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A simulation model to compare opportunistic maintenance policies

Bachelor's Thesis
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Abstract of bachelor's thesis

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Abstract

All technical systems require maintenance to stay operative. Components' lifetime is unknown, and their condition deteriorates constantly. In large technical plants, maintenance can be costly since keeping the system down causes production losses. The fixed cost that must be paid every time the system is maintained results in that it is useful to group maintenance activities. In addition, there can be economical or structural dependencies between components, which effect on the total maintenance cost. Therefore, opportunistic maintenance is often useful strategy to schedule and plan maintenance. In opportunistic maintenance, components can be maintained whenever an opportunity arrives, and the maintenance schedule can be updated when new information is received.

In this work, we implemented a simulation model to compare different maintenance policies using information about the total cost, the number of maintenance sessions and the number of failures. Components' condition is modelled as probability distributions that describe the probability that a component will fail by a certain time. The system forms a directed graph so that economic and structural dependencies can be modelled. The arcs describe dependencies between components and the weights of the arcs determine maintenance costs. In addition, the system has a fixed cost that is paid every time the system is maintained. In the simulation model, it is possible to implement different maintenance policies and compare them using Monte Carlo simulation.

We implemented four maintenance polices: age-based policy with and without inspections, and simple opportunistic maintenance policy with and without inspections. In the age-based policy, the maintenance is scheduled only based on the elapsed time since last maintenance and the threshold value is predetermined for each component individually. The inspections mean that we can do observations about components' condition every time the system is maintained, and update maintenance schedule based on new information. In simple opportunistic maintenance, the components can be maintained before originally planned, if the system is maintained. The results show that inspections decrease the amount of total costs especially in age-based policy. The opportunistic maintenance policies were more cost efficient than the age-based policies and the difference became more significant when the fixed cost was increased.

Keywords opportunistic maintenance, age-based maintenance, grouping of maintenance activities, simulation model, Monte Carlo simulation, graph

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Tiivistelmä

Kaikkia teknisiä systeemejä pitää huoltaa, jotta ne pysyvät toimintakunnossa. Komponenttien elinikä on tuntematon ja niiden tila heikkenee koko ajan. Suurissa laitoksissa huoltaminen voi olla kallista, sillä systeemin pysäyttäminen huollon ajaksi aiheuttaa tappioita. Etenkin tämän kiinteän kustannuksen takia huoltotoimenpiteitä on tarpeellista ryhmitellä. Lisäksi komponenttien välillä voi olla taloudellisia tai rakenteellisia riippuvuuksia, jotka osaltaan vaikuttavat huollon kokonaiskustannuksiin. Tämän vuoksi opportunistinen huolto on usein hyödyllinen strategia, kun suunnitellaan huollon aikataulua. Opportunistisessa huollossa hyödynnetään jokainen tilaisuus huoltaa komponentteja ja päivitetään huoltosuunnitelmaa sitä mukaan, kun uutta informaatiota löytyy.

Tässä työssä kehitettiin simulointimalli, jolla voidaan vertailla erilaisia opportunistisia huoltostrategioita tutkimalla syntyviä kokonaiskustannuksia, huoltokertojen määrää ja tapahtuneiden vikojen määrää. Komponenttien tilaa mallinnetaan todennäköisyysjakaumalla, joka kuvaa komponentin todennäköisyyttä vikaantua tiettyyn aikaan mennessä. Komponenttien välisiä taloudellisia ja rakenteellisia riippuvuuksia kuvataan muodostamalla systeemistä suunnattu graafi. Graafissa nuolet kuvaavat, miten muiden komponenttien huoltaminen vaikuttaa kyseisen komponentin huoltamiseen, ja niiden painot kertovat kustannuksen tällöin. Lisäksi systeemillä on kiinteä kustannus, joka maksetaan joka kerta, kun systeemiä huolletaan. Simulointimallissa on mahdollista rakentaa erilaisia huoltostrategioita, joita voidaan vertailla Monte Carlo -simuloinnilla.

Simulointimalliin toteutettiin neljä eri huoltostrategiaa: ikään perustuva huoltostrategia tarkastuksilla ja ilman sekä yksinkertainen opportunistinen huoltostrategia tarkastuksilla ja ilman. Ikään perustuva huolto määräytyy komponentin käyttöiän perusteella ja jokaiselle komponentille on määritelty aikaväli, jonka jälkeen se huolletaan. Tarkastukset tarkoittavat sitä, että huoltokertojen yhteydessä on mahdollista tehdä havaintoja komponenttien tilasta ja uuden tiedon perusteella päivittää huoltosuunnitelmaa. Yksinkertaisessa opportunistisessa huollossa komponentti on mahdollista huoltaa aikaisemmin kuin suunniteltu, jos systeemi menee huoltoon jostain muusta syystä. Tuloksista nähtiin, että tarkastusten tekiminen laskee kokonaiskustannuksia etenkin ikään perustuvassa strategiassa. Opportunistinen huolto oli kustannustehokkaampaa kuin ikään perustuva ja sen merkitys korostui entisestään, kun kiinteää kustannusta nostettiin.

Avainsanat opportunistinen huolto, ikään perustuva huolto, huoltotoimenpiteiden ryhmittely, simulointimalli, Monte Carlo -simulointi, graafi

Contents

1 Introduction

Industrial plants, machines and vehicles consist of multiple subsystems and even hundreds of components, and they all require maintenance to stay operative. All components are wearing out at their own rate. We would like to use all the components as long as possible to reduce costs from unnecessary maintenance. However, the lifetime of a component is typically unknown and random failures can occur all the time. A system failure causes production losses and can even be a serious safety hazard. According to Ab-Samat and Kamaruddin (2014), in chemical plants, production losses due to equipment failures can be tens of thousands of dollars per hour. To prevent failures and reduce costs, we need maintenance planning and scheduling.

In many technical systems, *a fixed cost* occurs whenever a system is maintained. In addition, a component-specific cost occurs when the component is repaired or replaced. Consequently, we can profit from grouping of maintenance activities and from using *opportunistic maintenance policies.* In opportunistic maintenance, a component can be preventively repaired when the system is down because of an other component's failure or maintenance, that is to say whenever an opportunity arrives.

According to Bevilacqua and Braglia (2000) the maintenance costs can be $15-70\%$ of total production cost depending on the type of industry. Maintenance optimization can reduce maintenance costs significantly. Optimization is often done based on a maintenance policy and the problem is to find optimal way of executing that policy. The challenge is to create an efficient model that describes the system accurately enough. New technologies, like cheap sensors and efficient data processing, offer new ways to develop system modeling.

In this work, we build a simulation model to compare different maintenance policies and especially analyze their possibilities to schedule and group maintenance activities. We want to examine ways to schedule maintenance activities of a multi-component system so that expected mainenance costs would minimize. The objective is that the model works for systems of 5-15 different components with economic and structural dependencies.

The thesis starts with a theory in Section 2 where the main concepts of maintenance are defined and relevant earlier research results are discussed. Section 3 overviews the methods used in the work including the explanation of the used distributions, the system model, the simulation model and the implemented maintenance policies. Section 4 presents the results of the simulation and the last Section 5 concludes the work.

2 Theory

2.1 Component's condition

Technical systems consist of components which wear out at their own rate. Probability of a component failure is usually modeled as *a failure distribution*. They are cumulative probability distributions that describe the probability that a component fails by a certain time.

Reliability and *failure rate* are often used to describe a condition of a component. Barlow and Proschan (1996) define that the reliability is the probability of a device performing adequately. Failure rate represents the probability that an object of certain age will fail. Figure 1 presents failure distributions, reliabilities and failure rates of two different components.

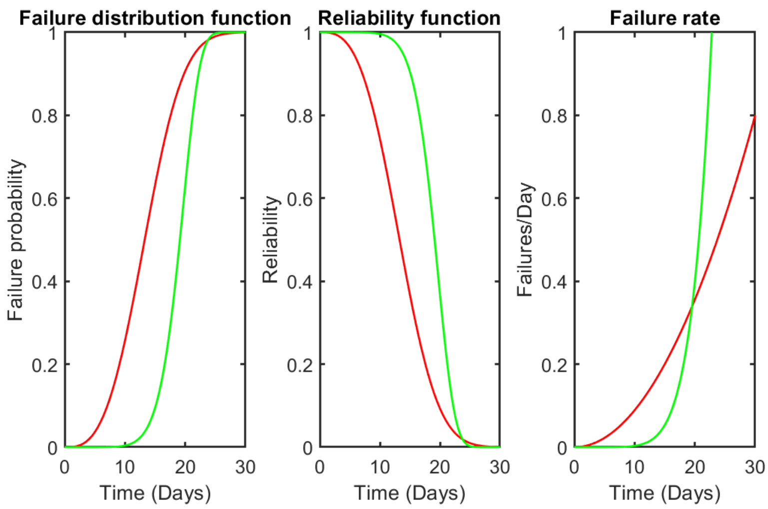

Figure 1: The figure presents failure distribution, reliability and failure rate of two different components. Both component's failure distributions are Weibull distributed.

Only some probability distributions are suitable for modeling technical systems. For example, the exponential distribution with a constant failure rate and the log normal distribution with decreasing failure rate are not useful for modeling technical systems, because components with constant or decreasing

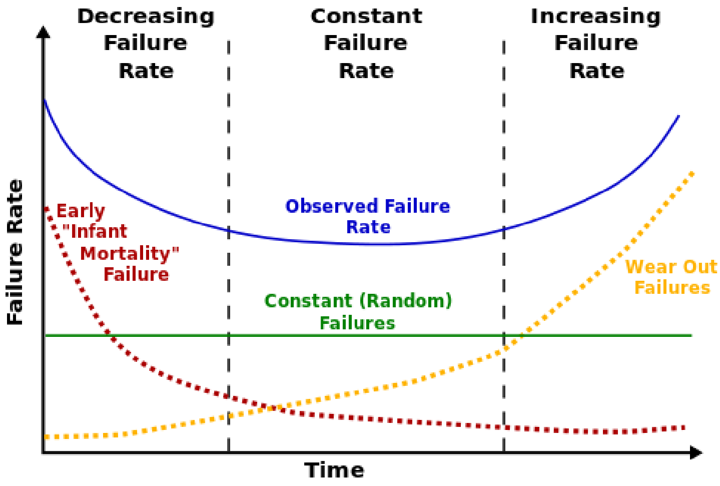

Figure 2: The figure present a bathtub curve. (The Figure is from the public domain.)

failure rates do not need preventive maintenance since we cannot reduce the risk of a failure by repairing or replacing the component.

In many applications, the failure rate is eventually increasing because of the inevitable deterioration of components. Commonly used distributions in technical context are the gamma distribution, the Weibull distribution and the bathtub curve. Modarres et al. (1999) tell that the bathtub curve has three regions as Figure 2 presents. At first, the failure rate is decreasing and it represents the early failures due to defects in design, manufacturing and construction. After that the failure rate is reasonably constant and only random failures occur. At the end, failure rate increases since the component is wearing out.

Amongst other methods these quantities can be used to determine the threshold for component to be too risky to fail. After the threshold time the component should be repaired or replaced. By using information from failure distributions we could obtain optimal maintenance schedule for each component.

2.2 Component dependencies

The individual maintenance schedules of components do not always result in an optimal plan for the whole system because of fixed costs and possible dependencies between components. Defined by Thomas (1986), dependencies are generally divided into three categories: *economic dependence*, *structural dependence* and *stochastic dependence*.

Economic dependence can be either positive or negative. Nicolai and Dekker (2006) explain that economic dependence implies that the cost of joint maintenance of components is not equal to the total cost of repairing these components individually. The most common reason for positive economic dependence is *the set-up cost*. The set-up cost consist of costs that occur when the system is prepared for maintenance.

The other often mentioned reason is *the downtime cost*. It consists of the production losses that are generated when system is put down and not operating. In some cases, this might result in negative dependence. For example, maintaining several components takes longer time and the production loss becomes too high. On the other hand, Nicolai and Dekker (2006) called this downtime opportunity which implies that it can result in positive dependence. When the component fails unexpectedly and the system has to be put down, we have an opportunity to execute other activities as well.

The set-up cost and the downtime cost are combined in many maintenance models and the combination is called *the fixed cost*. The fixed cost gathers together all the costs that have to be paid when the system is maintained and that are independent of the components under maintenance.

The set up cost and the downtime cost are not the only reasons for economic dependencies. Moreover, they can consist of multiple parts o subsystems. When the set-up cost concerns just a part of components, Nicolai and Dekker (2006) talked about multiple set-ups. In some cases the maintenance of different components might require different kind of set up activities. These dependencies can be seen as positive economic dependence between certain components. There are also several possible reasons for negative economic dependence. Nicolai and Dekker (2006) mentioned, for example, manpower restrictions and safety requirements.

Structural dependencies are a result of a system's physical appearance. For example, maintenance of component B can be a prerequisite for maintaining component A or component B has to be dismantled before component A can be handled. According to Nicolai and Dekker (2006), there may be several reasons for structural dependence. A as a result, there might be components that cannot be maintained individually at all.

Stochastic dependence describes components' failure interactions. This means that a component failure can increase the failure probability of another component. Because the condition of a component is often modeled using a probability distribution, stochastic dependence is sometimes called probabilistic dependence, Nicolai and Dekker (2006).

2.3 Maintenance types

The consequence of dependencies mentioned above is that there are constrains on what maintenance plans are feasible and that the total maintenance cost is not necessary linear in component-specific costs. *Maintenance policies or strategies* are ways to find optimal maintenance plan to schedule maintenance adequately and cost-efficiently. According to Pargar et al. (2017) there are three types of maintenance that we can use to create a functional maintenance strategy.

- *Corrective maintenance* is repairing already failed components. This is necessary for all maintenance policies because unexpected failures can be impossible to avoid. Considering unscheduled downtime, production losses and delays, corrective maintenance is rarely enough to keep the system operating cost-efficiently.
- *Preventive maintenance* includes scheduling of maintenance activities. The idea is to prevent failures to keep the system operating. A maintenance schedule can be planned based on component's age, reliability or some other quantity. Probability distributions can be used to decide when to repair or replace components but all methods are still based on statistical information.
- *Predictive maintenance* takes preventive maintenance a step further. It is also called a prognostic approach because maintenance schedule is made based on a predicted condition of the system. To predict the future condition of the system, we can use real-time information of component's condition and health status. These can be obtained by using sensors and IoT. Since this kind of technology is developing fast, so called *condition-based maintenance* is becoming more and more popular. In condition based maintenance, the up-to-date information about a component's condition is used for optimizing maintenance schedule.

2.4 Opportunistic maintenance policy

In *opportunistic maintenance policy* maintenance sessions caused by a failure can be used to execute preventive maintenance. The policy can also be combined with predictive methods. According to Zhang and Zeng (2015) an opportunistic maintenance is performed when an opportunity arrives. Opportunity can be anything from planned shutdown to a failure in a system. Because of the dependencies between components, a cost-efficient maintenance strategy often uses opportunistic maintenance. This usually includes grouping of maintenance activities and updating the maintenance plan when a failure occurs.

Figure 3 presents an example system of five components, for which we have used an opportunistic maintenance policy. Preventive maintenance has been used to group activities, corrective maintenance to repair the failed component and predictive maintenance to react to real-time information of the component's failure. The first two maintenance sessions are executed normally. At the time 13 component 3 fails and the maintenance plan is changed so that the planned maintenance of components 1, 2 and 5 is carried out earlier than initially planed.

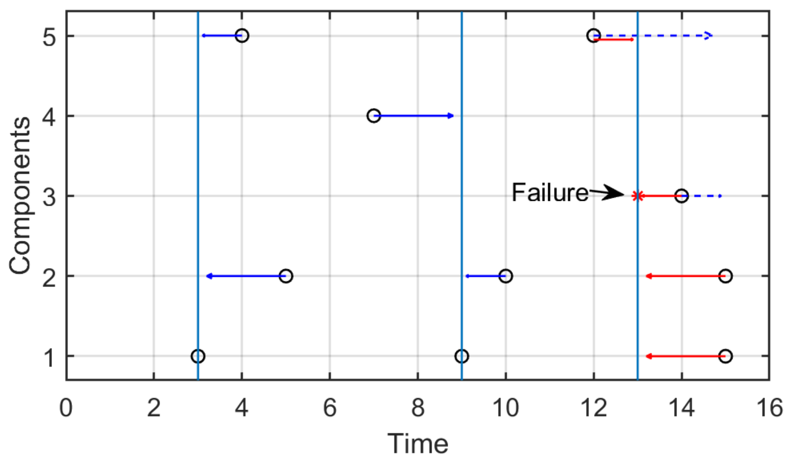

Figure 3: The individual maintenance plans are determined for each component (black dotes). Then maintenance plan is made by grouping the maintenance activities (blue arrows).

Opportunistic maintenance policies take advantage of the downtime opportunity mentioned by Nicolai and Dekker (2006). In their review of multi-component maintenance models Dekker et al. (1997) pointed out that the advantage of opportunistic maintenance is that the set-up costs can be saved. However, they though that planning and preparation for maintenance was difficult because the maintenance plan could be changed whenever something unexpected occurs.

2.5 Literature review

A survey of maintenance models for multi-unit systems by Cho and Parlar (1991) gathers together the works done before 1991 and gives an overview of the topic. They categorize opportunistic maintenance models together with group, block and cannibalization models. All of those models use grouping of activities but different policy to determine groups. Most of the models that Cho and Parlar (1991) present are build for two component systems or multi-component systems with one monitored component.

According to Ab-Samat and Kamaruddin (2014) opportunistic maintenance policies are initially build by combining *age replacement policy* and *block replacement policy*. In the age-based replacement or maintenance, decisions are made based on a component's age or the time since the last maintenance. Block replacement means maintaining components in groups which reduces the number of maintenance sessions.

By new techniques, it is possible to use more advanced methods for decision-making. In the approach of Hu and Zhang (2014), maintenance decisions based on an age and a risk of a component. They concluded that their method reduced losses and costs as well as helped on fault early-warning control. They concentrated especially on complex mechanical systems that have high production losses in case of a system failure.

There have also been plenty of studies on condition-based maintenance where components can be monitored and maintenance can be executed just in time. For example, Koochaki et al. (2012) studied condition-based maintenance in an opportunistic maintenance context. They considered maintenance costs and line productivity to compare their results. They concluded that even though condition-based maintenance policy minimized costs, it did not maximize line productivity. The reason for this was that their policy grouped less maintenance activities than simple age-based maintenance.

Bouvard et al. (2011) used condition-based maintenance for dynamic planning and grouping considering commercial heavy vehicle. They concluded that using of online information for decision making reduced costs and dynamic maintenance plans should be investigated further. Shi and Zeng (2016) presented a dynamic opportunistic condition-based maintenance strategy with economic and stochastic dependence. They used real-time information to predict the remaining useful life of a component. Their results showed that the proposed strategy maximized production efficiency, reduced costs and improved security.

Maintenance models' planning horizon can be finite (short or medium term) or infinite (long term). Wildeman et al. (1997) combined different time horizons and created a dynamic grouping policy using rolling time-horizon. The idea of a rolling time-horizon approach is to update the maintenance plan for finite horizon when new information is available or something unexpected occurs. Work of Wildeman et al. (1997) has been a baseline for many other articles about dynamic grouping, which can be used in advanced opportunistic maintenance. Vu et al. (2014) added more detailed positive and negative economic dependencies to the model and allowed the system to be more complex. Other works have also been done focusing on different aspects. For example, Do et al. (2015) concentrated on the downtime of the system and limited resources.

Pargar et al. (2017) used simulations to compare different maintenance policies. They concentrated on grouping of maintenance activities and balancing. Balancing was a strategy where the lifetime of a component was considered when making individual maintenance plan so that the component would not be maintained close to its removal from the system. Their result showed that the integrated balancing and grouping method was slightly better than strategy with only grouping. The article showed that even simple grouping policy is a competitive method against simple age-based policy.

Van Horenbeek and Pintelon (2013) compared different maintenance policies for complex multi-component systems. They created a dynamic predictive maintenance policy that was compared to five other policies: block-based maintenance, age-based maintenance, age-based maintenance with grouping, inspection condition-based maintenance and continuous condition-based maintenance. Their results showed that their dynamic predictive maintenance policy reduced costs significantly.

Economic dependencies are commonly used in maintenance models since the objective is often minimizing the total cost. Geng et al. (2015) considered a multi-component system with structural and economic dependencies and created an opportunistic maintenance model using Monte Carlo simulation. They concluded the article by showing that the opportunistic maintenance strategy was more cost-efficient when structural dependencies were included. Nguyen et al. (2015) also considered the system's structure in their model. They created a two-level decision-making process where the system and the components were considered in two steps. The system had complex structure and economic and structural dependencies between components were taken into account.

Laggoune et al. (2009) considered opportunistic policy of a system in continuously operating units, like chemical plants. In these kind of systems, production losses are often very large when a system failure occurs and the safety requirements are strict. Their method was applied to the refinery centrifugal compressor to test their model with real data. They concluded that their method based on Monte Carlo simulation was effective and found the optimal solution for grouping structure.

Rao and Bhadury (2000) presented various opportunistic maintenance models for a thermal power unit. Their case study showed that opportunistic policies with single opportunistic maintenance threshold for each component does not work as well as model with multiple thresholds. Different maintenance thresholds were obtained by taking into account dependencies between components so that the decision about opportunistic maintenance depended on the component that the system was taken down for.

Zhang and Zeng (2015) created a general modeling method for opportunistic maintenance. They showed how the method worked for single to three-component systems but also for general case. They concluded that even though the general method worked for multi-component systems the numerical solutions could be hard to find due to large computing time.

In their review on opportunistic maintenance, Ab-Samat and Kamaruddin (2014) discussed future work that should be done on the field. They thought that challenges are maintenance planning of complex multi-component system and translating that into industrial needs. In addition to, minimizing total cost, models should take into account system's availability, components' reliabilities and failure rates. More case studies should be done to test models with real data.

3 Methodology

3.1 Failure distributions

Components' failure distributions can differ significantly from each other. We cannot know exactly how fast or slow deterioration proceeds so we have to choose the distribution carefully for each component. The decision on what distribution to use and with which parameters can be done based on failure and maintenance data or expert experience. Barlow and Proschan (1996) explain that the choice between probability distributions can be difficult since the differences between most of them become significant only in the tails of the distributions. Furthermore, the observations can be sparse in the tails because of limited sample sizes.

Failure distribution and reliability were defined in Section 2.1. Barlow and Proschan (1996) defines that failure rate represents the probability that a component of age t will fail in the interval $[t, t + dt]$. The function for failure rate is defined as

$$r(t) = \frac{f(t)}{R(t)} = \frac{f(t)}{1 - F(t)}$$

where $R(t)$ is the reliability of the component, $F(t)$ is the failure distribution and $f(t)$ its probability density function.

In this work, we use *Weibull distributions* to model the components' failure probability. According to Bedford and Cooke (2001), the Weibull distribution is flexible in modeling failure rates, easy to calculate and describes well many physical life processes. The failure distribution function and the probability density function of component i are

$$F_i(t) = 1 - e^{-(t/\eta_i)^{\beta_i}}, \tag{1}$$

$$f(t) = \frac{\beta_i}{\eta_i}(\frac{t}{\eta_i})^{\beta_i - 1} e^{(-t/\eta_i)^{\beta_i}}, \tag{2}$$

$$\beta_i > 1, \eta_i > 0, \tag{3}$$

where t is the elapsed time since last maintenance, β_i is a shape parameter and η_i is a scale parameter. The scale parameter describes how wide the distribution is. In general, the shape parameter β_i is greater than zero.

It orders the shape of the failure rate so that it is decreasing when $\beta_i < 1$, constant when $\beta_i = 1$ and increasing when $\beta_i > 1$. When considering systems that need maintenance, it is reasonable to use increasing failure rates so we define that $\beta_i > 1$. The failure rate function for component i is

$$r_i(t) = \frac{\beta_i}{\eta_i}(\frac{t}{\eta_i})^{\beta_i - 1}. \tag{4}$$

Figure 4 presents the failure distribution and the failure rate of four different components. Component B wears out significantly faster than the others and this means that it has a smaller scale parameter and thus a shorter lifetime than the others. There are also differences between the shapes of the failure rates. The failure rate of component A increases slower in time which means that it deteriorates faster when its new. Component D has almost an exponentially growing failure rate so its deterioration accelerates. Component C lasts long and has a quite steadily growing failure rate.

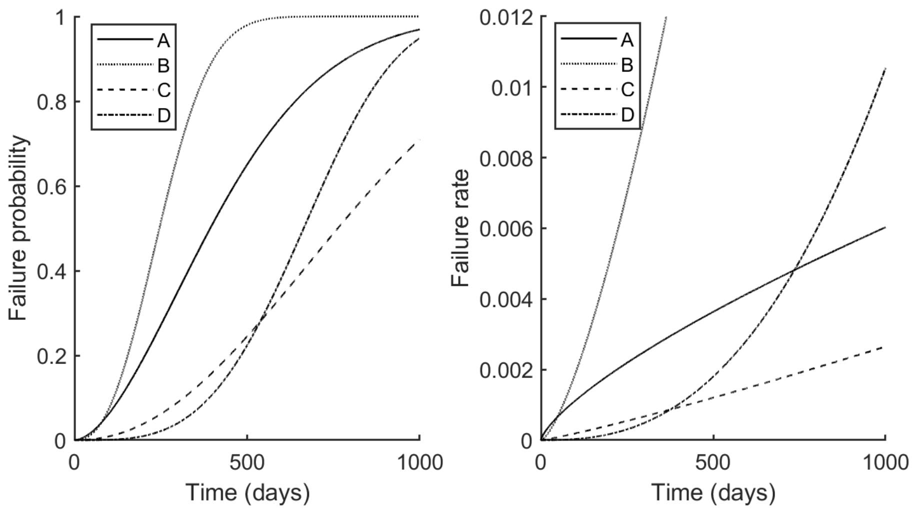

Figure 4: Failure distributions and failure rates of four different components, using Weibull distributions.

3.2 System model

The aim is to model a technical system with following constrains:

- The system consists of 5-15 components.

- All the components are critical which means that a failure of any component will cause a system failure.
- The system has a fixed cost, c_0, that has to be paid before every maintenance session.
- Corrective maintenance causes more costs than preventive maintenance.
- There may be more detailed economic dependencies between components than just the fixed cost.
- There may be structural dependencies between components that effect on what components can be or must be maintained simultaneously.

The system is modeled as *a directed graph*. According to Bertsimas and Tsitsiklis (1997), a directed graph consist of a set of nodes N and a set of directed arcs A. The graph is denoted as $G = (N, A)$.

The nodes represent components' maintenance activity and are denoted by letters A, B, C... The starting of the maintenance session is represented as node 0 which is the root node of the graph. Thereby, the number of nodes is one higher than the number of components, n, in the system. For example, Figure 5 presents a system of eight components.

The nodes are connected by directed arcs that represent dependencies between components. Directed arcs are noted as (i, j) where i is a start node and j is an end node. The arcs can be divided into incoming and outgoing arcs for each node.

$$I(i) = \{j \in N | (j, i) \in A\}$$
$$O(k) = \{k \in N \setminus \{0\}\} | (k, j) \in A$$

where $I(i)$ is the set of start nodes ans $O(i)$ the set of end nodes. The nodes that represent the maintenance activities can have both incoming and outgoing arcs depending on the structure of the system. An incoming arc indicates that the component can be maintained with certain cost after or jointly with the start node. There might be several ways to maintain a component as well as infeasible combinations of components to be maintained. In the system presented in Figure 5, all combinations are possible because there is an arc from the root node to each of the other nodes. As we can see in Figure 5, the root node has only outgoing arcs since it represents the start point of the maintenance session.

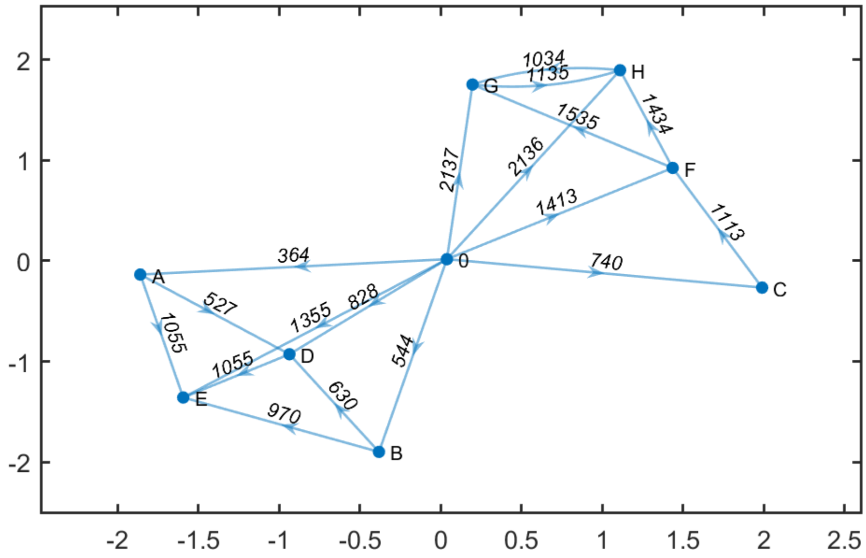

Figure 5: The system is modeled as a directed graph.

The arcs' weights indicate the cost of maintaining the component on the condition that the start node is maintained. The weight of (i, j) is denoted by c_{ij}. The arcs incident to the root node tell the cost of maintaining the component individually. The rest of the arcs tell the cost when the activity of the start node is executed as well. In this way, the economic dependencies can be modeled in detail since all the incoming arcs might have different weights.

In Figure 5, we can see that the cost of maintaining component D is $c_{0D} = 828$, but if it is maintained simultaneously with A the cost is $c_{AD} = 527$. The reason for this might be that maintaining D requires taking component A apart from the system first. If components A and B are maintained the cost of maintaining D is still the weight of (A, D) because $c_{AD} < c_{BD}$.

The costs of corrective maintenance are given as extra cost on top of the maintenance cost defined in the graph. This corrective maintenance surplus for component i is denoted by c_i^{cm}. Therefore, the total cost of repairing a failed component i is

$$c_i^{cm} + c_{ji}$$

where j depends on what other components are maintained during the same session. The corrective maintenance surplus can be different for every component. The fixed cost is given as a scalar c_0. The system's preventive maintenance costs for an arbitrary subset of maintenance activities are not as straightforward to calculate and they are discussed in Section 3.3.

For example, if component A fails in the system of Figure 5, the total cost can be calculated as follows. The fixed cost is $c_0 = 1000$ and the corrective maintenance surplus $c_A^{cm} = 1092$. When only A is maintained, the total cost is

$$c_0 + c_{0A} + c_A^{cm} = 1000 + 364 + 1092 = 2456.$$

If component D is preventively maintained during the same session, the total cost is

$$c_0 + c_{0A} + c_A^{cm} + c_{AD} = 1000 + 364 + 1092 + 527 = 2983.$$

3.3 Computing maintenance costs

The maintenance session always forms *a tree* in the graph. The tree is a connected graph with no cycles. When the graph is directed, a tree has a root that has a path to other nodes so that the directions of the arcs are taken into account. In our model, node 0 is the root and we need to find a tree that connects all the wanted nodes to the root with minimum cost.

According to Kleinberg and Tardos (2006), in a directed graph this kind of tree is called *a minimum-cost arborescence.* An arborescence is defined so that there is a path from the root node to every other node and there is only one incoming arc to every node except the root. Therefore, there are no cycles in an arborescence. Figure 6 presents a minimum-cost arborescence in red.

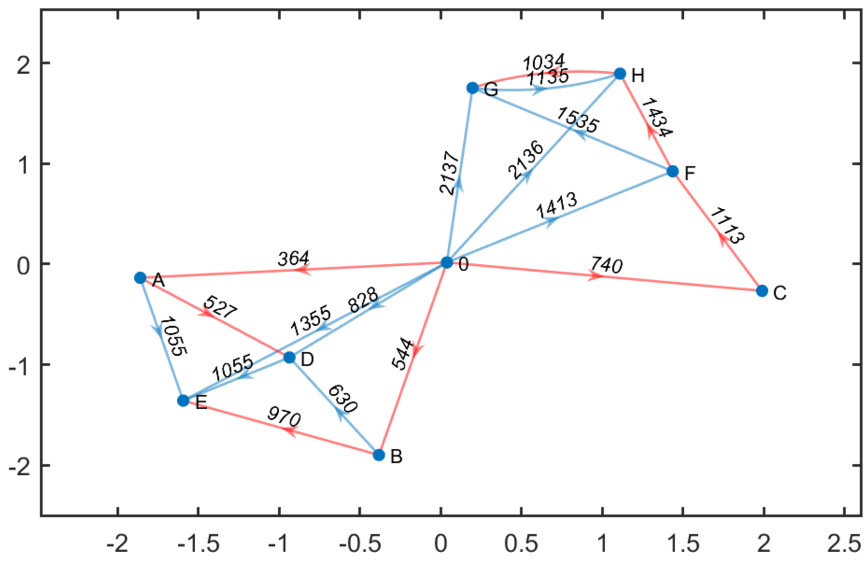

Figure 6: The minimum-cost arborescence of the graph is marked with red arcs.

Definition 3.1. Subgraph $T = (N, F)$ is an arborescence of the graph $G = (N, A)$ with root r if there is exactly one path from r to each node $v \neq r$. Now $r, v \in N$ and $F \subset A$.

The minimum-cost arborescence can be found using *Edmond's algorithm*. The algorithm and the theorems used are explained and proofed by Kleinberg and Tardos (2006). The optimum way to maintain all the components can be found using the algorithm for whole system. If we want to maintain a subset of components we need to calculate the minimum arborescence for the subgraph that contains the desired components.

Let $G = (N, A)$ be a directed graph in which the node $r \in N$ is a root. Now we want to find the minimum arborescence $T = (N, F)$. The pseudocode underneath shows how the algorithm proceeds.

Algorithm 1 Edmond's algorithm

procedure EDMOND(G)
    Delete $I(r)$
    Delete multiple arcs and choose (i, j) *with* $min(w_{ij})$
    for *for each* $v \neq r$ **do**
        $min(I(v)) = y_v$
        $F* = F * \cup y_v$
    if $P = (N, F*)$ *is acyclic* **then return** P
    else
        Find cycle C
        for *for each* $v \in C$ **do**
            $c'_{jv} = c_{jv} - y_v$
        Contract C *into supernode* v_C
        Form $G' = (N', A')$
        $P' = \text{EDMOND}(G')$
        Reconstruct $P' = (N, A*)$ **return** P'

First, all the arcs entering the root can be deleted. Also multiple arcs between two nodes are deleted so that only the cheapest arc remains. For each node $v \neq r$, we select the cheapest incoming arc and store them into the set of arcs $F*$. If $P = (N, F*)$ is an arborescence, it must be the minimum-cost arborescence. In that case, the directed graph P is returned.

If P is not an arborescence, it must contain at least one cycle. Let C be the set of nodes in N that forms a cycle.

Since every arborescence contains only one incoming arc to each node $v \neq r$, we can subtract the same quantity from the cost of every incoming arc to one node and the total cost of every arborescence changes the same amount. This means that we can modify the costs and still find the same optimum arborescence.

Let y_v be the smallest cost of the arcs incoming to node $v \in C$. We modify the costs of the edges incoming to nodes in C, so that

$$c'_{jv} = c_{jv} - y_v, \qquad v \in C,$$

where $j \in N$ is the start node of the incoming arcs to v.

Now nodes in C form a cycle in G consisting of arcs of cost 0 because the cycle was formed by the cheapest arcs entering the nodes. An optimum arborescence has exactly one arc entering C. We want to contract the nodes in

C into one super node v_C and then recursively find a minimum arborescence for the smaller graph G'.

The graph $G' = (N', A')$ includes nodes $N \backslash C$ and the contracted node v_C. All arcs incoming to the cycle C are modified so that the end node becomes the node v_C and its weight is the corresponding modified weight c'_{jv}. All arcs outgoing from the cycle get the start node v_C, but stay otherwise the same.

Recursion continues until acyclic P' is found. This always happens since at the end G' becomes a graph with one node. When an arborescence P' is found it is converted back to G by including the nodes in C. Arcs in P' are included so that arcs incident to v_C are the corresponding arcs in G. Arcs forming the cycle between the nodes in C are included except one. This removed arc is chosen so that it is the arc that enters the node that has an incoming arc that connects the cycle to the graph. Node v_C is removed and the resulting graph P' is a minimum-cost arborescence.

Figure 6 presents the minimum-cost arborescence for the system in Figure 5. This describes a situation where we want to maintain all the components in the system during the same maintenance session.

The total maintenance cost of all possible component combinations can be found by calculating the minimum-cost arborescence of subgraphs that include the root and desired components. For example, if we want to maintain only components A, D and E in the system in Figure 5, then we need to create the subgraph in Figure 7. Edmond's algorithm is then applied for this subgraph. The result is shown in Figure 7.

All subgraphs might not be connected and thus are not feasible component combinations. For the infeasible combinations, the cost is defined as infinity.

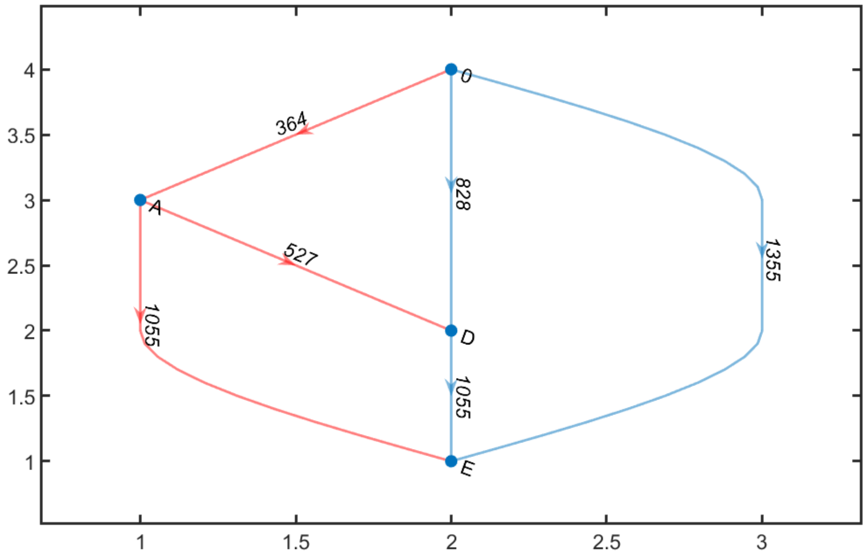

Figure 7: Figure presents a subgraph where the optimum arborescence is marked in red.

3.4 Simulation model

We created a simulation model to compare different maintenance policies. Simulation for each maintenance policy was run for several rounds and for each round information about the total cost, the number of maintenance sessions and the number of failures was saved. The number of maintenance sessions could be used to compare reasons behind the total cost. In some cases failures can be severe and they need to be avoided, hence it is useful to be able to examine and compare how the maintenance policy affects on the number of failures.

Our Monte Carlo simulation model was build using Matlab. One simulation lasts T time units which represents the time that the system is in use. For our example system $T = 2000$ days which means that the system is in use about five and a half years. The simulation was run for S times so that the result were near enough the real expected value. It appeared that close enough value could be get using $S = 1000$ and still run multiple simulation in a reasonable time.

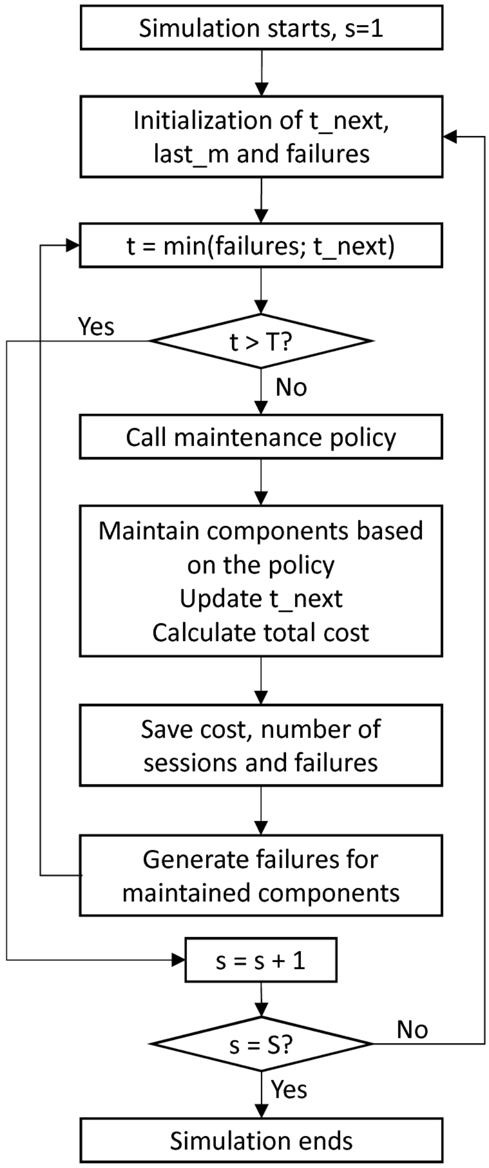

Figure 8: Chart presents how the simulation proceeds.

The simulation proceeds as the chart in Figure 8 presents. It starts by calling the function *simulateS*. The function requires as input the maintenance policy function, T, S, the fixed cost, the information about the corrective and preventive maintenance costs, the number of components in the system, the failure distributions of the components and additional arguments that different maintenance policies might require. Before the simulation starts, vectors for total costs, number of maintenance sessions and number of failures are initialized.

When the first simulation starts, $s = 1$. The next maintenance time, t_{next}, is calculated according to used maintenance policy and upcoming failure times, $failures$, are generated from the components' failure distributions. Time t is the minimum of upcoming maintenance time and failure times.

Next, the maintenance policy function is called to decide which components should be maintained at time t. It returns the list of maintained components, list of failed components, next maintenance time, updated last maintenance times and the cost of the maintenance session. The total cost is saved and the number of sessions and the number of failures is upgraded. This information is saved in vectors at place s. New failure times are generated randomly for the maintained components. More detailed explanation on maintenance policy functions are given in Section 3.5.

This loop is repeated until t reaches the lifetime of the system, T. New simulations rounds are repeated until $s = S$. Finally function *simulateS* returns three vectors including information about total costs, number of maintenance sessions and number of failures for each simulation round.

3.5 Maintenance policies

Maintenance policies are used to decide when components are maintained preventively in the simulation implementation. Maintenance policies are functions which are called in the beginning of a maintenance session. Corrective maintenance is executed always when a component fails since all the components are critical.

Age-based maintenance policy

In age-based policy (AB policy), a component is maintained when its age exceeds its threshold age. Each component has a predetermined value that determines how often the component is repaired.

In this policy, the threshold values were determined using a formula for optimal maintenance interval derived by Wildeman et al. (1997). The interval is optimal for the single component but not for the whole system and it takes into account only the cost of maintaining the component individually.

Wildeman et al. (1997) used Weibull distribution and the system had a fixed cost. The optimal maintenance interval for component i is

$$x_i = \sqrt[\beta_i]{\frac{(c_{0i} + c_0)\eta_i^{\beta_i}}{c_i^{cm}(\beta_i - 1)}}, \tag{5}$$

where β_i and η_i are parameters of the component's Weibull distribution and c_{0i}, c_i^{cm} and c_0 are the preventive, the corrective and the fixed cost. Each component is maintained every x_i time units independently of other components.

Age-based policy with inspections

During a maintenance session, we might be able to observe the current condition of the system. If a component has been damaged it can be maintained earlier based on these observations. In the age-based policy with inspections, we inspect all components during a maintenance session and reschedule their next maintenance if necessary. This policy is closer to condition-based maintenance but the information of the system's condition is available only at certain times.

The policy has a parameter t_{insp} that indicates how far into the future we can foresee the component's failure. This means that we know exactly the time of the failure, if it will happen in the interval $[t, t + t_{insp}]$. Using this information we can schedule maintenance just in time. On the other hand, we can postpone maintenance, if a component is not going to fail in the interval but it is scheduled to be maintained during that time.

The next scheduled maintenance for component i is denoted by t_i^m and the next failure is denoted by t_i^f. During a maintenance session we can reschedule maintenance based on the component's observed condition as follows

$$t_i^m = \begin{cases} t_i^f - 1 & t < t_i^f \leq t + t_{insp} \\ t + t_{insp} & t_i^f > t + t_{insp} \cap t < t_i^m < t + t_{insp} \\ t_i^m & \text{otherwise.} \end{cases}$$

This also means that components that are about to fail at the time $t + 1$ are maintained instantly.

Simple opportunistic maintenance policy

In the simple opportunistic maintenance policy (SOM policy) each component has two threshold values, x_i and x_i^{op}. The first value is the same as in the age-based policy and it determines the time when component is maintained at the latest. The policy is build based on the model of Hu and Zhang (2014) so that the opportunistic threshold is determined similarly.

The value x_i^{op} determines the threshold for opportunistic maintenance. If the elapsed time since the last maintenance is grater than x_i^{op} and the system is maintained, then also component i is maintained. The reason for the maintenance session can be a failure or a maintenance of some other component.

The policy is called simple since the opportunistic maintenance is determined simply as a percentage of x_i. This percentage p is same for all components and it tells how much earlier component can be maintained compared to preventive maintenance interval. Therefore the threshold value x_i^{op} is defined by

$$x_i^{op} = (1 - p)x_i. \tag{6}$$

This policy is useful when the fixed cost is large since it groups maintenance activities and reduces the number of maintenance sessions. However, components that are maintained opportunistically are not used as long as possible which might cause more expenses.

Simple opportunistic maintenance policy with inspections

This policy is similar to simple opportunistic maintenance policy but it is expanded with inspections. The next maintenance is scheduled the same way as in the age-based policy with inspections and the threshold for opportunistic maintenance is determined by the percentage p.

Inspections are done before opportunistic maintenance is executed so that we can maintain more components during the session based on the observations.

4 Results

The simulation model was tested using the example system of Figure 5. The system has eight components, $n = 8$, and its lifetime is $T = 2000$. Values for costs were chosen based on the values that were used by Urbani (2017), Laggoune et al. (2009) and Geng et al. (2015). The fixed cost and the corrective maintenance surplus are

$$C_0 = 1000$$
$$C_{cm} = [1092 \quad 1617 \quad 2235 \quad 2284 \quad 2711 \quad 2826 \quad 3201 \quad 2368]^T.$$

All components are different and their failure distributions and failure rates are presented in Figure 9. The failure distributions were modeled as Weibull distributions and in this work we use same values for shape parameters β_i and for scale parameters η_i as Laggoune et al. (2009). Table 1 presents the values of the parameters.

i	1	2	3	4	5	6	7	8
β_i	1.73	1.88	2.43	2.53	2.14	3.55	2.68	2.09
η_i	486	507	286	898	905	736	1094	1388

Table 1: The table presents shape and scale parameters of each components failure distribution.

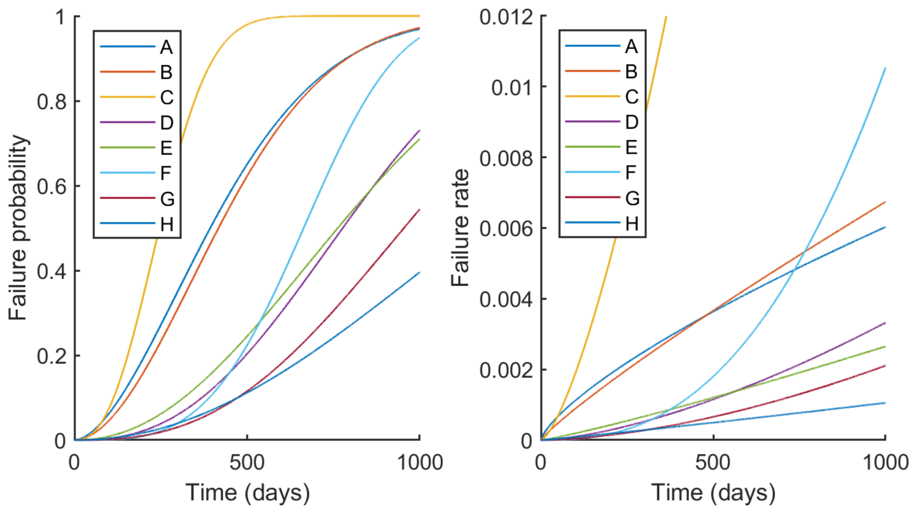

Figure 9: Figure shows failure distributions and failure rates of all components in the example system.

4.1 Choosing parameters

Simple opportunistic maintenance policy has parameter p that determines the opportunistic maintenance threshold. Age-based policy with inspection has parameter t_{insp} that determines how long into the future we can predict the condition of the component. Next we examine how they should be chosen for the example system and how varying them affect the results. For simple opportunistic maintenance with inspections we did not do the examination but used same values for the parameters.

The simulation was run for different values of p form 0 to 1 using step 0.02. Figure 10 presents the mean values of simulations. Total cost reaches its smallest value 79100 when $p = 0.4$. This means that the best results using this policy for this particular system can be obtained by maintaining components opportunistically 40% beforehand.

The number of maintenance sessions as well as the number of failures decreases when opportunistic maintenance is used earlier. Maintenance activities are executed simultaneously so the number of sessions decreases. Each component is used lesser time when p increases and thus the number of failures decreases. The total cost begins to increase after 0.4 because components are maintained too early compared to costs. When $p = 1$ the whole system is maintained during each maintenance session. In this case, the total cost reaches the value 112000 which is significantly more than the corresponding value in age based policy. The case were $p = 0$ presents age-based policy when the total costs are 99900.

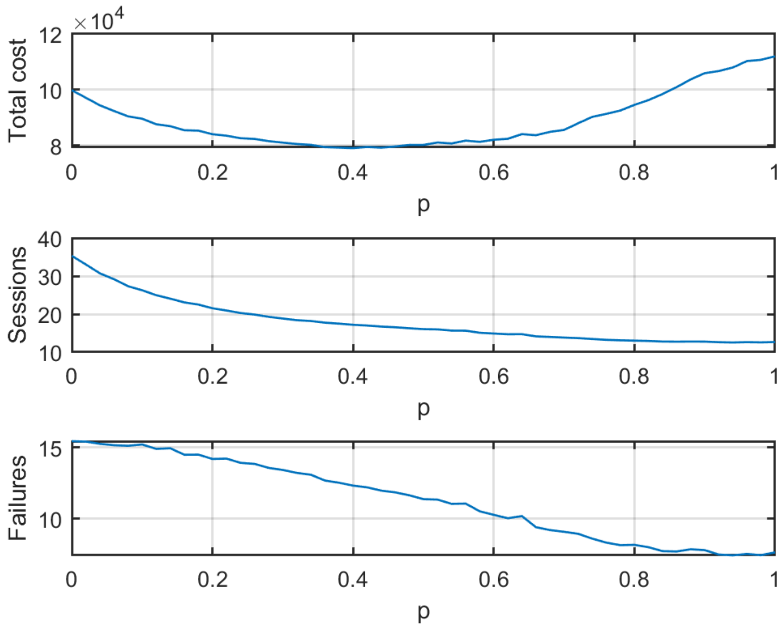

Figure 10: Mean values of simulations for the total cost, the number of sessions and the number of failures when simulation was run 1000 times for each p.

Age-based policy with inspections has a parameter t_{insp} that describes how long into the future we can predict failures during maintenance sessions. The simulation was run for values from 0 to 40 days. Figure 11 shows the the mean values of the total cost, the number of sessions and the number of failures. As expected, the total costs and the number of failures decrease when we are able to foresee failures earlier and we can schedule maintenance activities just in time. The number of maintenance sessions varies slightly.

Parameter t_{insp} cannot be chosen same way as parameter p because it is not realistic that we could foresee failures accurately far into the future. However, the result can be used to evaluate how much resources it is useful to allocate to inspections of components' conditions. In this work, we use value $t_{insp} = 30$ which means that we can determine upcoming failures 30 days in advance.

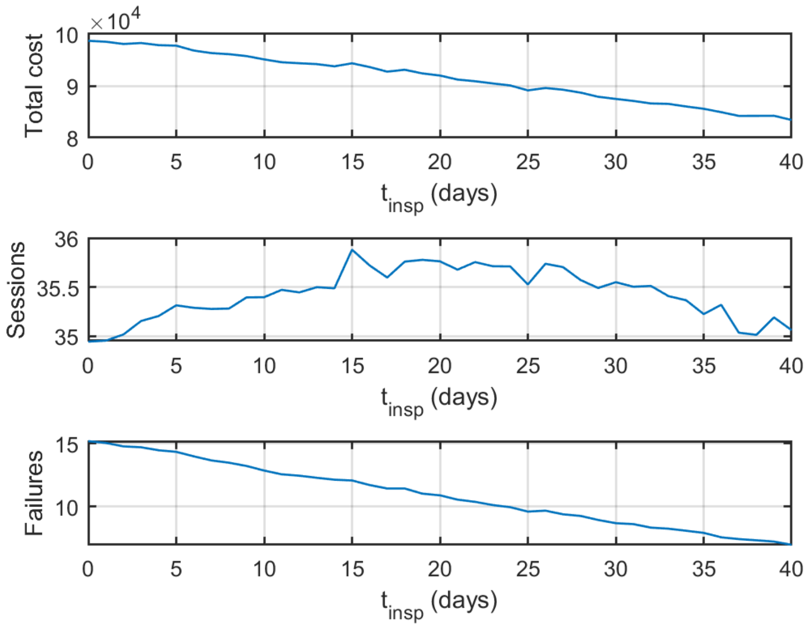

Figure 11: Mean values of simulations for the total cost, the number of sessions and the number of failures when simulation was run 1000 times for each t_{insp}.

4.2 Comparing maintenance policies

All policies were simulated 1000 rounds for the example system. We used values $p = 0.4$ and $t_{insp} = 30$ for parameters. Table 2 presents the mean values and standard deviation of the simulations for each maintenance policy. Figure 12 presents total costs for each policy as a histogram.

The policies without inspections result in wider distributions thus standard deviations are greater for all quantities. The policies with inspections are more stable since failures can be avoided. Age-based policy (AB policy) results in approximately same amount of maintenance sessions with and without inspections. In AB policy with inspections total costs and number of failures are clearly lower because maintenance schedule can be revised based on updated information.

Policies that use opportunistic maintenance (SOM policies) result in lower total costs because the number of maintenance sessions decreases significantly. The number of failures is on average 12.4 for SOM policy and 10.7 for SOM policy with inspections. Both values are smaller than the number of failures for normal AB policy but AB policy with inspections results in even smaller number of failures. When opportunistic maintenance is used and number of sessions reduced, possibilities to make observations is also reduced. This means that inspections do not help as much with opportunistic maintenance than with AB policy.

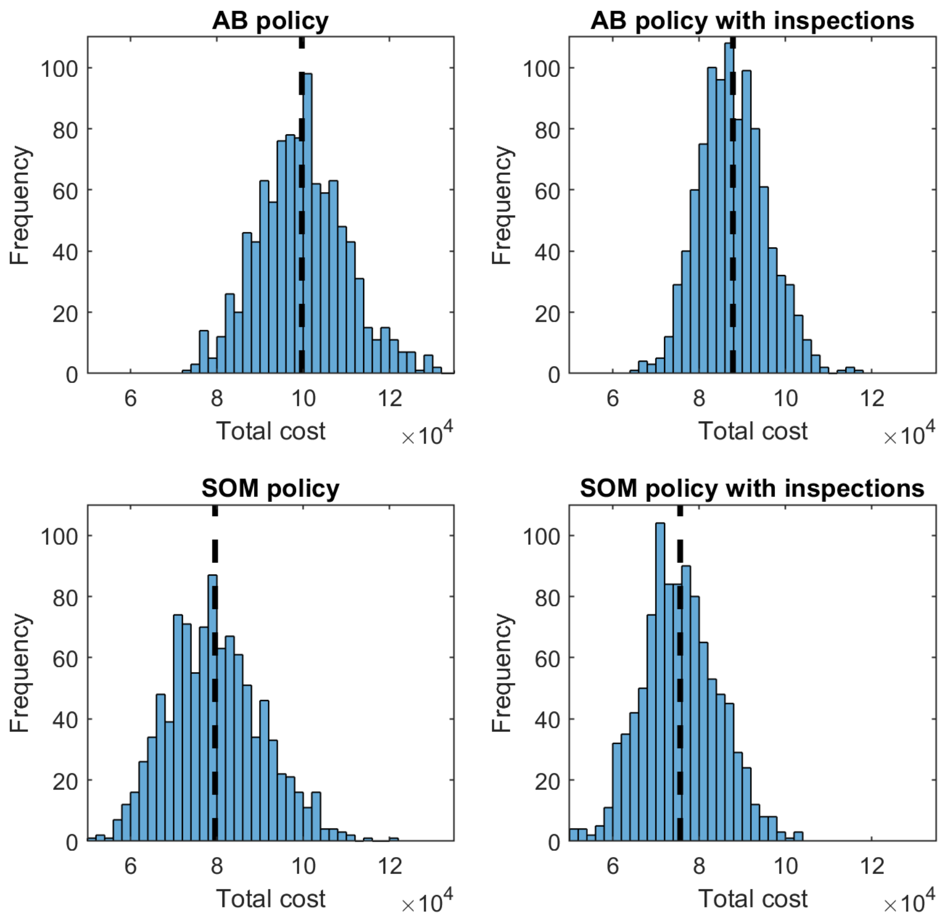

Figure 12: Distributions for total cost when simulation was run 1000 times. The mean value is marked as dashed line for each figure.

Table 2: Table presents mean values and standard deviations of total costs, number of maintenance sessions and number of failures for the example system.

	Total cost		Sessions		Failures	
	mean	std	mean	std	mean	std
AB policy	99 700	10 300	35.3	1.9	15.5	3.5
AB policy with insp.	87 800	7 800	35.5	2.3	8.9	2.5
SOM policy	79 600	10 900	17.3	2.5	12.4	3.2
SOM policy with insp.	75 600	8 900	16.4	2.0	10.7	2.6

The simulation can also be done without considering dependencies between components. In this case, only individual maintenance costs are taken into account and Figure 13 shows the system graph. The fixed cost and the corrective maintenance costs remain the same as in the earlier simulation.

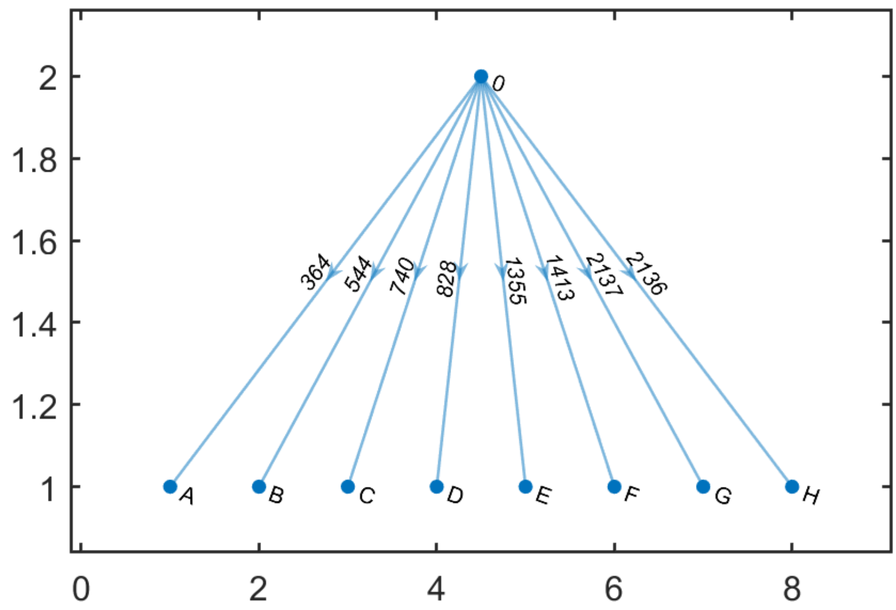

Figure 13: The system graph for the example system without any dependencies between components.

The results are shown in Table 3. We can see that age based policy with and without inspections does not change. However, leaving dependencies out does effect on results from opportunistic maintenance policies. Figure

14 shows that total costs are greater when dependencies are not considered. This means that modeling components' dependencies in detail is useful when we use opportunistic maintenance policies or otherwise group maintenance activities. The number of sessions and the number of failures are similar to values than in the results for the original system.

Table 3: Mean values and standard deviations of the results for the system where the dependencies are not taken into account.

System without dependencies	Total cost mean	std	Sessions mean	std	Failures mean	std
AB policy	100 000	10 500	35.3	2.0	15.5	3.5
AB policy with insp.	88 100	8 000	35.6	2.3	8.9	2.6
SOM policy	84 100	10 400	17.2	2.4	12.4	3.0
SOM policy with insp.	81 100	9 100	16.4	2.1	10.7	2.7

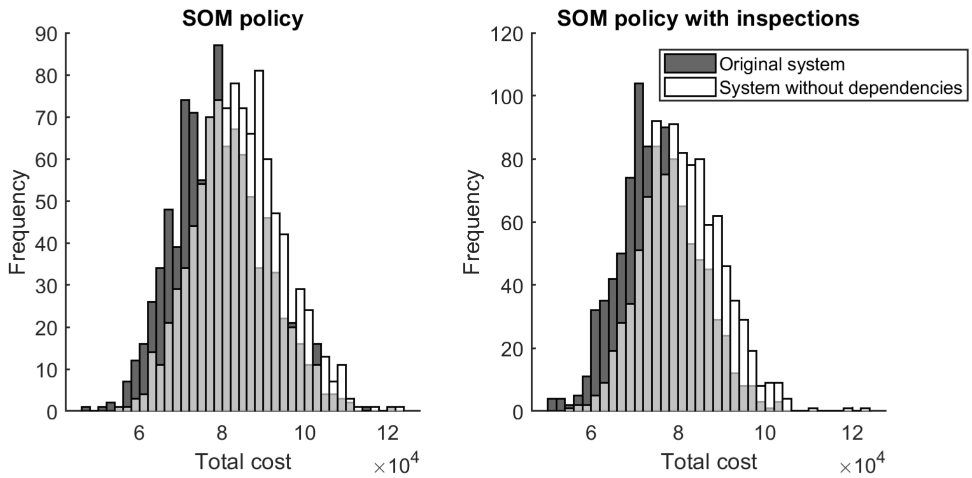

Figure 14: Distributions for total maintenance cost in system with and without component dependencies.

4.3 Varying the fixed cost

The original simulation was run with $c_0 = 1000$. The fixed cost is an important factor in economic dependence as could be seen in Section 4.2 when the simulation was run without other dependencies between components. The simulation was run for different values of the fixed cost form 0 to 5000.

The fixed cost did not effect on the number of failures or on the number of maintenance sessions. As could be expected, varying the fixed cost changed the total costs. Figure 15 shows the mean value of each simulation for all maintenance policies. The total costs increase linearly for all policies and whether there is inspections or not the slope does not change. Simple opportunistic maintenance policy with and without inspections increases significantly slower than age based policies. Therefore, opportunistic maintenance is more useful when the system has a great fixed cost. When the fixed cost is 5000 simple opportunistic maintenance policies saves approximately 38% of the total cost compared to age based policies.

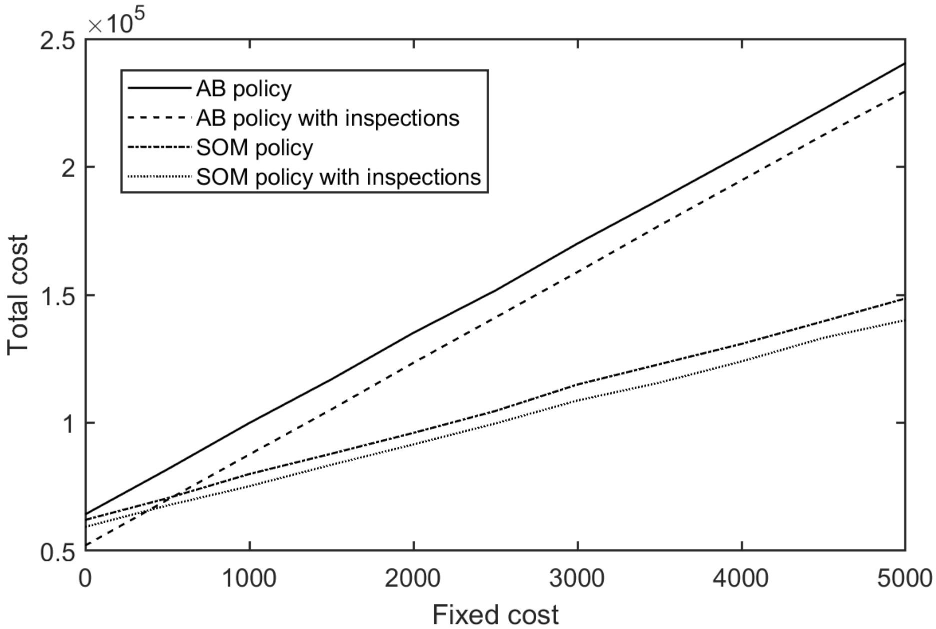

Figure 15: Total costs when the fixed cost is varied.

5 Conclusion

We developed a simulation model for comparing different maintenance policies. The focus was on opportunistic maintenance but the model should be flexible in implementing different policies. We should be able to describe multi-component systems with economic and structural dependencies.

The modeling consisted of three levels: component, system and simulation. The component level modeling was done using failure distributions and failure rates. The validity of the results is dependent on the accuracy of the failure distributions. Thus it is important to choose them carefully and that they describe the components as well as possible. In this work, Weibull distributions were used to model components' deterioration but the model is not fixed for this particular distribution.

The system was modeled as a directed graph which allows us to model dependencies in detail. The system has a fixed cost that causes dependencies but using the graph results in that economic and structural dependencies can be taken into account better. Maintenance costs are computed with Edmond's algorithm.

The simulation model was created as Monte Carlo simulation and it combined all the methods mentioned above. The model allows us to implement different maintenance policies that can be compared by saving the total costs, number of failures and number of maintenance sessions. The simulation model was tested using an example system and four different maintenance policies: age based policy (AB policy), AB policy with inspections, simple opportunistic maintenance policy (SOM policy) and SOM policy with inspections. Inspections allow us to reschedule maintenance just in time and opportunistic policies to maintain components when an opportunity arrives.

The results show that opportunistic policies reduce costs especially when the fixed cost is high. The inspections made results more stable, since failures could be avoided. In AB policies, inspections reduce costs significantly even though the number of maintenance sessions does not change. The results indicate that the simulation model works and different maintenance policies give reasonable results.

One of the greatest challenge in using this model is to choose the failure distributions correctly. In many practical cases, we do not have enough information about the components' deterioration that we could reliably determine failure distributions. Thus it would important to develop methods to monitor components' conditions.

The developed model makes it possible to account for different types of dependencies but in real life situations the information about them might not be available. The model can be used without economic or structural dependencies but, as the results show, they are not as reliable. We need to be able to examine the system dependencies to fully benefit from this model.

The next step would be to test this model using real data. The validity of the simulation model could be tested further. The policies that are currently used for maintaining technical systems could be implemented and the model could be used to compare them with new alternatives. It is also possible to implement more complex maintenance policies such as risk-based maintenance policy, condition-based maintenance with real-time data from the system or even policies that use dynamic programming and grouping of maintenance activities.

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