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# Comparison of Simulation and Performance Models of a Tracking System

Bachelor's thesis

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| AALTO UNIVERSITY<br>SCHOOL OF SCIENCE<br>PO Box 11000, FI-00076 AALTO<br>http://www.aalto.fi | ABSTRACT OF THE BACHELOR'S THESIS |
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Abstract:

Tracking systems are used to compute estimates of the state of a flying target, such as an airplane, based on measurements from air surveillance radars. Models of a tracking system are used for performance assessments, i.e. to determine the accuracy of the state estimates. Performance assessments of tracking systems commonly involve time-consuming Monte Carlo simulations. A simple performance model allows for fast computations while employing approximations and simplified assumptions of the tracking system.

In this thesis, a simulation model and a simple performance model of a tracking system are compared. Differences between the models are explained and the results produced by each model are compared numerically and graphically. The motivation of the comparison is to determine if the time-consuming simulations could be replaced by a reliable, but faster method. The performance model is modified for use in multisensor scenarios where asynchronous radars and the use of multiple kinematic models need to be taken into account.

The tracking errors given by the models are compared in six scenarios using different target trajectories, radar configurations and tracker parameter settings. The models are found to give similar results in most scenarios. The correction made for asynchronous radars is found to result in more realistic performance assessments. The computation times of the performance model and the simulation model are compared, and the performance model is found to be significantly faster in all scenarios.

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# Contents

# 1 Introduction

Tracking systems are used to estimate the state of an airborne target. Measurements of the position of the target are obtained using air surveillance radars. The tracking system combines the measurements from the radars and computes the state estimates in real time, including the position, the speed, and/or the acceleration of the target, commonly employing Kalman filters [Blackman, 1986]. The estimates are computed based on assumptions, or a priori information, on the measurement accuracy and the target dynamics, such as the maximum acceleration.

Several different approaches to performance assessment of tracking systems have been developed, including Monte Carlo simulation models, analytic models and error bounding techniques [Li and Bar-Shalom, 1994]. Monte Carlo simulations are generally simple to implement but time consuming. Analytic models rely on many assumptions and approximations, and their usefulness depends on the particular application. The tightness of the theoretical bounds obtained with error bounding techniques is generally unknown. They are also generally not applicable to specific scenarios [Blair and Miceli, 2012]. This thesis focuses on two particular models, the simulation model by Pousi et al. [2014] and the analytic performance model by Blair and Miceli [2012], both of which allow performance assessment in multisensor scenarios with predetermined target trajectories.

Pousi et al. [2014] present a model for assessing the performance of a tracking system as a part of performance assessment of air surveillance networks. A Monte Carlo simulation model is used to predict the accuracy of the position estimates in multisensor settings. The true trajectory of the target is first generated. The measurements are generated by adding random errors to the true position of the target. The mean squared errors of the state estimates are obtained directly by comparing the estimates to the true state of the target. Gating and track scoring are used in the simulations. Gating refers to ignoring measurements that are far from the predicted position of the target. This is done to avoid associating the target with measurements that are originated from other sources. In track scoring the estimated track of the target is given a numerical value (score) based on how accurate it is considered. A track may be eliminated if the measurements are not received at the expected rate and/or the measurements often lie outside the gate.

Blair and Miceli [2012] present a simplified model for performance assessment of a tracking system. The simplified model is justified by the need for, e.g., confirmation of computer simulations and systems engineering of complex multisensor systems. This approach does not involve simulation, but the mean squared errors of the state estimates are obtained without generating observations. The model is based on steady-state considerations of an $\alpha$-$\beta$ filter, which result in analytic expressions for the bias and the variance, i.e. the components of the mean squared error of the estimates. This model, where the Kalman filter of the tracking system is approximated by an $\alpha$-$\beta$ filter, is here referred to as the performance model.

Neither the simulation model nor the performance model necessarily produce performance assessments that correspond to reality. No data on the true performance of the tracking system exists, and gathering of such data may be practically infeasible. In addition, the Kalman filter used in the tracking system may be tuned for monitoring different types of targets in different situations as needed, and no single model will correspond to reality in all scenarios.

In this thesis, the parameters of the performance model are tuned in order to obtain results that are comparable with those given by the simulation model. The method used to combine measurements from different radars is modified to take into account the time differences between the measurements. Kalman filters with multiple kinematic models are considered by tuning the parameters of the performance model separately for each type of target trajectory. The type of the true trajectory is given as input to the performance model. The parameter tuning is done manually with a small number of test scenarios using the simulation results as a reference.

The modified performance model and the simulation model are compared in different scenarios with different target positions and radar configurations. The parameters of the performance model are not further tuned for these comparisons. An important motivation for the comparison is to determine if computationally expensive simulations could be replaced by the use of a faster method, at least in some scenarios. For this reason, the computation times are also compared. A fast and reliable performance model could be used in the design of an air surveillance network and optimization models where a Monte Carlo simulation approach would be both unreliable and computationally infeasible.

In literature, simulations of tracker models are commonly done in small numbers of chosen scenarios. The total number of performance evaluations done in this thesis is considerably larger. This results from the large number of target positions used in order to form graphical presentations of the performance of tracking systems.

This thesis is organized as follows. Filters used for state estimation are briefly introduced in Section 2. The performance model and the simulation model are described in Section 3. The modifications made to the performance model are introduced in Section 4. The approach used to compare the models is explained in Section 5. The process used to tune the parameters of the performance model is explained in Section 6. The numerical and graphical comparisons of the results provided by the models are presented in Section 7. The results and conclusions are summarized in Section 8.

# 2 Filters for State Estimation

The state of a target is commonly estimated using different types of filters [Blackman, 1986]. As the radar measurements of the position of the target are corrupted by noise, it is necessary to use assumptions and a priori knowledge of the target dynamics to obtain accurate state estimates. The two filters explained here are the Kalman filter and the $\alpha$-$\beta$ filter.

## 2.1 Kalman Filtering

The Kalman filter [Kalman, 1960] provides an optimal way to obtain state estimates of the target using the obtained measurements and a kinematic model of the target. Kalman filters are used in the simulation model, and the computations in the performance model are based on steady-state considerations of a Kalman filter. The following presentation is based on [Grewal and Andrews, 2008] and [Särkkä, 2013].

Assume that the dynamics of the target can be described as a linear system of the form

$$\begin{aligned} X_{k+1} &= F_k X_k + \nu_k, \\ \nu_k &\sim N(0, Q_k), \end{aligned} \tag{1}$$

where $X_k$ is the unknown state vector at time step $k$, $F_k$ is the known transition matrix that defines the dynamics of the target and $\nu_k$ is the random process noise with covariance matrix $Q_k$. The process noise models the deviations from the dynamics described by $F_k$. Assume that the measurements can be modelled as

$$\begin{aligned} Y_k &= H_k X_k + w_k, \\ w_k &\sim N(0, R_k), \end{aligned} \tag{2}$$

where $Y_k$ is the measurement vector at time step $k$, $H_k$ is the observation model and $w_k$ is measurement noise with known covariance matrix $R_k$. Starting from the initial state estimate $X_{0|0}$ and the initial error covariance matrix $P_{0|0}$, the following predictions and state estimates are computed recursively as new measurements are obtained. Given the state estimate at time step $k-1$, the state prediction is given by

$$\hat{x}_{k|k-1} = F_k \hat{x}_{k-1|k-1}. \tag{3}$$

The error covariance matrix of the predicted state, denoted by $P_{k|k-1}$, is then

$$P_{k|k-1} = F_k P_{k-1|k-1} F_k^T + Q_k. \tag{4}$$

When a new measurement $Y_k$ is received, the updated state estimate and its covariance matrix are obtained recursively by

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(Y_k - H_k \hat{x}_{k|k-1}), \tag{5}$$

$$P_{k|k} = (I - K_k H_k) P_{k|k-1}, \tag{6}$$

where the innovation covariance $S_k$ and the Kalman gain $K_k$ are defined as

$$S_k = H_k P_{k|k-1} H_k^T + R_k, \tag{7}$$

$$K_k = P_{k|k-1} H_k^T S_k^{-1}. \tag{8}$$

## 2.2 $\alpha$-$\beta$ Filter

The $\alpha$-$\beta$ filter uses a simple model to estimate the position and velocity of the target [Blair and Miceli, 2012]. The model can be defined with the state and measurement equations of (1) and (2) by defining

$$X_k = \begin{bmatrix} x_k \\ v_k \end{bmatrix}, \tag{9}$$

$$F_k = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \tag{10}$$

$$Q_k = \sigma_\nu^2 \begin{bmatrix} \frac{T^3}{3} & \frac{T^2}{2} \\ \frac{T^2}{2} & T \end{bmatrix}, \tag{11}$$

$$H_k = \begin{bmatrix} 1 & 0 \end{bmatrix}, \tag{12}$$

$$R_k = \sigma_w^2, \tag{13}$$

where $T$ is the sampling period, $\sigma_w^2$ is the measurement noise variance and $\sigma_\nu^2 T$ describes the expected deviations from the linear motion. The process noise covariance matrix $Q_k$ results from the Continuous White Noise Acceleration model. In [Bar-Shalom et al., 2002] it is suggested that as a guideline for choosing $\sigma_\nu^2$, $\sqrt{\sigma_\nu^2 T}$ should be of the order of the change in the velocity over one sampling period.

If the measurement rate is constant and $\nu_k$ and $w_k$ are stationary processes, the $\alpha$-$\beta$ filter is equivalent to the Kalman filter in steady state. When the process noise covariance matrix is chosen as in (12), the steady state Kalman gains are

$$K_k = \begin{bmatrix}\alpha & \frac{\beta}{T},\end{bmatrix} \tag{14}$$

where $\alpha$ and $\beta$ are parameters of the filter that are given by ([Blair and Miceli, 2012])

$$\Gamma^2 = \frac{\sigma_\nu^2 T^3}{\sigma_w^2}, \tag{15}$$

$$\mu = \frac{1}{3} + \sqrt{\frac{1}{12} + \frac{4}{\Gamma^2}}, \tag{16}$$

$$\alpha = \beta\sqrt{\mu} \tag{17}$$

$$\beta = \frac{12}{6(\mu + \sqrt{\mu}) + 1}. \tag{18}$$

# 3 Models of Tracking Systems

The simulation model and the performance model are introduced in this section. The flow graph in Figure 1 illustrates the differences between the simulation and performance model approaches. In the simulation approach, the performance of the tracking system is assessed by comparing the state estimates to the true state of the target [Pousi et al., 2014]. The root mean squared errors are obtained by averaging over a large number of simulations. The performance model gives theoretical RMSEs [Blair and Miceli, 2012]. As there are no random components in the performance model, it is enough to compute the RMSEs once for a given scenario.

The methods for combining the measurements from different radars (i.e., sensor fusion) in the two models are different, as illustrated in Figure 2. These sensor fusion methods are treated in detail in [Mitchell, 2012]. The performance model considers separate $\alpha$-$\beta$ filters that output independent position estimates. The position estimates are fused using linear squares estimation weighted by the error covariance matrices of the $\alpha$-$\beta$ filters. The simulation model uses measurement level fusion, i.e. all the measurements from the radars are filtered by a single filter with a varying time interval $T$. Some differences between the results produced by the models can be expected due to the differences in the sensor fusion methods.

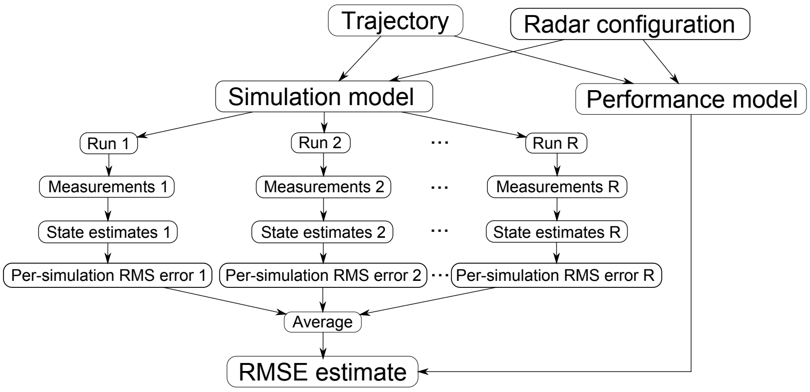
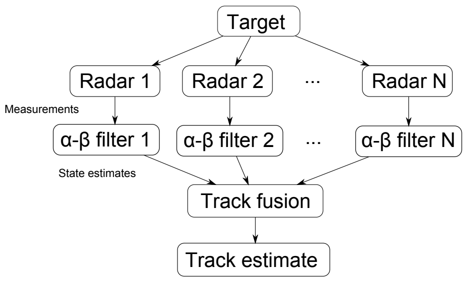
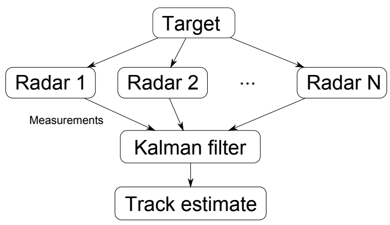



Figure 1: A flow graph of the simulation and performance model approaches.




(a) Track Fusion




(b) Measurement level fusion

Figure 2: Flow graphs of sensor fusion using track fusion and measurement level fusion.

## 3.1 Simulation Model

The simulation model presented by Pousi et al. [2014] uses repeated Monte Carlo simulations to assess the performance of a tracking system. The simulation model uses an IMM KF (Interacting Multiple Model Kalman Filter) that combines different kinematic models to account for different types of target movement. The kinematic models are weighted based on how closely their estimates match the realized measurements, and on transition probabilities between the kinematic models. These probabilities are contained in a parameter called the model transition matrix. IMM Kalman filters and the related computations are treated in detail in [Bar-Shalom et al., 2002].

Two kinematic models are used in the IMM KF. Singer's model [Singer, 1970] is used to account for targets with little to no maneuvering. In this kinematic model, the acceleration is estimated in addition to position and velocity. A parameter called the maneuver correlation time is used to describe the assumptions on the changes in acceleration. A coordinated turn model [Li and Jilkov, 2003] with a Wiener velocity model for altitude changes [Bar-Shalom et al., 2002] is used to account for maneuvering targets. In the coordinated turn model, the position and the velocity are estimated. Predictions are made assuming a turn in the $xy$-plane with a fixed angular velocity and a constant velocity in the altitude. The assumed steepness of the turn is given as a predetermined parameter.

The simulation is carried out as follows. The measurement error variances are given for each radar $i$=1...$N$ in range ($\sigma^2_{r,i}$), azimuth angle ($\sigma^2_{az,i}$) and elevation angle ($\sigma^2_{el,i}$) in a planar grid. The true trajectory of the target is generated as discrete points. The true state of the target between these points is obtained using linear interpolation. The Kalman filter is initialized with the correct values for the position, velocity and acceleration of the target. The times of the first measurements from the radars are randomized from uniform distributions and further measurements are obtained at constant time intervals. Measurements are generated by adding a random error to the true position of the target using the error variances $\sigma^2_{r,i}$, $\sigma^2_{az,i}$ and $\sigma^2_{el,i}$.

A measurement is ignored if the difference between the predicted position of the target and the measurement is too large. This is referred to as gating. If the measurement is ignored, the state of the target at the next time step is predicted using the standard Kalman prediction equations. The purpose of gating is to avoid associating the target with observations originating from other sources. An elliptic gate (e.g., Blackman [1986]) is used in the simulation model. The orientation and the size of the gate are computed using a gate size parameter, the measurement error covariance matrices and the error covariance matrix $P$ of the Kalman filter.

A track scoring system is used to determine if the tracking of the target should be continued. Measurements that lie inside the gate increase the track score, not exceeding a given maximum score. If no associated measurements are received within a given time, the track score decreases. The tracking ends when the track score is zero or negative.

The mean squared error for the position estimates in a given coordinate is obtained as follows:

$$\begin{aligned} p &= \text{coordinate of interest } (x,\ y,\text{ or } z) \\ r &= \text{simulation run index } (1...R) \\ k_{max,r} &= \text{number of measurements in simulation } r \\ x_{p,r}(k) &= \text{true position of the target in coordinate } p \text{ at step } k \text{ in run } r \\ \hat{x}_{p,r}(k) &= \text{the estimated position in coordinate } p \text{ at step } k \text{ in run } r \\ MSE_{p,r} &= \frac{1}{k_{max,r} - k_0} \sum_{k=k_0}^{k=k_{max,r}} (\hat{x}_{p,r}(k) - x_{p,r}(k))^2, \end{aligned} \tag{19}$$

where $k_0$ is chosen to be large enough so that the tracker has mostly forgotten the initial state. The RMS error in the $xy$-plane and altitude are then obtained as

$$R = \text{number of simulations}$$

$$RMSE_{xy} = \frac{1}{R} \sum_{r=1}^{R} \sqrt{MSE_{x,r} + MSE_{y,r}}, \tag{20}$$

$$RMSE_{alt} = \frac{1}{R} \sum_{r=1}^{R} \sqrt{MSE_{z,r}}. \tag{21}$$

The parameters of the simulation model can be adjusted for tracking different types of targets. To summarize, these parameters are the the process noise variances in Singer's model and the coordinated turn model, the maneuver correlation time in Singer's model, the assumed steepness of the turn in the coordinated turn model, the gate size parameter, and the model transition matrix in the IMM KF.

## 3.2 Performance Model

### 3.2.1 Single-Sensor Case

The model presented by Blair and Miceli [2012] uses known analytic formulas for steady state errors in $\alpha$-$\beta$ filters. First assume that an $\alpha$-$\beta$ filter is used to track the target in a single coordinate using a single radar. The mean squared error of the estimates in steady state is decomposed into sensor noise only (SNO) covariance $S^{\alpha\beta}_{k|k}$ and bias $B^{\alpha\beta}_{k|k}$, that is

$$MSE = S^{\alpha\beta}_{k|k} + B^{\alpha\beta}_{k|k}(B^{\alpha\beta}_{k|k})^T. \tag{22}$$

When there is no acceleration, the errors of the estimates result from the measurement errors. When the target is accelerating, the errors of the estimates result from both the measurement errors and the bias. It was shown in [Blair, 1992] that the SNO error covariance matrix of an $\alpha$-$\beta$ filter is

$$S^{\alpha\beta}_{k|k} = \frac{\sigma_w^2}{\alpha(4-2\alpha-\beta)} \begin{bmatrix} 2\alpha^2 + \beta(2-3\alpha) & \frac{\beta}{T}(2\alpha-\beta) \\ \frac{\beta}{T}(2\alpha-\beta) & \frac{2\beta^2}{T^2} \end{bmatrix}, \tag{23}$$

where $\alpha$ and $\beta$ are the parameters of the filter, $T$ is the time interval between measurements and $\sigma_w^2$ is the variance of the measurement errors in the coordinate in which the target is being tracked. The steady-state bias resulting from acceleration is

$$B^{\alpha\beta}_{k|k} = \begin{bmatrix} (1-\alpha)\frac{T^2}{\beta} \\ (\frac{\alpha}{\beta} - 0.5)T \end{bmatrix} A_k, \tag{24}$$

where $A_k$ is the acceleration in the coordinate of interest at time step $k$. The SNO covariance of $m$-step prediction errors is given by

$$S^{\alpha\beta}_{k+m|k} = F(m) S^{\alpha\beta}_{k|k} F(m)^T, \tag{25}$$

where

$$F(m) = \begin{bmatrix} 1 & mT \\ 0 & 1 \end{bmatrix}. \tag{26}$$

It can be shown that the bias of an $m$-step prediction is

$$B^{\alpha\beta}_{k+m|k} = \begin{bmatrix} (1-\alpha+(\alpha-0.5\beta)m + 0.5\beta m^2)\frac{T^2}{\beta} \\ (\alpha + (m-0.5)\beta)\frac{T}{\beta} \end{bmatrix} A_k. \tag{27}$$

Thus, the $m$-step prediction MSE can be obtained as

$$MSE(m) = S^{\alpha\beta}_{k+m|k} + B^{\alpha\beta}_{k+m|k}(B^{\alpha\beta}_{k+m|k})^T. \tag{28}$$

When the MSE is needed in several dimensions, the same equations can be used by replacing the matrices with block diagonal matrices, each block representing one of the coordinates. The measurement errors are generally defined in terms of range, azimuth angle, and elevation angle, and the process noise is defined in the $xyz$-frame. Thus, the acceleration of the target $A_k$, measurement error $\sigma_w^2$ and process noise variance $\sigma_\nu^2$ are converted to a common frame.

### 3.2.2 Multisensor Case

When multiple radars are used, the mean squared error of the state estimates is obtained using the following approach, also presented in [Blair and Miceli, 2012]. Each radar $i$=1...$N$ is assumed to use a separate $\alpha$-$\beta$ filter, producing $N$ tracks. Each filter only uses the measurements obtained by the corresponding radar. The errors in the estimated tracks are assumed to be independent. The fusion of the tracks is treated as a linear least squares estimation problem. The estimates from the filters are weighted using the track covariance matrices $P^i_{k|k}$ from each filter. The SNO covariance of the fused track is then found to be

$$S^f_{k|k} = P^f_{k|k} \left[ \sum_{i=1}^{N} (M^i_k)^T (P^i_{k|k})^{-1} S^i_{k|k} (P^i_{k|k})^{-1} M^i_k \right] (P^f_{k|k})^T, \tag{29}$$

where $S^i_{k|k}$ is the SNO covariance as derived in (23) and $M^i_k$ denotes a rotation matrix from the common $xyz$-frame to the local frame, which is aligned with the vector pointing from the radar to the target.

The process noise variance is obtained in the local frame of the radar $i$ at time $k$ using [Wasserman, 2010]

$$\begin{bmatrix} w^2_{r,i} \\ w^2_{u,i} \\ w^2_{v,i} \end{bmatrix} = M^i_k \begin{bmatrix} w^2_x & 0 & 0 \\ 0 & w^2_y & 0 \\ 0 & 0 & w^2_z \end{bmatrix} (M^i_k)^T, \tag{30}$$

where $r$, $u$, and $v$ refer to the coordinates of the local frame. As the measurement error variances are defined in radians squared, they are converted to meters in the local cartesian frame by

$$\begin{bmatrix} \sigma^2_r \\ \sigma^2_u \\ \sigma^2_v \end{bmatrix} = \begin{bmatrix} \sigma^2_{range} \\ d^2\sigma^2_\phi \\ d^2\sigma^2_\theta \end{bmatrix}, \tag{31}$$

where $d$ is the euclidean distance between the radar and the target. The bias of the fused track is given by

$$B^f_{k|k} = P^f_{k|k} \left[ \sum_{i=1}^{N} (M^i_k)^T (P^i_{k|k})^{-1} B^i_{k|k} \right]. \tag{32}$$

where $B^i_{k|k}$ is the bias as in (24). For a steady state $\alpha$-$\beta$ filter the error covariance is

$$P^i_{k|k} = \sigma^2_w \begin{bmatrix} \alpha & \frac{\beta}{T} \\ \frac{\beta}{T} & \frac{\beta(2\alpha-\beta)}{2(1-\alpha)T^2} \end{bmatrix}. \tag{33}$$

This error covariance matrix is derived based on the assumption of stationary measurement noise and process noise. It does not depend on the realized measurements. In the three-dimensional case, Equations (29), (32) and (33) are used with $S^i_{k|k}$ representing a block diagonal matrix of the SNO covariance matrices in the local coordinates, computed with Equation (23), and $B^i_{k|k}$ representing a column vector of the biases in the local frame, computed using Equation (24). The mean squared errors are then obtained as the diagonal elements of the matrix

$$MSE = S^f_{k|k} + B^f_{k|k} (B^f_{k|k})^T. \tag{34}$$

The formulas presented so far allow assessment of the tracking system at a single point with a given target acceleration. To obtain meaningful measures of accuracy in a given trajectory, the RMSEs are averaged over discrete points. Assuming that the trajectory is discretized at constant time intervals, the average planar RMSE and the average altitude RMSE are computed as

$$RMSE_{xy} = \frac{1}{K} \sum_{k=0}^{K} \sqrt{(MSE_x(x(k\Delta t), A(k\Delta t)) + MSE_y(x(k\Delta t), A(k\Delta t))}, \tag{35}$$

$$RMSE_{alt} = \frac{1}{K} \sum_{k=0}^{K} \sqrt{MSE_z(x(k\Delta t), A(k\Delta t))}, \tag{36}$$

where $MSE_x$, $MSE_y$ and $MSE_z$ refer to the diagonal elements of the steady-state error covariance matrix (34) obtained using the 3x1 true position vector $x$ and the 3x1 acceleration vector $A$ at time $k\Delta t$, where $\Delta t$ is the sampling interval of the discretized trajectory.

To summarize, the use of the performance model requires values for the time intervals between the measurements $T_i$, the process noise variances $w^2_x$, $w^2_y$, and $w_z^2$, the measurement noise variances $\sigma^2_{range,i}$, $\sigma^2_{\phi,i}$, and $\sigma^2_{\theta,i}$, and the target acceleration $A_k$. If prediction errors are needed, the prediction time steps $m_i$ need to be set.

# 4 Improvements to the Performance Model

The original performance model by Blair and Miceli [2012] is modified for use in scenarios where the time differences between the measurements and the use of multiple kinematic models has to be taken into account. The modifications are based on adjustments to the parameters of the performance model.

## 4.1 Correction for Asynchronous Radars

In the performance model, the accuracy of the track is computed using the errors of the state estimates from each filter (Equations (29) and (32)). As the errors of the state estimates are computed for the update phase of the Kalman filter, this fusion model corresponds to a scenario where all radars are synchronized and have the same measurement rate. If the measurement rates differ, or the radars are not synchronized, this approach underestimates the true error.

To correct for this, the prediction errors of the $\alpha$-$\beta$ filters (28) are used instead of the errors of the estimates (22). In [Blair and Miceli, 2012], it is noted that if the track fusion requires prediction, the prediction errors could be used instead of the estimation errors. It seems that this approach has not been applied in the literature. In the correction suggested here, the prediction errors are used to compensate for the time differences between the measurements in measurement level sensor fusion.

The prediction time in the performance model ($mT$ in Equation (28)) is computed for each radar. Assuming that a new measurement from radar $j$ is received at time $t$, the expected time between $t$ and the latest measurement from radar $i$ is obtained as follows:

$$\begin{aligned} t &= \text{current time} \\ j &= \text{radar making the measurement at time } t \\ u_i &= \text{time when radar } i \text{ last made a measurement} \\ T_i &= \text{time interval between measurements for radar } i \\ I &= \text{number of radars} \end{aligned}$$

$$E(t-u_i) = E(t-u_i|i=j)P(i=j) + E(t-u_i|i \neq j)P(i \neq j) \tag{37}$$

$$= E(t-u_i|i \neq j)P(i \neq j) \tag{38}$$

$$= \frac{T_i}{2}\left(1 - \frac{\frac{1}{T_i}}{\sum_{k=1}^{I}\frac{1}{T_k}}\right). \tag{39}$$

The expected time interval (39) is converted to a fractional number of time steps by

$$m_i = \frac{E(t-u_i)}{T_i} \tag{40}$$

$$= \frac{1}{2}\left(1 - \frac{\frac{1}{T_i}}{\sum_{k=1}^{I}\frac{1}{T_k}}\right). \tag{41}$$

Thus, the number of time steps given by (41) is used in place of $m$ in the equations for prediction covariance (25) and prediction bias (27). The weighting matrices are obtained with equations (4) and (33) using the corresponding time interval $m_i T_i$.

Note that this correction does not give the exact expected value of the RMSE, which would require computing the expected value of (34) over $m$. However, it will be shown in Section 7 that the correction of Equation 41 gives more reasonable results than the original performance model.

## 4.2 Combining Multiple Kinematic Models

In the simulation model, the IMM Kalman filter uses two kinematic models: one for little to no maneuvering (Singer's model) and one for coordinated turns (coordinated turn with Wiener velocity model for altitude changes). When the IMM Kalman filter determines that the target is flying along a straight-line trajectory, Singer's model is used. When the IMM Kalman filter determines that the target is maneuvering, a combination of the Singer model and the coordinated turn model is used.

The use of multiple kinematic models is taken into account by choosing the parameters of the performance model based on the true trajectory. The true trajectory type (straight-line/circular and the steepness of the turn) is given to the performance model as a parameter. Note that no attempt is made to model the weighting the IMM Kalman filter uses for the kinematic models. In the literature, the Hybrid Conditional Averaging method is used for such models [Li and Bar-Shalom, 1994]. However, this would make the computations considerably more complex and increase the computation times. In addition, it is not clear if this approach would work in multisensor scenarios.

The result of using multiple kinematic models is that the tracking error is smaller than when using either of the models alone. The kinematic model for coordinated turns is very specific, whereas Singer's model is a more general purpose kinematic model. Thus, the error of the estimates is the smallest when the target performs a coordinated turn and the true angular velocity equals the assumed angular velocity. The performance model, on the contrary, predicts that the error of the estimates increases when the acceleration is not zero. Thus, the performance model needs to be modified in order to make meaningful comparisons with the simulation model.

When the true trajectory is a straight line, the performance model is used as described in Section 3.2. When the true trajectory is circular, the following correction is made to the acceleration given to the performance model (Equations (24) and (27)).

$$A_k^C = \begin{bmatrix} A_{x,k} \\ A_{y,k} \\ A_{z,k} \end{bmatrix} = 9.81(g_{assumed} - g_{true})\frac{A_k}{|A_k|}, \tag{42}$$

where $A_k^C$ is the corrected acceleration at time $k$, $A_k$ is the true 3x1 acceleration vector and $g_{assumed}$ and $g_{true}$ are the assumed and true absolute values of the accelerations in g-force units. This forces the performance model to give the smallest error when the true acceleration of the target equals the assumed acceleration.

# 5 Performance Assessment of Tracking Systems

In this thesis, the performance of a tracking system is assessed based on the root mean squared errors of the position estimates of the target. A good understanding of the performance is gained by using sufficiently different trajectories and target types. Two types of target trajectories are used in the performance assessments: a straight-line trajectory with constant velocity, and a circular trajectory with a turn of 1080 degrees at constant angular velocity. The steepness of the turn and the altitude of the target is fixed for each scenario. The true trajectory is generated as discrete points with a discretization time step of one second.

The parameters of the simulation model and the performance model describe the assumptions on the target dynamics. If the tracking system is well tuned, the assumed maneuvering matches the true maneuvering of the target. However, performance assessments for poorly tuned tracking systems may also be of interest. The parameters are considered given for the simulation model and are not modified. Suitable parameters for the performance model are found as described in Section 6.

As the tracking accuracy depends on relative positions of the radars and the target, a large number possible target positions are considered. In each scenario, the map is divided into 20 km x 20 km cells. For each cell and each radar, the three measurement noise variances (for range, azimuth angle and elevation angle) and the time interval between the measurements are given. The target is placed at the center of the grid cell, from which it moves according to the predetermined trajectory. The performance model or the simulation model is run and an estimate of the tracking error is obtained for each cell. Not every radar is able to obtain measurements from every position of the target. Thus, the number of radars that can be used varies.

To summarize, for each scenario the target altitude, the true trajectory type (g-force of the turn or no turn), the assumed maneuvering (aggressive or slow), and the radar configuration are fixed. The simulation model and the performance model are then run for a large number of target positions in a grid.

# 6 Parameter Tuning

The parameters of the performance model are tuned to obtain results that are comparable to those given by the simulation model. This is necessary, as the objective is to compare the models, not the particular filter designs. The parameters of the simulation model are considered given and are not modified. The simulation model uses large values for the parameters related to maneuvering, described in Section 3.1, when it is assumed that an aggressively maneuvering target is being tracked. When it is assumed that the target maneuvers slowly, low values are used for the parameters.

The parameters of the performance model that are being tuned are the process noise variances $w_x^2$, $w_y^2$, and $w_z^2$. The possible scenarios are divided into four categories based on the type of the true trajectory and the assumed maneuvering of the target. One parameter set, comprising $w_x^2$, $w_y^2$, and $w_z^2$, is tuned for each scenario type. The parameter sets and the corresponding scenario types considered are summarized in Table 1.

| Parameter set | True trajectory type | Assumed maneuvering |
|---|---|---|
| 1 | Straight | Slow |
| 2 | Straight | Aggressive |
| 3 | Circular | Slow |
| 4 | Circular | Aggressive |

Table 1: The parameter sets tuned for the performance model.

The turn parameter of the performance model $g_{assumed}$ (in Equation (42)) is set to the same value as in the simulation model in parameter sets 3 and 4. For slowly maneuvering targets, this value is 2 g, and for large assumed accelerations the value is 6 g. The scenarios used to tune the parameters are summarized in Table 2.

| Parameter set | True trajectory type | Assumed maneuvering | Target altitude |
|---|---|---|---|
| 1 | Straight | Slow | 4000 m |
| 2 | Straight | Aggressive | 10 000 m |
| 3 | Circular (4 g) | Slow | 1000 m |
| 4 | Circular (7 g) | Aggressive | 10 000 m |

Table 2: The scenarios used to tune the parameters. A single radar is used in each scenario.

The parameters are tuned by hand, while comparing the results given by the performance model and the simulation model. The RMSEs given by the models are made to match adequately both near and far from the radar. No attempt was made to do this formally. A formal treatment would require a measure of similarity between the results given by the models.

Gating and track scoring are not modelled explicitly in the performance model. However, they are implicitly contained in the parameters of the performance model, as the parameter tuning is done using the simulation results as a reference.

# 7 Comparisons of the Results

The simulation model and the performance model with modifications described in Section 4 are compared in six different scenarios. The parameters of the performance model are tuned as described in Section 6. First, an example scenario is shown with a single radar and a straight-line trajectory. The models are then compared in multisensor scenarios with different target types. The scenarios used for the comparisons are summarized in Table 3. The assumed maneuvering determines if large or small values are used for the parameters of the simulation model, and if the parameter sets 1 and 3 or the parameter sets 2 and 4 are used in the performance model (see Table 1).

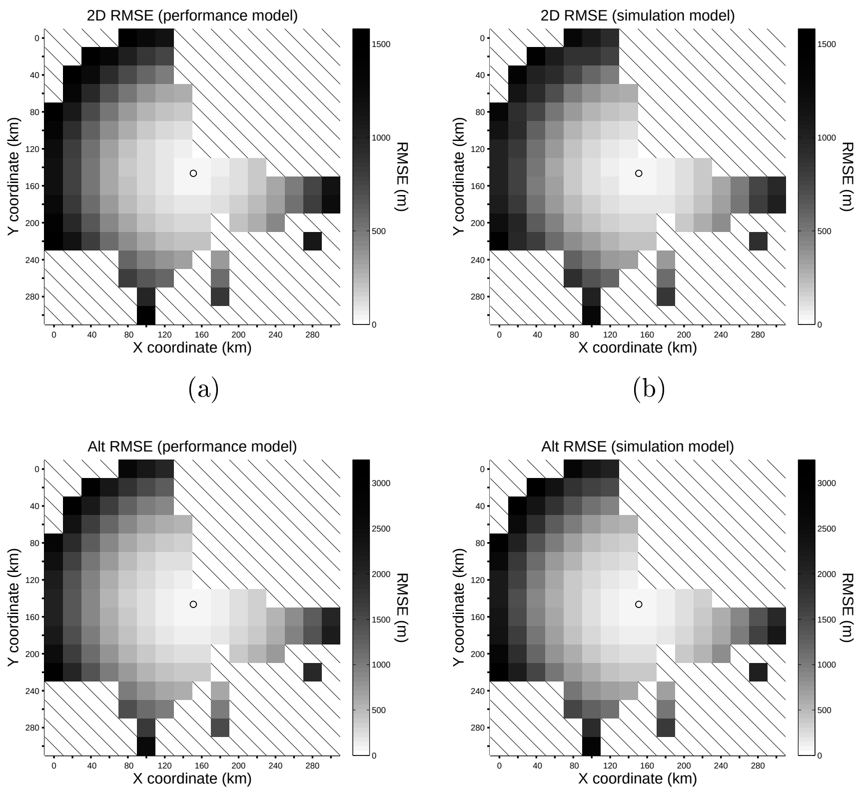
| Scenario no. | Trajectory type | Target altitude | Assumed maneuvering | No. of radars |
|---|---|---|---|---|
| 1 | Straight | 4000 m | Slow | 1 |
| 2 | Straight | 10 000 m | Slow | 4 |
| 3 | Circular (3 g) | 4000 m | Slow | 4 |
| 4 | Straight | 1000 m | Aggressive | 4 |
| 5 | Circular (9 g) | 4000 m | Aggressive | 4 |
| 6 | Circular (6 g) | 4000 m | Slow | 4 |

Table 3: A summary of the scenarios used to compare the models.

The measurement error variances are considered to be approximately constant in each trajectory. This decreases the computation times for both models, as the covariance matrices of the measurement errors are not rotated for each time step. For the straight-line trajectories, the performance model is only run once at the starting point in each grid cell. For the circular trajectories, the performance model is run at different points along a half-circle, as the errors only depend on the squared accelerations in each coordinate. In each scenario, the simulation model is run repeatedly until 5000 position estimates are obtained in each grid cell. This allows for visual comparisons of the results with mostly negligible Monte Carlo error.

## 7.1 Scenario 1: A Simple Single-Radar Scenario

The performance model and the simulation model are first compared in a simple single-radar scenario. A simple scenario is used to see to what degree the models match when it is not necessary to model sensor fusion or bias due to acceleration. Note that this is also one of the scenarios used for tuning the parameters. The altitude of the target is 4000 meters and the parameters of the simulation model and the performance model are set for slowly maneuvering targets.




(a) (b) (c) (d)

Figure 3: Average planar and altitude RMSEs of the position estimates given by the two models in Scenario 1.

Figure 3 shows the RMSEs given by each model in plane and in altitude. The radar is drawn as a circle. Areas where no measurements are received from are drawn with diagonal stripes. It is seen that the RMSEs in plane and altitude given by the models are very similar, considering that the measurement errors and the measurement rates are different in each grid cell.

## 7.2 Scenario 2: Multiple Radars

In this scenario the models are compared using multiple radars. The altitude of the target is 10 000 meters. Four radars are used, and the target moves at constant velocity. The parameters of the simulation model and the performance model are set for slowly maneuvering targets.

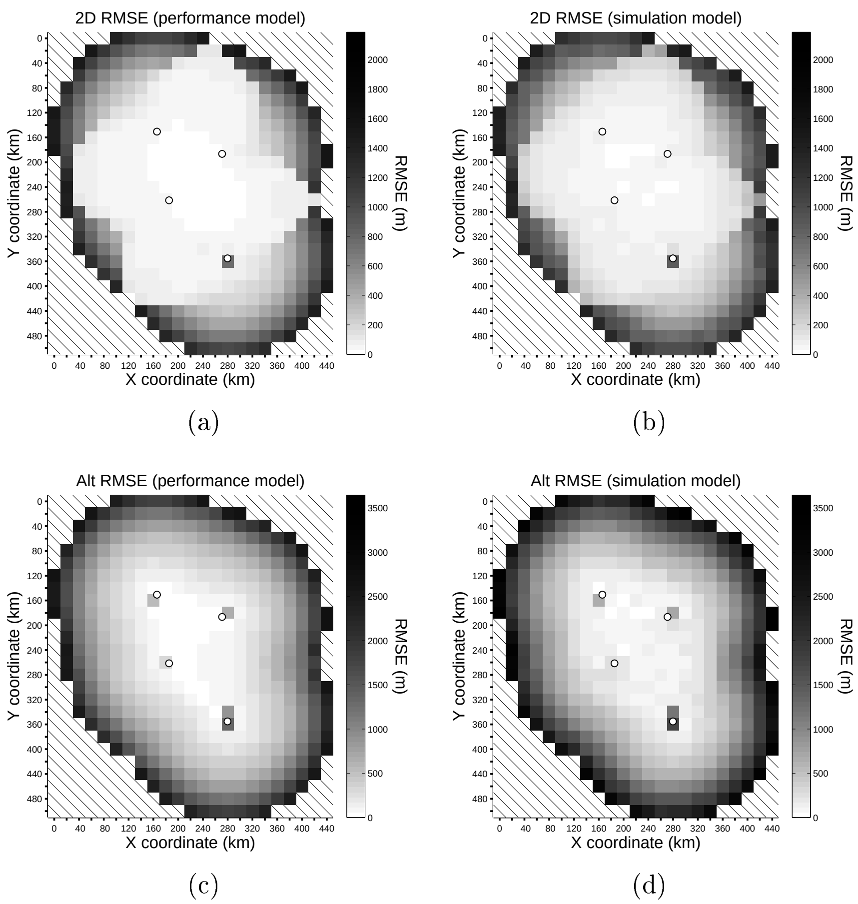



Figure 4: Average planar and altitude RMSEs of the position estimates given by the two models in Scenario 2.

Figure 4 shows the results given by the two models. The performance model gives slightly more optimistic results, but the areas with low RMSE are similar in shape.

The large RMSEs close to the radars are due to the inability of the radars to measure the position of a high-altitude target that is directly, or almost directly, above them. Figures 4(b) and 4(d) seem to contain some noise due to Monte Carlo errors.

## 7.3 Scenario 3: A Slowly Maneuvering Target

In the third scenario, the models are compared when a maneuvering target is being tracked. The correction for acceleration in Equation (42) is used, unlike in the first two scenarios where there was no acceleration. The target moves along a circular trajectory with an acceleration of 3 g at an altitude of 4000 meters. The assumed acceleration in the simulation model and the performance model is 2 g.

The results are shown in Figure 5. According to the performance model, the errors are lowest near the radars, whereas the simulation model also gives low errors in areas between the radars. The results of the simulation model show a sharp change in the RMSE in the upper left corner (x, y) = (80 km, 120 km) in Figure 5(b). In Figure 5(a), the change in the RMSE given by the performance model is more gradual.

In general, the RMSEs given by the models are similar both near and far from the radars, and neither model seems to be obviously better than the other. This suggests that the correction of Equation (42) can be used in scenarios where a slowly maneuvering target is assumed, and the true acceleration is close to 4 g, the value that was used when tuning the parameters.

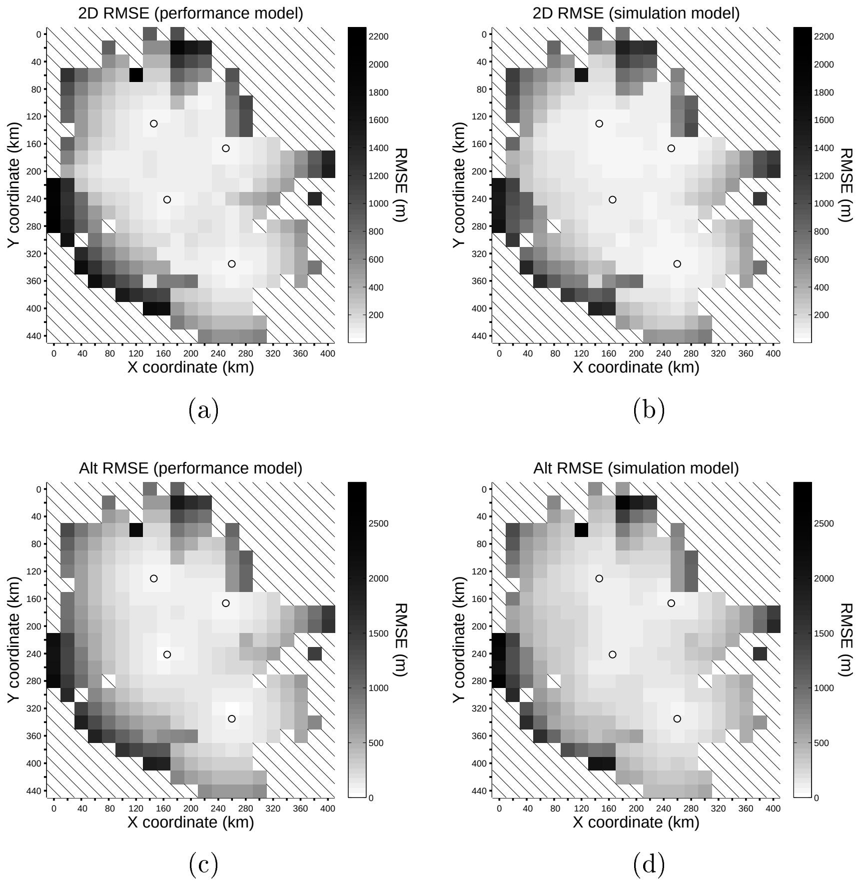
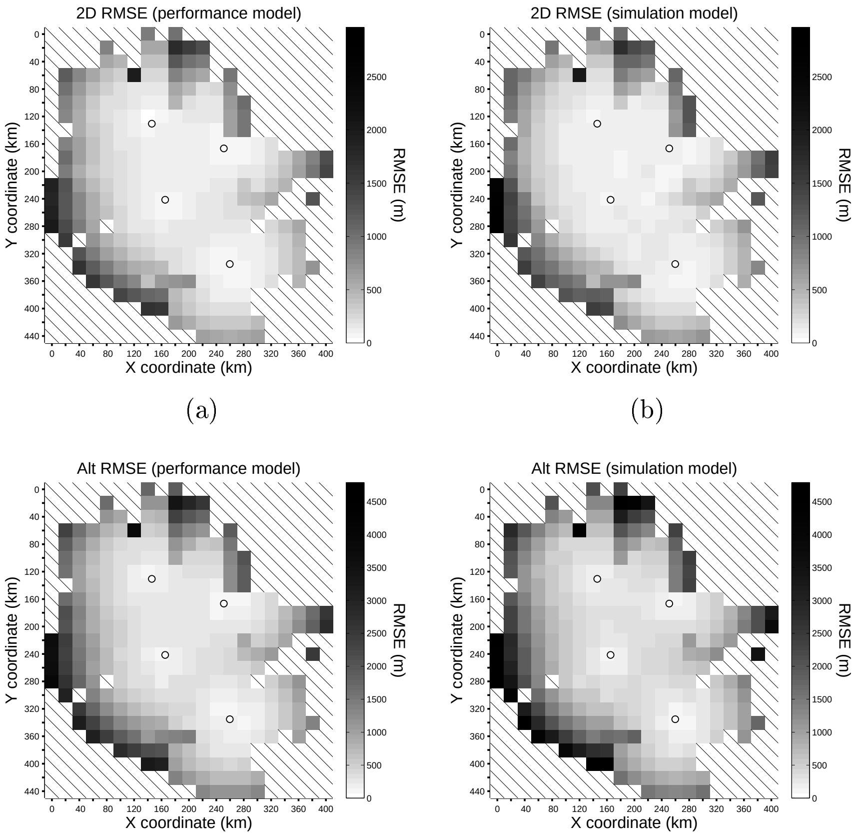



Figure 5: Average planar and altitude RMSEs of the position estimates given by the two models in Scenario 3.

## 7.4 Scenario 4: Aggressive Assumed Maneuvering

The motivation of the fourth scenario is to determine if the results are still comparable when the parameters of the models are set for tracking aggressively maneuvering targets. The target moves along a straight-line trajectory at an altitude of 1000 meters.

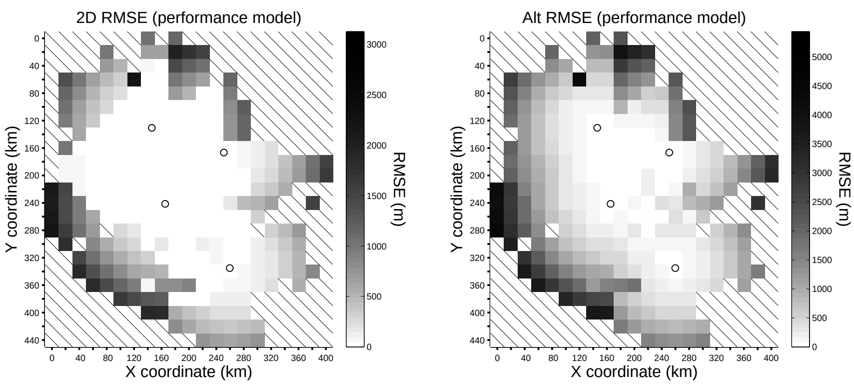
The results are shown in Figure 6. The performance model predicts smaller errors in altitude between the radars than the simulation model, as seen in Figures 6(c) and 6(d). The performance model also predicts slightly smaller 2D RMSEs near the radars. In general, the differences are small in both planar and altitude RMSE. Thus, the performance model can be used when aggressive maneuvering is assumed and the true trajectory is a straight line.




(a) (b)

(c) (d)

Figure 6: Average planar and altitude RMSEs of the position estimates given by the two models in Scenario 4.




(a) (b)

Figure 7: RMSEs given by the performance model in Scenario 4 when run without the correction explained in Section 4.1.

Figure 7 shows that the performance model fails to predict the planar errors with multiple radars if the time correction explained in Section 4.1 is not used. The planar errors are undersestimated where measurements are received from multiple radars. The RMSEs in altitude are also underestimated, although in a less obvious way.

## 7.5 Scenario 5: An Aggressively Maneuvering Target

The purpose of the fifth scenario is to determine if the correction to the acceleration in Equation 42 can be used with parameter settings for aggressively maneuvering targets. The target moves along a circular trajectory with an acceleration of 9 g at an altitude of 4000 meters.

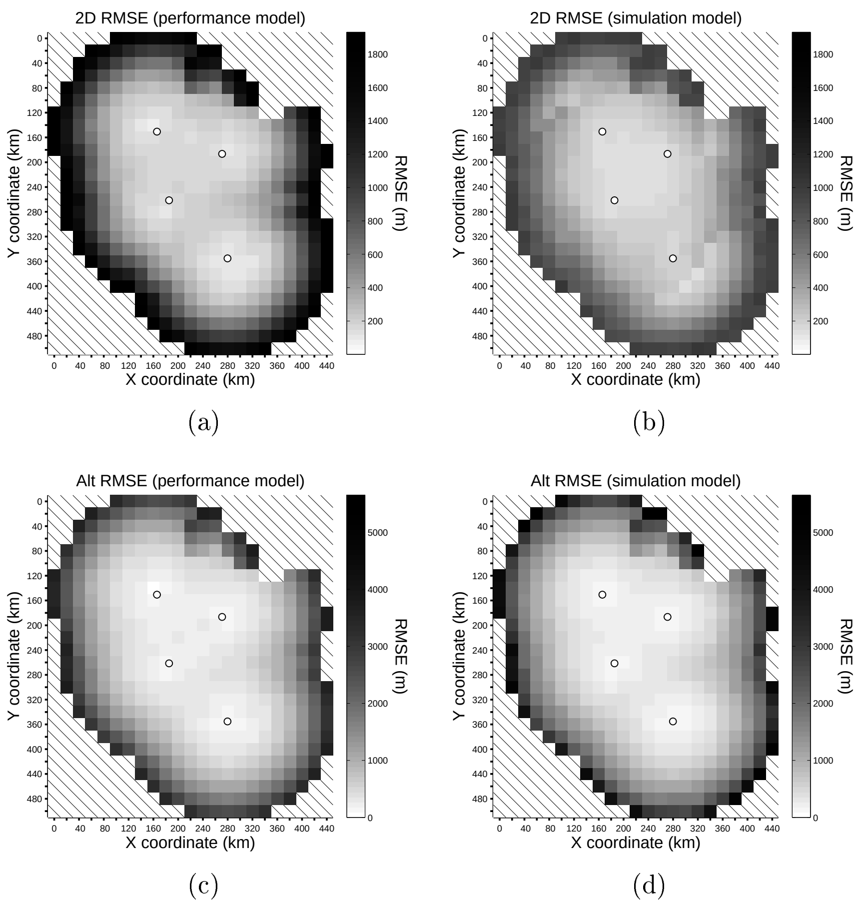
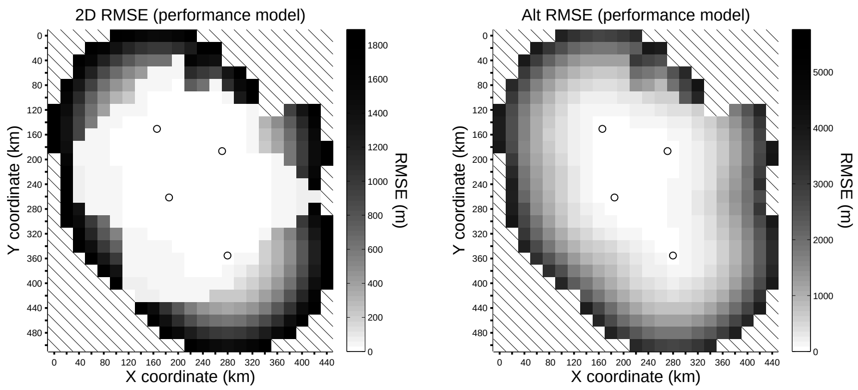



Figure 8: Average planar and altitude RMSEs of the position estimates given by the two models in Scenario 5.

The results are shown in Figure 8. The models seem to agree about the errors in altitude. The planar errors are clearly different, but still of the same order of magnitude. The simulation model gives small planar errors in areas where only one radar is used at (100 km, 100 km), (340 km, 160 km) and (300 km, 400 km) in Figure 8(b). This is not necessarily wrong, as more data does not necessarily improve the estimates when the tracking system is tuned for aggressively maneuvering targets (as noted in [Blair and Bar-Shalom, 1996]). However, in a well tuned system the errors should be smaller where more measurements are available. Thus, the results given by the performance model seem more reasonable.

For comparison, Figure 9 shows the results given by the performance model when the correction explained in Section 4.1 is not used. The 2D RMSE is underestimated when multiple radars are used.




(a) (b)

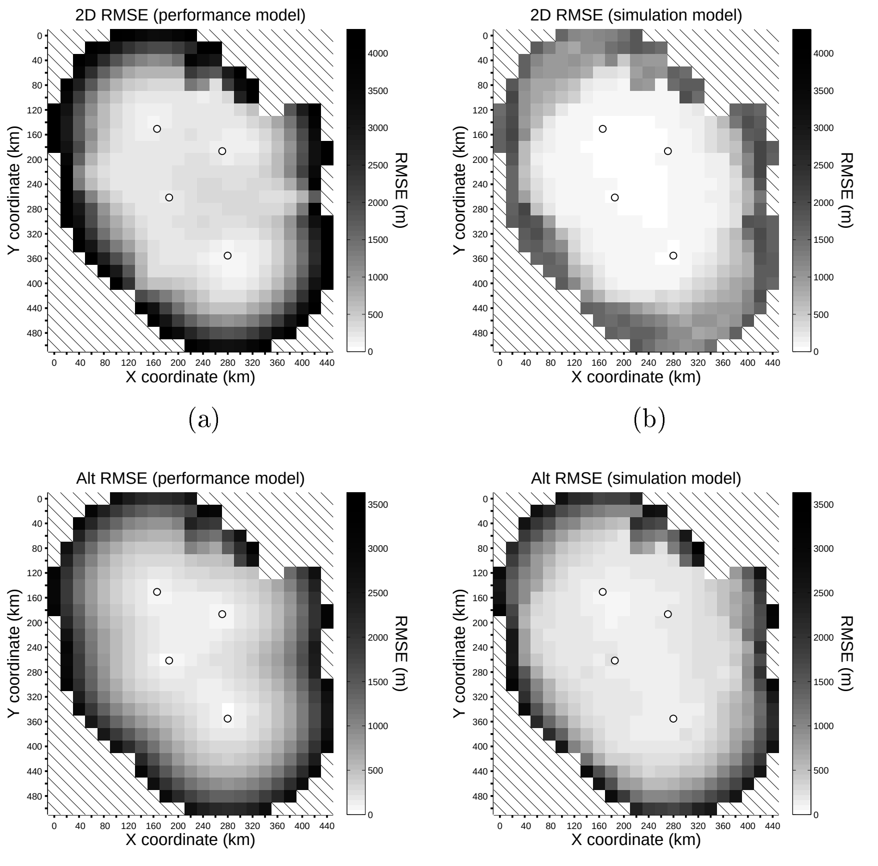
Figure 9: RMSEs given by the performance model in Scenario 5 when run without the correction explained in Section 4.1.

## 7.6 Scenario 6: A Poorly Tuned Tracking System

In the sixth scenario the parameters of the simulation model and the performance model are suited for slowly maneuvering targets, but an aggressively maneuvering target is being tracked. Thus, the parameters are poorly suited for tracking this type of target. Results from such scenarios do not reveal typical performance of the tracking system, but can be used for, e.g., risk analyses. The true acceleration of the target is 6 g, while the assumed acceleration in the simulation model and the performance model is 2 g.

The results are shown in Figure 10. The shapes of the areas with small errors given by the models are different. The performance model gives small planar errors in areas where only one radar is used, which is especially noticeable at the upper and lower parts of Figure 10(a). The planar errors given by the simulation model seem more realistic than those given by the performance model.

The differences in the RMSEs may be related to gating and track scoring. In the simulation model, the tracking is stopped before the errors of the estimates become too large, as the large errors in the estimates result in measurements that lie outside the gate.




(a) (b) (c) (d)

Figure 10: Average planar and altitude RMSEs of the position estimates given by the two models in Scenario 6.

## 7.7 Numerical Comparisons

The computation times and the differences in the results given by the simulation model and the performance model are summarized in Table 4. The absolute differences between the RMSEs given by the models are averaged over the grid in each scenario.

| Scenario no. | Comp. time (perf. model) | Comp. time (sim. model) | Avg. abs. diff. in 2D RMSE | Avg. abs. diff. in alt RMSE |
|---|---|---|---|---|
| 1 | 0.4 s | 268.4 s | 47.9 m | 133.1 m |
| 2 | 1.4 s | 2187.3 s | 51.1 m | 154.2 m |
| 3 | 7.3 s | 1978.1 s | 79.2 m | 134.1 m |
| 4 | 1.1 s | 1430.2 s | 59.3 m | 209.1 m |
| 5 | 4.4 s | 2098.2 s | 256.9 m | 181.7 m |
| 6 | 6.0 s | 2905.7 s | 654.5 m | 245.1 m |

Table 4: The computation times and the average absolute differences between the RMSEs given by the models.

The computation times in each scenario mostly depend on the number of cells in the grid and the type of the true trajectory. The performance model is especially fast in the scenarios with straight-line trajectories (1, 2, and 4), as it is only necessary to run the performance model at a single point. When a circular trajectory is used, the computation times of the performance model are several times longer, although still considerably shorter than the simulation model. The computation times of the simulation model naturally depend on the number of simulations. According to the simulation results presented in Sections 7.1-7.6, these computation times seem to be enough for creating useful visualizations with little noise caused by Monte Carlo errors.

The differences between the 2D RMSEs given by the models depend heavily on the particular scenario, whereas the differences in the altitude RMSEs remain remarkably constant. This may be related to the constancy of the altitude of the target, and that the elevation angles between the target and the radars have little variation throughout the map, whereas the azimuth angles have large variations. It appears that the differences between the 2D RMSEs are larger when aggressively maneuvering targets are being tracked. The differences in the altitudes of the targets (10 000 meters in Scenario 2, and 1000 meters in Scenario 4) do not seem to cause significant differences between the results given by the models.

# 8 Summary

The performance model and the simulation model of a tracking system were compared. Modifications to a performance model of a tracking system were suggested for use in performance assessments where asynchronous radars and the use of multiple kinematic models have to be taken into account. The parameters of the performance model were tuned by using the results given by the simulation model as a reference. The modified performance model and the simulation model were compared in six chosen scenarios using the RMSEs of the position estimates given by each model.

The results given by the models were found to be similar in most scenarios. The differences were larger in scenarios with an aggressively maneuvering target. The largest differences were found in a scenario where the parameters of the models were chosen poorly for the particular target. When the correction for asynchronous radars was not used, the errors given by the performance model appeared to be too small where multiple radars were used. The performance model was found to be significantly faster than the simulation model in all scenarios. Typical computation times of the performance model seem to be less than 10 seconds. The differences in the computation times were the largest in scenarios where the target moved along a straight-line trajectory.

According to the comparisons, the models give similar altitude RMSEs in all scenarios. The 2D RMSEs are similar at least when straight-line trajectories are used. If the simulation model was considered more reliable, it could be used only in the scenarios where the performance model is known to give different results. Thus, it would be possible to choose the model depending on the scenario to speed up the computations.

The correction made for asynchronous radars is intuitive and works reasonably well according to the comparisons. However, further studies would be needed to draw general conclusions about how well this correction works. As the $\alpha$-$\beta$ filter in the performance model and the IMM Kalman filter in the simulation model are quite different, it is possible that the correction for asynchronous radars also corrects for the differences between the filters. Further comparisons could be made using a simulation model that uses an $\alpha$-$\beta$ filter without gating or track scoring.

The correction made for the accelerations during coordinated turns was necessary to obtain results that are comparable with those given by the simulation model. As the correction is made specifically for coordinated turns, the performance model may not give reasonable results for arbitrary target trajectories. This would require modelling the way the IMM Kalman filter weighs the different kinematic models.

In the comparisons made in this thesis, the errors of the position estimates were considered. In practice, it is not necessarily clear how the performance of a tracking system should be measured. In some applications, it is of importance to assess the accuracy of the predicted positions rather than the estimated positions, or a combination of the two [Blackman, 1986]. The performance model could be used for such applications by adjusting the prediction time in the equations of the RMSEs. If the RMSEs of the velocity estimates were needed, the equations are presented in [Blair and Miceli, 2012].

Further research is required for modelling gating and track scoring. A more accurate model of the IMM Kalman filter, or another Kalman filter with multiple kinematic models, might result in RMSEs that are closer to the simulation results. For maneuvers with low accelerations, it could be useful to consider an interpolation between the RMSEs of the straight-line trajectory and the circular trajectory given by the performance model.

The performance model uses a very simple method to transform the measurement errors from spherical coordinates to cartesian coordinates. The results might be more realistic if a different transformation was used to take into account the nonlinear relationship of the coordinate systems. Several transforms have been suggested in the literature, e.g., the unscented transform [Julier and Uhlmann, 2004] and the Gauss-Hermite transforms [Ito and Xiong, 2000]. Taking into account the possibility of missing measurements might be useful for performance assessments in difficult conditions, as has been done in studies of error bounding techniques, such as [Farina et al., 2002]. For the performance model to be a useful alternative for simulation, the possible improvements should not cause an exceedingly large increase in the computational cost.

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# A Summary in Finnish

Seurantalaskin yhdistää ilmavalvontatutkien tekemät mittaukset lentävän kohteen paikasta ja estimoi kohteen tilan, joka voi sisältää kohteen paikan, nopeuden ja/tai kiihtyvyyden. Estimointi suoritetaan tavallisesti hyödyntämällä Kalman-suotimia, jotka perustuvat oletuksiin kohteen liikehdinnästä sekä mittausvirheiden suuruuksista.

Seurantalaskimen malleja käytetään ilmavalvontajärjestelmien suorituskyvyn arviointiin. Tässä työssä keskitytään kahteen kirjallisuudesta löytyvään malliin, Monte Carlo -simulointimalliin [Pousi ym. 2014] ja suorituskykymalliin [Blair & Miceli 2012], jotka perustuvat Kalman-suodinten käyttöön. Simulointimallin etuja ovat helppo muokattavuus erilaisia tilanteita varten sekä melko vähäiset oletukset seurantalaskimen toiminnasta. Simulointi kuitenkin vaatii usein pitkiä laskenta-aikoja, koska se perustuu satunnaishavaintojen generointiin. Tarkkojen suorituskykyarvioiden saaminen vaatii useita simulaatioita, joista saadaan keskiarvoistamalla lopullinen virhearvio. Suorituskykymalli ei perustu satunnaishavaintojen generointiin, vaan se on täysin deterministinen. Suorituskykymallin etuna ovat siten nopeat laskenta-ajat. Malli perustuu kuitenkin erilaisiin yksinkertaistuksiin, eikä sen soveltuvuutta erilaisiin tarkasteluihin ole tutkittu kattavasti.

Kandidaatintyön tavoitteena on muokata suorituskykymallia siten, että sitä voidaan käyttää samanlaisiin suorituskykyarviointeihin kuin simulointimallia. Lisäksi malleja vertaillaan erilaisilla lentoradoilla ja tutkakonfiguraatioilla, jolloin saadaan käsitys mallien eroista. Simulointimallia ei muokata, vaan sitä käytetään vertailukohtana. Malleja vertaillaan graafisesti sekä numeerisesti. Graafisissa esityksissä ilmavalvontatutkat sijoitetaan kartalle ja kukin kartan piste väritetään sen mukaan, kuinka tarkasti kyseisessä paikassa sijaitsevaa kohdetta voidaan seurata. Numeerisissa vertailuissa verrataan simulointimallin ja suorituskykymallin laskenta-aikoja sekä virhearvioiden erotuksia. Suuret erot mallien antamissa tuloksissa viittaisivat jommankumman mallin epätarkkuuteen.

Suorituskykymalli mahdollistaa huomattavasti aiempaa nopeamman suorituskykyarviointeihin liittyvän laskennan. Nopeaa suorituskykyarviointia voidaan hyödyntää esimerkiksi ilmavalvontajärjestelmän suunnittelussa sekä osana siihen liittyviä optimointiongelmia.

Suorituskyvyn arviointiin liittyvässä laskennassa lentävälle kohteelle luodaan joko suora tai kaartuva lentorata. Simulointimallissa kohteen todelliseen paikkaan lisätään satunnaista mittausvirhettä ja näin generoidut havainnot syötetään Kalman-suotimelle. Kalman-suodin tuottaa paikkaestimaatin, jonka virhe saadaan vertaamalla sitä kohteen todelliseen paikkaan. Suorituskykymalli perustuu yksinkertaisen $\alpha$-$\beta$-suotimen steady-state-tarkasteluihin. Mallille syötetään kohteen paikka ja kiihtyvyys, jolloin saadaan arvio paikkaestimaatin virheestä.

Simulointimalli sisältää kaksi seurannassa käytettävää menetelmää, portituksen ja radan pisteytyksen. Portituksella tarkoitetaan havaintojen jättämistä huomiotta, jos ne ovat liian kaukana estimoidusta lentoradasta. Jos havainto jätetään huomiotta, kohteen uusi paikkaestimaatti muodostetaan ennustamalla siten, että hyödynnetään aiempia havaintoja ja kohteen liikehdintämallia. Portituksen tavoitteena on liittää kohteeseen vain ne havainnot, jotka todella ovat peräisin kohteesta. Radan pisteytyksellä tarkoitetaan sitä, että kohteen seuranta lopetetaan, jos kohteen paikasta ei saada riittävän usein mittauksia tai jos ne jäävät liian usein portituksen vuoksi huomiotta. Näin voidaan poistaa virheelliset lentoradat.

Simulointimallissa käytettävät oletukset kohteen liikehdinnästä ovat huomattavan erilaisia suorituskykymallin oletuksiin nähden. Simulointimallissa käytettävä IMM Kalman -suodin (Interacting Multiple Model Kalman filter) sisältää kaksi liikehdintämallia. Toinen liikehdintämalli vastaa kohteen etenemistä vakiokiihtyvyydellä ja toinen kaartamista vakiokulmanopeudella. Näitä liikehdintämalleja painotetaan sen mukaan, kuinka hyvin ne vastaavat havaintoja. Yksinkertaiseen $\alpha$-$\beta$-suotimeen perustuvassa suorituskykymallissa oletuksena on kohteen eteneminen vakionopeudella.

Osana kandidaatintyötä suorituskykymallin parametrit viritetään siten, että ne voisivat vastata samankaltaisia oletuksia kohteen liikehdinnästä, kuin mitä simulointimallissa on käytetty. Tämä tehdään säätämällä parametreja valituissa tapaustarkasteluissa siten, että suorituskykymalli tuottaa virhearvioita jotka ovat samaa suuruusluokkaa kuin simulointimallin antamat virhearviot. Alkuperäinen suorituskykymalli ei huomioi sitä, että eri tutkat tuottavat mittauksia eri ajanhetkinä. Tästä seuraa paikkaestimaatin virheen aliarviointi. Osana kandidaatintyötä suorituskykymallia muokataan siten, että mittausten väliajat otetaan huomioon. Suorituskykymallia muokataan lisäksi siten, että se kykenee ottamaan huomioon usean liikehdintämallin käytön kohteen seurannassa. Tällöin suorituskykymallille syötetään kohteen todellisen lentoradan tyyppi. Yksinkertaistuksena eri liikehdintämallien painotusta siis ei pyritä mallintamaan.

Suorituskykymallia ja simulointimallia vertailtiin kuudessa esimerkkitapauksessa. Mallien antamat tulokset vastasivat toisiaan hyvin neljässä tapauksessa, yhdessä kohtalaisesti ja yhdessä heikosti. Tulokset vaikuttivat vastaavan toisiaan hyvin suorien lentoratojen tapauksessa ja heikommin tapauksissa, joissa kohde kaartaa. Kohteen lentokorkeuden ei havaittu aiheuttavan merkittäviä eroja mallien antamien tulosten välille. Vertailuissa todettiin, että suorituskykymallin antamat tulokset vastaavat simulointimallin antamia tuloksia huomattavasti paremmin, kun havaintojen eriaikaisuus huomioidaan siten, että käytetään työssä esitettyä korjausta.

Suorituskykymallin todettiin olevan huomattavasti simulointimallia nopeampi. Tarkastelluissa tapauksissa suorituskykymallin laskenta-aika oli noin 1-10 sekuntia eli alle 1 % simulointimallin käyttämästä laskenta-ajasta. Suorituskykymallin laskenta-ajan todettiin riippuvan tutkittavan kartan koon lisäksi lentoradan tyypistä. Laskenta on nopeinta suoraa lentorataa käytettäessä, koska tehtyjen yksinkertaistusten vuoksi on riittävää käyttää suorituskykymallia vain yksittäisessä lentoradan pisteessä. Kaartuvien lentoratojen tapauksessa laskenta tulee suorittaa useissa lentoradan pisteissä.

Suorituskykymalliin esitetyt parannukset vaikuttavat mahdollistavan samankaltaiset suorituskykytarkastelut, kuin mihin simulointimallia voidaan käyttää. Parannusten toimivuudesta voitaisiin tehdä yleisempiä päätelmiä esimerkiksi soveltamalla niitä tilanteisiin, joissa simulointimalli vastaisi paremmin suorituskykymallissa tehtyjä yksinkertaistuksia. Tällaisessa simulointimallissa voitaisiin esimerkiksi olla käyttämättä portitusta ja radan pisteytystä.

Jatkotutkimuksissa suorituskykymallia voitaisiin kehittää mallintamalla portituksen ja radan pisteytyksen vaikutuksia paikkaestimaattien virheisiin. Usean liikehdintämallin yhdistäminen voitaisiin pyrkiä mallintamaan tarkemmin arvioimalla niiden painotusta IMM Kalman-suotimessa. Suorituskykymallissa syntyy jonkin verran epätarkkuutta, joka liittyy mittausvirheiden muuntamiseen koordinaatistosta toiseen. Kirjallisuudessa on esitetty lukuisia muunnoksia, joita soveltamalla voitaisiin pyrkiä saamaan paremmin todellisuutta vastaavia suorituskykyarvioita.