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Modeling space elasticity of demand to support retail replenishment planning

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ABSTRACT OF THE
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The demand of a product is affected by the shelf space allocated for it. This dependency is called space elasticity and the magnitude of it depends on multiple product and store-specific attributes. The objective of this thesis is to review the literature related to this relationship between shelf space and demand, and develop guidelines with which the space elasticity can be accurately estimated for a large group of products in multiple stores. Also, ways to utilize the space elasticity information to support retail replenishment planning are studied and similar guidelines for optimizing shelf space are presented.

An important finding regarding the estimation of space elasticity is that the effect of shelf space is minor compared to other variations in demand. Another significant conclusion is that the products' position in the shelf, as well as the store at which the product is sold, affects the space elasticity estimates. Therefore, these two factors need to be controlled when estimating the space elasticity values.

Regarding the operational applications of space elasticity information, this thesis concludes that multiple models to optimize shelf space allocation exists, proposing significant profit increases compared to traditional allocation methods. These models however contain components and assumptions that complicate their application to practical use in large retail chains. Therefore, a simplified model, where profit related to shelf space allocation is maximized with restrictions assuring the rationality of the result, is proposed for further investigation. Using shelf space information to optimize other operations, such as promotion planning is also proposed to be the subject of future studies.

Keywords: space elasticity, shelf space allocation, decision support

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Tuotteen kysyntään vähittäiskaupassa vaikuttaa sen saama hyllytilan määrä. Tätä riippuvuutta kutsutaan kysynnän tilajoustoksi ja sen voimakkuus riippuu useista tuote- ja myymäläkohtaisista ominaisuuksista. Työn tarkoituksena oli kirjallisuuskatsauksen avulla tutkia hyllytilan ja kysynnän välistä yhteyttä sekä kehittää perussääntöjä kysynnän tilajouston luotettavaan arvioimiseen suurelle määrälle tuotteita monissa eri myymälöissä. Tavoitteena oli myös löytää tapoja hyödyntää kysynnän tilajoustoa vähittäiskauppojen täydennyksen suunnittelussa ja tarjota samankaltaiset perussäännöt hyllytilan jakamisen optimointiin.

Tärkeä tulos kysynnän tilajouston estimointiin liittyen oli, että hyllytilan vaikutus kysyntään on pieni suhteessa muihin kysyntään vaikuttaviin tekijöihin. Tutkimuksessa selvisi myös, että tuotteen sijainti hyllyssä, kuten myös myymälä, jossa tuotetta myydään, vaikuttavat kysynnän tilajoustolle saataviin arvoihin. Näin ollen nämä tekijät tulee ottaa huomioon tilajoustoa arvioitaessa.

Kysynnän tilajoustotiedon operationaalisen hyödyntämisen suhteen tutkimuksessa selvisi, että on olemassa useita hyllytilan jakamista optimoivia malleja, jotka lupaavat merkittäviä lisäyksiä myymälöiden tuottoihin normaaleihin hyllytilajakoihin verrattuna. Näissä malleissa on kuitenkin ominaisuuksia ja oletuksia, jotka vaikeuttavat niiden hyödyntämistä suurten vähittäiskauppaketjujen tilanhallinnan jatkuvassa optimoinnissa. Tästä syystä työssä ehdotetaan lisätutkimusten kohteeksi yksinkertaistettua mallia, jossa optimoidaan vain hyllytilajakoon liittyvää tuottoa, ratkaisun järkevyyden varmistavien rajoitusehtojen vallitessa. Työssä ehdotetaan jatkotutkimuksen kohteeksi myös tilajouston hyödyntämistä muissa sovelluksissa, kuten kampanjoiden suunnittelussa.

Avainsanat: kysynnän tilajousto, hyllytilan jako, päätöksenteko

Contents

1 Introduction

In retailing, the demand of a product is affected by a large number of factors. One of these is the shelf space allocated for the product. If the shelf space of the product is increased, consumers are more likely to notice the good and the product is bought more often. Naturally, it is usually not possible to increase the shelf space of every product. Therefore, defining the products and the situations in which the space increases would be beneficial need to be defined. The demand of different products reacts differently to altering their shelf space, meaning that products have different space elasticities. This can be due to multiple factors, such as the category to which the product belongs, location and size of the store it is sold at and numerous other factors. Having knowledge of how the demand of a product, or product category, will react to variations in shelf space is useful in making decisions about different store operations, such as space allocation, replenishment and promotion planning.

Most retailers acknowledge the space elasticity phenomenon on a general level and have developed rules of thumb to allocate shelf space, for example so that the most popular products, or products having the highest profit margins, get more shelf space. There is however a potential to draw more significant benefits by determining accurate space elasticity estimates for each product and optimizing different store operations based on this information. However, the problem is that forecasting the effect of shelf space on the demand of the product is difficult. The relationship is not similar for every product category and intuitive dependencies between product attributes and the space elasticity are hard to find. The different factors affecting the space elasticity of a product have been a subject of investigation in a number of studies where different relations between different factors and space elasticities have been found. The findings are in some sense consistent, but often ambiguous, which leads to the conclusion that the different factors and relations may not yet be completely clear. Knowing the space elasticity of a product is in itself useless, if the knowledge is not used to support decision making in store operations. Therefore, multiple studies have been conducted focusing also on practical use-cases for the space elasticity information. These studies focus mainly on different models to optimize shelf space allocation in order to maximize profit or sales.

This thesis reviews the studied dependencies between different product attributes and product's space elasticity, and aims to find a generally good way to estimate space elasticities for a large group of SKU's belonging to different categories in different stores. After this, the purpose is to review some of the developed operational applications exploiting the space elasticity information. Finally, based on those findings, we determine guidelines for estimation and usage of this information in large retail chains.

2 Estimating space elasticity

2.1 Depencency between shelf space and demand

Curhan [1972] defines the shelf space elasticity as a ratio of the percentage change in sales to percentage sales in shelf space with a formula

$$E = \frac{(C_{t_1} - C_{t_0})/C_{t_0}}{(S_{t_1} - S_{t_0})/S_{t_0}}, \tag{1}$$

where C is unit sales and S is the shelf space. t_0 represent the time period observed before the change in the shelf space and t_1 is the time period observed after the change in the shelf space. To study the space elasticity, Curhan constructed an experiment where he aimed to detect the space elasticity of different products by altering the shelf space of a sample set of products and measuring the unit sales 5 to 12 weeks before or after the change. After this, the author explained the measured elasticities with different variables using linear regression. The experiment was conducted with a regional supermarket chain. Four of the chain's stores acted as a test stores where the different products shelf space was altered and 24 other stores of the chain in the same area were used as control stores. Curhan chose this approach in order to remove the seasonal, promotional and other variations from sales. In practice, it was done by multiplying the sales of the test product with the ratio of the base period unit sales to test period unit sales of the control product. This was done with formula

$$U_{t_1} = U_{observed\ t_1}(U_{control\ t_0}/U_{control\ t_1}), \tag{2}$$

where U is the unit sales. The unit sales data in the study was obtained from the warehouse withdrawal records, which can cause some additional variation in the results.

Curhan explained the observed space elasticities with the following linear regression model.

$$E_i = a + b_1x_1 + b_2x_2 + b_3x_3 \; ... \; b_{11}x_{11} + e, \tag{3}$$

where E_i is the space elasticity coefficient for product i, x_j are the predictors, a and b_j are constants and e is the error term. The constants b_j represented different product attributes such as package size, retail price, rate of sales, extent of unplanned purchasing and availability of substitutes.

The average observed space elasticity across all of the products in Curhan's experiment was 0.212. However, the regression analysis resulted in a low R^2 of 0.012 and high standard error meaning that the predictors were not able to explain the observed space elasticities with the defined linear regression model. In additional tests, Curhan examined the predictors one at a time by dividing the products into two subsets based on the distribution of the values of the currently examined predictor. The author then calculated the mean elasticity values and variations for both subset to see whether they differ significantly. Significant differences against an alpha level of 0.25 was found for predictors x_3 (brand type, indicating whether the brand was private or national brand) and x_{10} (extend of unplanned purchasing). Also, a significant difference was found so that the space elasticity coefficient would be higher for space increases that for decreases. However, as pointed out by Eisend [2014], these findings were tested against a very high alpha level of 0.25, meaning that none of the differences can be conclusively considered significant. In the conclusions of the study, Curhan states that "important payoffs are unlikely from investigations of space elasticity". [Curhan, 1972]

One reason that might have distorted the analysis and contributed to the inconclusiveness of the findings is that Curhan did not choose any store attributes as predictors in the regression model. It has been proposed in later studies that the different attributes of stores might also have an effect on the space elasticity of products. Curhan noted the importance of cleaning the sales data, since the effects of shelf space might otherwise be left unnoticed because the sales have variation caused by many other factors. The approach of using control stores however might be prone to inaccuracies, since it is not certain that different stores follow same seasonal patterns and other variation sources. Due to this inconsistency between the stores, multiplying the sales with the control stores ratio may not have cleaned the sales data well enough, and it might even have a negative effect to the results.

A similar type of experiment was also conducted by Cox in 1964, where they manipulated the shelf space of four different products during the time of 6 weeks. The experiment was conducted in four stores of a regional supermarket chain and in two stores of a local supermarket chain in Austin, Texas. The products studied were baking soda, hominy, Tang and powdered coffee cream. The shelf space independent variation in sales was controlled using Latin square design with which the variation between stores and time periods could be handled. They also made sure that the price and the shelf level were kept constant during the experiment and that there were not any other promotional activities for the products. The idea of the Latin square test was to divide the total variation of the sales into variations between stores, time periods, shelf space adjustments and residuals with the formula

$$\sum(y-\bar{y})^2 = \sum(\bar{y}_s-\bar{y})^2+\sum(\bar{y}_t-\bar{y})^2+\sum(\bar{y}_a-\bar{y})^2+\sum(y-\bar{y}_s-\bar{y}_t-\bar{y}_a+2\bar{y})^2, \tag{4}$$

where y is a vector of the actual sales, $\bar{y}$ is the mean of all of the sales, $\bar{y}_s$, is a vector of the means of the sales in each store, $\bar{y}_t$ is a vector of the means of the sales of each time period and the $\bar{y}_a$ is a vector of the means of the sales corresponding to each shelf space adjustment. The summations are done along the components of the vectors. Cox [1964] then used the corrected sales to determine the effects of the shelf space changes to sales. Cox used regression analysis to determine the significance of the results. The only product for which the shelf space elasticity was found significant was hominy, and all of the others were interpreted as non-responsive regarding the relationship between the shelf space and sales.

The relevance of Cox's study can be questioned due to it being from the 1960's. The consumer behavior and retail store characteristics have changed so drastically that also the relationship between the shelf space and unit sales have most likely also changed. However, Cox had an interesting way of handling other variations with the Latin square design, but as the inconclusive findings indicate, it may not have served its purpose optimally and is probably not the most efficient approach.

Desmet and Renaudin [1998] estimated space elasticities per product category using data from a French variety store chain. They had sales, space and margin data for each category for a year. The data was gathered from over 200 stores, but Desmet and Renaudin excluded stores that did not have both a non-food and food departments, leaving a total of 126 stores. They used the store classification system of the retail chain to divide the stores into three categories (plus, standard and essential) to take differences in stores customer profiles into account. They measured demand as monthly turnover while shelf space was measured with linear meters allocated for each product category. The position of the product in the shelf, or in the stores, was not taken into account, so all of the shelf space was considered as equally valuable. They chose a proportional model, in which share of sales are compared with share of space, to eliminate the possible effects that the store size may have on the space elasticities. They also justified the proportional model as being closer to retailers' view, since all of the stores space must be allocated between different product categories. The model for explaining the share of sales of product category c in store s of type g with the space allocated for the same product category in the same store was defined as

$$\mathrm{SSALES}_c = \exp(\alpha_{0_c}) \times \mathrm{SSPACE}_c^{\beta_{cg}} \times \exp(\sum_{i=1}^{125} \delta_{i_c} s_i + \sum_{j=1}^{11} \gamma_{j_c} m_j), \quad (5)$$

where SSALES_c is the percent of the total turnover covered by the product category c, SSPACE_c is the percentage of the total shelf space allocated for product category c in linear meters, β_{cg} is the space elasticity of the product category c for the store group g, δ_{i_c} is a dummy variable for the store s_i and the product category c, and γ_{i_c} is a dummy variable for the month m_i and for the product category c. [Desmet and Renaudin, 1998]

Desmet and Renauding had data of 43 different product categories but decided to exclude highly seasonal categories and the categories that were sold only in small number of stores, leaving 24 categories to be considered in the analysis. The parameters of the model were estimated using ordinary least squares method (OLS). As a general result, Desmet and Renauding obtained an average space elasticity of 0.2138. They also found out that the space elasticities of different categories vary, which is in consonance with the other studies related to space elasticities. Their results also indicated that for most product categories, space elasticities are significantly non-zero and that the elasticities are higher for impulse buying products. For two of the store categories, standard and plus, they found rather similar space elasticities for each product category, while for the category essential, they obtained a very different profile. As the essential stores were the smallest, having less data and therefore the data being more unreliable, they were not able to confirm their hypothesis of the store characteristics affecting the space elasticities.

The study of Desmet and Renaudin can be considered relevant, since they were able to find significantly non-zero space elasticities and define the coefficients for different categories. One difference, and possible reason for their success, in comparison with previous studies, were that they accounted for multiple variables regarding the store characteristics in their study, even though they were not able to confirm their significance. Also, the proportional model that they used eliminated the effects that store size may have, leaving them less variance to explain, therefore making the estimates more accurate. One interesting fact is that they did not correct or clean the sales data from seasonal, promotional or other variation, but simply left the categories they assumed to suffer more from these external variations out from the analysis. This may have biased the findings and, as the space elasticities are relevant also for seasonal product categories, this limits the possibilities to generalize the findings and methods of the study. Also, it is likely that the categories included in the study are also, to a small extent, affected by external factors such as seasonality. This may have caused errors in the estimations.

Eisend [2014] took a meta analytic approach to evaluate previous studies regarding the space elasticity and to find out how much the study and method characteristics affects the obtained space elasticity information. One of the purposes was to determine whether the method selection affects the space elasticity estimates. The idea of the study was to take the estimated space elasticities from multiple studies and explain them with different characteristics of the study and the data used in it. The purpose of this was to determine the influencing factors in the space elasticity estimation and obtain generalized estimates for the space elasticity. Curhan was able to find data from 31 studies presenting 1268 estimates for space elasticity in 57 different stores. To explain the space elasticities with these characteristics, the author used a three level hierarchical linear model (HLM).

Curhan obtained a mean space elasticity of 0.17. The variables found to be significant were product category, whether the shelf space is increased or decreased, whether the study is published or unpublished and whether the price variables were included in the study. The product category was found to affect the space elasticities so that the more impulse buying the product was, the higher space elasticity it had, which is in line with the literature. One interesting finding was that the space increase results in higher space elasticity estimates than decrease. If this is really the case, it can create interesting options for retailers to take advantage of the space elasticity. The effect of the manuscript status was explained by publication bias, which states that the studies with more significant findings get published more probably and thus the general results in published papers might have a bias. In this case the bias was found to be so that published studies resulted in higher space elasticities. However, it needs to be noted that from the 31 studies included in the meta-analysis, only 4 was unpublished. Therefore, because of the sample size, the finding cannot be considered conclusive. The significance of including the price variables was explained in the study with a hypothesis that the space variations are sometimes linked with price reductions, meaning that the stores tend to allocate more, and better, shelf space to products in promotion, which increases the space elasticity estimates if the price data is not considered. This is something that should be considered when estimating space elasticities.

One of the most interesting non-significant variables in Eisend [2014] was the store size, which has been found significant in other studies. The study also found that whether the space elasticities were estimated on a brand or category level was not significant in the model. However, they found that these two variables had an interaction effect. While the aggregate level did not have significant effect in small or medium sized stores, in large stores the brand level resulted in significantly lower estimates than the category level. The study conjectures that in larger stores where the assortment sizes are larger, there are more different brands in the same category so the changes in the brand level assortment are not noticed that clearly and, therefore they do not affect the customer behavior so drastically.

Eisend [2014] offered a good overall look to the literature considering this subject and examined multiple possible variables that could affect the space elasticity. Interesting finding was that the estimation methods did not seem to have as radical effect as previously have been thought. This would mean that the parameters considered and the correctness and relativeness of the sales data are far more important than the estimation methods itself, indicating that the space elasticity is in fact a significant and measurable phenomenon and could thus be estimated reliably and used in operational planning of retail chains.

2.2 Significance of store-specific estimation

Van Dijk et al. conducted a study in 2004 where they had a hypothesis that the relationship between the unit sales and shelf space of the product are affected by unobservable actions performed by the retailer. They stated that space elasticity is therefore endogenous and cannot be estimated accurately with OLS based estimates that assume exogeneity. Van Dijk et al. assume that the error term in OLS estimated models is correlated with shelf space, since retailers are assumed to perform also other marketing activities when they increase the products shelf space. Thus, the models might erroneously attribute changes in sales to changes in shelf spaces. This would result in systematically too high space elasticity estimates. Hypothesis of the study is that managers make these unobserved decisions based on store profiles and thus attempt to model the correlation between space elasticity and the error term with different store profiles. However, they note that the space elasticity estimates obtained in studies that have used experimental approach, such as Curhan [1972], may not have the bias because of the experiment setups. In their study, Van Dijk et al. examine different methods of estimating space elasticities, of which one is the regular OLS estimated regression model. By doing so, they aim to prove that the space elasticities estimated with OLS are biased. The reference model for regular OLS estimates is

$$\ln(\text{SALES}_j) = \alpha_j l_K + \beta_j \ln(\text{SHELF}_j) + \epsilon_j, \quad \epsilon_j \sim N(0, \sigma^2_{\epsilon_j} \times I_k) \tag{6}$$

where K is the number of stores, SALES_j is a vector with length K representing the average sales of the shampoo brand j in store $k \in [0, K], k \in \mathbb{N}_0$. SHELF_j is a vector with length K representing the shelf space of brand j in store $k \in [0, K], k \in \mathbb{N}_0$, α_j is the models intercept for each brand j. l_K is a vector of ones with length K, β_j is the space elasticity coefficient for brand j and ϵ_j is the error term.

In the study Van Dijk et al. considered price, promotion and sales data from five different brands in shampoo category over 107 weeks during years 1995-1997, gathered from 44 stores from a large retailer in Netherlands. They also had three shelf space measurements for each brand in each store, so that the effects of the shelf space variations could be measured. These shelf space measurement points positions at weeks 45, 69 and 107 and they used the average sales of preceding and succeeding 6 weeks of each data point to represent the sales corresponding to each shelf space measure. Since the periods include promotions and other irrelevant variation regarding the space elasticity estimates, they corrected the sales by estimating a regression model with promotion related regressors for each brand of shampoo and then used the store-week specific baseline sales forecasts obtained by the model as the sales variable. They estimated the models with purely cross-sectional data (only the first data point) as well as with data including time-variation (first and second data points). The third data point was used only for validation. For cross sectional data, they used three different methods in order to capture the endogeneity. The first approach was to use a model with control variables. In this approach, the authors added control variables to equation (6) with which they try to explain the correlation of the shelf space and the error term. They call this model OLS with control variables (OLSC) and it is defined as follows

$$\ln(\text{SALES}_j) = \alpha_j l_K + \beta_j \ln(\text{SHELF}_j) + \pi_{j1} P_{1j} + \pi_{j2} P_{2j} \epsilon_j, \tag{7}$$

where P_{1j} and P_{2j} are vectors with length K consisting of the store profile variables, π_{j1} and π_{j2} are parameters associated with the P vectors. Other parameters are same as in the equation (6).

The second approach was to use two different spatial models, which accounts the endogeneity of space elasticity with spatial structure based on store profiles (SPATIAL-CHAR model) or based on geographical distance between the stores (SPATIAL-GEO model). The spatial models are presented in Appendix A.

In the case of longitudinal time variation, Van Dijk et al. presents three different approaches. Fixed effect (FE) model and, similar to the cross-sectional data, two different spatial models that are presented in appendix A. (SPATTEMP-GEO and SPATTEMP-CHAR) The fixed effect model adds a store-specific parameter to OLS model with which the endogeneity is attempted to be captured. The model is defined as

$$\ln(\text{SALES}_{jp}) = \mu_j + \beta_j \ln(\text{SHELF}_{jp}) + \tau_{jp} + \epsilon_{jp}, \tag{8}$$

where μ_j a vector representing the store intercepts τ_{jp} is a vector of length K consisting of the period effects for brand j in period $p \in \{1, 2\}$.

The store profiles used in all spatial models are based on eight variables, as illustrated in Table 1. With the models and data described above, Van Dijk et al. estimated and validated the space elasticities for the 5 different brands of shampoo. As a result for the purely cross-sectional data, they found out that the elasticities obtained with the basic OLS model were systematically higher (average 0.85) than the ones estimated with other models, for example SPATIAL-CHAR, giving estimates averaging with space elasticity of 0.21, which matches the average of the experiment conducted by Curhan [1972]. The OLSC model positioned between the spatial and OLS models. However, none of the estimates obtained with the SPATIAL-CHAR or SPATIAL-GEO model were statistically significant. When the predictive capabilities of the cross-sectional data were tested with the third shelf space measure point, the SPATIAL-CHAR model was found to be the best, obtaining 44% more accurate estimates than OLS model, when the accuracy measured with MSE (Mean Squared Error), and 27% better when measured with MAE (Mean Absolute Error). In the case of longitudinal data, similar results were obtained. Both the FE and SPATTEMP-CHAR models yielded average space elasticities of 0.22. However, in this case for 3 out of five

Table 1: Variables Van Dijk et al. used in store profile classification [Van Dijk et al., 2004]

Variable	Description
SCLASS	Five variables capture the number of customers in different social classes. Classes are based on combinations of education level and profession
FLF	Five variables capture the number of customers in different family life phases. Households are classified according to the age of the oldest child or the age of the wage earner.
DRUG	Presence of a drug store within a radius of 100 meters (yes = 1, no = 0)
SUPDIST	Average distance to the five nearest stores (in meters)
SUPSIZE	Sales area of the store divided by the sales area of the nearest five competitors (in squared meters)
AREA	Sales area in squared meters
ACV	Total annual turnover (in guilders)
THC	Number of checkouts

brands, SPATTEMP-CHAR model was able to obtain statistically significant results. Also, the SPATTEMP-GEO model delivered significant results, averaging the space elasticity of 0.19. In the predictive capability test, the models using longitudinal data positioned rather close to each other's, improving the accuracy of the estimated elasticities 45-48% when measured in MSE and 26-29% in MAE. The FE model presented the lowest improvements (45% and 26%) and SPATTEMP-CHAR the highest (48% and 29%) in both cases.

As a conclusion, Eisend state that the results show that the space elasticity estimates obtained with OLS are biased upwards as was hypothesized. Another finding was that the geographical locations of the stores had little power to explain the endogeneity of the shelf space, but the store profile based classifications seemed more promising in that sense.

The study's hypothesis that the endogeneity of the space elasticities can be accounted for using only information about store profiles is interesting. The fact that they were able to obtain similar space elasticity estimates as Curhan [1972], who tackled the endogeneity with the experimental design, Desmet and Renaudin [1998] and Eisend [2014] who both took store profiles into account, may indicate that the hypothesis could be correct. However, the spatial models they used were complex, and the results obtained were not conclusive in every case. A better approach could be to recognize the sources of variations unrelated to shelf space more specifically and control those with more case-specific variables than store profiles. These variables could be related to price, promotion or season and using them in a regression model could result in more accurate estimates with less complicated models. The scope of the study was rather limited, focusing only on different shampoo brands, so any generalized findings or conclusions cannot be drawn conclusively, but it can be noted that there most likely are many other factors contributing to demand of a product that should be controlled when estimating the effect of shelf space.

2.3 Effect of the position of product

Dreze et al. [1994] conducted a study discussing also the effect the position of the product has on products' demand. To study the effects of shelf space and position of a product, they estimated a log-log model to explain the logarithm of unit sales with variables related to space and location elasticities and control variables. For the location effect, they decided to model the position of the product in the shelf considering two axes. They modeled the sales dependency to horizontal movement as quadratic and to vertical movement as cubic in order to be able to model the different shelf shapes accurately (i.e the refrigerator wells). The model for the effect of position was thus the following

$$\text{Position} = a_4 X_{ijk} + a_5 X_{ijk}^2 + a_6 Y_{ijk} + a_7 Y_{ijk}^2 + a_8 Y_{ijk}^3, \tag{9}$$

where X_{ijk} represents the products distance from the left edge of the shelf measured from center of the products facings and Y_{ijk} represents the products distance from the foot of the shelf measured similarly as the X coordinate. i is a brand, j a store and k a week index. For the space elasticity effect, they used a Gompertz growth model because it follows the similar S-shape they assumed the space elasticity effect to follow. The model they used for the space elasticity effect was

$$\text{Space} = a_9 e^{-kA}, \tag{10}$$

where A is the shelf space allocated for the product and k is the shelf space elasticity coefficient. In order to eliminate the effect of variations in sales caused by other than location and space, they added control variables to the model. They had dummy variables for each brand and store and a price coefficient to incorporate variation those factors cause on sales. The control portion of the model was

$$\text{Control} = a_0 + a1_{1i}B_i + a_{2j}S_j + a_3\log(P_{ijk}), \tag{11}$$

where B_i is a dummy for brand i, $_j$ is a dummy for store j, and k is a index for week. The complete model to explain the logarithm of sales was of form

$$\log(\text{U}_{ijk}) = \text{Control} + \text{Position} + \text{Space}, \tag{12}$$

where the U_{ijk} is the unit sales of product i if it is allocated j facings of space in shelf k.

They estimated the model with data from 8 product categories obtained from 60 stores for time period of 32 weeks in the middle of which the planograms (a visual presentation of products in the shelf according to which the products are arranged to the shelves) were changed. All promotional sales were removed from the dataset. The R^2 of the model ranged from 0.53 to 0.86. The importance of independent variables remained consistent across categories and the order based on the proportion of variance explained was brand and store, price and position, and shelf space as the least significant variable. As for the position effect, they found that it was statistically significant in every category, and best vertical positions were at eye-level. Horizontally, the best positions varied across categories and the best positions were either on middle of the shelf or near the edges. They estimated an average increase of 39% in sales if the product was moved from the worst possible vertical position to the best, and average increase of 15% when moving the product similarly in horizontal direction. The estimated average was 59% when moving from worst to best in both directions. For shelf space, they found significant relations with sales for all except one category. However, the study found out that in many categories, most of the products had so much shelf space that they end up to the upper flat part of the S-curve, meaning that increasing or decreasing the shelf space would not affect the sales significantly.

The findings in this study are significant since they offer data about the effects of the products' position on the shelf. As it may be intuitive, the position of the product has notable effects on sales, and according to the study, even more than the plain shelf space itself. The vertically best position being on eye-level sounds realistic and rather intuitive and, therefore, builds confidence in the credibility of the results. Also, the horizontal results sound believable and the difference in the best position could be explained for example with positions of the shelfs in the store, or some other factors. Based on the results of this study, it seems important to account for the position of the product when estimating the effects of shelf space allocation. Dreze et al. estimated percentages to demonstrate the benefits of good positioning of products and the increases in demand seemed very high. However, it must be noted that, as they were estimated for a case where the product is moved from the worst position to the best, the presented increases can be considered only as upper bounds for the benefit. Because of this, no further conclusions can be drawn from them. Furthermore, since this was a single study with a very limited scope, more research on this subject is needed before the findings can be seen as conclusive.

2.4 Summary

Based on the reviewed articles, the shelf space elasticities can be estimated by explaining the changes in sales with regression model having shelf space as one regressor. Curhan [1972] tried to estimate the effect by formulating the space elasticity as a function of shelf space and unit sales but findings with this approach were inconclusive. Therefore, the regression model estimation is preferred. The regression models reviewed that explained the demand with space were multiplicative and the relationship with demand and space were most often modeled as

$$D = as^{\alpha}, \tag{13}$$

where D represents demand, a is a scaling factor, s is the allocated space and α is the space elasticity. Most of the conclusive findings were obtained using this form of dependency, so this approach could be considered to be the most reliable. [Desmet and Renaudin, 1998] [Van Dijk et al., 2004] [Hansen and Heinsbroek, 1979] [Corstjens and Doyle, 1981]

Important point that arises from the reviewed studies is that multiple different product and store-specific characteristics affect the space elasticity, but there is no conclusive definition of the affecting variables. However, if the space elasticity could be estimated separately for each product or product category in each location, these product category and store-specific variables could be disregarded in the estimation. This will however require large amount of shelf space changes for each product-location and in a case of modern retail chains, sophisticated big-data technology and availability of calculating power to handle the amount of data needed. Nevertheless, this approach is preferred since the studies conducted so far have not been able to determine definitely the specific store and product group-specific variables affecting the space elasticity of the product in certain store.

Another consistent finding in the studies was also that the effect of space variation in sales is often less significant than other variation sources such as price, seasonality, promotion or trend-based variations. Therefore, these variations must be controlled either in a regression model with extra predictors, or already in the sales data used for the estimation. For example, the promotional variation can be controlled by correcting the sales of the campaign period to estimated normal sales of the period, or the campaign related variables can be added into the regression model. Both ways should produce equally accurate results and the preferred way depends on the data available. Taking the variables causing the external variations in sales into account in the regression model requires usually more data, since the variation in the explained variable, in this case sales, increases. One possible approach to estimate the space elasticities would be to try to explain the error in the forecasts with shelf space changes, if accurate enough forecasts are available. This approach assumes that all other variations in demand are already taken into account in the existing forecast and all remaining variation is caused by shelf space changes. This would be however suboptimal if the forecasts are not accurate, since the model would try to explain variations unrelated to shelf space changes with the shelf space changes.

As Dreze et al. [1994] observed that the position of the product in the shelf can be more significant factor in sales than mere shelf space amount, the position should also be included in the shelf space elasticity estimations. There are multiple different approaches on how to handle this effect. The first one is to calculate space elasticities separately for each product category in each store in each shelf position. The major disadvantage of this approach are the massive data requirements. Another, more practical, way to take this effect into account is to add regressors to the model to capture the effect of the position. Since taking the exact position into account, like Dreze et al. [1994] did, would complicate any operational applications unreasonably, the horizontal effect could be disregarded because the effect of it was less significant than vertical, and there were not significant conclusions about the horizontal position.

One discussion point could be that whether the shelf position affects also the space elasticities, or only the demand of the product. If the position affects only the demand, then it would be sufficient only to control the variation caused by it. But if the position also affects the space elasticity values, then the space elasticities should be estimated separately for each product category in each store in each shelf position. Another way to handle the dependency between the space elasticity and position could be to estimate an interaction between the shelf space and position of a product and add that to the model.

3 Using space elasticity information

3.1 Existing optimization models

The information about different space elasticities is itself useless, if it is not used to support decision making in different retail operations. As there are many articles concentrating on how the space elasticity can be estimated, there are also multiple articles that present ways to utilize the information in practice. The most intuitive application is to use space elasticity information to support space allocation decisions in stores and majority of the literature seems to concentrate on this.

Corstjens and Doyle [1981] developed an algorithm to optimize shelf space allocations for each product in a way that takes both space and cross elasticities as well as operating costs, caused by the shelf space, into account. The cross elasticities were included because they stated that increasing sales in one brand of a category will almost always result in a decrease in some other brand's sales in that category. This is a justifiable claim, and is often referred to as the cannibalization effect. They formed an optimization problem in which they maximized the difference between the overall gross margin resulting from the space allocation and the total operating costs related to it. The total gross margin was formulated as

$$\sum_{i=1}^{K} w_i \left[\alpha_i (s_i)^{\beta} \prod_{j=1, j \neq i}^{K} s_j^{\delta_{ij}} \right], \tag{14}$$

where w_i is the margin rate of product i, β_i is the direct space elasticity of product i, s_i is the amount of shelf space allocated for product i, δ represents the cross-space elasticity for products i and j, α is a product-specific constant and K is the total amount of products in the assortment. The expense related to the shelf space allocation was formulated as

$$\sum_{i=1}^{K} \gamma_i \left[\alpha_i^{\tau_i} s_i^{\beta_i \tau_i} \prod_{j=1, j \neq i}^{K} s_j^{\delta_{ij} \tau_i} \right], \tag{15}$$

where γ_i is cost rate of product i and τ_i is the cost elasticity, indicating the costs associated with the increased demand of product i. The optimization problem was constructed as follows

$$\max \sum_{i=1}^{K} w_i \left[\alpha_i (s_i)^{\beta} \prod_{j=1, j\neq i}^{K} s_j^{\delta_{ij}} \right] - \sum_{i=1}^{K} \gamma_i \left[\alpha_i^{\tau_i} s_i^{\beta_i \tau_i} \prod_{j=1, j\neq i}^{K} s_j^{\delta_{ij}\tau_i} \right] \tag{16}$$

s.t

$$\sum_{i=1}^{K} s_i \leq S^* \tag{17}$$

$$\alpha_i s_i^{\beta_i} \prod_{j=1, j\neq i}^{K} s_j^{\delta_{ij}} \leq Q_i^* \qquad i = 1, ..., K, \tag{18}$$

$$s_i^L \leq s_i \leq s_i^U \qquad i = 1, ..., K \tag{19}$$

$$s_i \geq 0, \qquad i = 1, ..., K \tag{20}$$

where the first constraint limits the total allocated space to the total capacity S^* of the store. The second constraint relates to the production or other availability limit Q_i^*, the demand of the product should not be higher than this limit. Third constraint allows retailer to decide lower and upper limits for the space allocated for each product. The last constraint ensures the non-negativity of the allocated space for each product.

Corstjens and Doyle tested the model in practice, as they had data from a retailer that had 140 stores. The authors decided to estimate the direct and cross space elasticities for five different product categories that were chocolate confectionary, toffee, hard-boiled candy, greeting cards, and ice cream. The stores were divided into two categories, small and large based on the size of the store. They used equation (14) without the product margin rate w_i to model the relationship between the unit sales and the allocated space of the product. With this method, they obtained statistically significant direct space elasticities and the values of them were similar to ones obtained in previous studies. For obtaining the cost elasticities (τ_i) and the cost rates (γ_i), they consulted the group managers of the retail chain who evaluated the procurement, carrying and out of stock costs related to sales for each product category in the case study. They then fitted equation (15) to that data and obtained estimates for the both cost coefficients.

With these product-specific coefficients, Corstjens and Doyle then solved the optimization problem in reasonable time using signomial geometrical programming procedure and the branch-and-bound method. The results were reasonable in a practical sense and differed significantly from the existing allocations, which indicates that significant increases in profit can be achieved by optimizing the allocation. They also state that the profit increases were substantial when the allocation patterns were adapted in stores. In order to study the effect of the cross elasticities, the authors also estimated the direct space elasticities without the cross-effect terms and evaluated the model with the new direct space elasticities. They found that the allocation result differed significantly from the previous result and the proposed profit increases were smaller. Therefore, they concluded that the cross elasticities had a significant effect in the model.

Hansen and Heinsbroek [1979] proposed a similar model to use space elasticity information, but without the effects of cross elasticities. They modeled the profit of product i with a formula

$$\pi_i = m_i A_i r_i^{\alpha_i} - c_i r_i, \tag{21}$$

where m_i represents the gross margin of a product i, r_i is the space allocated for product i, A_i is a scale factor, α_i denotes the space elasticity and c_i is the unit cost of space. They formulated an optimization problem

$$\max \quad \sum_{i=1}^{n} \pi_i - f(N, L) \tag{22}$$

s.t

$$\sum_{i=1}^{n} r_i \leq R \tag{23}$$

$$r_i \geq r_i^m y_i, \qquad i = 1, 2, ..., n \tag{24}$$

$$r_i \leq R y_i, \qquad i = 1, 2, ..., n \tag{25}$$

$$y_i \in \{0, 1\}, \qquad i = 1, 2, ..., n \tag{26}$$

$$\frac{r_i}{l_i} \in N^+, \qquad i = 1, 2, ..., n \tag{27}$$

where $f(N, L)$ is a function of a replenishment frequency N (times per week) and the lead time L (days), when $L = 0$, there is backroom stock for the product and the shelf can be replenished from the backroom, when $L > 0$, backroom stock is not available and it will take L days for the new stock to get to the store. Term l_i represents the length of one facing of product i, y_i is a boolean variable and is 1 when the product is part of the assortment and 0, otherwise; r_i^m is the minimum amount of space to be allocated for product i, if it is a part of assortment and R is a total shelf space of the store.

The constraints ensure the sensibility of the results. They solved the optimization problem by using the Generalized Lagrange Multipliers technique. [Everett III, 1963]

To test the model, they obtained data from a store audit of LOEB-IGA [Limited, 1970]. From there, they were able to collect all of the required information about 6443 products, except the space elasticity (α), expenses related to replenishment ($f(N, L)$) and minimum shelf space for products if they are in the assortment (r^m). To attain the space elasticity information, they reviewed 20 experiments related to space elasticities and defined a distribution according to which the space elasticities have usually been distributed among different products. Then, they assigned a space elasticity for each product using a random process based on that distribution. The authors justified this by stating that conclusive relation between product characteristics and space elasticities have not been found. The values of $f(N, L)$ were estimated by a manager of one LOEB-IGA store. The values of r^m were estimated by calculating a minimum quantity with which the product would have an availability of 98% with different values of N and L, taking the sales variation into account. With this dataset, the authors found theoretical profit increases varying from 83$ to 167$ per week in one store. This would mean increases from 4300$ to 8600$ per store in one year. However, these increases were only theoretical and not verified in practice so they should be considered only as illustrative values. They also estimated the model without the individual values, having a median space elasticity value of 0.15 for all of the products and found out that the estimated profits were 51 $ per week less in one store than with the individual values. Because of this finding, they state that procedures of estimating specific values of α are worth studying.

After studying the effects of the position to the shelf elasticity, Dreze et al. [1994] developed a model to optimize the shelf space and position of different product categories to maximize the total profit gained from them. Their plan was not to develop a complete model to be used in practice but rather to learn more about the magnitude of the potential that better space allocation could have. They formulated the optimization problem as

$$\max \quad \sum_{i=1}^{N} \text{profit}_{ijk} \times \text{usage}_{ijk} \tag{28}$$

s.t

$$\sum_{j=1}^{J}\sum_{k=1}^{K}\text{usage}_{ijk} = 1, \qquad i = 1, ..., N \tag{29}$$

$$\sum_{i=1}^{N}\sum_{j=1}^{J}\text{usage}_{ijk} \times \text{size}_{ij} \leq 1, \qquad k = 1, ..., K \tag{30}$$

where i is the index of the product, j represents the amount of facings (one facing represents the shelf space that one unit of the product occupies) and k is the index of the shelf level in vertical direction. $profit_{ijk}$ represents the profits obtained when j facings of product i is allocated to shelf level k. $usage_{ijk}$ is a binary variable with a value of 1 when j facings of product i is allocated to shelf level k, 0 otherwise. $size_{ij}$ represents the proportion of the shelf levels total width that is populated if j facings of product i is allocated to it. To model the profit, they used the function (12) described in the previous section, but decided to disregard the horizontal position of the product, as its effect was minor, and taking it into account would have complicated the optimization problem drastically. It is notable that the authors do not comment on how the costs of product are related to shelf space in the model, or whether they have approximated them as independent of shelf space. They solved the optimization problem for the same product categories for which they estimated the regression model (12). In order to study the effects of both shelf space and position, they estimated the model first while optimizing both variables, and then for both individually by holding other dimension constant. The profit increase when optimizing both was 15%, when optimizing only position 10% and when optimizing only shelf space 3%. This amplifies the conclusions that the products position in the shelfs has a significant effect on its sales and the most optimal solution can be obtained when both are optimized. However, these findings were not verified in practice, so the profit increases are purely theoretical and can be considered only directive.

Urban [1998] proposed a generalized optimization model to allocate shelf space and select products to compose the assortment while taking into account the effect the inventory level has on the demand. They modeled the displayed inventory level so that it is kept at the allocated quantity if there is backroom inventory, and when the products' backroom inventory level is used, also the displayed inventory level starts to decrease, thus affecting the demand rate of the product until the replenishment order arrives. This, however, is not always an optimal way to operate and, since the inventory level complicates the optimization, it may be reasonable to assume that retailers who seek to gain advantage from optimized space allocation already uses replenishment systems that allows them to replenish their products so that they are at almost all times able to keep their displayed inventory level at the allocated quantity. If we disregard the inventory level effect from their demand function, the shelf space related demand function they discussed was rather similar to the previously reviewed studies and of form

$$d_j = \alpha_j \phi_j^{\beta_j} \left[\prod_{k \in N^+} \phi_k^{\delta_{jk}} \right] \left[1 + \sum_{i \in N^-} (1 - \lambda_{ij}) f(\alpha_i, \delta_{ij}) \right], \tag{31}$$
$$\alpha_j > 0, 0 < \beta_j < 1, 0 \leq \lambda_{ij} \leq 1,$$

where β_j is the space elasticity coefficient for product j, ϕ_j is the shelf space allocated for product j and α_j is a space scale parameter for product j. δ_{ij} represents the cross-space elasticity between products i and j, λ_{ij} represents the magnitude of resistance that is expected from the customer to switch their purchase choice from a product that is not included in the assortment to a product included in it. N^+ is the set products composing the assortment, while N^- is the set of products not included in the assortment plan of the store. If we examine the equation in three parts, the first one represents the demand of the products responding to the products own shelf space, second part represents the effect the other products included in the assortment plan has on it and the last part incorporates the potential additional demand gained when consumers change their purchase decision from the products not included in the assortment plan to ones that are included. The part that makes this equation different from other formulations is the inclusion of the effect that the products not chosen in assortment might have. This possible effect on demand has not come up in previously reviewed studies and the approximated effect of the phenomenon does not seem significant. Furthermore, obtaining reliable estimates of λ_{ij} values of different products would be difficult, making it hard to take the effect into account in any operational applications. Excluding the last part of the demand function results in a demand function similar to ones discussed in other literature.

Hariga et al. [2007] proposed a similar joint model, in which they tried to find an optimal way to select the products to compose the assortment, the locations of the shelves in which they would be located, allocated shelf space quantities for each product and an order quantity for each product. Once again, the optimal order quantity is not the subject of interest in this thesis, since it complicates the optimization problem, and since there exists already several tools with which retailers can optimize the replenishment. As stated before, it is assumed that the retailers who are seeking to draw benefits from the space allocation, should have their other operations, such as ordering, well optimized. They also assume a similar inventory cycle than Urban [1998], with the exception that the backroom stock is always assumed to go to zero just before the new order arrives to store, making the situation rather unrealistic regarding the operational use cases. For the demand of different products, they used an equation similar to (14), without the margin rate and with inclusion of the shelf index k, so that the demand, allocated space and direct space elasticities are always investigated for item i in shelf k. They used this function to model the space dependent demand in their joint optimization model, but as the model focused also on replenishment, it is not itself applicable to this thesis. However, the similarity of the demand function to previous studies contributes to verifying the previously formed equation to model the relationship between shelf space and demand.

Yang [2001] had a different approach on solving the shelf space allocation problem. He interpreted the problem as a multi-constraint knapsack problem and developed an algorithm similar to the ones used to solve traditional knapsack problems. The knapsack problem is usually concerned with finding a set of items with a limit for the total weight of the items. Each item has a weight and a utility factor, which tells how much it is worth if chosen in the knapsack. Yang interpreted the shelf space allocation problem so that the products estimated profit per length of one facing is considered as its length and the ranking order of those weights as a utility. He does not comment on the estimation of the profit per length of facing and focuses mainly on the performance of the algorithm, which is not the main interest of this thesis. However, modeling the space allocation problem as knapsack problem is interesting and could be usable when the optimal solution method is developed, since there exist fast and accurate algorithms for solving knapsack problems.

3.2 Summary

The optimization models developed to utilize space elasticity in shelf space allocation have good elements but none of them are completely suitable for operational applications as such. The studies seem to be unanimous about the relationship between direct space elasticities and demand, but conclusive findings about what else should be included in the model are not yet fully understood. Most optimization models take cross-space elasticities between different products into account, but in operational applications, acquiring reliable information about the relationship between products is both difficult and time consuming. The cannibalization effect is much stronger when the shelf space effect is studied for each product individually, since consumers can easily substitute one brand with another from the same category, but if they are estimated only for each product category, the effect of cross space elasticities depreciate. The effect of cross elasticities can thus be minimized by evaluating the space elasticities only for each product category, and optimizing the space allocation on product category level. This approach should justify the exclusion of the effect of cross elasticities, thus making it easier to obtain the data needed in continuous operational application.

As discussed in the previous section, the position of the product in the shelf plays an important role in the relationship between shelf space allocation and demand. This effect, therefore, needs to be taken into account for the optimization model to deliver actually beneficial results. The remaining question is how to take the effect of space into account in the optimization models. One approach could be to divide the shelf in different blocks and value the shelf space in them differently. The optimization model could then allocate the shelf space separately for each block, similarly to approach of Dreze et al. [1994] in equation (28). However, this will complicate the optimization model, since the amount allocated to each block needs to be handled separately. Another approach is to include the position to the problem as another decision variable. Since taking the exact position into account would result in a really complex set of possible solutions, the space measure should be simplified somehow, for example by dividing the shelf into blocks.

Many of the papers also used profit functions that included shelf space related costs for each product in the assortment. The assumption that the shelving and other shelf space related costs are different for each product is valid and could result for example from different sales packages. Regarding the operational applications, the information required to estimate the different costs reliably for each product is hard, since it can rarely be derived from data. Obtaining the data would therefore require interviewing the store managers, and the estimates could still be inaccurate. In operational applications, it would be desirable to be able to draw all necessary information from available data, so taking these different costs into account would complicate the application substantially. However, when the space allocation is optimized on a product category level, the different space allocation related costs of products could be equalized, so that the costs between product categories could be estimated as constant. However, this should be validated in future studies. If the costs could be estimated as constant for all categories, the space related costs could be disregarded from the optimization problem and the function to be maximized could be the demand related profit of all product categories.

Regardless of being the most intuitive option, optimizing the shelf space allocation is not the only way to use shelf space elasticity information in operational applications. Together with information about price elasticities, space elasticity could be utilized in promotion planning. For example, a demand of a product or product category that has a high space elasticity and low price elasticity benefits more from promotions that focuses on the placement of the products rather than large discounts. This way the retailer would obtain a higher return from the promotions, as they would be able to hold up a higher profit margin. Similarly, by using higher discount percent with less shelf space for products/product categories that have high price elasticity maximizes the demand of the product while saving more and better shelf space to products that benefit more from it. However, determining the exact methods on how to operationalize this information reliably in promotion planning requires more information about the subject and should therefore be a subject of further studies. Another possible approach for retailers on exploiting the space elasticity would be to use it in discussions with the manufacturers and importers. Manufacturers often negotiate deals with retailers to ensure sufficient amount of shelf space for their products, by selling their products with smaller margins if the retailers agree to allocate more shelf space to it. However, if retailers have accurate information about how the products demand depends on its shelf space, they are able to price their shelf space more effectively. Similarly, operationalizing this strategy requires more research on the subject.

4 Conclusions

This thesis conducted a literature review about the studies discussing the estimation and usage of space elasticity information. Based on the review, we constructed guidelines to generalize and operationalize the estimation and utilization of the space elasticity information.

It can be concluded from the reviewed articles that a significant relationship between shelf space and demand exists and the magnitude of it varies across product categories and stores. The space elasticity coefficient averages somewhere below 0.2, according to most recent studies. The value of space elasticity of a product is affected by multiple variables related to its category and store it is sold in, but the studies have not been able to determine the affecting variables conclusively. Therefore, space elasticity should be estimated separately for each product category in each store. The products' position in the shelf was also found to affect its demand, but the relationship between the space elasticity and the position have not been studied. It should therefore be investigated further and if the position is shown to affect the space elasticity of the product, the space elasticities should be estimated separately for each product in each store in each shelf position. Even though there is a significant relationship between shelf space and demand, it accounts for only a minor part of the variation in the sales of product. To estimate the space elasticity, all the other variation, such as seasonal, promotional, trend, price or shelf position related variation should be controlled in the estimation. Controlling can either be done by cleaning the sales data from these variations, or by adding appropriate control variables to the regression model. The preferred way of doing this depends on the available data.

Regarding the usage of the space elasticity information, the most studied application is to optimize the shelf space allocation so that the obtained profit is maximized. Also, other application areas seem promising, such as supporting promotion planning or retailers' negotiations with manufacturers. However, developing useful ways to utilize the information regarding these areas requires further investigation. Based on this study, the preferred way to optimize the shelf space allocation would be to estimate the space elasticities for each product category, and then maximize the total profit for each store by determining the optimal way to allocate shelf space and position between different product categories.

A set of restrictions is required to ensure a realistic allocation of shelf space in a shelf space optimization. At minimum, the restrictions should limit the total allocated space to the total capacity of the store, enable manual lower and upper bounds for each product category and ensure the non-negativity of each allocation decision. The model could also be used to determine which products to include in the assortment. If the model is for these assortment decisions, a binary variable should be included in the model and in the restrictions, to indicate whether the product is a part of the assortment or not. Also, other restrictions may need to be applied to ensure sensible allocation, but these restrictions need to be defined based on the data used and the needs of the retailer. Optimizing store-specific shelf space allocations is reasonable since the store was found to affect the product categories' space elasticities, and optimizing the allocations for each store ensures the optimality of the allocation. However, in a case of large retail chain, maintaining store-specific space allocations may not be the best policy. In this case, it could be desirable to group the stores based on the product category and store-specific space elasticities and use the average of each category's space elasticity as a representative of each product category in a store group. Then, the allocations could be calculated for each store group, so that only a few different space allocations should be maintained at the same retail chain.

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A Spatial models

Spatial model for cross-sectional data

$$\ln(\text{SALES}_j) = \alpha^*_{\text{SALES},j} l_K + \beta^*_j \ln(\text{SHELF}_j) + \epsilon^*_{\text{SALES},j} \tag{32}$$

$$\ln(\text{SHELF}_j) = \alpha^*_{\text{SHELF},j} l_K + \epsilon^*_{\text{SHELF},j} \tag{33}$$

$$\epsilon^*_{\text{SALES},j} = \lambda^*_j W \epsilon^*_{\text{SALES},j} + \upsilon^*_{\text{SALES},j} \tag{34}$$

$$\epsilon^*_{\text{SHELF},j} = \gamma^*_j \epsilon^*_{\text{SALES},j} + \upsilon^*_{\text{SHELF},j}, \tag{35}$$

where $\alpha^*_{\text{SALES},j}$ is the constant term for the sales equations, $\alpha^*_{\text{SHELF},j}$ is the constant term for the shelf space equations, W is a standardized spatial weight matrix of dimension $K \times K$, consisting of elements connecting the stores, for SPATIAL-GEO this means the geographical proximity, and for SPATIAL-CHAR the store profiles. It is standardized so that the rows of the matrix sums to one. $\epsilon^*_{\text{SALES},j}$ and $\epsilon^*_{\text{SHELF},j}$ are vectors of size K consisting of error terms with the spatial structure. $\upsilon^*_{\text{SALES},j}$ and $\upsilon^*_{\text{SHELF},j}$ are vectors of size K consisting of error terms and variances that are assumed to be independently distributed. β^*_j, γ^*_j and λ^*_j are store-specific parameters.

The spatial model for longitudinal data

$$\ln(\text{SALES}_{jp}) = \alpha^{***}_{\text{SALES},j} l_K + \beta^{***}_j \ln(\text{SHELF}_{jp}) + \epsilon^{***}_{\text{SALES},jp} \tag{36}$$

$$\ln(\text{SHELF}_{jp}) = \alpha^{***}_{\text{SHELF},j} l_K + \epsilon^{***}_{\text{SHELF},jp} \tag{37}$$

$$\epsilon^{***}_{\text{SALES},jp} = \mu^{***}_j + \upsilon^{***}_{\text{SALES},jp} \tag{38}$$

$$\mu^{***}_j = \lambda^{***}_j W \mu^{***}_j + \upsilon^{***}_j \tag{39}$$

$$\epsilon^{***}_{\text{SHELF},jp} = \gamma^{***}_j \mu^{***}_j + \xi^{***}_{\text{SHELF},jp} \tag{40}$$

$$\xi^{***}_{\text{SHELF},jp} = \rho^{***}_{\text{SHELF},j} \xi^{***}_{\text{SHELF},j,p-1} + \upsilon^{***}_{\text{SHELF},jp}, \tag{41}$$

where the index $p \in \{1, 2\}$ indicates the period of the shelf space measurement, μ^{****}_j is a vector of random intercepts constant over time and spatially related to each other. $\xi^{***}_{\text{SHELF},jp}$ is a vector of length K consisting of autocorrelated error terms with autocorrelation parameter $\rho^{***}_{\text{SHELF},j}$. The vectors $\upsilon^{***}_{\text{SALES},jp}, \upsilon^{***}_j$ and $\upsilon^{***}_{\text{SHELF},jp}$ represents the error terms and variances that are assumed independently distributed. [Van Dijk et al., 2004]