Testing the Random Walk Hypothesis for Helsinki Stock Exchange

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School of Science

Bachelor's thesis

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Abstract of the bachelor's thesis

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Abstract
This Bachelor's thesis examines the random walk hypothesis for weekly returns of two indices, OMXHPI and OMXH25, and eight stocks in Helsinki Stock Exchange. The returns run from January 2000 to February 2018. In order to test the null hypothesis of a random walk, the study employs three variance ratio tests: the Lo–MacKinlay test with the assumption of heteroscedastic returns, the Chow–Denning test and the Whang–Kim test. The variance ratio estimates produced by the Lo–MacKinlay test are analyzed for various lag values. The results indicate that both indices and all stocks, except for UPM–Kymmene, follow a random walk at the 5 percent level of significance. Furthermore, the variance ratios are found to be less than unity for shorter lags, which implies that stock returns may be negatively autocorrelated for short return horizons. Some stocks and both indices show a high variance ratio estimate for larger lag values, contradicting a mean reverting model of stock prices. The results demonstrate, in contrast to previous studies, that Helsinki Stock Exchange may be an efficient market and thus, that predicting future returns based on historical price information is difficult if not impossible.

Keywords random walk hypothesis, market efficiency, variance ratio test, Helsinki Stock Exchange

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Sammandrag

Enligt slumpvandringshypotesen bildar avkastningarna på aktier en följd av oberoende slumpvariabler, vilket implicerar att det är svårt eller omöjligt att förutspå kommande avkastningar på basis av tidigare prisinformation. Huruvida aktiekurserna är autokorrelerade eller inte har såväl praktiska implikationer för investerare som teoretiska implikationer för effektiviteten av marknaden. Detta kandidatarbete undersöker slumpvandringshypotesen för två index, OMXHPI och OMXH25, samt åtta aktier på Helsingforsbörsen. Undersökningsmaterialet består av veckoavkastningar från januari 2000 till februari 2018. Nollhypotesen om en slumpvandring undersöks med tre olika varianskvottest: Lo–MaKinlay–testet med antagandet av heteroskedastiska avkastningar, Chow–Denning–testet samt Whang–Kim–testet. Vidare undersöks varianskvotestimaterna för diverse tidsförskjutningar för att få en bättre bild av hur tidsserierna är autokorrelerade.

Tidigare studier tyder på att Helsingforsbörsen inte följer en slumpvandring. Få av studierna använder sig dock av moderna varianskvottest och veckoavkastningar: en stor del av forskningarna undersöker dagliga avkastningar och utnyttjar sig huvudsakligen av seriella korrelationstest eller enbart ett varianskvottest. Därmed finns det ett behov att testa slumpvandringshypotesen i Finland med nyare data, veckoavkastningar och med flera olika varianskvottest.

Resultaten av denna forskning tyder på att båda indexen och alla de undersökta aktierna, med undantag av UPM–Kymmene, följer en slumpvandring på fem procents signifikansnivå. Både Lo–MacKinlay–testet och Whang–Kim–testet förkastar slumpvandringshypotesen för UPM–Kymmene. Vidare indikerar varianskvotestimaterna att avkastningarna är svagt negativt autokorrelerade för korta tidsförskjutningar, vilket delvis kunde bero på den låga omsättningen i Helsingforsbörsen. För längre tidsförskjutningar verkar varianskvoterna vara i medeltal större än ett, vilket strider mot teorin om att aktiepriserna tenderar att återgå till det historiska medelvärdet på lång sikt.

I motsats till tidigare studier visar denna forskning att Helsingforsbörsen kan vara en effektiv marknad och att det därmed är svårt för investerare att utveckla köpstrategier med vars hjälp högre avkastningar än den väntade avkastningen uppnås. Huruvida tidsperioden för avkastningarna och undersökningsperioden inverkar på testresultaten på Helsingforsbörsen är delvis öppna frågor.

Nyckelord slumpvandringshypotes, marknadseffektivitet, varianskvottest, Helsingforsbörsen

Contents

Symbols and abbreviations

Abbreviations

CD	Chow–Denning
EMH	Efficient market hypothesis
LM	Lo–MacKinlay
RWH	Random walk hypothesis
SMM	Studentized maximum modulus
VR	Variance ratio
WK	Whang–Kim

Symbols

CD_{crit}	Chow–Denning test critical value
$M_1(k)$	Lo–MacKinlay test statistic evaluated at lag k
MV_1	Chow–Denning and Whang–Kim test statistic
$V(k)$	Estimator of the variance ratio for lag k
WK_{crit}	Whang–Kim test critical value

1 Introduction

The predictability and nature of stock returns have long been topics of both interest and controversy in academic and business circles. If stock returns have predictable patterns, then it is possible to develop a quantitative model of stock price movements and hence, increase expected earnings. On the other hand, if stock prices move randomly, then forecasting stock returns using historical price information is no easier than forecasting a sequence of randomly generated numbers. The question whether stock prices are predictable is thus very intriguing from an investor's point of view.

The random walk hypothesis (RWH) states that stock returns are independent of previous returns and thus, that predicting future stock prices based on previous price information is impossible. According to the efficient market hypothesis (EMH), stocks are in some sense always correctly priced, meaning that it is not possible for an investor to "beat the market" [1]. In such a market, stock prices reflect all available price information at any time and instantaneously adjust to new information. Bachelier [2] and Osborne [3] theorize that, if information affecting the stock's price is generated randomly or if there is uncertainty concerning the stock's intrinsic value, stock prices should follow a random walk in an efficient market. The random walk hypothesis is thus of great interest because it gives insight into whether or not a stock market is efficient. However, as noted by Stephen [4, p. 111], random walk tests cannot be considered as tests of market efficiency, as RWH is neither a sufficient nor a necessary condition for EMH.

RWH has been tested extensively in various markets. Among others, Fama [5] has studied dependence in financial time series using sample serial correlation coefficients and runs test for successive returns. The results show that common US stocks do not show statistically significant dependence, supporting the random walk assumption. Furthermore, Kendall and Hill [6] have studied British industrial share prices and other financial time series and found that the prices show little serial correlation. The consensus is that RWH holds for large and developed markets, such as the US and UK stock markets.

Later studies on RWH in thin or emergent markets show opposite results. Jennergren's and Korsvold's 1974 study on Swedish and Norwegian stock markets indicates that the said markets may be inefficient [7]. They use the serial correlations test and the runs test to test their hypothesis. Moreover, recent studies reveal mean reversion tendency in returns. For instance, Fama and French [8] establish that for long holding periods, returns are significantly negatively correlated.

Older studies seem to suffer from restrictive testing methods or strict assumptions. The variance ratio (VR) test presented by Lo and MacKinlay (LM) in 1988, introduces a more modern way of testing RWH [9]. One version of their test allows the time series to have heteroscedastic increments, which is considered to be commonplace in financial time series. Frennberg and Hansson [10] apply the LM test to Swedish stock prices from the period 1919–1990 and reject RWH for the whole period.

Few studies examine RWH in Finland. Shaker [11] demonstrates that the stock market indices OMXH25 and OMXS30 in Helsinki and Sweden do not follow random walks. The study uses the variance ratio test by Lo and MacKinlay, an autocorrelation test and the Dickey–Fuller unit root test. Shaker tests RWH for daily returns of the aforementioned indices from the period 2003 to 2012. Although the study gives convincing evidence against RWH in the case of stock market indices, it does not reveal whether individual stocks follow random walks. Furthermore, Shaker's study covers a fairly short time period and utilizes only one individual variance ratio test and no multiple variance ratio tests. Therefore, there is a need for a reexamination of RWH in Finland, using more recent data and newer variance ratio tests.

This thesis examines whether RWH holds for weekly returns of the indices OMXHPI and OMXH25 as well as eight highly traded stocks in Helsinki Stock Exchange. The returns run from January 2000 to February 2018, and they are collected from Yahoo! Finance [12] and Investing [13]. Three different variance ratio tests are being used: the Lo–MacKinlay test, and two enhanced versions of it: the Chow–Denning (CD) test and the Whang–Kim (WK) test. The data is processed and the tests are performed in RStudio with the R programming language. A secondary objective of this thesis is to investigate the nature of the variance ratios of the data. In particular, the magnitudes of the variance ratio estimates for various lags are of major interest. There might be differences between the variance ratio profiles of the stocks and the indices – these differences are analyzed as well.

This thesis is structured as follows. Section 2 discusses previous research and definitions of random walks. Section 3 describes the variance ratio tests used in this thesis and the related test statistics. The latter part of section 3 presents the data and describes how the data is processed and what software is used. Section 4 discusses the results and section 5 concludes the study and gives suggestions for further research.

2 Background

2.1 The random walk hypothesis

To test the random walk hypothesis, one has to give a meaningful definition of it first. The essence of RWH is captured in the idea that stock prices are unpredictable, which means that future stock returns are independent of previous returns, that is $\mathsf{P}[r_t = r \mid r_{t-1}, r_{t-2}, \dots] = \mathsf{P}[r_t = r]$, where r_t is the return of time period t. This assumption of uncorrelated returns is the central assumption in RWH.

Let us denote by p_t the stock price at time t and define the logarithmic price process as $x_t = \log p_t$. Then the time series x_t is said to follow a random walk if it is generated by the following process:

$$x_t = \mu + x_{t-1} + \epsilon_t, \tag{1}$$

where μ is a constant parameter and ϵ_t is the random term [14]. A classical definition of RWH is that the terms ϵ_t are independent and identically distributed random variables. Some authors even consider the disturbance terms ϵ_t to be normally distributed. However, the assumption that ϵ_t are i.i.d. is a very strong assumption and a test of this hypothesis does not necessarily tell much about the predictability of returns. For instance, if the conditional variances of returns are time–varying (which is often considered to be the case for financial time series), then the null hypothesis is rejected even though the time series may be unpredictable [4, p. 110]. Thus, for this study, the classical definition of RWH presented above is too restrictive.

Lo and MacKinlay [9] use the following common assumption H as the basis for RWH:

- H1: $E[\epsilon_t] = 0$ for all t and $E[\epsilon_t \epsilon_{t-\tau}] = 0$ for all $\tau \neq 0$.
- H2: $\{\epsilon_t\}$ is ϕ–mixing with coefficients $\phi(m)$ of size $r/(2r-1)$ or is α–mixing with coefficients $\alpha(m)$ of size $r/(r-1)$, where $r > 1$, such that for all t and for any $\tau \geq 0$, there exists some $\delta > 0$ for which $E[|\epsilon_t \epsilon_{t-\tau}|^{2(r+\delta)}] < \Delta < \infty$.
- H3: $\lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} E[\epsilon_t^2] = \sigma_0^2 < \infty$.
- H4: $E[\epsilon_t \epsilon_{t-j} \epsilon_t \epsilon_{t-k}] = 0$ for all t and for any $j, k \neq 0$ where $j \neq k$.

These assumptions are more complex than the classical assumptions, but they allow for different forms of heteroscedasticity while still maintaining the key assumption that the process x_t has uncorrelated increments [9]. Therefore, the assumption H is an appropriate description of a random walk for financial time series. All tests used in this study assume H as the common assumption.

2.2 Previous research

There is a vast amount of research that examines the random walk nature of stock prices. One of the first stochastical models of stock returns was presented in 1900 by Bachelier [2], who modeled stock returns as a Brownian motion with linearly increasing variance. Samuelson [15] put forward the idea that stock prices should be unpredictable in an efficient market. One of the first empirical tests on RWH was conducted by Fama [5], who tested the validity of RWH in the US stock market and concluded that stock returns show little to no serial correlation.

Later studies on RWH in smaller stock markets and developing markets show evidence against RWH. For instance, Jennergren's and Korsvold's study from 1974 on 45 Swedish and Norwegian stocks, using serial correlations tests and runs tests, show that the said stock markets may not be efficient [7]. They hypothesize that the results might be due to infrequent trading. Solnik [16] investigates serial correlation coefficients and their stability for European stock prices. The main findings are that there are slight deviations from RWH, but the serial correlation coefficients are still small from an investor's point of view. Solnik [16] presents some possible explanations for why European markets, in particular, show inconsistencies with RWH. These are, among others, the thinness of the markets, discontinuous trading, loose requirements for the announcement of information and little control over insiders' trading.

Mean reversion of stock prices is a theory in finance which states that a stock's price will tend to revert to its historical average over time. Fama and French [8] find mean reverting tendencies in US stock prices over longer time periods of 3–5 years. Similarly, Frennberg and Hansson [10] demonstrate that Swedish stock prices show negative autocorrelations for longer investment horizons, supporting the theory of mean reversion. In contrast, they show that Swedish stock prices show significant positive autocorrelations for one to twelve month periods [10].

Early research primarily relies on serial correlations tests, runs tests and similar methods to test RWH. Lo and MacKinlay [9] introduce a new test methodology: the method of variance ratios. The variance ratio test is based on the fact that the variance of a k–period return of a random walk process increases linearly with respect to k. The LM test has two versions: one with the assumption of homoscedastic increments and another with the assumption of heteroscedastic increments. The latter allows for different forms of time–varying volatility, giving somewhat less restrictive assumptions than some of the earlier tests. A detailed description of the LM test is presented in section 3.2.

In their experiment, Lo and MacKinlay [9] reject RWH for weekly returns of US stock portfolios. They show that the said portfolios show significant positive autocorrelation for all return horizons, casting doubt on the mean reversion model. What is interesting is that empirical variance ratios computed by Lo and MacKinlay are on average significantly different one, indicating that the test gives less ambiguous results than some of the earlier tests. Indeed, Lo and MacKinlay [17] show in a 1989 article, using Monte Carlo simulations, that their method is more reliable than the Dickey–Fuller and Box–Pierce tests.

Few studies investigate RWH for Finnish stocks. Berglund et al. [18] test the weak–form efficiency of the Finnish and Scandinavian stock exchanges, using daily returns from the period 1970–1981. The results indicate that all markets seem to follow an adjustment process, with positively autocorrelated returns for lags 1–3, negative autocorrelations for lags 3–5 and zero autocorrelation thereafter. The study reveals that Helsinki Stock Exchange is the most inefficient of the Scandinavian markets.

Shaker [11] examines RWH for Finnish and Swedish stock indices OMXH25 and OMXS30 using daily returns from the period 2003–2012. Shaker uses the ADF test, an autocorrelation test, and the LM test. The results are quite striking: RWH is strongly rejected for all tests, with the p–value of the LM test being very close to zero for all k–values. The autocorrelations are significantly negative for short time periods of 1–3 days.

Narayan and Smyth [19] test the presence of a unit root, which is a necessary condition for a random walk, against the alternative of mean reversion in the market indices of 22 OECD countries, including Finland. They use the ADF and Phillips–Perron unit root tests as well as the sequential trend break test by Zivot and Andrews [20] and a panel data unit root test by Im et al. [21]. The findings are that almost all indices possess a unit root, supporting the theory of random walks. It seems, based on the test statistics, that the Helsinki Stock Exchange show smaller deviations from RWH than the average index.

In conclusion, it seems that the results on whether RWH holds in European markets and Finland are highly contradictory. Overall, it appears that RWH holds in large markets with high trading volumes, such as the US and UK stock markets, but not necessarily in emergent or thin markets such as the Middle Eastern, the Latin American or emergent European markets. Based on the few studies that there exist, Helsinki Stock Exchange appears not to be efficient in the random walk sense. However, there are not many recent studies that seriously investigate RWH for both indices and individual stocks in Helsinki Stock Exchange with multiple VR tests and adequately large datasets. The chosen assumptions of RWH, the test methodologies used and the return horizon profoundly affect the results. Therefore, this thesis aims at choosing proper return intervals, testing methods and motivating the choice of the null hypothesis.

3 Research material and methods

3.1 The variance ratio

Tests based on variance ratios have gained popularity in recent years [14]. The VR methodology relies on the fact that the variance of k–period random walk increments is linear with respect to the difference k, that is $\mathrm{Var}[x_t - x_{t-k}]$ is k times $\mathrm{Var}[x_t - x_{t-1}]$. This motivates the following definition of the variance ratio:

$$\mathrm{VR}(k) = \frac{1}{k} \frac{\mathrm{Var}[x_t - x_{t-k}]}{\mathrm{Var}[x_t - x_{t-1}]}. \tag{2}$$

With the assumption that x_t is a random process, one gets that $\mathrm{VR}(k)$ should be equal to unity for all differences k. Let us denote the estimator of $\mathrm{VR}(k)$ as $V(k)$:

$$V(k) = \frac{\hat{\sigma}^2(k)}{\hat{\sigma}^2(1)}. \tag{3}$$

Here, $\hat{\sigma}^2(k)$ is the unbiased estimator of $(1/k)\mathrm{Var}[x_t - x_{t-k}]$. There are several valid choices for $\hat{\sigma}^2(k)$, but Lo and MacKinlay [9] use an estimator based on overlapping k–period returns, which according to their simulations yield desirable properties for finite samples. Their estimator is defined as

$$\hat{\sigma}^2(k) = m^{-1} \sum_{t=k}^{T} (x_t - x_{t-k} - k\hat{\mu})^2, \tag{4}$$

where $(x_0, \ldots, x_T)$ is the data, $\hat{\mu} = T^{-1} \sum_{t=1}^{T} x_t$ is the estimator of the mean, $T + 1$ is the sample size and $m = k(T - k + 1)(1 - kT^{-1})$. If the logarithmic stock price process x_t is a random walk, the expectation of the test statistic $V(k)$ should be equal to one for all differences k.

3.2 The Lo–MacKinlay test

Lo and MacKinlay [9] present two test statistics based on variance ratios, one of which is robust under the assumption of homoscedasticity and another which is robust under different forms of heteroscedasticity. This study uses the latter one to accommodate for time–varying volatility. The heteroscedastic LM test uses the test statistic

$$M_1(k) = \frac{V(k) - 1}{\sqrt{\phi(k)}}, \tag{5}$$

where

$$\phi(k) = \sum_{j=1}^{k-1} \frac{4(k-j)^2}{k^2} \delta(j) \tag{6}$$

and

$$\delta(j) = \frac{\sum_{t=j+1}^{T} (x_t - x_{t-1} - \hat{\mu})^2 (x_{t-j} - x_{t-j-1} - \hat{\mu})^2}{\left[\sum_{t=1}^{T} (x_t - x_{t-1} - \hat{\mu})^2\right]^2}. \tag{7}$$

Under the null assumption that $V(k) = 1$ for all k, the test statistic $M_1(k)$ follows the standard normal distribution asymptotically, that is $M_1(k) \sim \mathrm{N}(0,1)$ for a fixed k and $T \to \infty$. As noted by Charles et al. [14], the test statistic proposed by LM is asymptotic, meaning that the sampling distribution is approximated by the limiting distribution. For large values of k relative to T, the test statistic is right skewed and non–normal [17]. Due to this phenomenon, Lo and MacKinlay [9] propose selecting k–values no larger than half of the sample size T. Here, k–values in the range $\{2, 4, 8, 16, 32, 64\}$ are used for the LM test.

3.3 The Chow–Denning test

The LM test is an individual variance ratio test, meaning that it tests the null hypothesis for a given difference k. The null hypothesis H as defined in section 2.1 holds if and only if the individual LM tests pass for all values of k that are selected for the test. This method of several individual tests can increase the probability of a type I error, that is over rejection of the null hypothesis [14]. Furthermore, Chow and Denning [22] argue that the choices of the k–values play a significant role on the results of the LM test: focusing on extreme statistics may lead to over rejection of the null hypothesis.

The CD test considers the joint null hypothesis that the VR estimates are equal to one for all chosen differences k in a set of m different k–values. The CD test uses the maximum absolute value of the LM statistics as its test statistic:

$$MV_1 = \max_{1 \le i \le m} |M_1(k_i)|, \tag{8}$$

where $M_1(k_i)$ is defined in (5). By applying the results obtained by Sidak [23], Hochberg [24] and Richmond [25], Chow and Denning give an upper bound to the critical value of MV_1 and show that it follows the studentized maximum modulus distribution SMM(α, m, T), where α is the level of significance of the test. Chow and Denning [22] show that the null hypothesis is rejected at α level of significance if and only if MV_1 exceeds the $\frac{1}{2}(1 + (1-\alpha)^{1/m})$:th percentile of the standard normal distribution. At the 5 percent level of significance and k–values $\{2, 4, 8, 16\}$, the critical value of MV_1 is calculated as 2.491.

3.4 The Whang–Kim test

The CD test statistic is approximated by an upper bound on the exact critical value, which can in some cases be too conservative of an approximation. Thus, Whang and Kim [26] propose a strategy that directly approximates the exact critical value. Their method is based on a subsampling technique. The WK test uses the same test statistic as the CD test, namely:

$$MV_1 = g_T(x_0, \ldots, x_T) \tag{9}$$

where $g_T(x_0, \ldots, x_T) = \max_{1 \leq i \leq m} |M_1(k_i)|$. Whang and Kim [26] then approximate the cumulative distribution function of MV_1, denoted as G_T, by looking at b–sized subsamples $(x_t, \ldots, x_{t-b+1})$ and calculating $g_{T,b,t} = g_b(x_t, \ldots, x_{t+b-1})$ for all $t = 0, \ldots, T-b+1$. The distribution of G_T is then approximated by the formula

$$\hat{G}_{T,b}(x) = (T-b+2)^{-1} \sum_{t=0}^{T-b+1} \mathbb{1}(g_{T,b,t} \leq x), \tag{10}$$

where $\mathbb{1}$ is the indicator function [26]. The critical value of the test is the $(1-\alpha)$–percentile of $\hat{G}_{T,b}$, meaning that RWH is rejected if MV_1 exceeds this critical value. Note that one must choose an appropriate value for the subsample size b in order to perform the test successfully. Whang and Kim use in their Monte Carlo simulations six different b–values equal distances apart in the range $[2.5T^{0.3}, 3.5T^{0.6}]$. In this thesis, three subsample sizes in the aforementioned range are used: $b \in \{50, 100, 150\}$. The k–values are the same as in the CD test.

3.5 Data and software

The data consist of weekly prices for two indices, OMXHPI and OMXH25, as well as eight stocks. All prices are from the time period January 3, 2000 to February 28, 2018. Stock prices and related information are collected from Yahoo! Finance [12] and the prices of the two indices are retrieved from Investing [13]. The stocks are randomly selected from the OMXH25 index, with the restriction that there are less than ten missing data points. Nokia is included by default, since it is the most traded stock in Helsinki Stock Exchange and thus of special interest. The data, including further details, is presented in table 1. Key financial information appears in table 2.

All prices are close prices, which are adjusted for possible splits. Dividends are not included in the returns, and there are some missing data points that are simply ignored. The problem with not including dividends in the return history is that, since dividends are predictable, there is a deterministic adjustment of the stock price that corresponds to the size of the dividend. Ignoring missing data points can cause volatility spikes in the time series.

Lo and MacKinlay [9] suggest using weekly returns, as their sampling method is based on an asymptotic approximation requiring a large number of observations. Daily returns would give more observations, but there are many problems related to it. As noted by Lo and MacKinlay [9], the unwanted effects of infrequent trading, the bid–ask spread and asynchronous prices may become emphasized for daily returns. Thus, weekly returns are a good compromise: Lo and MacKinlay follow this strategy in their own experiment [9].

Table 1: Summary of the data. Note that the number of observations does not include missing data points.

Stock	Ticker symbol	Observations	Missing data points
OMXHPI index	OMXHPI	947	1
OMXH25 index	OMXH25	945	3
Nokia Oyj	NOKIA.HE	939	9
Wärtsilä Oyj Abp	WRT1V.HE	942	6
Amer Sports Oyj	AMEAS.HE	948	0
UPM–Kymmene Oyj	UPM.HE	940	8
Fortum Oyj	FORTUM.HE	940	8
Stora Enso Oyj	STERV.HE	940	8
Elisa Oyj	ELISA.HE	940	8
Nokian Renkaat Oyj	NRE1V.HE	945	3

The data is imported into the RStudio software, where it is analyzed, missing data points are removed and the time series are logarithmized and differentiated. The test statistics and the auxiliary functions are written as R functions. The actual source code used in the tests, including comments, are presented in appendix A.

Table 2: Key financial information about the data.

Time series	Avg. price (€)	Avg. return (€)	Avg. weekly vol. (M€)
OMXHPI index	7915.96	-5.29	342.80
OMXH25 index	2416.39	0.80	–
Nokia Oyj	13.54	-0.04	124.62
Wärtsilä Oyj Abp	21.80	0.06	2.62
Amer Sports Oyj	12.35	0.02	1.33
UPM–Kymmene Oyj	14.71	0.01	11.56
Fortum Oyj	14.27	0.02	9.84
Stora Enso Oyj	9.19	0.00	16.58
Elisa Oyj	19.14	0.00	2.65
Nokian Renkaat Oyj	19.58	0.04	3.00

4 Results

4.1 Results of the Lo–MacKinlay test

The results of the LM test are presented in table 3. The RWH is accepted for both indices and all stocks, except for UPM–Kymmene, at the 5 percent level of significance. The main rows contain the VR estimates $V(k)$ for each parameter value k, with the LM heteroscedastic test statistic $M_1(k)$ reported in parenthesis under the corresponding VR estimate. The null hypothesis is rejected for UPM–Kymmene, as $M_1(2)$ exceeds the 5 percent critical value of 1.96.

Most substantial deviations from unity variance ratio are observed for UPM–Kymmene, Amer Sports, Fortum, Nokian Renkaat and Stora Enso. Their maximum absolute values of the LM statistic are 2.01, 1.86, 1.84, 1.55 and 1.53, respectively. The smallest deviations from unity variance ratio are observed for Nokia and the OMXHPI index – their maximum absolute values of the LM test statistic are 0.74 and 0.92, respectively.

The variance ratios are less than one for all stocks and indices for $k = 2$, and for most stocks and indices for $k \in \{4, 8, 16, 32\}$. It can be shown that VR(2) corresponds to $1 + \rho_1$, where ρ_1 is the first–order autocorrelation coefficient of the return process $r_t = x_t - x_{t-1}$. Therefore, the results show that all return processes have a negative first–order autocorrelation coefficient. Lo and MacKinlay [9] report that this is a common symptom of infrequent trading. It appears that the VR estimates increase as k increases: only UPM–Kymmene, Stora Enso and Nokian Renkaat has it the opposite way around.

Table 3: The results of the LM test.

	Aggregation parameter k					
Time series	2	4	8	16	32	64
OMXHPI index	0.96 (-0.92)	0.93 (-0.80)	1.00 (-0.02)	0.96 (-0.20)	0.99 (-0.02)	1.15 (0.37)
OMXH25 index	0.94 (-1.30)	0.92 (-1.02)	0.96 (-0.34)	1.01 (0.05)	1.22 (0.82)	1.43 (1.15)
Nokia Oyj	0.99 (-0.24)	1.03 (0.49)	1.08 (0.74)	1.03 (0.19)	0.92 (-0.34)	1.02 (0.06)
Wärtsilä Oyj Abp	0.97 (-0.80)	0.98 (-0.18)	0.99 (-0.08)	0.98 (-0.11)	1.12 (0.47)	1.31 (0.87)
Amer Sports Oyj	0.91 (-1.86)	0.86 (-1.59)	0.76 (-1.77)	0.74 (-1.36)	0.81 (-0.73)	0.99 (-0.04)
UPM–Kymmene Oyj	0.91 (-2.01)*	0.90 (-1.23)	0.87 (-1.05)	0.76 (-1.33)	0.75 (-1.00)	0.78 (-0.64)
Fortum Oyj	0.92 (-1.54)	0.83 (-1.84)	0.81 (-1.34)	0.84 (-0.74)	0.92 (-0.26)	1.15 (0.39)
Stora Enso Oyj	0.94 (-1.40)	0.89 (-1.37)	0.81 (-1.45)	0.70 (-1.53)	0.72 (-1.01)	0.76 (-0.65)
Elisa Oyj	0.96 (-0.78)	1.00 (0.02)	1.00 (-0.04)	1.00 (0.00)	1.12 (0.42)	1.39 (0.97)
Nokian Renkaat Oyj	0.97 (-0.70)	0.98 (-0.27)	1.12 (0.92)	1.28 (1.55)	1.25 (0.99)	1.10 (0.29)

The VR estimates $V(k)$ are reported without parenthesis and the LM statistics $M_1(k)$ are given in parenthesis. An asterisk (*) indicates that the test statistic exceeds the 5 percent critical value of 1.96.

A more detailed profile of the VR estimates for the given time series are presented in figures 1, 2 and 3. The plots show the variance ratio estimates for all k–values up to 128. Indeed, it seems that the VR estimates tend to increase as k increases, up until approximately $k = 80$, where the estimate starts to decline slightly. This pattern is observable for both indices, and for some of the stocks. The results are in agreement with the findings of Lo and MacKinlay [9], who report positive serial correlation for US stock market indices for long holding periods. The results give more evidence against the mean reversion model proposed by Poterba and Summers [27]. However, it should be emphasized that the observed variance ratios are insignificantly different from one, both statistically and economically, except perhaps for UPM–Kymmene.

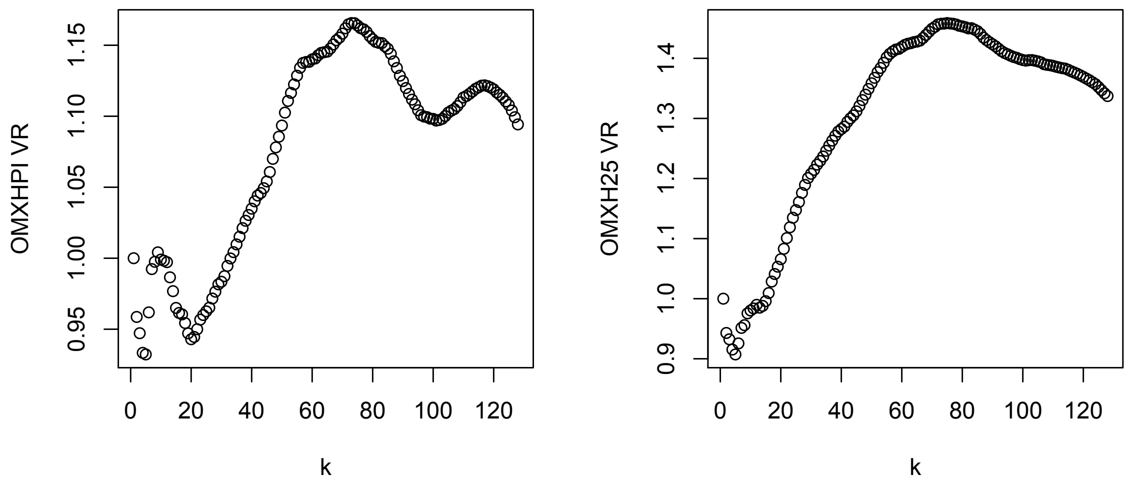

Figure 1: The LM variance ratio estimates for the OMXHPI and OMXH25 indices plotted against k–values in the range 0–128.

An interesting observation is that the OMXHPI index shows smaller deviations from unity variance ratio than the OMXH25 index. This is not expected, since the OMXH25 index contains the most highly traded stocks in Helsinki Stock Exchange, whereas OMXHPI contains also less frequently traded stocks. Another observation is that the results for the indices do not differ significantly from the results for individual stocks. Lo and MacKinlay [9] report smaller autocorrelation coefficients for individual securities than for portfolios, and point out that this could be because individual stocks carry much firm–specific noise.

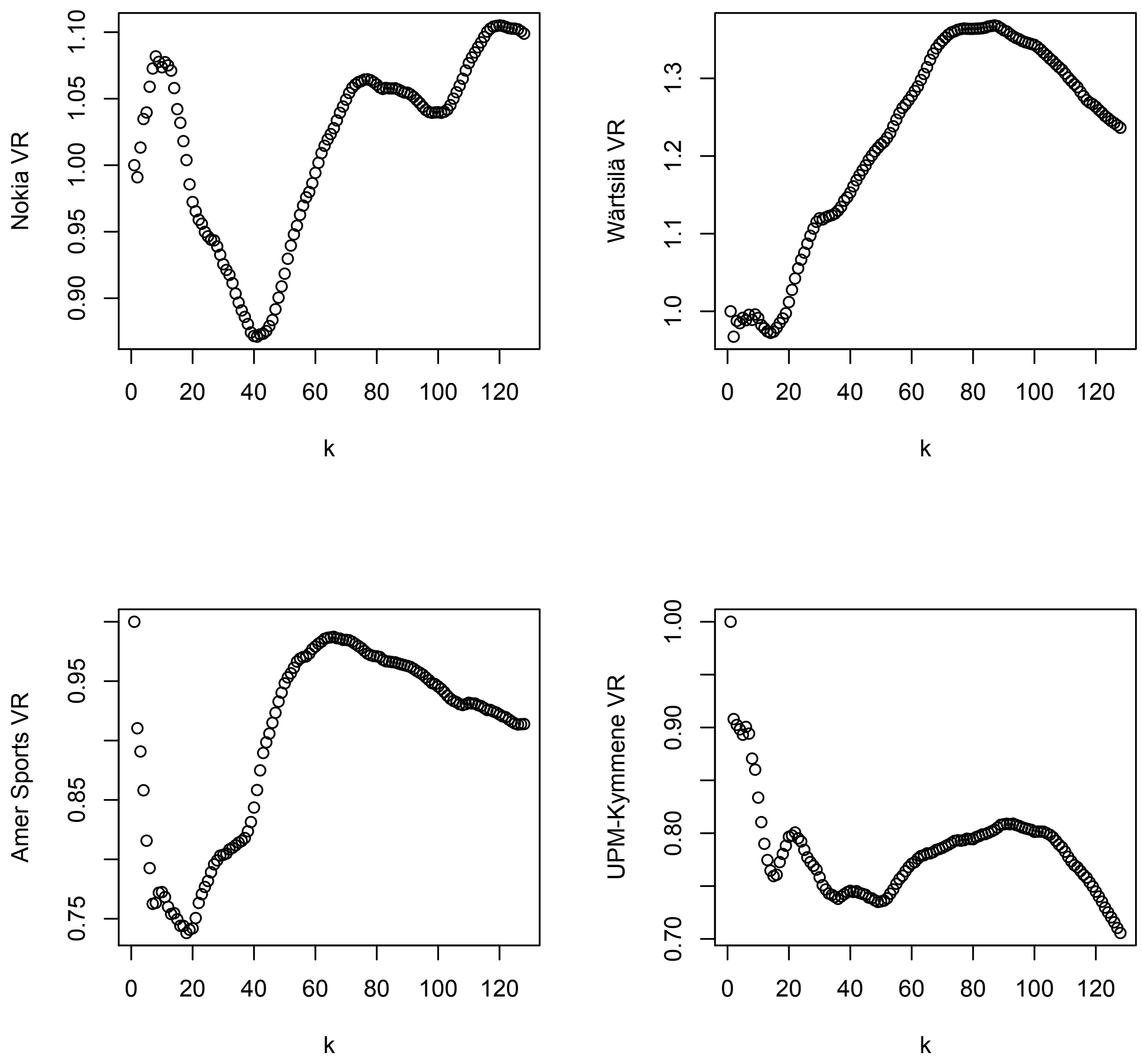

Figure 2: The LM variance ratio estimates for Nokia, Wärtsilä, Amer Sports and UPM–Kymmene plotted against k–values in the range 0–128.

There are clear differences between the VR profiles for different stocks. Nokia's variance ratio estimates are on average closest to one, implying that its price history agrees best with the random walk model. This is not unexpected since Nokia is the overwhelmingly most traded stock in Helsinki Stock Exchange (see table 2). On the other hand, infrequently traded stocks such as Amer Sports or Wärtsilä do not show radical departures from RWH, either.

UPM–Kymmene and Stora Enso differ from all other stocks in that their VR estimates are notably less than one and their VR estimates decrease as k increases. The fact that the said stocks have similar VR profiles is not surprising, as both companies operate in the forest industry and thus have similar price history. What is quite remarkable is that the two stocks show negative serial correlation for all return horizons. This could be a coincidence, or it could be an indication of mean reversion.

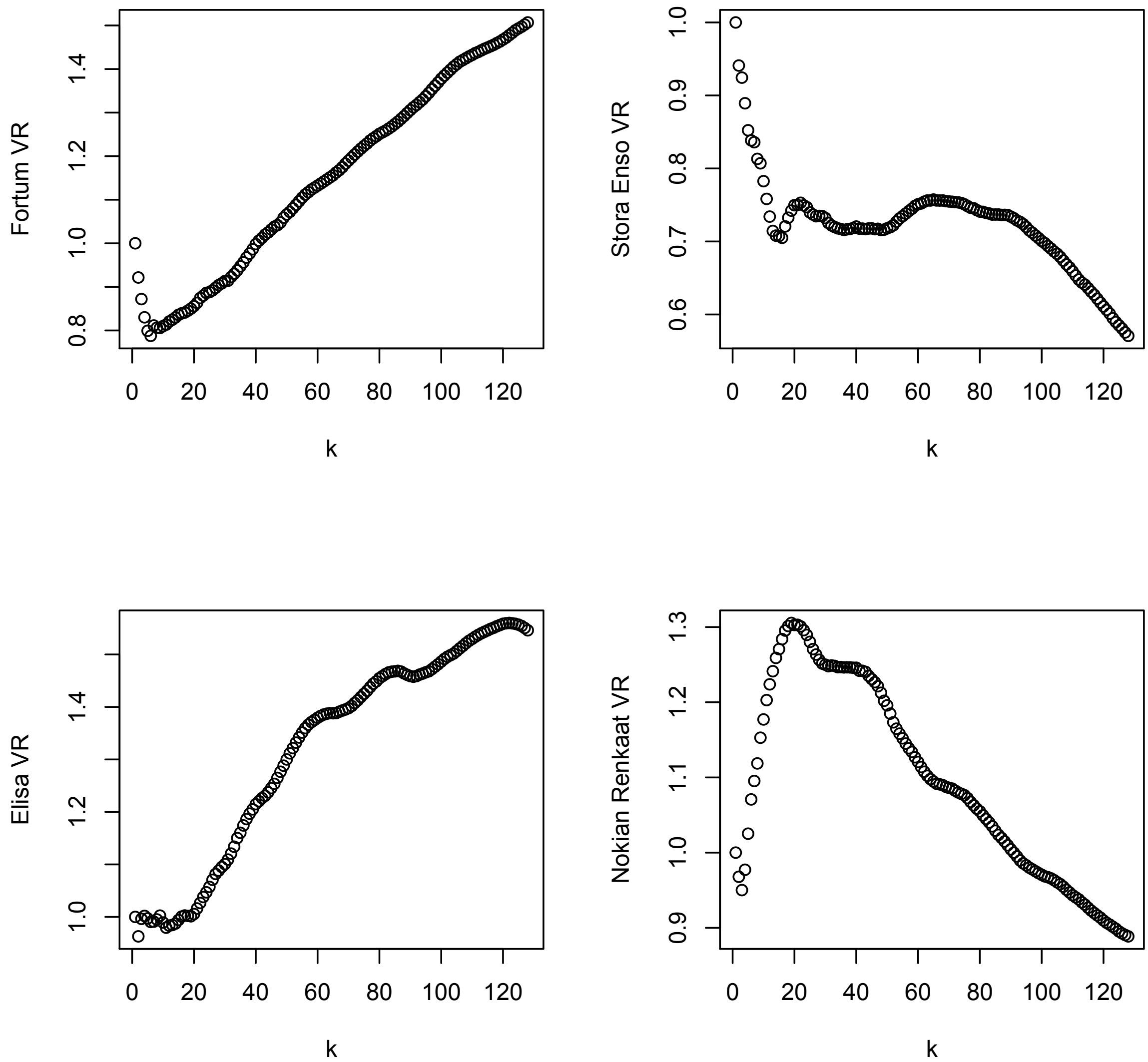

Figure 3: The LM variance ratio estimates for Fortum, Stora Enso, Elisa and Nokian Renkaat plotted against k–values in the range 0–128.

4.2 Results of the Chow–Denning and Whang–Kim tests

The results of the CD and WK tests are presented in table 4. The common test statistic MV_1 is reported in the first column, with the critical value of the CD test, abbreviated CD_{crit}, given in the second column. The critical values of the WK test, denoted WK_{crit}, are reported in the three rightmost columns for the chosen subsample sizes b. All critical values are 5 percent critical values.

Table 4: The results of the CD and WK tests.

			WK_{crit} for subsample size b		
Time series	MV_1	CD_{crit}	50	100	150
OMXHPI	0.92	2.491	2.26	2.00	2.14
OMXH25	1.30	2.491	2.16	2.24	2.45
Nokia Oyj	0.74	2.491	2.64	2.07	1.87
Wärtsilä Oyj Abp	0.80	2.941	2.04	2.05	2.19
Amer Sports Oyj	1.86	2.491	2.05	2.06	2.13
UPM–Kymmene Oyj	2.01	2.491	2.26	2.27	1.91*
Fortum Oyj	1.84	2.491	2.02	2.02	2.06
Stora Enso Oyj	1.53	2.491	2.17	2.25	2.17
Elisa Oyj	0.78	2.491	2.39	2.39	2.50
Nokian Renkaat Oyj	1.55	2.491	2.85	3.89	3.59

An asterisk (*) indicates that the test statistic exceeds the given critical value.

The CD test accepts RWH for all time series, whereas the WK test rejects RWH for UPM–Kymmene with subsample size $b = 150$ at the 5 percent level of significance. The results are in agreement with the LM test and give further support that the time series follow a random walk. As expected, WK_{crit} is on average less than CD_{crit}. This shows that the WK test indeed gives a sharper bound for the critical value compared to the CD test, at least on average.

5 Conclusion

The results of this study indicate that stocks and the main indices in Helsinki Stock Exchange follow random walks. Only UPM–Kymmene is found to violate the random walk assumption at the 5 percent significance level. The results contradict some of the previous studies on RWH in Finland and Scandinavia. In particular, the results are not consistent with Shaker [11], who strongly rejects RWH for daily returns of the OMXH25 index for the period 2003 to 2012. Furthermore, the results are not in accordance with the results obtained by Berglund et al. [18], who show that Helsinki Stock Exchange and other Scandinavian stock exchanges are inefficient in the random walk sense.

It must be emphasized that the results obtained in this study are not entirely comparable with some of the previous studies. Firstly, this study uses only variance ratio tests, whereas Berglund et al. [18], Jennergren and Korsvold [7] and some other authors use primarily serial correlation tests and runs tests. Secondly, many of the previous studies examine daily returns, while this study uses weekly returns. As noted by Lo and MacKinlay [9], daily sampling can induce many side–effects: biases related to infrequent trading or the bid–ask spread may become emphasized. Since Helsinki Stock Exchange is a relatively thin market, these biases might be even more pronounced than in larger stock markets.

One should also note that studies on RWH in Finland cover vastly different time periods of varying lengths. For instance, Shaker [11] considers price data from 2003 to 2012, which is a fairly short time period, whereas this study covers more recent data from 2000 to 2018. It could well be that the market efficiency of Helsinki Stock Exchange has evolved over time. Indeed, according to the adaptive market hypothesis, as market ecology, institutional environment, regulations and taxes change over time, so does the efficiency of the market [28]. The trading volume in Helsinki Stock Exchange has not changed notably during the past 20 years, so the market can still be characterized as a thin stock market.

If stock prices in Helsinki Stock Exchange follow a random walk, what are the implications for investors and the efficiency of the market? As stated in section 1, RWH is not a sufficient condition for market efficiency and thus, the results do not imply market efficiency. However, the results suggest that Helsinki Stock Exchange might be efficient, at least in the weak sense. The implication for investors is that technical analysis of historical returns may not improve expected earnings significantly – especially when transaction costs are considered. However, there may be a handful of stocks, including UPM–Kymmene, that have somewhat predictable patterns. Nevertheless, since the LM test does not give an alternative model to RWH, constructing a model for the stock price generating process could be difficult.

This thesis raises many interesting questions regarding RWH and its validity in Helsinki Stock Exchange. How do the results of variance ratio tests and other tests of RWH compare for returns of different time periods: are the results significantly different for daily, weekly and monthly returns? Has the efficiency of Helsinki Stock Exchange evolved over time? Perhaps a more extensive study including many, if not all stocks in Helsinki Stock Exchange could yield a better understanding of which stocks possess a random walk nature and which do not. There are undoubtedly many open questions for further research to address.

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A Appendix

A.1 Test statistics and related functions in R

```
# sigma_hat returns the unbiased estimator of (1/k):th of the k-period return
# variance, see definition (4). Parameters: k = aggregation parameter,
# T = sample size - 1, ts = time series.

sigma_hat <- function(k, T, ts) {
  cur <- 0
  mu <- mean(diff(ts))
  m <- k*(T-k+1)*(1-k/T)
   for(t in k:T){
    cur <- cur + (ts[t+1]-ts[t+1-k]-k*mu)^2
    }
   cur/m
}

# V returns the LM estimator of the variance ratio. See equation (3).
# Parameters: k = aggregation parameter, T = sample size - 1, ts = time series.

V <- function(k, T, ts) {
  sigma_hat(k, T, ts)/sigma_hat(1, T, ts)
}

# VTest plots the variance ratio estimates V for the given time series for
# k-values 1...128. Parameters: T = sample size - 1, ts = time series,
# label = y-axis label.

VTest <- function(T, ts, label) {
  v <- rep(0,128)
  for (k in 1:128) {
    v[k] = V(k, T, ts)
  }
  plot(v, xlab='k', ylab=label)
}

# delta returns delta(j) for the given time series, which is needed to
# compute phi(k), see equation (7). Parameters: j = see equation (7),
# T = sample size - 1, ts = time series.

delta <- function(j, T, ts) {
  numerator <- 0
  denominator <- 0
  mu <- mean(diff(ts))
  for(t in (j+1):T){
    numerator <- numerator + (ts[t+1]-ts[t]-mu)^2 * (ts[t+1-j]-ts[t-j]-mu)^2
    }
  for(t in 1:T){
    denominator <- denominator + (ts[t+1]-ts[t]-mu)^2
  }
  numerator/denominator^2
}
```

```
# phi returns phi(k) for the given time series, see equation (6).
# Parameters: k = see equation (6), T = sample size - 1, ts = time series.

phi <- function(k, T, ts) {
  cur <- 0
  for(j in 1:(k-1)){
    cur <- cur + (4*(k-j)^2/(k^2)) * delta(j, T, ts)
  }
  cur
}

# M_1 returns the LM test statistic for the given time series and aggregation
# parameter k. Parameters: k = aggregation parameter, T = sample size - 1,
#  ts = time series.

M_1 <- function(k,T,ts) {
  (V(k, T, ts)-1) / sqrt(phi(k, T, ts))
}

# data returns the subsample (x_t, ..., x_(t+b-1)) of the given time series.
# Parameters: t = first data point, b = subsample size, ts = time series.

data <- function(t, b, ts) {
    ts[t:(t+b-1)]
}

# gTbt returns g_b(x_t, ..., x_(t+b-1)), i.e the CD statistic computed for the
# subsample (x_t, ..., x_(t+b-1)) of the given time series.
# Parameters: t = first data point, b = subsample size, ts = time series.

gTbt <- function(t, b, ts) {
    M <- c()
    i <- 1
    for (k in c(2,4,8,16)) {
    M[i] = M_1(k, length(data(t, b, ts))-1, data(t, b, ts))
    i <- i + 1
    }
    max(abs(M))
}

# GTb returns the the cumulative distribution function of the CD statistic
# MV_1, as approximated by WK, evaluated at x. Parameters: T = sample size - 1,
# b = subsample size, ts = time series, x = argument of the cumulative distribution
# function.

GTb <- function(T, b, ts, x) {
    cur <- 0
    for(t in 1:(T-b+2)) {
        cur <- cur + ifelse(gTbt(t, b, ts) < x, 1, 0)
    }
    cur / (T-b+2)
}
```