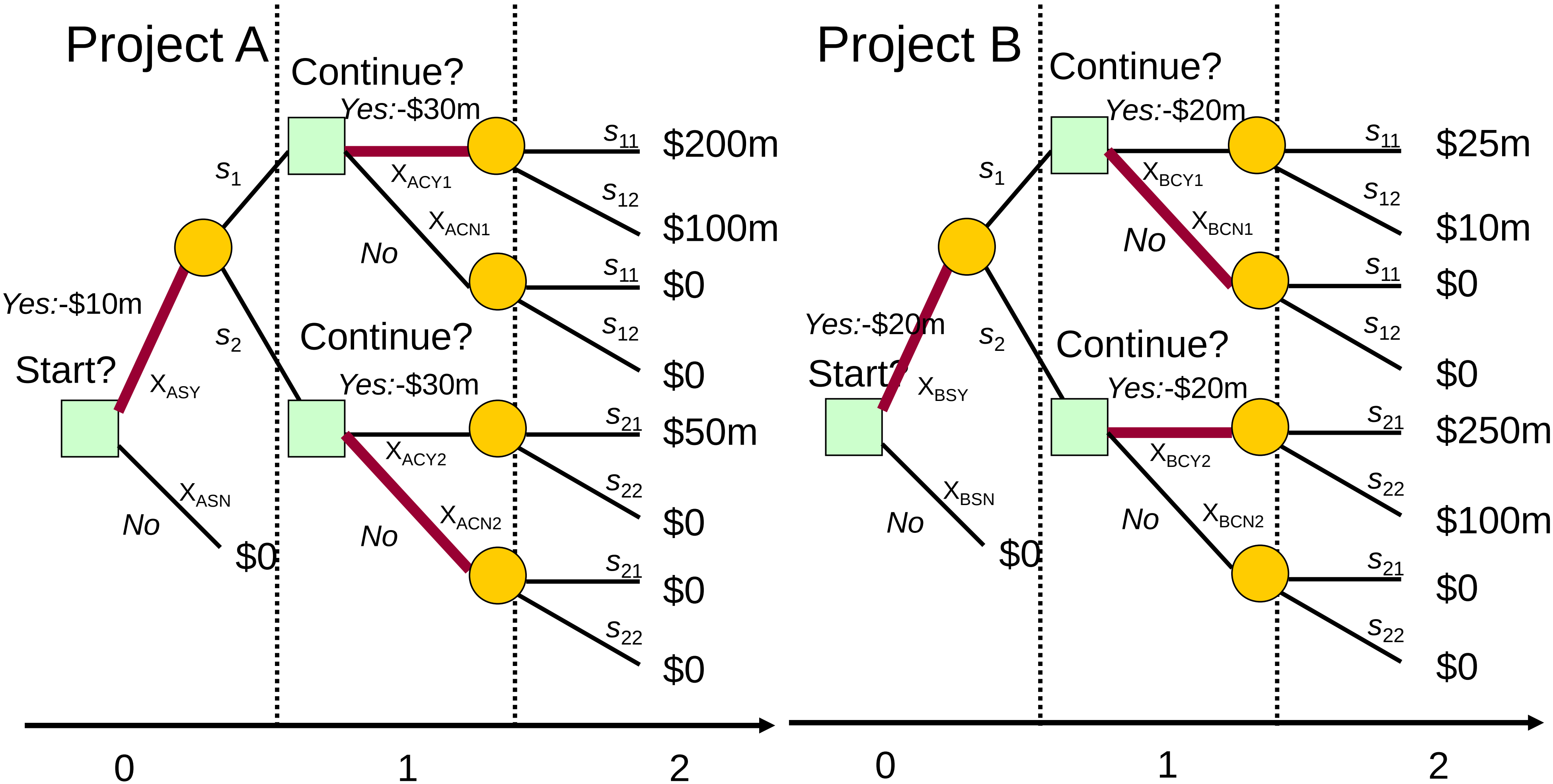
$$k \longrightarrow \; - 0.25 \cdot [$$

$$absdev \begin{cases} 50\%\cdot 30\% \cdot (\Delta NPV_{sp11}^{+} + \Delta NPV_{sp11}^{-}) + \\ 50\%\cdot 70\% \cdot (\Delta NPV_{sp12}^{+} + \Delta NPV_{sp12}^{-}) + \\ 50\%\cdot 40\% \cdot (\Delta NPV_{sp21}^{+} + \Delta NPV_{sp21}^{-}) + \\ 50\%\cdot 60\% \cdot (\Delta NPV_{sp22}^{+} + \Delta NPV_{sp22}^{-})] \end{cases}$$

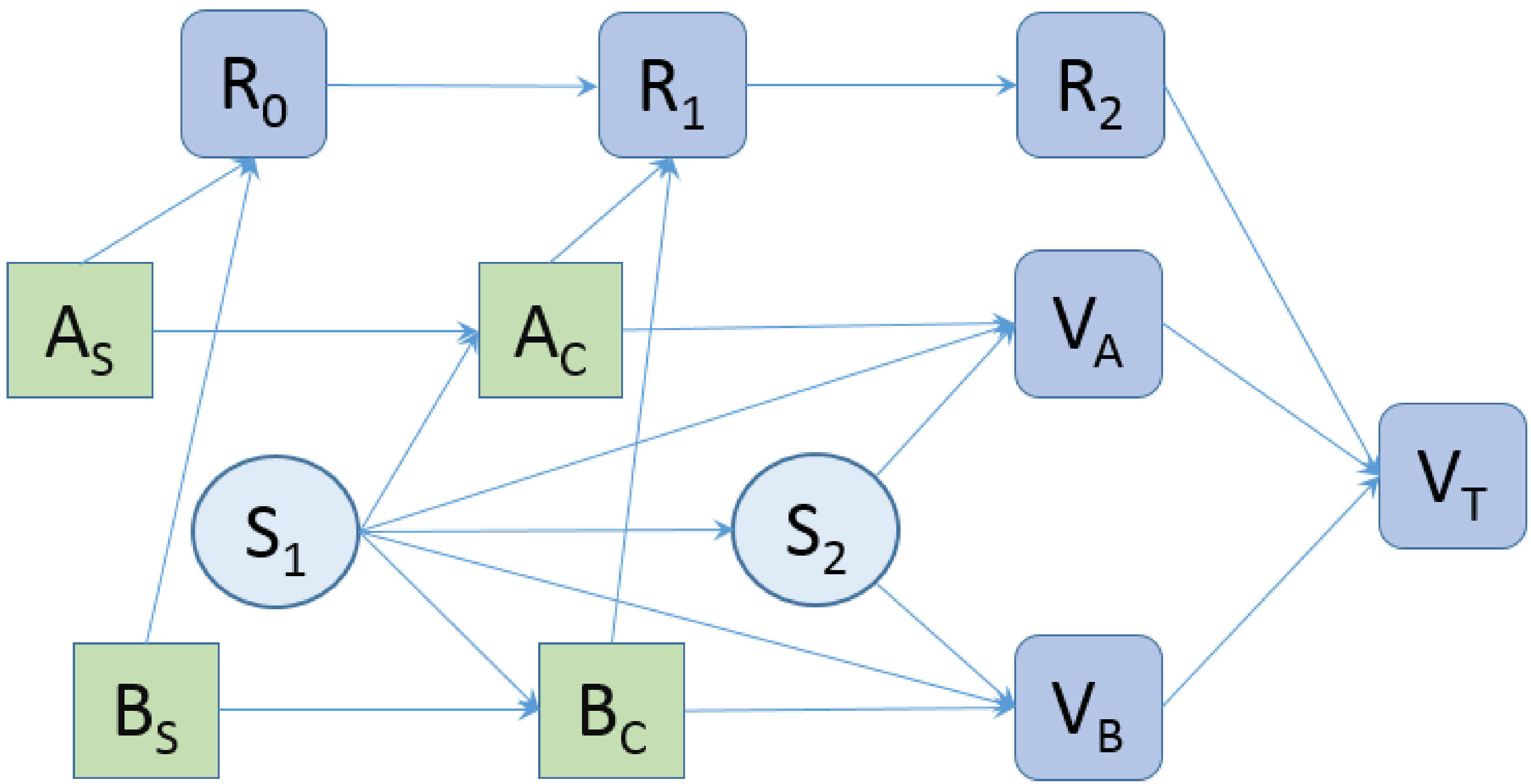
Solution in decision trees



Solution in decision trees



Influence diagram for this CPP example



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Decision Support

Scenario-based portfolio model for building robust and proactive strategies

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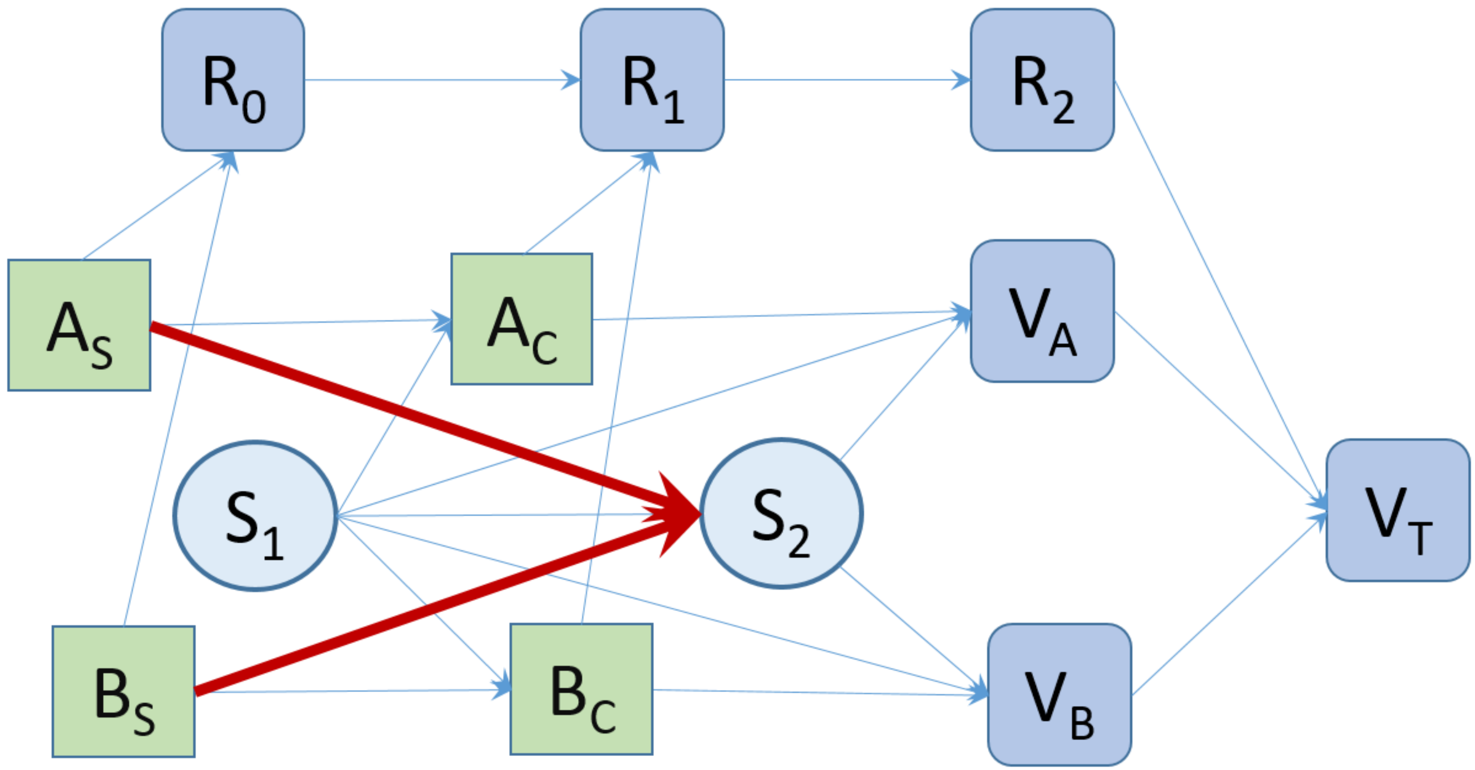
ABSTRACT

In order to address major changes in the operational environment, companies can (i) define scenarios that characterize different alternatives for this environment, (ii) assign probabilities to these scenarios, (iii) evaluate the performance of strategic actions across the scenarios, and (iv) choose those actions that are expected to perform best. In this paper, we develop a portfolio model to support the selection of such strategic actions when the information about scenario probabilities is possibly incomplete and may depend on the selected actions. This model helps build a strategy that is *robust* in that it performs relatively well in view of all available probability information, and *proactive* in that it can help steer the future as reflected by the scenarios toward the desired direction. We also report a case study in which the model helped a group of Nordic, globally operating steel and engineering companies build a platform ecosystem strategy that accounts for uncertainties related to markets, politics, and technological development.

The CPP example with endogenous uncertainties



Decision analysis + mathematical programming

- A framework for modelling endogenous uncertainties in CPP and stochastic optimization
- … but **more generally**, an approach for using **mixed integer linear programming (MILP)** to solve multi-stage decision problems which can be represented as influence diagrams with limited information
- Extensions
  - The 'no-forgetting' assumption can be forgotten
  - Many kinds of logical, resource and risk constraints can be handled
- Does not depend on dynamic programming as a solution approach
- Problems of realistic size can be solved thanks to the MILP formulation

Definitions (1/3)

- Decision problem represented as an acyclic network $G=(V,A)$
- V consists of chance $c \in C$, decision $d \in D$ and value nodes $u \in U$
- There are $n = |C| + |D|$ chance and decision nodes
- Chance and decision nodes $i \in C \cup D$ have a finite set of states $s_i \in S_i$
- Arcs $(i,j) \in A$ represent dependencies between nodes
- **<u>Information set</u>** $I(j)$ consists of nodes from which there is an arc to j
- **<u>Information state</u>** $s_{I(j)} \in S_{I(j)} = \Pi_{i \in I(j)} S_i$ is a combination of states s_i for nodes in the information set of predecessors $i \in I(j)$

Definitions (2/3)

- G is acyclic $\Rightarrow$ nodes labelled so that $(i,j) \in A \Rightarrow i < j$
- Each node $i \in C \cup D$ is associated with a variable X_i
- At chance nodes $c \in C$, the states of X_c occur based on conditional probabilities

$$\boldsymbol{P}(X_c = s_c \mid X_i = s_i, i \in I(c))$$

- At decision nodes $d \in D$, **local decision strategies** map information states to decisions $Z_d : S_{I(d)} \rightarrow S_d$

$$\boldsymbol{P}(X_d = s_d \mid X_i = s_i, i \in I(d), Z_d) = 1 \Leftrightarrow Z_d(s_{I(d)}) = s_d$$

- A **(decision) strategy** $Z = \Pi_{d \in D} Z_d$ is a combination of local decision strategies for all decision nodes $d \in D$

Definitions (3/3)

- At the value node $u \in U$, the consequences of decisions and chance events are given by function $X_u: S_u \rightarrow \mathfrak{C}$
- Utility function $\boldsymbol{U}: \mathfrak{C} \rightarrow \mathfrak{R}$ gives the real-valued utility of consequences
- A **path** of length $k \leq n$ is a sequence of states $(s_1, s_2, \ldots, s_k)$ such that $s_i \in S_i$, $i = 1, \ldots k$
- Paths of length n are denoted by $s \in S = \Pi_{i \in C \cup D} S_i$
- If $s \in S$, then $s_i \in S_i$ is the state of node i along this path s

Path probabilities

- For strategy Z, the probability of path $s \in S$ is

$$\boldsymbol{P}(X_i{=}s_i,\ i{=}1,\ldots k \mid Z) = \boldsymbol{P}(X_k{=}s_k \mid X_i{=}s_i,\ i{=}1,\ldots,k\text{-}1, Z) \times \boldsymbol{P}(X_i{=}s_i,\ i{=}1,\ldots,k\text{-}1 \mid Z),$$

where for chance nodes $k \in C$ the first term is

$$\boldsymbol{P}(X_k{=}s_k \mid X_i = s_i,\ i \in I(k))$$

and for decision nodes $k \in D$ it is

$$\boldsymbol{P}(X_k{=}s_k|X_i{=}s_i,\ i \in I(k), Z) = 1 \Leftrightarrow Z_k(s_i,\ i \in I(k)) = s_k$$

Towards an optimization formulation

- Fix any decision strategy Z and scenario path $s \in S$
- Define binary variables $z(s_d \mid s_{I(d)}) \in \{0,1\}$ so that

$$Z_d(s_{I(d)}) = s_d \iff z(s_d \mid s_{I(d)}) = 1 \qquad (1)$$

- Put $\pi_0(s) = 1$ and define **path probabilities** $\pi_k(s)$ recursively so that for chance nodes $k \in C$

$$\pi_k(s) = \boldsymbol{P}(X_k = s_k \mid X_i = s_i,\ i \in I(k))\ \pi_{k-1}(s) \qquad (2)$$

and for decision nodes $k \in D$

$$\pi_k(s) = \pi_{k-1}(s), \qquad \text{if } z(s_k \mid s_{I(k)}) = 1$$
$$\pi_k(s) = 0, \qquad \text{otherwise}$$

Towards a MILP formulation

Theorem: Let Z be a decision strategy and choose a path $s \in S$.
If the constraints (1), (2) and

$$0 \leq \pi_d(s) \leq \pi_{d-1}(s) \qquad (3)$$

$$\pi_d(s) \leq z(s_d \mid s_{I(d)}), \qquad (4)$$

$$\pi_d(s) \geq \pi_{d-1}(s) + z(s_d \mid s_{I(d)}) - 1 \qquad (5)$$

hold for $z(s_d \mid s_{I(d)})$, $d \in D$, $s_d \in S_d$, $s_{I(d)} \in S_{I(d)}$ and $\pi_k(s)$, $k=1,\ldots,n$, then

$$\pi_k(s) = \boldsymbol{P}(X_i = s_i,\ i=1,\ldots k \mid Z),\ k=1,\ldots,n$$

Optimal decision strategies

Corollary: The strategy Z^* which maximizes the decision maker's expected utility is the solution to the optimization problem

$$\max \sum \pi_n(s)\, \boldsymbol{U}[X_u(s_i,\, i \in I(u))],$$

where the summation is taken over all paths $s \in S$ subject to constraints (1)-(5).

Notes:

- Path probabilities needed for paths of full length n only
- Utilities for consequences can be pre-evaluated
- The chance component $p(s) = \Pi_{c\in C}\, \boldsymbol{P}(X_c = s_c \mid X_i = s_i,\, i \in I(c))$ of $\pi_n(s)$ can be pre-evaluated, too

Constraints on full paths only

Proposition: Let Z be a decision strategy and choose a path $s \in S$. If the constraints (1), (2) and

$$0 \leq \pi(s) \leq p(s), \qquad (3')$$

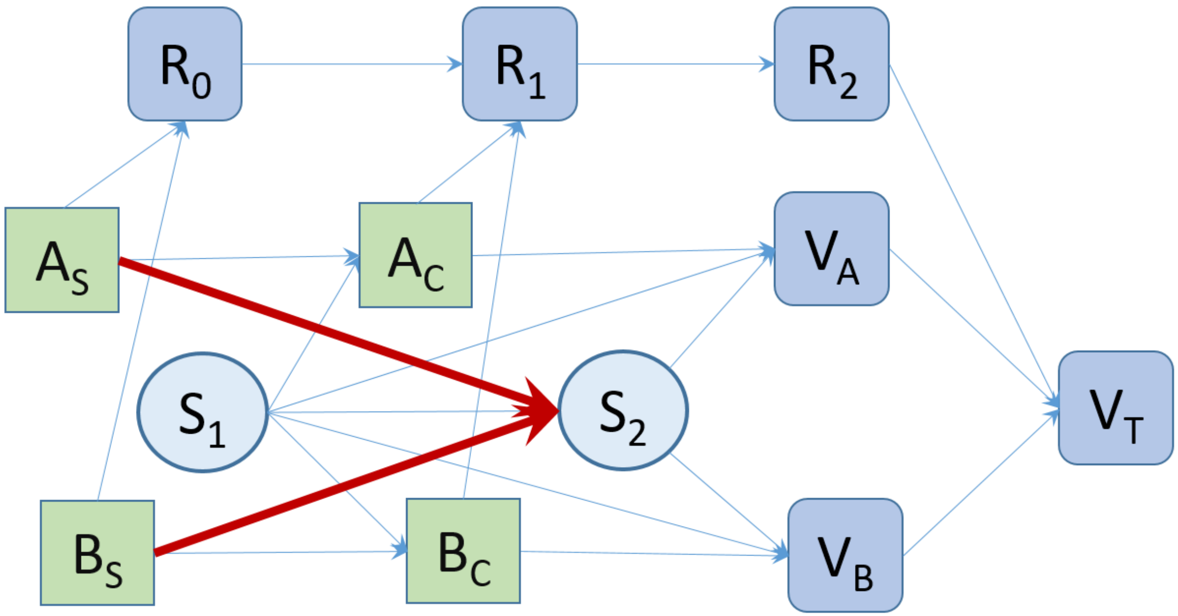
$$\pi(s) \leq z(s_d \mid s_{I(d)}) \qquad (4')$$

$$\pi(s) \geq p(s) + \sum_{d \in D} z(s_d \mid s_{I(d)}) - |D| \qquad (5')$$

hold for $s \in S$, then

$$\pi(s) = \boldsymbol{P}(X_i = s_i,\ i=1,\ldots k \mid Z).$$

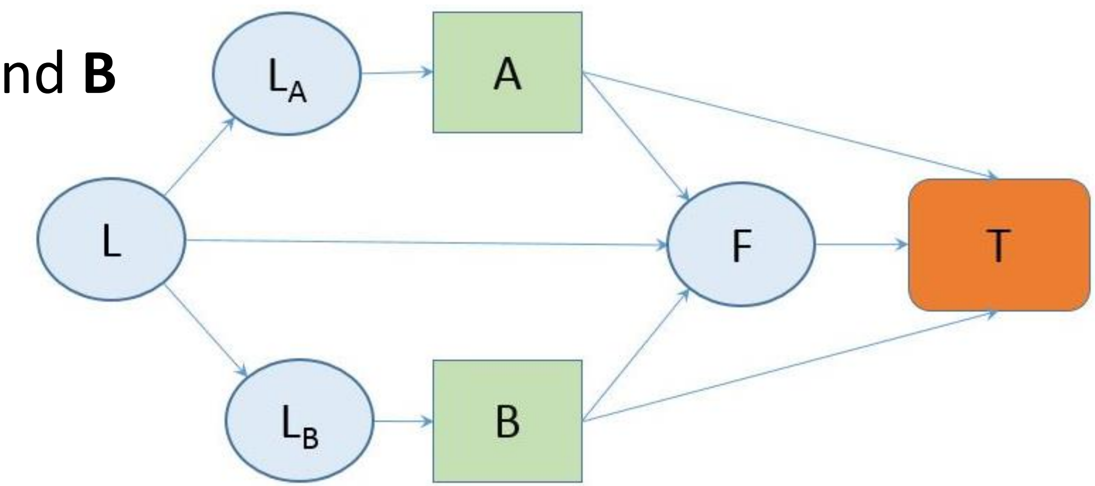
Endogenous uncertainties in CPP

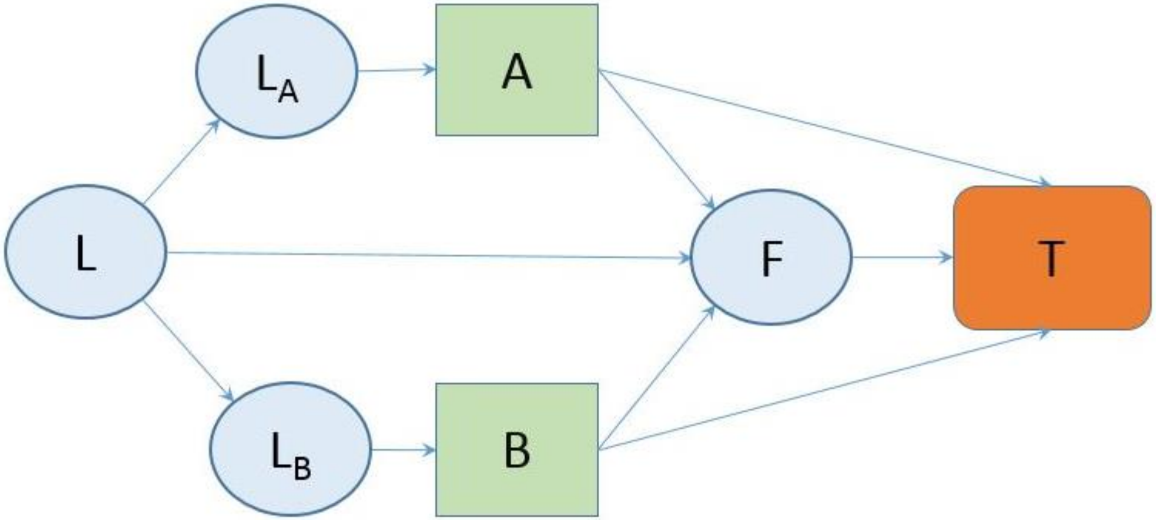


- Decision and chance nodes in sequence $(A_S, B_S, S_1, S_2, A_C, B_C)$
- Scenario paths are sequences of states $(a_S, b_S, s_1, s_2, a_C, b_C)$
- Probabilities $\boldsymbol{Pr}(s_2 | a_S, b_S, s_1)$ needed in defining $\pi_4(\boldsymbol{s})$

Forgetting the “no forgetting” assumption

- An uncertain load **L** on a structure which may fail **F**
- The structure can be fortified through actions **A** and **B**, informed by conditionally independent measurements $\mathbf{L_A}$ and $\mathbf{L_B}$
- Decision **A** is <u>not known</u> when making decision **B** and vice versa
  $\Rightarrow$ There is no decision tree formulation for this problem
- 4 local strategies at **A** and **B**
- 4x4 = 16 strategies





Priors		Conditionals for A			Conditionals for B			Utilities		Costs	
L		L	LA+	LA-	L	LB+	LB-	F	U(F)		Costs
+	0,6	+	0,8	0,2	+	0,9	0,1	ok	100	CA	60
-	0,4	-	0,1	0,9	-	0,05	0,95	not ok	0	CB	40

P(F|L,A,B)	(+,+,+)	(+,+,-)	(+,-,+)	(-,+,+)	(+,-,-)	(-,+,-)	(-,-,+)	(-,-,-)
ok	0,90	0,80	0,75	0,96	0,02	0,90	0,95	0,70
not ok	0,10	0,20	0,25	0,04	0,98	0,10	0,05	0,30

- The optimum can be computed with linear programming

L			LA			A			LB			B			F		A?	B?	Decision	p	U(F)-CA-CB	E[U(F)-CA-CB]	CA+CB
L	+	0,6	LA	+	0,8	A	Yes	0	LB	+	0,9	B	Yes	1	ok	0,90	0	1	0	0,39	60	0,00	0
															not ok	0,10	0	1	0	0,04	-40	0,00	0
													No	0	ok	0,80	0	0	0	0,35	100	0,00	0
															not ok	0,20	0	0	0	0,09	0	0,00	0
										-	0,1	B	Yes	0	ok	0,90	0	0	0	0,04	100	0,00	0
															not ok	0,10	0	0	0	0,00	0	0,00	0
													No	1	ok	0,80	0	0	0	0,04	100	0,00	0
															not ok	0,20	0	0	0	0,01	0	0,00	0
							No	1	LB	+	0,9	B	Yes	1	ok	0,75	0	1	1	0,32	60	19,44	40
															not ok	0,25	0	1	1	0,11	-40	-4,32	40
													No	0	ok	0,02	0	0	0	0,01	100	0,00	0
															not ok	0,98	0	0	0	0,42	0	0,00	0
										-	0,1	B	Yes	0	ok	0,75	0	0	0	0,04	100	0,00	0
															not ok	0,25	0	0	0	0,01	0	0,00	0
													No	1	ok	0,02	0	0	1	0,00	100	0,10	0
															not ok	0,98	0	0	1	0,05	0	0,00	0
				-	0,2	A	Yes	1	LB	+	0,9	B	Yes	1	ok	0,90	1	1	1	0,10	0	0,00	100
															not ok	0,10	1	1	1	0,01	-100	-1,08	100
													No	0	ok	0,80	1	0	0	0,09	40	0,00	0
															not ok	0,20	1	0	0	0,02	-60	0,00	0
										-	0,1	B	Yes	0	ok	0,90	1	0	0	0,01	40	0,00	0
															not ok	0,10	1	0	0	0,00	-60	0,00	0
													No	1	ok	0,80	1	0	1	0,01	40	0,38	60
															not ok	0,20	1	0	1	0,00	-60	-0,14	60
							No	0	LB	+	0,9	B	Yes	1	ok	0,75	0	1	0	0,08	60	0,00	0
															not ok	0,25	0	1	0	0,03	-40	0,00	0
													No	0	ok	0,02	0	0	0	0,00	100	0,00	0
															not ok	0,98	0	0	0	0,11	0	0,00	0
										-	0,1	B	Yes	0	ok	0,75	0	0	0	0,01	100	0,00	0
															not ok	0,25	0	0	0	0,00	0	0,00	0
													No	1	ok	0,02	0	0	0	0,00	100	0,00	0
															not ok	0,98	0	0	0	0,01	0	0,00	0

- The lower part for the less heavy load (**L** = -) is similarec

L			LA			A			LB			B			F		A?	B?	Decision	p	U(F)-CA-CB	E[U(F)-CA-CB]	CA+CB
L	-	0,40	LA	+	0,10	A	Yes	0	LB	+	0,05	B	Yes	1	ok	0,96	0	1	0	0,00	60	0,00	0
															not ok	0,04	0	1	0	0,00	-40	0,00	0
													No	0	ok	0,90	0	0	0	0,00	100	0,00	0
															not ok	0,10	0	0	0	0,00	0	0,00	0
										-	0,95	B	Yes	0	ok	0,96	0	0	0	0,04	100	0,00	0
															not ok	0,04	0	0	0	0,00	0	0,00	0
													No	1	ok	0,90	0	0	0	0,03	100	0,00	0
															not ok	0,10	0	0	0	0,00	0	0,00	0
							No	1	LB	+	0,05	B	Yes	1	ok	0,95	0	1	1	0,00	60	0,11	40
															not ok	0,05	0	1	1	0,00	-40	0,00	40
													No	0	ok	0,70	0	0	0	0,00	100	0,00	0
															not ok	0,30	0	0	0	0,00	0	0,00	0
										-	0,95	B	Yes	0	ok	0,95	0	0	0	0,04	100	0,00	0
															not ok	0,05	0	0	0	0,00	0	0,00	0
													No	1	ok	0,70	0	0	1	0,03	100	2,66	0
															not ok	0,30	0	0	1	0,01	0	0,00	0
				-	0,90	A	Yes	1	LB	+	0,05	B	Yes	1	ok	0,96	1	1	1	0,02	0	0,00	100
															not ok	0,04	1	1	1	0,00	-100	-0,07	100
													No	0	ok	0,90	1	0	0	0,02	40	0,00	0
															not ok	0,10	1	0	0	0,00	-60	0,00	0
										-	0,95	B	Yes	0	ok	0,96	1	0	0	0,33	40	0,00	0
															not ok	0,04	1	0	0	0,01	-60	0,00	0
													No	1	ok	0,90	1	0	1	0,31	40	12,31	60
															not ok	0,10	1	0	1	0,03	-60	-2,05	60
							No	0	LB	+	0,05	B	Yes	1	ok	0,95	0	1	0	0,02	60	0,00	0
															not ok	0,05	0	1	0	0,00	-40	0,00	0
													No	0	ok	0,70	0	0	0	0,01	100	0,00	0
															not ok	0,30	0	0	0	0,01	0	0,00	0
										-	0,95	B	Yes	0	ok	0,95	0	0	0	0,32	100	0,00	0
															not ok	0,05	0	0	0	0,02	0	0,00	0
													No	1	ok	0,70	0	0	0	0,24	100	0,00	0
															not ok	0,30	0	0	0	0,10	0	0,00	0
																						28,31	

Computation times ($n = 9 \Rightarrow 4^9 = 262\ 144$ strategies)

Table 1 Results on the 10 randomly generated n-monitoring instances.

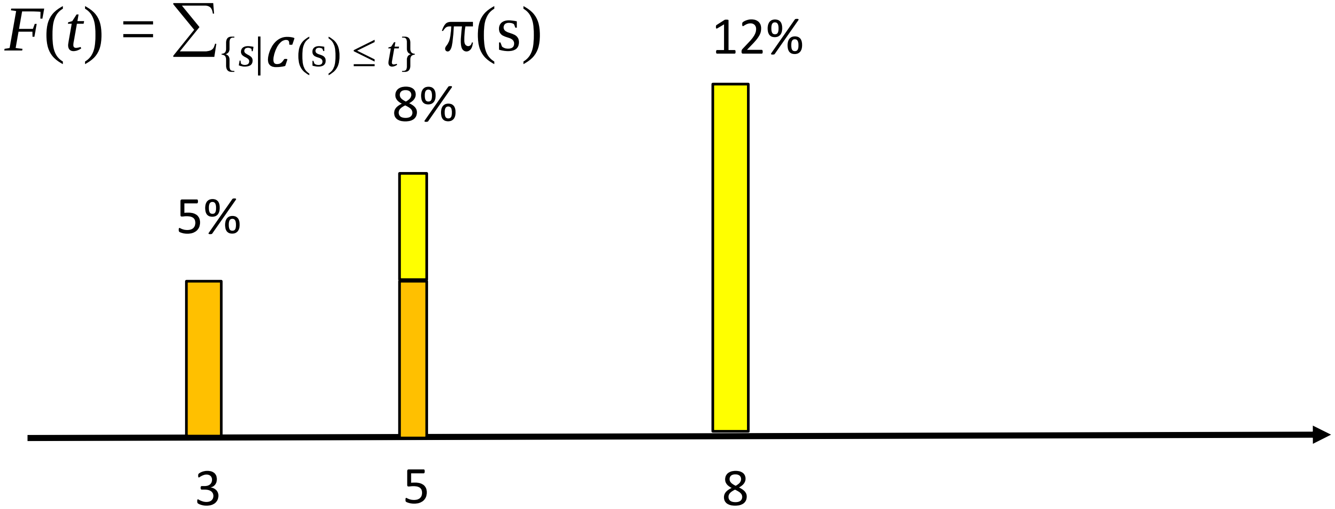
	Number of variables		No probability cuts		With probability cuts	
# Nodes	Binary	Real	A	SD	A	SD
2	8	64	0.01	0.00	0.02	0.00
3	12	256	0.12	0.13	0.04	0.01
4	16	1024	2.73	1.64	0.09	0.01
5	20	4096	34.60	30.02	0.44	0.14
6	24	16384	540.95	277.34	3.39	0.75
7	28	65536	10337.94	4370.09	42.09	20.44
8	32	262144	-	-	231.52	133.58
9	36	1048576	-	-	2256.50	1951.08

(Conditional) Value-at-Risk constraints

- Value-at-Risk defined (see e-g. Artzner et al 1999)

$$\mathrm{VaR}(Z) = F^{-1}_Z(\alpha) = \sup\,\{t \mid \boldsymbol{P}(\{s \mid \boldsymbol{C}(s) \leq t\}) < \alpha\}$$

where $F(t) = \sum_{\{s \mid \boldsymbol{C}(s) \leq t\}} \pi(s)$



Liesiö & Salo (2012). Scenario-Based Portfolio Selection of Investment Projects with Incomplete Probability and Utility Information, *European Journal of Operational Resesearch* 217/1, 162-172.

PROPOSITION 1. *Let $\alpha \in (0,1]$ and assume that Z is a decision strategy. If η^* is the optimum to the minimization problem*

$$\min \quad \eta \tag{32}$$

$$\eta - \mathcal{C}(s) \leq M\lambda(s), \qquad \forall s \in S \tag{33}$$

$$\eta - \mathcal{C}(s) \geq (M+\epsilon)\lambda(s) - M, \qquad \forall s \in S \tag{34}$$

$$\eta - \mathcal{C}(s) \leq (M+\epsilon)\overline{\lambda}(s) - \epsilon, \qquad \forall s \in S \tag{35}$$

$$\eta - \mathcal{C}(s) \geq M(\overline{\lambda}(s) - 1), \qquad \forall s \in S \tag{36}$$

$$\overline{\rho}(s) \leq \overline{\lambda}(s), \qquad \forall s \in S \tag{37}$$

$$\pi(s) - (1 - \lambda(s)) \leq \rho(s) \leq \lambda(s), \qquad \forall s \in S \tag{38}$$

$$\rho(s) \leq \overline{\rho}(s) \leq \pi(s), \qquad \forall s \in S \tag{39}$$

$$\sum_{s \in S} \overline{\rho}(s) = \alpha, \tag{40}$$

$$\overline{\lambda}(s), \lambda(s) \in \{0,1\}, \qquad \forall s \in S \tag{41}$$

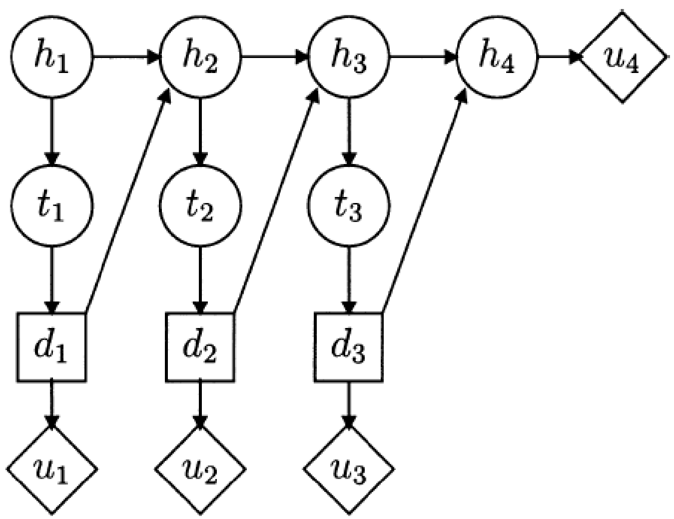
$$\overline{\rho}(s), \rho(s) \in [0,1], \qquad \forall s \in S \tag{42}$$

$$\eta \in [c^{\circ}, c^{*}], \tag{43}$$

then $\mathrm{VaR}_\alpha(Z) = \eta^*$ *and* $\mathrm{CVaR}_\alpha(Z) = \frac{1}{\alpha}\sum_{s \in S} \overline{\rho}(s)\mathcal{C}(s)$.

Revisiting the pig farm

- What if the pigs should have the chance to live longer?
- What if the farmer wishes to curtail risks in the lower part of the profit distribution?



Computational results without risk constraints

Table 2 Results for the pig farm problem for different numbers of decision periods.

# Months	Optimal value (DKK)	Solution time (s)
3	764	0.03
4	727	0.09
5	703	1.43
6	686	43.83
7	674	920.48

Note: With 7 decision periods, there are 4^7 = 16 384 strategies

Determining all non-dominated strategies

- With many value nodes $v \in V$, strategy Z is non-dominated iff there is no Z' such that

$$\mathbf{E}[U_v|Z'] \geq \mathbf{E}[U_v|Z]$$

for all $v \in V$ with at least one strict inequality

- Any multiple objective optimization algorithms for discrete optimization problems can used (Holzmann Smith 2018)

Alternative algorithm

- Choose a single set of positive weights $w_v > 0, \sum w_v = 1$
- Exclude previously generated non-dominated solutions Z' through

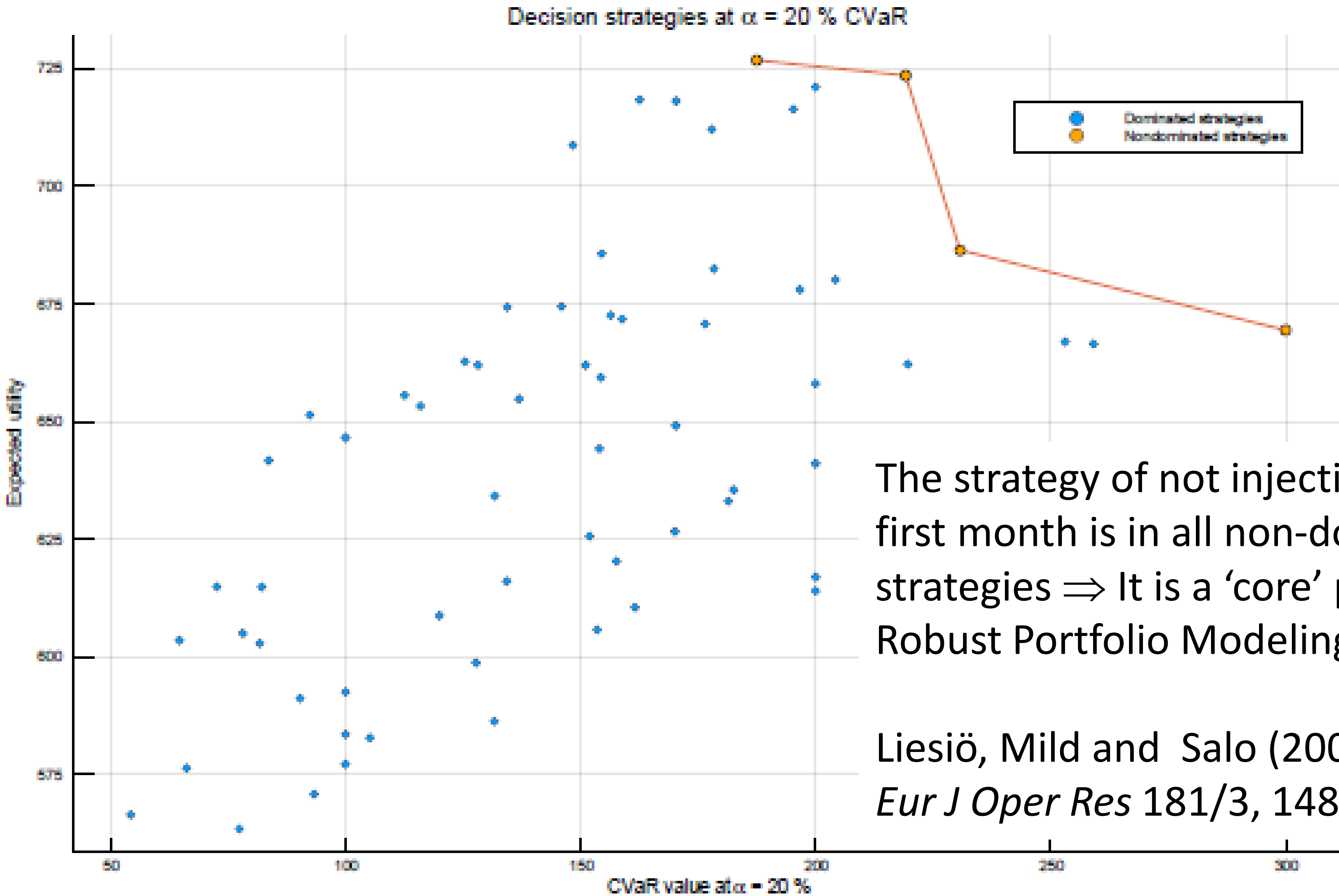
$$\sum_{\{(s_i,s_{I(i)})\,|\,z'(s_i\,|\,s_{I(i)})=0\}} z(s_i\,|\,s_{I(i)}) + \sum_{d\in D}\prod_{i\in I(d)}|S_i| - \sum_{\{(s_i,s_{I(i)})\,|\,z'(s_i\,|\,s_{I(i)})=1\}} z(s_i\,|\,s_{I(i)}) \geq 1$$

- Prune dominated solutions through the necessary conditions

$$\lambda^{+}_{Z',v}(Z) + \lambda^{-}_{Z',v}(Z) = 1 \qquad \lambda^{+}_{Z',v}(Z), \lambda^{-}_{Z',v}(Z) \in \{0,1\}, v \in V$$

$$\mathbb{E}[U_v\,|\,Z] \leq \mathbb{E}[U_v\,|\,Z'] + M\lambda^{+}_{Z',v}(Z) \qquad \sum_{v\in V}\lambda^{+}_{Z',v}(Z) \geq 1,\ Z' \in Z_{ND}$$

$$\mathbb{E}[U_v\,|\,Z'] \leq \mathbb{E}[U_v\,|\,Z] + M\lambda^{-}_{Z',v}(Z)$$



The strategy of not injecting in the first month is in all non-dominated strategies ⇒ It is a 'core' project in Robust Portfolio Modeling

Liesiö, Mild and Salo (2007), *Eur J Oper Res* 181/3, 1488-1505.

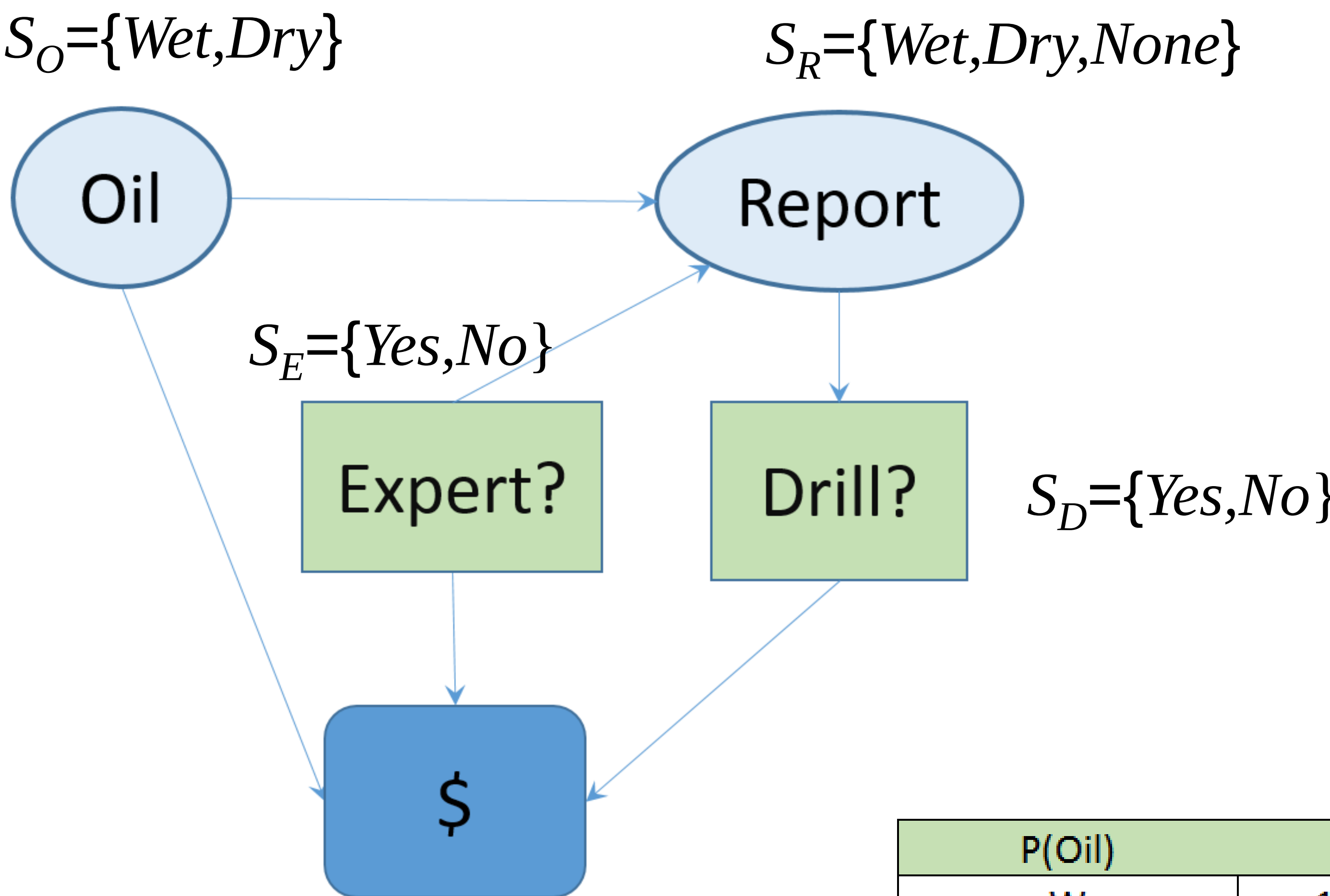
Figure 5 Expected utilities and conditional expectations in the lower $\alpha = 0.20$ tail for all 64 strategies of the 4-month pig problem. The four non-dominated strategies are connected and marked with orange circles.

Decision analysis + mathematical programming

- Leverages MILP to solve multi-stage optimization problems which can be represented as extended influence diagrams
- The 'no-forgetting' assumption can be forgotten
- Many types of deterministic and probabilistic constraints can be handled (resources, risks, logical dependencies)
- Value-at-Risk / Conditional VaR measures can be accommodated
- Computationally tractable (but problems can become large)
- Paper available at **https://arxiv.org/abs/1910.09196**

Ongoing and planned applications

- Planning and operational implementation of **state-dependent risk management actions** in safety critical systems (with Alessandro Mancuso, Michele Compare, Enrico Zio)
- **Identification of most critical scenarios** in the safety assessment of nuclear repositories (with Edoardo Tosoni, Enrico Zio)
- Multi-criteria evaluation of **diagnostic strategies** in healthcare (with Ellie Dillon, Eeva Vilkkumaa)
- Extensions to **adversarial risk management** (with Juho Roponen, David Ríos Insua)



P(Oil)	
W	15 %
D	85 %

Payoff	200
Drilling	20
Expert	7

P(Report|Oil)	W	D
W	95 %	15 %
D	5 %	85 %

Expert	**Y**	**Report**	*W*	*27 %*	**Drill(W)**	**Y**	**Oil**	*W*	*53 %*	173
16,1	*16,1*	16,1			**78,6**	*78,6*	*78,6*	*D*	*47 %*	-27
						N	**Oil**	*W*	*53 %*	-7
						-7,0	*-7,0*	*D*	*47 %*	-7
			D	*73 %*	**Drill(D)**	**Y**	**Oil**	*W*	*1 %*	173
					-7,0	*-24,9*	*-24,9*	*D*	*99 %*	-27
						N	**Oil**	*W*	*1 %*	-7
						-7,0	*-7,0*	*D*	*99 %*	-7
	N				**Drill(None)**	**Y**	**Oil**	*W*	*15 %*	180
	10,0				**10,0**	*10,0*	*10,0*	*D*	*85 %*	-20
						N	**Oil**	*W*	*15 %*	0
						0,0	*0,0*	*D*	*85 %*	0

P(Report=W)	27 %
P(Report=D)	73 %

P(W|W)=P(O=W&R=W)/P(R=W)	53 %
P(D|W)=P(O=D&R=W)/P(R=W)	47 %
P(W|D)=P(O=W&R=D)/P(R=D)	1 %
P(D|D)=P(O=D&R=D)/P(R=D)	99 %

Oil drilling with decision programming

Oil		Expert		Report		Drill		Decisions	p	Π	Payoff	Expert	Drilling	U	EU
Wet	15 %	Yes	1	Wet	95 %	Yes	1	1	14 %	14 %	200	-7	-20	173	24,65
	15 %				95 %	No	0	0	14 %	0 %	0	-7	0	-7	0,00
	15 %			Dry	5 %	Yes	0	0	1 %	0 %	200	-7	-20	173	0,00
	15 %				5 %	No	1	1	1 %	1 %	0	-7	0	-7	-0,05
	15 %	No	0	None	100 %	Yes	0	0	15 %	0 %	200	0	-20	180	0,00
	15 %				100 %	No	1	0	15 %	0 %	0	0	0	0	0,00
Dry	85 %	Yes	1	Wet	15 %	Yes	1	1	13 %	13 %	0	-7	-20	-27	-3,44
	85 %				15 %	No	0	0	13 %	0 %	0	-7	0	-7	0,00
	85 %			Dry	85 %	Yes	0	0	72 %	0 %	0	-7	-20	-27	0,00
	85 %				85 %	No	1	1	72 %	72 %	0	-7	0	-7	-5,06
	85 %	No	0	None	100 %	Yes	0	0	85 %	0 %	0	0	-20	-20	0,00
	85 %				100 %	No	1	0	85 %	0 %	0	0	0	0	0,00
										1,000					16,1

Decisions are in dark red cells

All other decisions are implied (in light orange)

Direct drilling is possible only if there is no consultation