An exact algorithm for the Green Vehicle Routing Problem

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Outline

Green Vehicle Routing Problem (G-VRP)

Defined on a complete graph $G = (N \cup F, A)$ where N is a set of n customers plus a depot 0, and F is a set of s refueling stations

- An unlimited number of vehicles with fuel level Q are available at 0
- Each vehicle can be assigned a trip (called route) that visits a subset of the customers and returns back to 0
- Traversing an arc $(i, j) \in A$ (i.e., traveling from i to j) consumes a fuel amount c_{ij} (cost of (i, j)) and a travel time $t_{ij} = \kappa c_{ij}$ (κ is a constant)
- A vehicle can visit a station during its route to restore its fuel level back to Q. Refueling consumes a refueling time δ

Constraints:

- The fuel level must remain positive during each trip (fuel constraints)
- Each customer must be visited by exactly one route. A customer visit consumes a service time τ
- The duration of a route cannot exceed a maximum driving time T

Objective A set of routes that minimize the total fuel consumption

Literature Review

The G-VRP was introduced by [Erdoğan and Miller-Hooks(2012)]

- Motivated by growing popularity of low-emission alternative-fuel vehicles (biodiesel, electricity, hydrogen..): limited fuel autonomy and limited refueling infrastructure
- Refueling stops, and resulting delays need to be modeled explicitly

Applications to electric vehicles: Routing of vehicles with replaceable batteries (battery swap systems)

Heuristic Algorithms

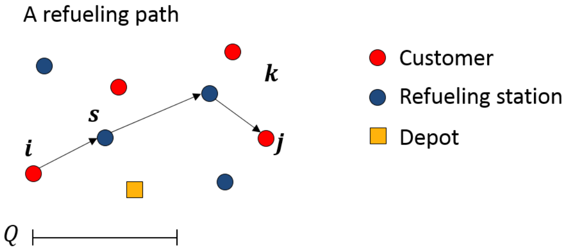
- Modified savings [Erdoğan and Miller-Hooks(2012)], VNS/TS [Schneider et al.(2014a), Schneider et al.(2014b)], modified MSH [Montoya et al.(2014)], local search and SA [Felipe et al.(2014)]

Exact Algorithms

- There are no exact algorithms for the G-VRP
- [Desaulniers et al.(2014)] develop an exact algorithm for a generalization of G-VRP called Electric VRPTW. They do not consider the G-VRP

Refueling paths

A refueling path is a simple path $P = (i, s, ..., k, j)$ from $i \in N$ to $j \in N$ such that $s, ..., k$ are refueling stations and all arcs (u, v) it traverses have cost $c_{uv} \leq Q$



The cost of a refueling path is the sum of the costs of its arcs

The time of a refueling path from i to j is the sum of the travel times of its arcs, plus the refueling times of its stations, plus the service time τ (if $i \neq 0$)

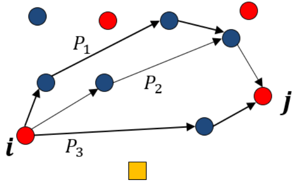
Refueling-path multigraph

Some refueling paths are dominated: We can assume they are not traversed by any vehicle in an optimal solution

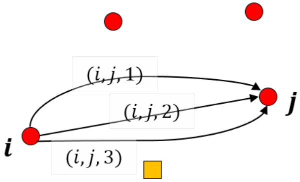
- Dominance of a refueling path depends on (i) the cost of its first and last arc, (ii) the total path cost (iii) the number of stations it visits
- The non-dominated refueling paths can be efficiently computed a-priori

We model the G-VRP on a multigraph $\mathcal{G}$ with an arc (i, j, p) for each non-dominated refueling path p, plus the arcs $(i, j) \in A$ with $c_{ij} \leq Q$

Three refueling paths from i to j



The three corresponding arcs in $\mathcal{G}$



G-VRP routes and Set Partitioning formulation

A G-VRP route is a simple circuit in $\mathcal{G}$ starting from 0, having duration less than or equal than T and satisfying the fuel constraints

Define:

- $\mathscr{R}$: index set of all G-VRP routes
- $x_\ell, \forall \ell \in \mathscr{R}$: 0-1 variables taking value 1 if route R_ℓ is in solution
- c_ℓ: cost of route R_ℓ
- $a_{i\ell}$: 0-1 coefficient equal to 1 if route R_ℓ visits $i \in N$

The G-VRP can be modeled as a Set Partitioning problem:

$$
\begin{aligned}
\text{(SP)} \quad z(SP) = \min & \sum_{\ell \in \mathscr{R}} c_\ell x_\ell \\
\text{s.t.} & \sum_{\ell \in \mathscr{R}} a_{i\ell} x_\ell = 1 \quad && i \in N \setminus \{0\} \\
& x_\ell \in \{0, 1\} && \ell \in \mathscr{R}
\end{aligned}
$$

We call LSP the LP relaxation of SP

Valid Inequalities

We use three types of valid inequalities to tighten LSP

- Subset Row inequalities (SR3) [Jepsen et al.(2008)]
- Weak Subset Row inequalities (WSR3) [Baldacci et al.(2011)]
- k-path cuts [Laporte et al.(1985)]

$$\sum_{\ell \in \mathscr{R}} b_{S\ell} x_\ell \geq r(S) \quad \forall S \subseteq N \setminus \{0\}, |S| > 2$$

where $b_{S\ell}$ equals 1 if route R_ℓ visits S, and 0 otherwise, and $r(S)$ is the minimum number of routes needed to visit S

Separation of k-path cuts I

Consider any set $S \subseteq N \setminus \{0\}$. If we can find a vector $\boldsymbol{\pi} \in \mathbb{R}^n_+$ satisfying

(i) $\pi_i = 0, \forall i \in N \setminus S,$

(ii) $\sum_{i \in N \setminus \{0\}} a_{i\ell} \pi_i \leq 1, \forall \ell \in \mathscr{R}$

(iii) $\sum_{i \in N \setminus \{0\}} \pi_i > r(S) - 1$

then the k-path cut defined by S can be rewritten as the following rank-1 (w.r.t. LSP) Chàvatal-Gomory cut (rank-1 CG cut)

$$\sum_{\ell \in \mathscr{R}} \left\lceil \sum_{i \in N \setminus \{0\}} a_{i\ell} \pi_i \right\rceil x_\ell \geq \left\lceil \sum_{i \in N \setminus \{0\}} \pi_i \right\rceil \tag{1}$$

Moreover, we have $S = \{i \in N : \pi_i > 0\}$, and $r(S) = \left\lceil \sum_{i \in N \setminus \{0\}} \pi_i \right\rceil$

Observation:

A violated CG cut (1) defined by a $\boldsymbol{\pi}$ satisfying (i) – (iii) provides a violated k-path cut defined by the set $S = \{i \in N : \pi_i > 0\}$

Separation of k-path cuts II

Separation problem: Given a fractional solution $\bar{\mathbf{x}}$, find a violated rank 1 CG cut defined by a $\boldsymbol{\pi} \in \mathbb{R}^n_+$ with $\sum_{i \in N \setminus \{0\}} \pi_i \leq 1, \forall \ell \in \mathscr{R}$

The problem of finding a maximally violated rank-1 CG cut can be modeled as a MILP and solved by a MILP solver [Fischetti, Lodi(2007)]. In our case, it is

$$
\begin{aligned}
(SEP) \quad & \max z - \sum_{\ell \in \bar{\mathscr{R}}} \bar{x}_\ell \, y_\ell \\
& \text{s.t.} \sum_{i \in N \setminus \{0\}} a_{i\ell} \pi_i \leq y_\ell, \quad \forall \ell \in \bar{\mathscr{R}} \\
& \quad \sum_{i \in N \setminus \{0\}} a_{i\ell} \pi_i \leq 1, \quad \forall \ell \in \mathscr{R} \\
& \quad \epsilon - 1 \leq \sum_{i \in N \setminus \{0\}} \pi_i - z \leq 0 \\
& \quad y_\ell \in \{0, 1\}, \quad \forall \ell \in \bar{\mathscr{R}} \\
& \quad \pi_i \geq 0, \quad \forall i \in N \setminus \{0\} \quad \text{and} \quad z \in \mathbb{Z}_+
\end{aligned}
$$

where $\bar{\mathscr{R}}$ is the index set of routes R_ℓ with $\bar{x}_\ell > 0$

Exact solution algorithm I

2-phase method based on the schema proposed by [Baldacci et al. 2008, 2011] for the Capacitated VRP and VRP with Time Windows

Phase I:

- Compute a lower bound $z(\mathrm{LB})$ as the cost of an optimal dual solution of LSP plus SR3, WSR3 and k-path cuts (called LSP_+)
- Compute an upper bound $z(UB)$ by running an ALNS heuristic which uses the multigraph $\mathcal{G}$

Phase II:

- Enumerate all routes $\mathscr{R}^*$ having reduced cost $\leq z(\mathrm{UB}) - z(\mathrm{LB})$ with respect to the dual solution obtained in Phase I
- $\mathscr{R}^*$ is computed by dynamic programming. It is guaranteed to contain an optimal set of routes
- An optimal solution is obtained by solving SP with $\mathscr{R}$ replaced by $\mathscr{R}^*$

Exact solution algorithm II

$z(\mathrm{LB})$ in Phase I is computed by cut-and-column generation methods

Pricing problem: Find a least cost G-VRP route:

- It is an elementary shortest path problem with resource constraints in the multigraph $\mathcal{G}$. Solved by a forward dynamic programming algorithm
- Bounding functions based on the ng-path relaxation [Baldacci et al.(2011)] are used to fathom sub-optimal states
- The same algorithm is used when solving the separation problem for k-path cuts to detect violated constraints $\sum_{i \in N \setminus \{0\}} a_{i\ell}\pi_i \leq 1, \ \forall \ell \in \mathscr{R}$

Computational Experiments

[Erdoğan and Miller-Hooks(2012)] proposed two sets of instances with 20 and 109–500 customers, respectively

- Based on data from an U.S. medical textile supply company in Virginia
- Max. driving time of $T = 11$ hours and max. travel distance without refuel $Q = 300$ miles. Vehicles travel at a constant speed of 40 miles/h.
- Customers service time is $\tau = 30$ min., refueling time is $\delta = 15$ min.
- Each vehicle incurs an initial refueling time at the depot before starting its route (i.e., in practice $T = T - \delta$)

We have considered all Erdogan, Miller-Hooks instances with up to 109 customers, and created an additional set of problems with $\sim$ 50, 75 and 100 customers by extracting customers from the larger ones

Computer used: Intel Xeon X3450, 2.67 GHz with 12 GB RAM (CPLEX is used as MILP and LP solver)

Preliminary Computational Results

Instances proposed by [Erdoğan and Miller-Hooks(2012)]

All instances with 20 customers are solved within a few seconds (Phase I always terminates with an integer solution after solving LSP_+)

Table: Instances of [Erdoğan and Miller-Hooks(2012)] with 109 customers

Inst.	n	s	Opt	$\%LB_0$	$\%LB_{SR}$	$\%LB$	T_{LB}	# kP	# SR	Time
111c_21s	109	21	$*$	97.59	98.09	99.82	17994	88	509	18714
111c_22s	109	22	$*$	97.59	98.09	99.92	19920	89	606	20223
111c_24s §	109	24		97.59	98.09	99.70	21912	98	662	22560
111c_26s	109	26	$*$	97.59	98.09	99.95	16419	81	533	16547
111c_28s	109	28	$*$	97.58	98.09	99.90	13737	85	423	14261
Average				97.59	98.09	99.86	17996			

§: found new best known upper bound

- LB_0: optimal cost of LSP without valid inequalities, ($\%LB_0$: % ratio of LB_0)
- LB_{SR}: optimal cost of LSP plus WSR3 and SR3 inequalities, ($\%LB_{SR}$: % ratio of LB_{SR})
- LB: optimal cost of LSP plus WSR3, SR3 and k-path cuts, ($\%LB$: % ratio of LB)
- T_{LB}: cpu time (sec.) to compute LB
- # kP and # SR: total number of k-path cuts and SR3 plus WSR3 inequalities added to LSP
- Time: Total cpu time (sec.)

Preliminary Computational Results

New instances

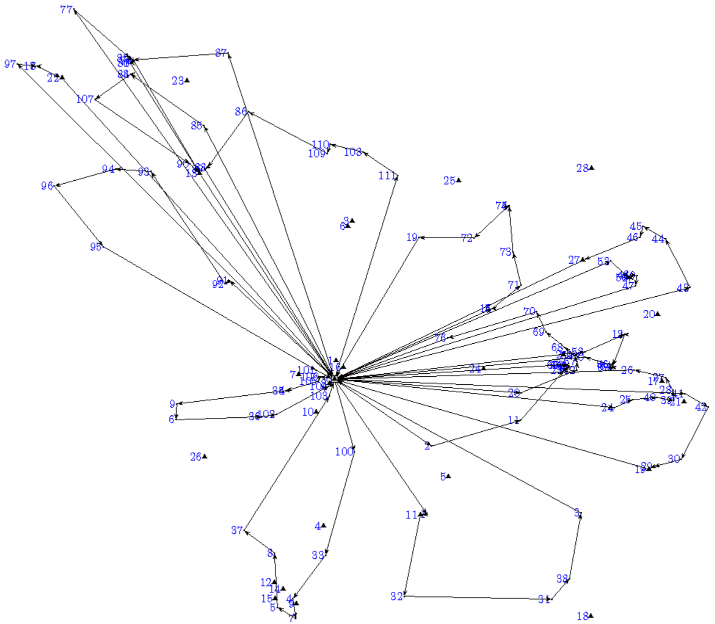
Table: Instances derived by taking the first 75 and 100 customers from 111c_21s and 111c_28s

Inst.	n	s	Opt	$\%LB_0$	$\%LB_{SR}$	$\%LB$	T_{LB}	# kP	# SR	Time
75c_21s	75	21	$*$	97.38	98.04	100.00	4860	41	140	4861
75c_28s	75	28	$*$	97.38	98.04	100.00	7999	63	140	8000
100c_21s	98	21	$*$	98.80	99.25	100.00	9850	74	124	9852
100c_28s	98	28	$*$	98.80	99.25	99.89	9894	17	335	10166
Average				98.09	98.64	99.97	8151			

Table: Average results on new instances derived by randomly extracting customers from the large [Erdoğan and Miller-Hooks(2012)] instances

# of inst.	n	s	$\%LB_0$	$\%LB_{SR}$	$\%LB$	T_{LB}	# kP	# SR	Time	Opt
7	50	21	95.98	97.87	99.84	2464	50	108	2477	7/7
8	75	22	97.24	99.16	99.67	2988	45	179	4384	8/8
8	98	24	97.62	99.11	99.56	7068	27	466	10409	7/8

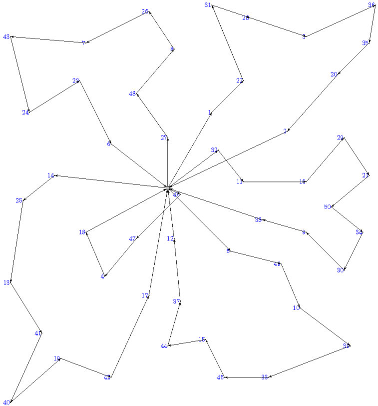
Optimal solution of instance 111c_28s



Conclusions

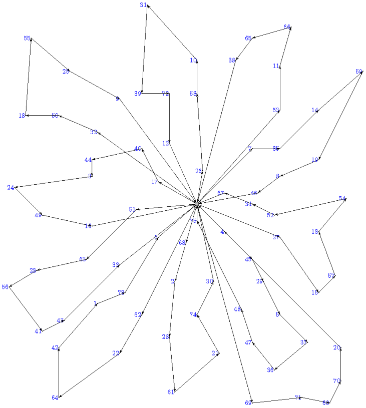
- We have proposed an exact algorithm for solving the Green Vehicle Routing Problem which can be viewed as a basic model for alternative-fuel vehicle routing optimization
- We have modeled the problem by using a multigraph which does not explicitly model the refueling stations and excludes a-priori sub-optimal refueling paths
- We have characterized a subset of the k-path cuts as Chàvatal-Gomory cuts of rank 1. This permitted to use k-path cuts within a cut-and-column generation algorithm
- We reported computational results on benchmark instances based on a case study from [Erdoğan and Miller-Hooks(2012)]
- The exact algorithm provides tight lower bounds and optimally solves instances with up to 109 customers

Optimal solution of the distance-constrained CVRP instance CMT6



Inst.	n	s	UB^*	Opt	$\%LB_0$	$\%LB_{SR}$	$\%LB$	T_{LB}	# kP	# SR	Time
CMT6	50	0	555.43	$*$	96.83	99.01	100.00	573	9	294	573

Optimal solution of the distance-constrained CVRP instance CMT7



Inst.	n	s	UB^*	Opt	$\%LB_0$	$\%LB_{SR}$	$\%LB$	T_{LB}	# kP	# SR	Time
CMT7	75	0	909.68	$*$	98.35	99.81	99.85	1272	3	317	1290

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