Optimising the Hybrid Energy System: A mathematical modelling approach considering uncertainties

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ABSTRACT OF THE
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This thesis explores the economic viability of Hybrid Energy Systems (HES) within Nordic energy markets, with a focus on Finnish data. It delves into the complexities of these markets, addressing the inherent uncertainties and utilizing Monte Carlo Simulations for scenario analyses. The research develops a linear optimization framework to maximize revenue and manage market volatilities. Results show significant revenue increases through HES, highlighting their resilience against market price fluctuations. Despite the model's strengths, limitations in solar production forecasting accuracy are noted, impacting decision-making effectiveness. The study emphasizes the need for advanced energy forecasting models and suggests incorporating diverse energy sources and market mechanisms to enhance HES adaptability. This research contributes to the understanding of HES's economic potential, paving the way for further investigations into more integrated and robust energy system modeling.

Keywords: Hybrid Energy Systems, linear optimization, energy markets

Contents

Abbreviations

HES	hybrid energy systems
DAM	day-ahead market
IDM	intraday market
MCP	market clearing price
MW	megawatt
MWh	megawatt hour
DER	distributed energy resources
MAE	mean absolute error
SOC	state of charge
PDF	probability density function

1 Introduction

The global transition towards sustainable and resilient energy systems is imperative in the face of escalating environmental concerns and the urgent need to mitigate climate change. Hybrid Energy systems (HES) stand out as prime candidates in facilitating a shift towards more sustainable and efficient energy practices. A hybrid energy system utilises an integration of two or more distinct energy sources which leads to more efficient system.[1] Currently HES provides a reliable power generation solution to areas where conventional energy grids are non-existent or impractical. Apart from that, a hybrid energy system might also open up market opportunities for emergent energy technologies that are still not fully developed. For example, if a new type of fuel cell is not yet efficient enough for a standalone electricity production system, it can still fit well in a hybrid framework, facilitating its market entry and adoption.[2]

Even so, the HES poses a few financial and technical challenges. The high capital costs involved in the setup phase require meticulous planning and strategic management to ensure that the system is not only operationally viable but also economically sustainable.[3] Given the multitude of components each energy system encompasses, the operational intricacy of the HES cannot be understated. From connections to utility markets and energy converters facilitating transitions between different energy sources, to battery storage units and additional revenue streams like hydrogen markets, a hybrid energy system is a combination of various moving parts, each with its own set of challenges and considerations.

Another point of consideration is the volatility of the electricity markets. The electricity markets tend to display abrupt and generally unanticipated extreme changes in the Day-ahead prices prices known as 'jumps' or 'spikes'.[4] Consider the graph (Fig. 1) that depicts spot prices at an hourly time interval taken from the Nord Pool system.

This thesis examines the investment possibilities of HES through an optimisation model in order to investigate their economic viability. Whether HES warrants major investment is the central research question in this analysis. The study intends to give a thorough assessment of HES and their role in the shift towards renewable energy sources by analysing the performance, cost-efficiency, and market adaptability of these systems.

Chapter 2 provides a comprehensive insight into the economic principles guiding the energy and hydrogen markets and delineate the uncertainties prevalent within these domains. Furthermore, this chapter will outline methodologies to quantify and address uncertainties in energy system modelling, with a particular emphasis on Monte Carlo Simulation. Chapter 3 presents a linear optimization framework for HES, detailing the step-by-step construction of an optimization model. It begins by formulating an objective function for revenue maximization and progresses through iterative stages, incorporating grid energy transactions, battery storage dynamics, hydrogen production and sale, and finally, the integration of imbalance settlement procedures to navigate solar production variability. Chapter 4 evaluates the benefits of HES and simulates scenarios using Monte Carlo simulations and discusses the results. Finally, Chapter 5 summarises the conclusions from this thesis and proposes further areas of research.

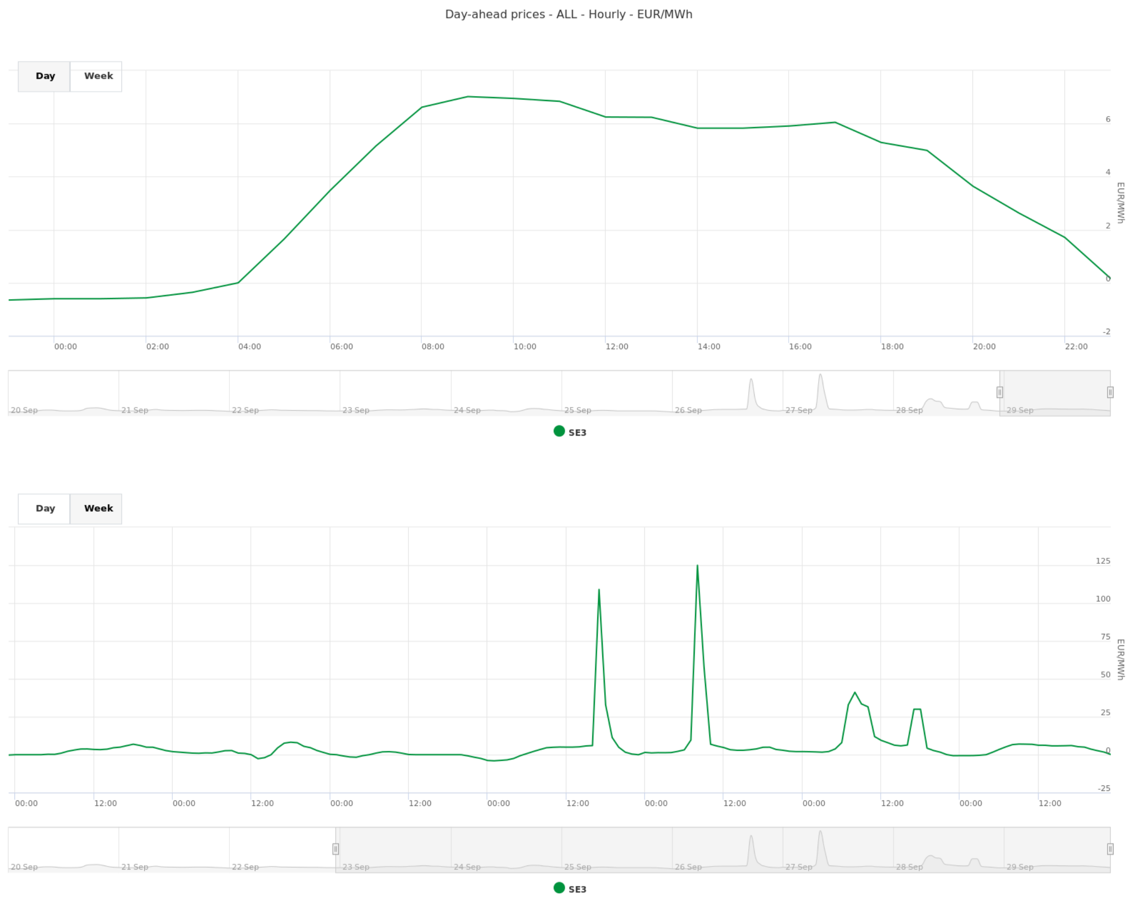

Figure 1: Nord Pool. (2023). Area Prices [Hourly].

2 Economic Principles and Uncertainties in Energy and Hydrogen Markets

2.1 Introduction

Understanding the economic principles driving the energy markets, alongside the pervasive uncertainties, is crucial for stakeholders, policy-makers, and researchers engaged in energy planning and management.

Economic principles form the backbone of energy markets, dictating the interactions between supply and demand, influencing pricing mechanisms, and shaping market players' strategies and responses [8]. These principles provide a framework to comprehend the operational dynamics within the energy and hydrogen markets, thereby facilitating more informed and strategic decision-making processes.

Energy prices, particularly in deregulated markets, have been subjected to significant fluctuations, resulting from a myriad of factors including geopolitical events, infrastructure constraints, and supply-demand imbalances [9]. In parallel, the hydrogen market, though in its infancy, has been influenced by its production costs, its potential in providing energy security, and its projected demand in various sectors, from transportation to industrial applications [10]. These uncertainties, if not adequately accounted for, may lead to sub-optimal planning and operation of energy systems, thereby risking economic viability and stability. Therefore, integrating an understanding of market economics with a robust approach to managing uncertainties is pivotal.

One such powerful tool is the Monte Carlo Simulation, a computational technique that provides a range of possible outcomes and the probabilities they will occur for any choice of action [11]. Applied to energy planning and forecasting, it offers stakeholders a probabilistic assessment, thereby aiding in robust decision-making in the face of uncertainty.

2.2 Economic Principles in Energy Markets

2.2.1 Supply and Demand Dynamics

Supply and demand serve as pivotal concepts in the field of economics, wielding considerable influence in the competitive marketplace where they jointly determine both the price and the quantity of goods, securities, and other tradeable commodities that are sold [12]. Understanding the supply and demand dynamics within energy and hydrogen markets is pivotal to accurately model and predict market behaviors and prices. These dynamics are shaped by numerous variables, including production capacity, consumption rates, market strategies, and external shocks.

- **Energy Market**
  The fundamental economic principles of supply and demand are embodied in the supply and demand curves. Fig. 2 provides graphical representations depicting the relationship between the unit price and the total quantity of goods.

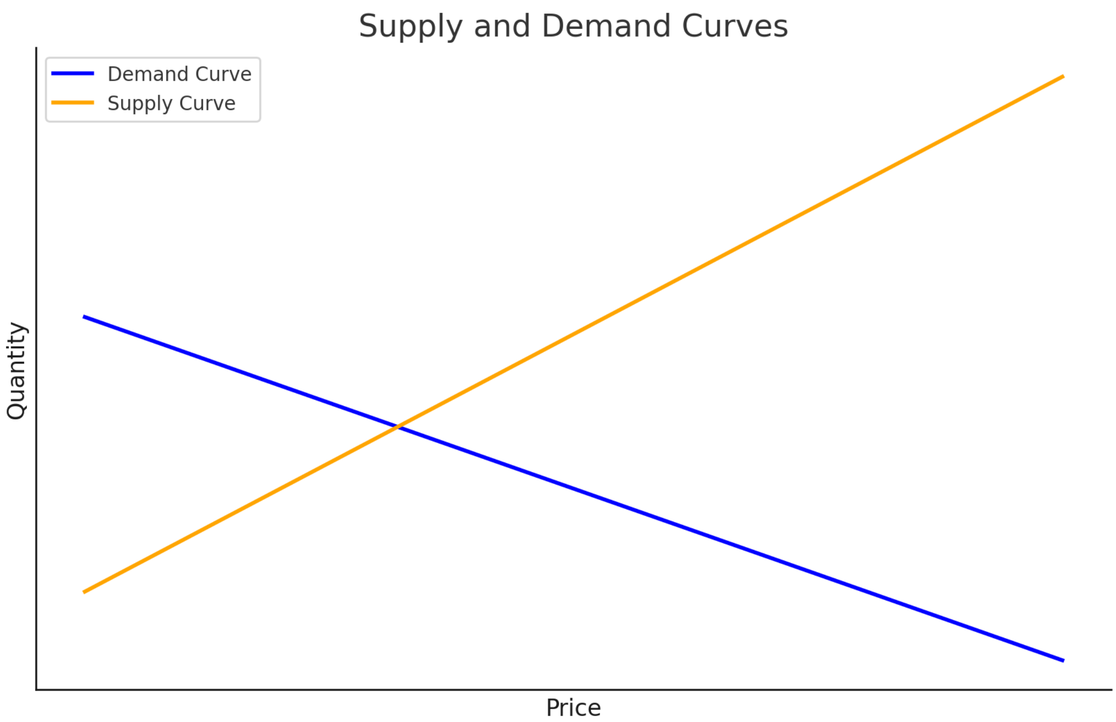

Figure 2: Typical supply and demand curves

The supply curve, with an upward slope, reveals how the unit price relates to the quantity of goods producers are willing to offer. This upward trajectory is especially observable in the electricity market and can be elucidated by examining the operational dynamics of power plants. Initially, supply is managed by efficient plants that can generate electricity at lower costs. However, as demand escalates, less efficient plants, characterized by higher operational costs, are utilized to augment supply, leading to an increase in the unit price of electricity[13]. Moreover, the supply side is further complicated by factors such as the availability of resources, production capacity, and governmental policies. The volatility is particularly pronounced with renewable energy sources whose supply is inherently contingent upon unpredictable factors like weather conditions.
On the other side, the demand curve is characterized by a downward slope, signifying an inverse relationship between the unit price and the quantity consumers are willing to purchase. As prices increase, demand from consumers tends to diminish. This principle is crucial in understanding market dynamics, as higher prices usually result in reduced demand [12]. The energy sector

witnesses significant demand surges during economic growth periods, resulting in price hikes if supply does not correspondingly increase. Furthermore, the global transition towards sustainable and renewable energy sources has led to an increase in demand for these resources, introducing additional volatility to market prices.

- **Hydrogen Market**
  The costs associated with producing hydrogen can vary significantly based on technology, energy prices (particularly electricity for electrolysis), and regional factors. This variability can translate into price volatility in the hydrogen market [14]. Electrolysis driven by renewable energy, especially wind and solar, is subject to the intermittency of these sources. As such, the inconsistent supply can result in volatile production rates, which can influence hydrogen prices [15]. Limited storage and transport infrastructure can create bottlenecks, affecting the regular supply of hydrogen and leading to price fluctuations [16]. Changes in government policies, subsidies, or incentives can impact the production, distribution, and consumption of hydrogen, leading to shifts in its pricing structure. Given the emerging nature of the hydrogen market, it can be influenced by the speculative behaviors of investors and traders, adding to its price volatility.

2.2.2 Price Mechanisms in Energy Markets

The energy markets are a complex ecosystem where prices are influenced by a variety of factors ranging from supply-demand dynamics, geopolitical events, technological advancements, and even environmental factors. Energy prices are established through several mechanisms, which reflect the interplay between market participants and the rules set by regulators and exchanges. Our focus is primarily on the Nordic Energy Markets. The Nordic energy market, comprising of Norway, Denmark, Finland, and Sweden, represents one of the most integrated and liberalized energy markets in the world. Within this market, electricity price determination and trade occur via two principal mechanisms: the Day-Ahead Market (DAM) and the Intraday Market (IDM). Understanding these two facets provides insight into how supply and demand equilibrium is established, and how market participants can manage their risk and operational strategies.

- **Day Ahead Markets**
  The DAM is a forward market where electricity is traded one day before the actual delivery. Participants submit their bids (both buying and selling) for each hour of the following day. Once all bids are submitted, they are aggregated, and an equilibrium price, commonly known as the Market Clearing Price (MCP), is determined for each hour based on supply and demand [17]. Price determination in the DAM is a result of the merit order principle. It ranks the available sources of energy based on the ascending order of their bid prices [18]. The renewable sources with low marginal costs (like wind and hydropower) are dispatched first, followed by fossil fuels and then peak reserves. The intersection

of cumulative supply and demand curves establishes the MCP for each hour. The DAM serves multiple purposes:

- **Price Transparency**: It offers a clear indication of expected electricity prices for the next day, enabling market participants to make informed decisions.
- **Risk Management**: By locking in prices a day in advance, producers and consumers can hedge against potential price fluctuations.
- **Operational Planning**: Utilities and energy-intensive industries can plan their operations based on anticipated energy costs.

To better understand the workings of the DAMs, we can take an example. Assuming that a cold winter day is approaching in Finland. The temperature is forecasted to drop sharply the next day. With most homes and many industries relying on electricity for heating in Finland, the demand for electricity is anticipated to be high [19].Various market participants, ranging from large power generation companies, renewable energy producers, to industrial consumers, prepare to submit their bids for the next day's electricity delivery.

- **Power Generation Companies**: A major Finnish utility, like Fortum, evaluates its portfolio. They decide they can provide a certain amount of megawatts (MW) at a specific price derived from their production costs, which includes the costs of fuels, operations, and other factors.
- **Renewable Energy Producers**: A wind farm operator in Ostrobothnia estimates the wind conditions for the next day. Based on the forecast, they calculate the potential electricity output and decide on a selling price. Given that wind energy has negligible marginal costs once the infrastructure is in place, they might bid at a very low price to ensure their electricity is dispatched.
- **Industrial Consumers**: A large pulp and paper factory in southern Finland calculates its electricity requirement for the next day. They submit a bid indicating how much electricity they are willing to buy and at what maximum price.

All bids are submitted to the Nord Pool power exchange, which operates the DAM in Finland and other Nordic countries.

1. Bids are aggregated, and the supply and demand curves are formed.
2. The intersection of these curves gives the hourly system price. This is the price at which electricity will be traded for each specific hour the next day.

For this cold winter day in our scenario, let us assume that due to the high demand and limited supply, the price spikes during peak demand hours in the evening.

- **Power Generation Companies**: Fortum, having successfully bid at a price below the system price, gets ready to deliver the promised amount of electricity the next day.
- **Renewable Energy Producers**: The wind farm operator, having bid at a very competitive price, finds that their electricity will also be dispatched. They monitor wind conditions closely to ensure they can meet their commitments.
- **Industrial Consumers**: The pulp and paper factory, based on the System Price, might decide to curtail some of its operations during the peak demand hours to save on electricity costs. Alternatively, if they had bid at a price higher than the system price, they would continue operations as usual.

This example demonstrates the intricacies of the DAM in Finland. It is a dynamic play of supply and demand, predictions, and strategies.

- **The Intraday Market**
The IDM allows for trading of electricity after the DAM has cleared and before actual delivery. Given the increasing share of renewable energy, particularly wind power which is inherently volatile, the IDM provides a platform to adjust positions closer to real-time as more accurate forecasts become available. Unlike the DAM which clears at a specific time, the IDM operates as a continuous market. This means participants can buy and sell electricity throughout the day until one hour before delivery. As forecasts for renewables and demand become more accurate closer to real-time, the IDM allows for continuous adjustments. The IDM's significance, especially in the Nordic context, cannot be understated:

  - **Integration of Renewables**: The IDM has become increasingly important with the rise of wind and solar energy in the Nordic mix. It allows for last-minute adjustments based on real-time weather forecasts.
  - **Operational Flexibility**: Utilities can optimize their generation schedules, and large consumers can adjust their consumption patterns based on intraday prices.
  - **System Reliability**: By allowing adjustments close to the delivery hour, the IDM plays a role in ensuring the stability and reliability of the grid.

Fig. 3 shows that in the DAM, participants commit to producing or consuming certain amounts of electricity based on forecasts and the DAM allows these positions to be adjusted in response to new information or unexpected events. Continuing from our cold winter day scenario in Finland, after the DAM has determined the system price for electricity for each hour of the next day, we now move closer to real-time operations. Even with the best forecasts, actual conditions can vary. The wind may not blow as predicted, a sudden technical fault might reduce generation from a power plant, or there could be an unexpected surge in electricity demand. The DAM helps mitigate such scenarios.

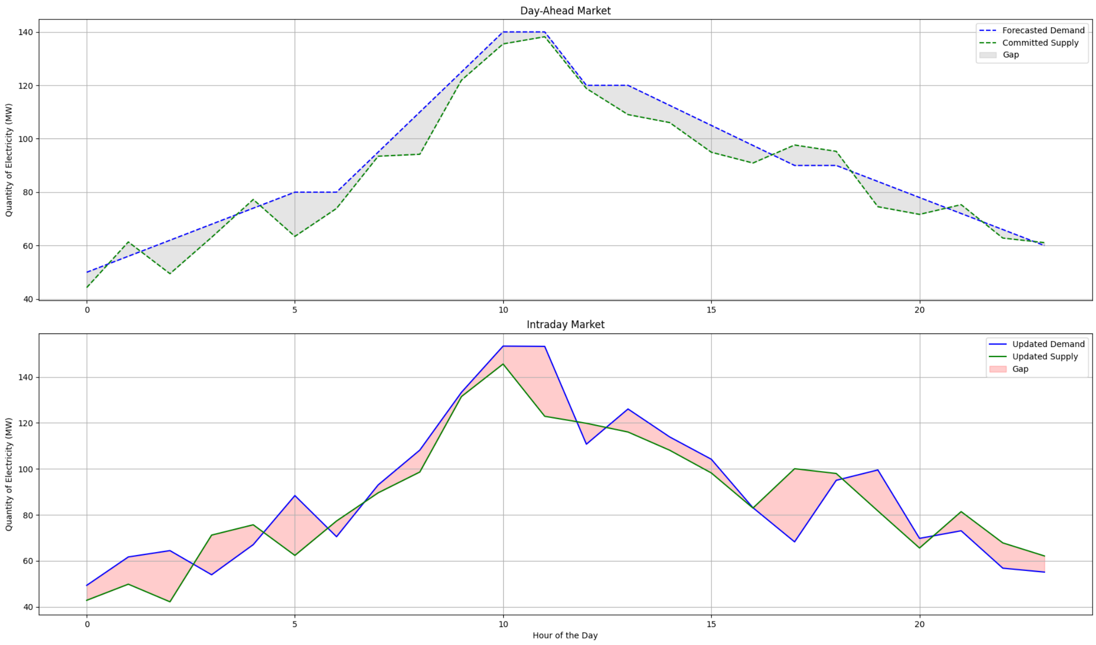

Figure 3: An Example Visualization of the Day-ahead and IDMs

The IDM, on the other hand, allows market participants to adjust their positions based on the latest information and forecasts. It provides an opportunity to buy or sell additional electricity if their production or consumption varies from their day-ahead commitments and hedge against the price differences that might arise due to these last-minute changes. Looking at it sequentially:

1. After the DAM has cleared, the IDM opens. Participants can submit updated bids or offers for electricity based on their new forecasts and needs.
2. The IDM is continuous, meaning transactions can occur anytime once the market is open until the delivery hour.
3. Prices in the IDM are determined by the immediate dynamics of supply and demand, which can be different from the Day-ahead prices.

Let us go back to our cold winter day in Finland:

- **Wind Farm Operator in Ostrobothnia**: By midday, they realize the wind is not blowing as strongly as forecasted. Their electricity production will be lower than what they committed to in the DAM. To fulfill their commitment, they decide to buy electricity in the IDM.
- **Pulp and Paper Factory in Southern Finland**: They unexpectedly land a big order and decide to ramp up production overnight. They will need more electricity than they initially bid for. They turn to the IDM to purchase the extra electricity they need.

  - **A Hydroelectric Power Plant in Northern Finland**: Thanks to recent rains, they have more water reserves than anticipated. They can produce more electricity and decide to sell this surplus in the IDM.

All these participants adjust their positions in real-time, ensuring a balance between electricity supply and demand, and minimizing imbalances which could destabilize the grid. The IDM, thus, plays a crucial role in the efficient operation of the electricity system in Finland.

2.2.3 Market Players and Regulations

The energy and hydrogen markets, like most markets, consist of a variety of players that interact within a framework shaped by regulations and policies. Understanding the dynamics of this framework is crucial for comprehending market functions, price mechanisms, and the overall behavior of the market.

- **Market Players**

  Producers: These entities are responsible for producing and sometimes selling energy or hydrogen directly to the market. Producers might range from large multinational corporations that engage in the extraction of raw materials, refining, and commercial sale, to smaller entities that may focus on a specific segment of the production process.

  Example: Major oil and gas companies can be producers in the energy market, while specialized companies might engage in the production of hydrogen through electrolysis or other means.

  Consumers: Consumers in the energy and hydrogen markets can be individuals, businesses, or industrial entities. They purchase energy or hydrogen for various needs, from powering homes and vehicles to serving as a raw material in certain industrial processes.

  Example: Households buying electricity, factories requiring hydrogen for refining processes, or transport companies buying fuel.

  Intermediaries: These players sit between producers and consumers and can include brokers, traders, and other entities that buy from producers and sell to consumers, often adding value in the process.

  Example: Energy trading firms, or hydrogen distribution companies that transport and sell hydrogen to end-users.

  Regulatory Bodies: Government agencies or international entities oversee and regulate the energy and hydrogen markets to ensure fairness, security, environmental protection, and more. They establish rules and guidelines and monitor market activities.

  Example: The European Energy Commission.

- **Role of Regulations and Policies**

  Safety and Environmental Concerns: Regulations often aim to ensure that the extraction, production, distribution, and consumption of energy or hydrogen do not compromise public safety or the environment. This could involve setting standards for safe operations or emission limits.

  Fair Market Operations: To avoid monopolistic practices or any other unfair market manipulations, regulatory bodies often establish guidelines to ensure fair competition, transparent pricing mechanisms, and protection of consumers.

  Promotion of Sustainable Technologies: Governments may enact policies or provide incentives to promote cleaner and sustainable energy technologies, which can shape the dynamics of the market. For example, subsidies for renewable energy sources or tax breaks for hydrogen production can influence market participation and pricing [20].

  Infrastructure Development: Regulations might also emphasize the need for adequate infrastructure such as pipelines for natural gas or refueling stations for hydrogen. Such policies can dictate the accessibility and distribution efficiency in the market.

  Price Stabilization: In some markets, regulatory bodies might intervene to stabilize prices, especially if there are severe fluctuations that could harm consumers or producers. This could be achieved through strategic reserves, tariffs, or subsidies.

2.3 Uncertainties in Energy and Hydrogen Markets

Energy and hydrogen markets, like many commodities markets, are exposed to various uncertainties. Looking at the previous section 2.2, we can get an idea of the different uncertainties affecting the energy and hydrogen market. These uncertainties stem from a combination of exogenous factors (e.g., geopolitical events, natural disasters) and endogenous factors (e.g., technological developments, market dynamics) and addressing them is crucial to ensure resilient and effective planning and decision-making in the sector.

2.3.1 Types of Uncertainties

- **Price Volatility:** Energy and hydrogen prices can be highly volatile, influenced by various factors. Fluctuations may arise from global events, such as sudden shifts in production decisions by major oil producers [21], policy changes, or shifts in supply and demand dynamics. Such volatility poses challenges for investors and operators in accurately forecasting revenues and costs.

- **Demand Fluctuations:** The demand for energy and hydrogen can vary based on several factors. Economic growth, energy efficiency measures, shifts in consumer behavior, and technological advancements play a significant role. For instance, during the pandemic, the global fuel consumption fell by more than 25%, leading to drastic changes in demand [21]. Additionally, the adoption rate of hydrogen-powered vehicles can also influence hydrogen demand.

- **Technological Uncertainties:** The energy sector is undergoing rapid technological evolution [22]. There is uncertainty regarding which technologies will dominate in the future, their efficiency rates, cost trajectories, and scaling potential. For hydrogen, uncertainties might revolve around production techniques (e.g., electrolysis vs. natural gas reforming), storage solutions, and distribution infrastructure.
- **Regulatory and Policy Shifts:** The transformation of the electricity industry worldwide has been influenced by regulatory changes aiming to replace state-owned monopolies with open and competitive markets [23]. Governments worldwide are setting aggressive climate targets, leading to evolving energy and environmental policies. Uncertainty in regulatory direction can affect investments and operations in both conventional energy sectors and emerging markets like hydrogen.
- **Supply Chain Disruptions:** Events like natural disasters, geopolitical tensions, or pandemics can disrupt energy and hydrogen supply chains, creating uncertainty in both availability and pricing.
- **Geopolitical Risks:** The energy market, especially oil and gas, has historically been influenced by geopolitical events. These can impact access to resources, dictate trade routes, and influence international energy prices [21].

2.3.2 Impact of Uncertainties

- **Operational Impact:** Uncertainties in the energy and hydrogen markets, especially those stemming from sudden shifts in supply or demand, can lead to significant operational challenges [24]. For instance, a hydrogen production facility that anticipates a certain level of demand might face underutilization if that demand unexpectedly drops [25]. Such scenarios can lead to operational inefficiencies and, consequently, financial losses. Moreover, uncertainties in renewable energy sources can also impact the operational security and efficiency of power systems. Managing these uncertainties becomes a major challenge in power systems with a high penetration of volatile generation and load [26].
- **Financial and Profitability Risks:** Factors such as price volatility, unpredictable demand, and rapid technological shifts can significantly affect both revenues and costs. This volatility directly impacts the profitability of companies operating within the sector. For investors and stakeholders, clear visibility of potential returns is crucial. However, these uncertainties can cloud judgment and hamper investment decisions.

- **Strategic Impacts:** In the long run, uncertainties can significantly influence the strategic direction of energy companies. Regulatory uncertainties, for example, might deter companies from investing heavily in fossil fuels [27]. Instead, they might opt to diversify their portfolio, incorporating more renewable energy or hydrogen solutions to hedge against potential regulatory shifts.

- **Reputation and Compliance Risks:** Anticipating and addressing uncertainties is crucial, especially those related to regulatory changes. Failure to do so can expose companies to compliance risks. Such oversights might not only result in financial penalties but can also tarnish a company's reputation. This can have long-term implications, affecting market standing and eroding stakeholder trust. For instance, energy companies face a growing volume of regulations, as well as stepped-up sanctions and regulatory actions. When asked about the negative impact of policy and regulatory developments, 41 per cent of energy companies say they are extremely concerned about environmental protection regulation [28].

- **Importance in Planning and Decision-making:** To effectively navigate these uncertainties, companies must incorporate flexibility into their decision-making processes. Rigid strategies or infrastructure investments can leave firms vulnerable to the ever-changing market dynamics. Advanced modeling techniques, such as Monte Carlo Simulations, scenario planning, and sensitivity analysis, are invaluable tools in this context. They help companies understand, quantify, and mitigate uncertainties, ensuring more informed decision-making.

2.3.3 Addressing Uncertainties in Energy System Modeling

As observed in the previous sections, an energy system and more precisely, a hybrid energy system deals with considerable uncertainty. From price volatility, to supply and demand related uncertainties, there are numerous areas to look at when trying to model an energy system. On top of that, there are different parts of the energy system, each of which deals with its own set of uncertainties. Modelling each and every uncertainty for each and every part of the energy system would fall out of the scope of the thesis, and therefore the focus has been put on one key aspect of the energy system, the DAM.
The growth of Distributed Energy Resources (DERs), such as solar panels and storage devices, adds uncertainty to demand forecasting which is an important part of the DAM. On the supply side, the increasing integration of renewable energy sources like wind and solar generates significant uncertainty in supply forecasts. These sources are highly volatile, with cloud cover and wind speed changes affecting the production of renewable energy. Unexpected equipment breakdowns and transmission failures might also lower the projected supply. In terms of pricing, the activity of electricity traders and expectations about real-time market prices can cause volatility in DAM prices. [29]. Quantifying these uncertainties can help market participants can make more informed decisions about bidding, purchasing, or selling energy in the DAM which can lead to improved profitability and risk management. It also aids in improving operational reliability, stakeholder confidence, and better risk management.

Quantifying the variability between the forecasted and actual solar production is crucial in managing and planning the operation of solar energy systems. This process helps in understanding, and mitigating the risks associated with unpredictability of solar energy production. Statistical methods such as standard deviation, expected values and mean absolute error (MAE) are commonly used in quantifying the variability of solar energy production. A graph can also be plotted to show the difference between the forecasted and actual solar production values.

Traditionally, forecasts have been point predictions representing the expected values at different times, however as the field of weather forecasting has long recognized, forecasts are essentially five dimensional spanning the three-dimensional space, time and probability [31]. This has led to the utilization of probabilistic forecasting for quantifying the variability of wind power. A popular method in the field of forecasting renewable energy production is the Monte Carlo Simulation. Monte Carlo simulation is a mathematical technique that predicts possible outcomes of an uncertain event. Using a probabilistic model to simulate the outcome will produce different results each time. For example, the distance between a train station and a school is fixed. However, a probabilistic simulation might predict different travel times by considering factors such as congestion, bad weather, and vehicle breakdowns. [32]

2.4 Monte Carlo Simulations

Probabilistic solar forecasting represents a significant shift in solar resource assessment and forecasting. Traditional methods relied on deterministic forecasts, providing single-valued predictions [33]. However, the variability and uncertainty in solar energy production have led to the adoption of probabilistic forecasts. These forecasts present possible outcomes in forms such as probability distributions, ensembles, quantiles, and/or prediction intervals. This approach allows for a more comprehensive understanding of potential scenarios and better decision-making in the face of uncertainty [33].

Monte Carlo Simulation [36] is a mathematical technique, which is used to estimate the possible outcomes of an uncertain event. In contrast to conventional forecasting models that use fixed input values, Monte Carlo Simulation forecasts a variety of outcomes based on a speculated spectrum of values (hence, probabilistic in nature). This method constructs a scenario of potential results by applying a probability distribution, such as uniform or normal, to each variable that carries inherent uncertainty. The model then runs multiple iterations, each employing a distinct set of random values within the defined minimum and maximum limits. Upon completion it provides an array of potential outcomes, each accompanied by the likelihood of its occurrence [34]. A classic example illustrating Monte Carlo Simulation involves estimating the probability of rolling two standard dice. With 36 possible dice combinations, the likelihood of any specific outcome can be calculated manually. However, by employing a Monte Carlo Simulation to replicate the dice roll 10,000 times (or even more), the model can attain a higher precision in its predictions.

3 Linear Optimization Framework for Hybrid Energy Systems

3.1 Introduction to Linear Optimization Framework

HES are complicated in nature to optimise for due to the need for integrating various energy sources, conversion technologies, and storage systems. Linear Programming, also known as Linear Optimization, is a widely used tool in the field of operations research and management science. It is characterized by a linear objective function, decision variables, and constraints exhibiting linear relationships. The result of a linear programming problem is obtained by either maximizing or minimizing the objective function[5]. A conventional linear programming problem can be briefly represented in a standard matrix form[6].

$$
\begin{aligned}
\text{maximize} \quad & \mathrm{c}^T\mathrm{x} \\
\text{subject to} \quad & \mathrm{Ax} \geq \mathrm{b} \\
& \mathrm{x} \geq 0,
\end{aligned}
\tag{1}
$$

where the objective function is represented as the dot product of the vectors c and x, and the result is obtained by the maximization of the said function. The constraints are represented as the matrix-vector product Ax, which should be greater than or equal to vector b, and the vector x being greater than or equal to 0. Each row in matrix A corresponds to a linear constraint on the decision variables.

3.2 Formulation of the Objective Function

The central tenet driving the linear optimization framework is revenue maximization, a critical factor for the sustainable operation of HES. With the volatile nature of energy markets, devising strategies for maximizing revenue ensures that the HES can consistently generate economic value while mitigating financial risks. It is pivotal to understand that the objective here is not merely technical efficiency but also economic sustainability, wherein the system is capable of generating sufficient revenues to cover operational costs and, potentially, drive profits.

The formulation process involves constructing an objective function, decision variables, and constraints that collectively represent the operations and limitations of the hybrid energy system. The objective function Z represents the daily operational profit and is defined as:

$$\begin{aligned} \max, Z = & \sum_{t=0}^{23} \text{day\_ahead\_vars}(t) \cdot \text{day\_ahead\_prices}(t) \\ & - \sum_{t=0}^{23} \text{grid\_buy\_vars}(t) \cdot \text{day\_ahead\_prices}(t) \\ & + \text{hydrogen\_cost\_kg} \cdot \sum_{t=0}^{23} \text{hydrogen\_sold\_vars}(t) \\ & - \text{hydrogen\_cost\_kg} \cdot \sum_{t=0}^{23} \text{hydrogen\_buy\_vars}(t), \end{aligned} \tag{2}$$

Each term in the objective function corresponds either to the cost or to the revenue component of the energy system's operations. It is constructed to capture the dynamics between energy sales, purchases, and conversions in the day ahead energy trading or hydrogen markets.

3.2.1 Revenue from Day Ahead Energy Sales

The first term of the equation:

$$\sum_{t=0}^{23} \text{day\_ahead\_vars}(t) \times \text{day\_ahead\_prices}(t)$$

corresponds to the revenue accrued from selling energy within the DAM. It calculates the total earnings for every hourly interval, considering the amount of energy sold at hour t scheduled for delivery at hour t of the following day.

3.2.2 Cost of Buying Energy from Grid

The second term of the equation:

$$-\sum_{t=0}^{23} \text{grid\_buy\_vars}(t) \times \text{day\_ahead\_prices}(t)$$

corresponds to the expenditure related to purchasing energy from the grid, calculated for each hourly interval.

3.2.3 Revenue from Hydrogen Sales

The third term of the equation:

$$+\sum_{t=0}^{23} \text{hydrogen\_sold\_vars}(t) \times \text{hydrogen\_cost\_kg}$$

represents the function aims to optimise revenue generated through the sales of hydrogen, computed for each hour within the operating period.

3.2.4 Cost of Buying Hydrogen

The fourth term of the equation:

$$-\text{hydrogen\_cost\_kg} \times \sum_{t=0}^{23} \text{hydrogen\_buy\_vars}(t)$$

corresponds to the total costs associated with procuring hydrogen, effectively calculated for each hour of operation.

3.3 Decision Variables

Decision variables are introduced to represent the amount of energy (in MWh) and hydrogen (in kg) sold, purchased, stored, or converted at each hour. The Decision variables used are:

- day_ahead_vars(t): The energy sold in the DAM. Each element t denotes energy sold for delivery at hour t. All the deals are made for the next day.
- grid_buy_vars(t): Energy bought from the grid at hour t.
- battery_charge_vars(t) and battery_discharge_vars(t): Battery charge and discharge amounts at hour t, respectively.
- electricity_to_hydrogen_vars(t) and hydrogen_to_electricity_vars$_t$: Energy converted to hydrogen and hydrogen converted to energy at hour t, respectively.
- hydrogen_buy_vars(t) and hydrogen_sold_vars(t): Hydrogen bought and sold at hour t, respectively.

These decision variables are constrained to be non-negative and continuous.

3.3.1 Constraints

Several constraints are implemented to ensure the feasibility and practicality of the optimization solution:

- **Grid Capacity Constraint:** The total energy bought from the grid and sold in the DAM at any hour is limited to 50 MWh.
- **Energy Balance:** At every hour, the energy inputs must equal the energy outputs.
- **Battery Constraints:** Constraints on the storage capacity and power rating of the battery as well as the condition of having a fully charged battery at the end of the day.
- **Hydrogen Constraints:** Constraints on hydrogen operations such as the conversion to and from energy along with the storage capacity constraint.

3.4 Modeling Assumptions and Parameters

3.4.1 Assumptions on the Hybrid Energy System

Solar Production

The model assumes a predetermined hourly solar energy output stored in solar_production. This assumption simplifies the optimization problem by treating solar production as a known parameter, effectively excluding variability and uncertainty in solar energy production from the model.

Battery System

The battery storage system integrated into the hybrid energy system has defined operational parameters. The maximum capacity (battery_capacity_MW), discharge rate (battery_c_value), and efficiency during charge-discharge cycles (battery_efficiency) are all predetermined and constant. The model also assumes that the initial state of charge (SOC) of the battery is zero, and the maximum energy that can be discharged at any given time is limited to 80% of the total battery capacity.

Hydrogen System

The hydrogen system's operating constraints and parameters are clearly defined. The model stipulates a fixed hydrogen storage capacity and assumes that the storage begins empty. The electrolyser has a predetermined maximum capacity and operates with specified efficiency and conversion rates. Additionally, the hydrogen fuel cell's size and conversion efficiency are set parameters within the model.

3.4.2 Trading Operation Assumptions

Day-ahead Market Trading

For the DAM trading operations, the model allows hourly energy sale transactions. It sets a maximum energy capacity limit that can either be bought from or sold to the grid each hour.

Battery Trading Operations

The battery can either be charged or discharged within a given hour but cannot perform both operations simultaneously. The model enforces constraints to ensure the SOC is within the permissible bounds at all times.

Hydrogen Trading Operations

Several constraints govern the hydrogen system's operation within the trading environment. These include limitations on the amount of electricity converted to and from hydrogen and constraints on hydrogen storage levels. The model also implements constraints to ensure that the hydrogen sold and used for electricity generation does not exceed the available hydrogen storage.

Limits on Hydrogen Sale

The model imposes restrictions on the number of times hydrogen can be sold within a day, with the current setup limiting this to once per day.

3.4.3 Additional Technical Assumptions

Linear programming is the chosen optimization technique, implying that the model assumes linearity in all relationships. It also employs a big-M method to linearize certain constraints, requiring a careful selection of a sufficiently large M to ensure model feasibility and accuracy.

3.5 Iterative Model Development Process

Due to the number of moving parts in a hybrid energy system, namely solar production, battery storage, hydrogen storage, electrolysers, and others, it is important to keep track of the workings of the optimization model. One way of doing that is by simplifying the model to only cover one or two decision variables to optimise. It enables us to manually check the model at the initial stages. After the implementation and verification of the base model, we can start adding more decision variables and the corresponding constraints iteratively to keep track of the changes in the optimal value of the Linear Optimization task.

3.5.1 Model Formulation - Stage 1

In the first stage of our iterative model development, our primary focus is on the basic structure of the model, which revolves around maximizing the revenue generated from the solar energy production by selling it in the DAM.

- **Decision Variables**
  The initial set of decision variables includes the amount of energy (in MWh) sold in the DAM. For each hour t of the day, energy is sold for delivery at hour t of the following day. Mathematically, this can be expressed as:

  $$\text{day\_ahead\_vars}(t), \quad \forall t \in \{0, 1, \ldots, 23\},$$

  where $\text{day\_ahead\_vars}(t) \geq 0$

- **Objective Function**
  The objective function is formulated to maximize the revenue from selling solar-produced energy in the DAM. For each hour t (of the previous day), energy is sold for delivery at hour t of the next day (of the current day), with special

consideration for the last hour due to the indexing limitation The objective function can be represented as follows:

$$\max, Z = \sum_{t=0}^{23} \text{day\_ahead\_vars}(t) \cdot \text{day\_ahead\_prices}(t) \tag{3}$$

- **Constraints**

  1. **Energy Balance Constraint**:
     For each hour t in the planning horizon $\{0, 1, \ldots, 23\}$, the energy sold in the day-ahead market must be equal to the energy produced from solar. This is captured by the following constraint:

     $$\text{day\_ahead\_vars}(t) = \text{solar\_production}(t), \quad \forall t \in \{0, 1, \ldots, 23\} \tag{4}$$

     Equation (4) is referred to as the Energy Balance Constraint.

  2. **Grid Capacity Constraint**:
     For each hour t in the planning horizon $\{0, 1, \ldots, 23\}$, the energy sold in the day-ahead market must be less than the grid capacity, represented by S. This is captured by the following constraint:

     $$\text{day\_ahead\_vars}(t) \leq \text{S}, \forall t \in \{0, 1, \ldots, 23\} \tag{5}$$

- **Model Solution**
  Upon solving the model using PuLP [7], the optimal solutions for the decision variables (i.e the optimal amount of energy to sell at the DAM at each hour) and the maximum revenue that can be generated are obtained.

Hour	**Energy Sold (MWh)**	**Intraday Price Hour**
6	1.20	7
7	3.60	8
8	7.00	9
9	9.60	10
10	12.00	11
11	16.20	12
12	20.80	13

Table 1: Example Output: Selling Decisions

3.5.2 Stage 2: Incorporation of Battery Storage

In Stage 2 of the model development, we introduce battery storage into the system, represented by various decision variables and constraints associated with the operation of a battery.

- **Decision Variables**
  In addition to the DAM decision variables (day_ahead_vars) from Stage 1, we introduce three new sets of decision variables:

  - battery_charge_vars(t) : Energy charged into the battery at hour t.
  - battery_discharge_vars(t) : Energy discharged from the battery at hour t.
  - battery_SOC_vars(t) : SOC of the battery at the end of hour t.

- **Battery Parameters**
  The parameters for the battery are set as follows:

$$
\begin{aligned}
&\text{Capacity} : 10\,\text{MW}\\
&\text{C-value} : 0.5\\
&\text{Efficiency} : 0.9\,(\textit{or}\ 90\%)\\
&\text{Discharge Limit} : 8\,\text{MW}\,(80\%\ \text{of Capacity})\\
&\text{Lifetime Cycles} : 5000\\
&\text{Initial State of Charge(SOC)} : 0
\end{aligned}
$$

- **Objective Function**
  The objective function remains the same as in Stage 1; to maximize revenue from selling solar-produced energy in the DAM.

- **Constraints**
  In addition to the constraints from Stage 1, we introduce the following constraints and their modifications:

  1. **Energy Balance Constraint**:

$$
\begin{aligned}
&\text{day\_ahead\_vars}(t) + \text{battery\_charge\_vars}(t) \\
&\leq \text{solar\_production}(t) + \text{battery\_discharge\_vars}(t) \times \text{battery\_efficiency}, \\
&\forall t \in \{0, \ldots, 23\}
\end{aligned} \tag{6}
$$

  Equation (6) ensures that the energy produced and energy discharged from the battery should be less than or equal to the energy stored in the battery and the energy sold in the DAM.

  2. **Charging Limit Constraint**:

$$
\begin{aligned}
&\text{battery\_charge\_vars}(t) \leq \text{battery\_capacity\_MW} \times \text{battery\_c\_value} \\
&\forall t \in \{0, \ldots, 23\}
\end{aligned} \tag{7}
$$

Equation (7) ensures that the charging rate at any time t does not exceed the product of the battery's capacity and the charging rate coefficient (C-value).

3. **Discharging Limit Constraint**:

$$\text{battery\_discharge\_vars}(t) \leq \text{battery\_discharge\_limit} \times \text{battery\_c\_value} \tag{8}$$

$\forall t \in \{0, \ldots, 23\}$

Equation (8) guarantees that the discharging rate at any time t is within the battery's specified discharge limit.

4. **SOC Constraint**:

$$\text{battery\_SOC\_vars}(t) - \text{battery\_discharge\_vars}(t) \geq 0 \tag{9a}$$
$$\text{battery\_SOC\_vars}(t) + \text{battery\_charge\_vars}(t) \leq \text{battery\_capacity\_MW} \tag{9b}$$

$\forall t \in \{0, \ldots, 23\}$

Constraints (9a) and (9b) ensure that the SOC remains within the bounds of 0 and the battery's capacity, post charging and discharging activities.

5. **Binary Charging and Discharging Constraints**:

$$\text{battery\_charge\_vars}(t) \leq M \times (1 - \text{battery\_action}(t)) \tag{10a}$$
$$\text{battery\_discharge\_vars}(t) \leq M \times \text{battery\_action}(t) \tag{10b}$$
$$\forall t \in \{0, \ldots, 23\}$$

These binary constraints, as indicated by (10a) and (10b), prevent simultaneous charging and discharging within the same time period.

6. **Initial SOC Constraint**:

$$\begin{aligned} \text{battery\_SOC\_vars}(0) = \text{battery\_SOC} &+ \text{battery\_charge\_vars}(0) \\ &- \text{battery\_discharge\_vars}(0) \times \text{battery\_efficiency} \end{aligned} \tag{11}$$

Equation (11) defines the initial SOC of the battery, considering initial conditions and efficiencies.

7. **SOC Dynamics Constraint**:

$$\begin{aligned} \text{battery\_SOC\_vars}(t) = {} & \text{battery\_SOC\_vars}(t-1) \\ & + \text{battery\_charge\_vars}(t) - \text{battery\_discharge\_vars}(t) \\ & \times \text{battery\_efficiency} \\ & \forall t \in \{1, \ldots, 23\} \end{aligned} \tag{12}$$

Equation (12) describes the evolution of SOC over time, taking into account the charging and discharging events and the battery's efficiency.

Hour	Energy Sold (MWh)	Day Ahead Price of Hour (€/MWh)
8	4.40	16.78
9	12.93	56.65
10	8.80	23.85
11	19.65	28.89

Table 2: Hourly selling decisions in Stage 2 (partial).

- **Selling Decisions**
  Table 2 shows the energy amounts sold at each hour, considering the day ahead prices.
- **Battery Charging Decisions**
  Table 3 outlines the hourly decisions to charge the battery, indicating the amount of energy (in MWh) stored at each hour.

Hour	Energy Charged (MWh)
7	1.20
8	3.60
9	2.60
11	3.15

Table 3: Hourly battery charging decisions in Stage 2 (partial).

- **Battery Discharging Decisions**

  Similarly, Table 4 presents the energy amounts discharged from the battery at each hour to be sold in the DAM. Note that the discharged amount is taken to be the amount available after loss during the discharge cycle, i.e battery efficiency.

Hour	Energy Discharged (MWh)
10	3.70
14	4.00
18	4.00
20	2.00

Table 4: Hourly battery discharging decisions in Stage 2 (partial).

- **Battery SOC**
  The SOC of the battery at each hour is essential to understand the battery's operation and performance throughout the day. Table 5 summarizes the energy available in the battery at each hour.

3.5.3 Stage 3: Introduction of Grid Energy Purchase

In Stage 3, we further refine the model by considering the possibility of purchasing energy from the grid. This stage allows the system not only to sell energy but also to buy it, providing additional flexibility and optimization potential in operations.

- **Decision Variables**
  Stage 3 introduces a new decision variable while retaining those from Stage 2:
  - $\text{grid\_buy\_vars}(t)$: Amount of energy bought from the grid at hour t.

- **Objective Function Modification**
  The objective function is updated to maximize the net revenue, considering both sales to and purchases from the grid:

$$\max, Z = \sum_{t=0}^{23} \text{day\_ahead\_vars}(t) \times \text{day\_ahead\_prices}(t) - \sum_{t=0}^{23} \text{grid\_buy\_vars}(t) \times \text{day\_ahead\_prices}(t) \tag{13}$$

- **New and Updated Constraints**
  New constraints are introduced, and previous constraints are updated to incorporate the grid energy purchase option. Let:

  1. **Energy Balance Constraint**:

$$\begin{aligned} &\text{day\_ahead\_vars}(t) + \text{battery\_charge\_vars}(t) \\ &\leq \text{solar\_production}(t) + \text{battery\_discharge\_vars}(t) \times \text{battery\_efficiency} \\ &+ \text{grid\_buy\_vars}(t), \\ &\forall t \in \{0, \ldots, 23\} \end{aligned} \tag{14}$$

  Equation (14) now has increased supply of electricity by now allowing buying from the grid.

Hour	Energy in Battery (MWh)
7	1.20
8	4.80
9	7.40
10	3.70
11	6.85
12	6.85

Table 5: Hourly SOC in the battery in Stage 2 (partial).

2. **Grid Capacity Constraint**:
For each hour t in the planning horizon $\{0, 1, \ldots, 23\}$, the energy sold in the day-ahead market and bought must be less than the grid capacity, represented by S. This is captured by the following constraint:

$$\text{day\_ahead\_vars}(t) + \text{grid\_buy\_vars}(t) \leq \text{S}, \forall t \in \{0, 1, \ldots, 23\} \quad (15)$$

This stage offers more operational flexibility by allowing energy purchases from the grid, optimizing the energy flow, and maximizing the revenue generated through strategic buying and selling decisions. With the integration of grid energy purchasing, the model can better navigate the dynamics of energy demand and supply, providing a robust framework for efficient energy management.

- **Buying Decisions**
Table 6 shows the energy amounts bought at each hour, considering the day ahead prices.

Hour	Energy Bought (MWh)	Day Ahead Price of Hour (€/MWh)
2	5.0	7.89
3	2.5	11.18
6	3.125	9.71
7	0.3625	10.33

Table 6: Hourly buying decisions in Stage 3.

3.5.4 Stage 4: Integration of Hydrogen Production and Sale

Stage 4 incorporates hydrogen production and sale into the model, enhancing its operational flexibility and revenue-generation capabilities by tapping into the hydrogen market.

- **Decision Variables**
In addition to previous decision variables, Stage 4 introduces the following:

  - $\text{hydrogen\_storage\_vars}(t)$: Amount of hydrogen stored at hour t.
  - $\text{electricity\_to\_hydrogen\_vars}(t)$: Amount of electricity converted to hydrogen at hour t.
  - $\text{allow\_hydrogen\_sale\_vars}(t)$: Binary variable allowing hydrogen sale at hour t.
  - $\text{hydrogen\_sold\_vars}(t)$: Amount of hydrogen sold at hour t.

- **Hydrogen Parameters**

The parameters for the battery are set as follows:

$$\begin{aligned}
\text{hydrogen_storage_capacity_kg} &: 1250\\
\text{electrolyser_size_MW} &: 6\\
\text{electrolyser_conversion_rate} &: 0.0333\\
\text{electrolyser_efficiency} &: 0.6\\
\text{electrolyser_conversion_efficiency} &: \frac{\text{electrolyser_efficiency}}{\text{electrolyser_conversion_rate}}\\
\text{hydrogen_demand_flat} &: 100\,\text{kg}\\
\text{hydrogen_market_price_EUR_per_kg} &: 6\\
\text{hydrogen_cost_price_EUR_per_kg} &: 6
\end{aligned}$$

- **Objective Function Modification**
  The objective function is updated to include revenue from hydrogen sales:

$$\begin{aligned}
\max, \mathrm{Z} = &\sum_{t=0}^{23} \text{day_ahead_vars}(t) \times \text{day_ahead_prices}(t) \\
&- \sum_{t=0}^{23} \text{grid_buy_vars}(t) \times \text{day_ahead_prices}(t)\\
&+ \sum_{t=0}^{23} \text{hydrogen_sold_vars}(t) \times \text{hydrogen_market_price_EUR_per_kg}
\end{aligned} \tag{16}$$

- **New and Updated Constraints**

  1. **Energy Balance Constraint**:

$$\begin{aligned}
&\text{day_ahead_vars}(t) + \text{battery_charge_vars}(t)\\
&+ \text{electricity_to_hydrogen_vars}(t)\\
&\leq \text{solar_production}(t) + \text{battery_discharge_vars}(t) \times \text{battery_efficiency}\\
&+ \text{grid_buy_vars}(t),\\
&\forall t \in \{0, \ldots, 23\}
\end{aligned} \tag{17}$$

     Equation (14) now has increased supply of electricity by now allowing buying from the grid.

  2. **Grid Capacity Constraint**:
     For each hour t in the planning horizon $\{0, 1, \ldots, 23\}$, the energy sold in the day-ahead market and bought must be less than the grid capacity, represented by S. This is captured by the following constraint:

$$\text{day\_ahead\_vars}(t) + \text{grid\_buy\_vars}(t) \leq \mathrm{S}, \forall t \in \{0, 1, \ldots, 23\} \tag{18}$$

3. **Electricity-to-Hydrogen Conversion Constraint:**
For each hour t in the planning horizon $\{0, 1, \ldots, 23\}$, the electricity being converted to hydrogen cannot be more than the capacity of the electrolyser installed.

$$\text{electricity\_to\_hydrogen\_vars}(t) \leq \text{electrolyser\_size\_MW}, \quad \forall t \in \{0, \ldots, 23\} \tag{19}$$

4. **Initial Hydrogen Storage:**

$$\begin{aligned} &\text{hydrogen\_storage\_vars}(0) = \text{initial\_hydrogen\_storage} \\ &+ \text{electricity\_to\_hydrogen\_vars}(0) \times \text{electrolyser\_conversion\_efficiency} \\ &- \text{hydrogen\_sold\_vars}(0) \end{aligned} \tag{20}$$

5. **Hydrogen Storage Dynamics:**

$$\begin{aligned} &\text{hydrogen\_storage\_vars}(t) = \text{hydrogen\_storage\_vars}(t-1) \\ &+ \text{electricity\_to\_hydrogen\_vars}(t) \times \text{electrolyser\_conversion\_efficiency} \\ &- \text{hydrogen\_sold\_vars}(t), \\ &\forall t \in \{1, \ldots, 23\} \end{aligned} \tag{21}$$

6. **Hydrogen Storage Bounds:**

$$0 \leq \text{hydrogen\_storage\_vars}(t) \leq \text{hydrogen\_storage\_capacity\_kg}, \quad \forall t \in \{1, \ldots, 23\} \tag{22}$$

7. **Hydrogen Sale Constraints:**

$$\text{hydrogen\_sold\_vars}(t) \leq 1500 \times \text{allow\_hydrogen\_sale\_vars}(t), \quad \forall t \in \{1, \ldots, 23\} \tag{23a}$$

$$\sum_{t=0}^{23} \text{allow\_hydrogen\_sale\_vars}(t) \leq 1 \tag{23b}$$

This stage further expands the model's capabilities, enabling it to optimise operations considering both the energy and hydrogen markets, thus providing a more comprehensive framework for energy management from more than a single source and revenue optimization in a hybrid energy system.

- **Electricity to Hydrogen Conversion Decisions**
The optimization decision includes converting specific amounts of electricity into hydrogen at various hours. Table 7 provides details on these decisions.

Hour	Electricity Converted to Hydrogen (MWh)
0	6.0
1	6.0
11	6.0
23	6.0

Table 7: Hourly Electricity to Hydrogen Conversion Decisions in Stage 4 (partial).

- **Hourly Hydrogen Storage Decisions**
  Table 8 provides details on the amounts of hydrogen stored hourly as per optimization decisions.

Hour	Hydrogen Stored (kg)
0	108.0108
1	216.0216
6	756.07561
8	972.09721
9	1080.108
21	1250.00
23	100.00

Table 8: Hourly Hydrogen Storage Decisions in Stage 4.

- **Total Hydrogen Sold**
  The total amount of hydrogen sold as per optimization decisions in Stage 4 is 1258.0108 kg.

3.5.5 Stage 5: Incorporating Hydrogen Market and Hydrogen-to-Electricity Conversion

In Stage 5, the model is extended to interact with the hydrogen market, both buying and selling, and to convert stored hydrogen back to electricity. This stage enhances the system's operational flexibility and allows more intricate energy management strategies.

- **Decision Variables**
  Stage 5 introduces new decision variables while retaining those from Stage 4:

  - hydrogen_to_electricity_vars(t) : The hydrogen (in kg) converted to energy at each hour t.
  - hydrogen_buy_vars(t) : Amount of Hydrogen bought from the Hydrogen Market in kg at hour t.

- **Objective Function Modification**
  The objective function is updated to maximize the net revenue, considering hydrogen sales, purchases, and energy trading:

$$\begin{aligned}
\max, \mathrm{Z} = &\sum_{t=0}^{23} \text{day_ahead_vars}(t) \times \text{day_ahead_prices}(t) \\
&- \sum_{t=0}^{23} \text{grid_buy_vars}(t) \times \text{day_ahead_prices}(t) \\
&+ \sum_{t=0}^{23} (\text{hydrogen_sold_vars}(t) \times \text{hydrogen_market_price_EUR_per_kg} \\
&- \text{hydrogen_buy_vars}(t) \times \text{hydrogen_cost_price_EUR_per_kg})
\end{aligned} \tag{24}$$

- **New and Updated Constraints**
  Constraints are added and updated to accommodate the new decision variables:

  1. **Energy Balance Constraint**:

$$\begin{aligned}
&\text{day_ahead_vars}(t) + \text{battery_charge_vars}(t) \\
&+ \text{electricity_to_hydrogen_vars}(t) \\
&\leq \text{solar_production}(t) + \text{battery_discharge_vars}(t) \times \text{battery_efficiency} \\
&+ \text{grid_buy_vars}(t) + \text{hydrogen_to_electricity_vars}(t), \\
&\quad \forall t \in \{0, \ldots, 23\}
\end{aligned} \tag{25}$$

  2. **Hydrogen-to-Electricity Conversion Constraint:**
     For each hour t in the planning horizon $\{0, 1, \ldots, 23\}$, the hydrogen being converted to electricity cannot be more than the capacity of the Fuel Cell installed.

$$\begin{aligned}
&\text{hydrogen_to_electricity_vars}(t) \times \text{fuel_cell_conversion_efficiency} \\
&\leq \text{hydrogen_fuel_cell_size_MW}, \\
&\forall t \in \{0, \ldots, 23\}
\end{aligned} \tag{26}$$

  3. **Initial Hydrogen Storage:**

$$\begin{aligned}
&\text{hydrogen_storage_vars}(0) \\
&= \text{initial_hydrogen_storage} - \text{hydrogen_to_electricity_vars}(0) \\
&+ \text{electricity_to_hydrogen_vars}(0) \times \text{electrolyser_conversion_efficiency} \\
&- \text{hydrogen_sold_vars}(0) + \text{hydrogen_buy_vars}(0)
\end{aligned} \tag{27}$$

4. **Hydrogen Storage Dynamics:**

$$\begin{aligned}
&\text{hydrogen_storage_vars}(t) \\
&= \text{hydrogen_storage_vars}(t-1) - \text{hydrogen_to_electricity_vars}(t) \\
&+ \text{electricity_to_hydrogen_vars}(t) \times \text{electrolyser_conversion_efficiency} \\
&- \text{hydrogen_sold_vars}(t) + \text{hydrogen_buy_vars}(t), \\
&\forall t \in \{1, \ldots, 23\}
\end{aligned} \tag{28}$$

3.5.6 Stage 6: Integration of Imbalance Settlement Procedure

Stage 6 introduces the incorporation of the imbalance settlement procedure into the model. This enhancement is pivotal in managing the discrepancies between actual and forecasted solar production, thereby optimizing revenue in the face of real-world uncertainties.

- **Objective Function Refinement**
  The objective function is adjusted to accommodate the financial implications of the imbalance between forecasted and actual solar production. It aims to maximize net revenue, factoring in the costs associated with energy imbalances:

$$\begin{aligned}
\max, Z = &\Bigg(\sum_{t=0}^{23} \text{day_ahead_vars}(t) \times \text{day_ahead_prices}(t) \\
&- \sum_{t=0}^{23} \text{grid_buy_vars}(t) \times \text{day_ahead_prices}(t) \\
&+ \sum_{t=0}^{23} (\text{hydrogen_sold_vars}(t) \times \text{hydrogen_market_price_EUR_per_kg} \\
&- \text{hydrogen_buy_vars}(t) \times \text{hydrogen_cost_price_EUR_per_kg}) \\
&+ \sum_{t=0}^{23} (\max(0, \text{solar_production_actual}(t) - \text{solar_production_forecast}(t)) \\
&\times \text{positive_imbalance_price}(t) \\
&+ \min(0, \text{solar_production_actual}(t) - \text{solar_production_forecast}(t)) \\
&\times \text{negative_imbalance_price}(t)) \Bigg)
\end{aligned} \tag{29}$$

- **New and Updated Constraints**

  1. **Grid Capacity Constraint**
     A constraint is set to ensure that the combined total of day-ahead market trades and grid purchases does not exceed the grid capacity limits, taking

into account the imbalance in solar production:

$$\begin{aligned}&\text{day\_ahead\_vars}(t) + \text{grid\_buy\_vars}(t)\\&+ |\text{solar\_production\_actual}(t) - \text{solar\_production\_forecast}(t)|\\&\leq \text{grid\_capacity\_limit\_MW},\\&\forall t \in \{0, \ldots, 23\}\end{aligned} \tag{30}$$

2. **Energy Balance Constraint**:
   The energy balance constraints are revised to align with the changes in solar production, ensuring that the total energy used or stored at any given time does not exceed the available resources and capacities:

$$\begin{aligned}&\text{day\_ahead\_vars}(t) + \text{battery\_charge\_vars}(t)\\&+ \text{electricity\_to\_hydrogen\_vars}(t)\\&+ |\text{solar\_production\_actual}(t) - \text{solar\_production\_forecast}(t)|\\&\leq \text{solar\_production}(t) + \text{battery\_discharge\_vars}(t) \times \text{battery\_efficiency}\\&+ \text{grid\_buy\_vars}(t) + \text{hydrogen\_to\_electricity\_vars}(t),\\&\forall t \in \{0, \ldots, 23\}\end{aligned} \tag{31}$$

- **Significance and Implications**
  The integration of the imbalance settlement procedure highlights the model's ability to adapt to real-world operational uncertainties. This stage underscores the significance of accurate solar production forecasting and its financial implications, offering insights into optimal energy management strategies in the presence of solar production variability.

4 Analysis of Optimization under Uncertainties

4.1 Asset Utilization

We will analyse the model's output in this subsection to determine how effective asset utilisation is in the long run. We will take into account the rates of utilisation, the financial advantages, and the operational robustness that our all-encompassing strategy provides. Our study will show the highs and lows of asset utilisation, dissect the model's optimisation reasoning, and offer a road map for further advancements.

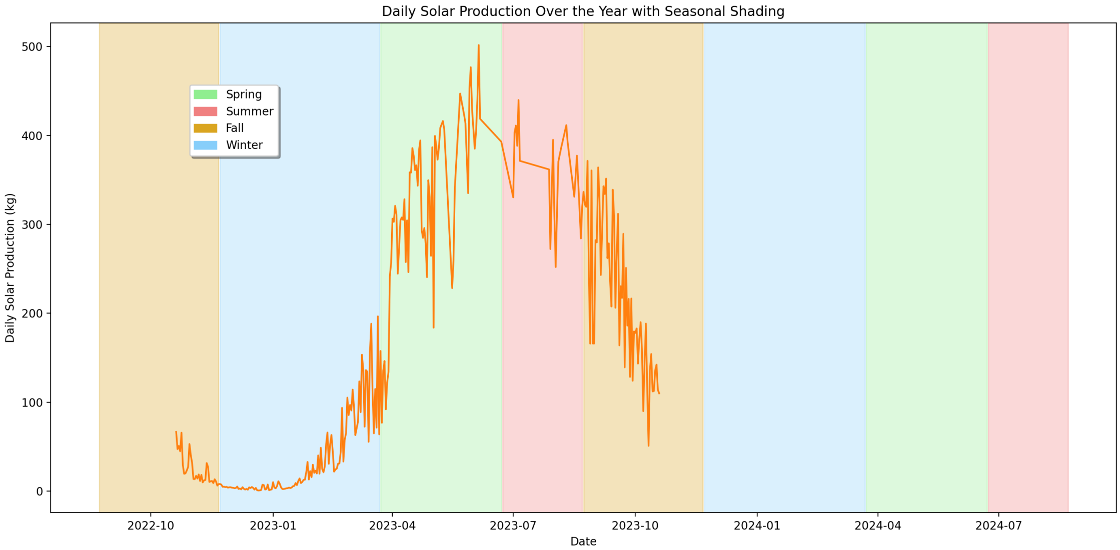

Figure 4: Daily Solar Production throughout the year.

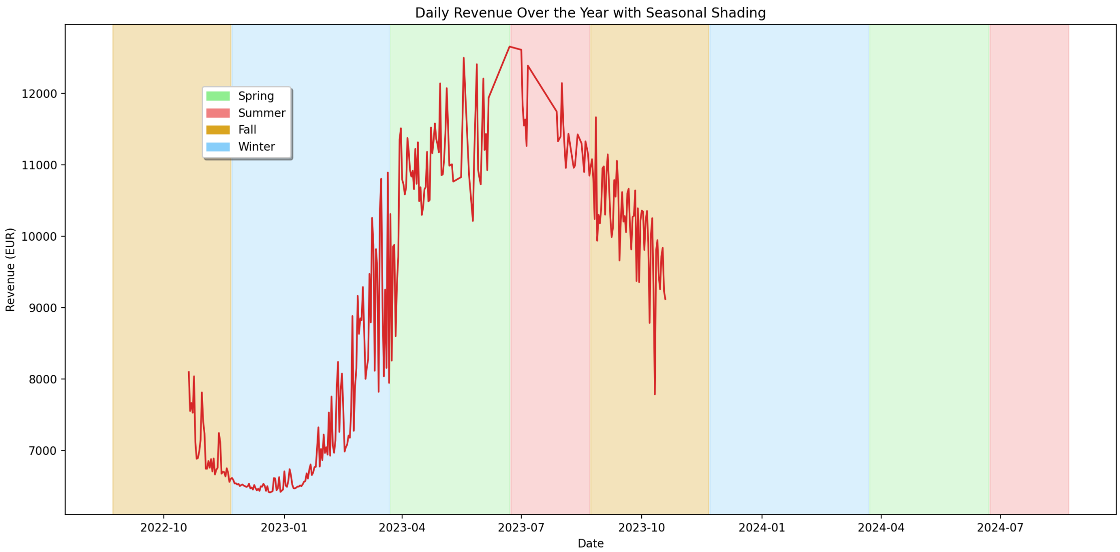

Figure 5: Daily Revenue throughout the year.

Fig. 4 illustrates the amount of energy produced from solar power every day for a year. The seasonal color-coding highlights the impact of seasons on solar energy production. A noticeable trend may show higher production in the summer months compared to winter, aligning with longer daylight hours. This graph informs stakeholders about the variability of solar energy production and the need for complementary energy solutions or storage systems to balance the supply throughout the year.

Fig. 5 shows how the revenue generated from energy production varies throughout the year. The fluctuations in the graph indicate that revenue is not constant; it rises and falls, possibly due to varying market prices and changes in production efficiency. A key takeaway is that certain times of the year might be more profitable than others, which could influence strategic planning for energy sales or marketing. Despite the presence of a battery and hydrogen storage along with the electrolysers and fuel cells, it can be seen that the outline of Fig. 5 resembles the outline of Fig. 4, indicating a clear dependency on solar production.

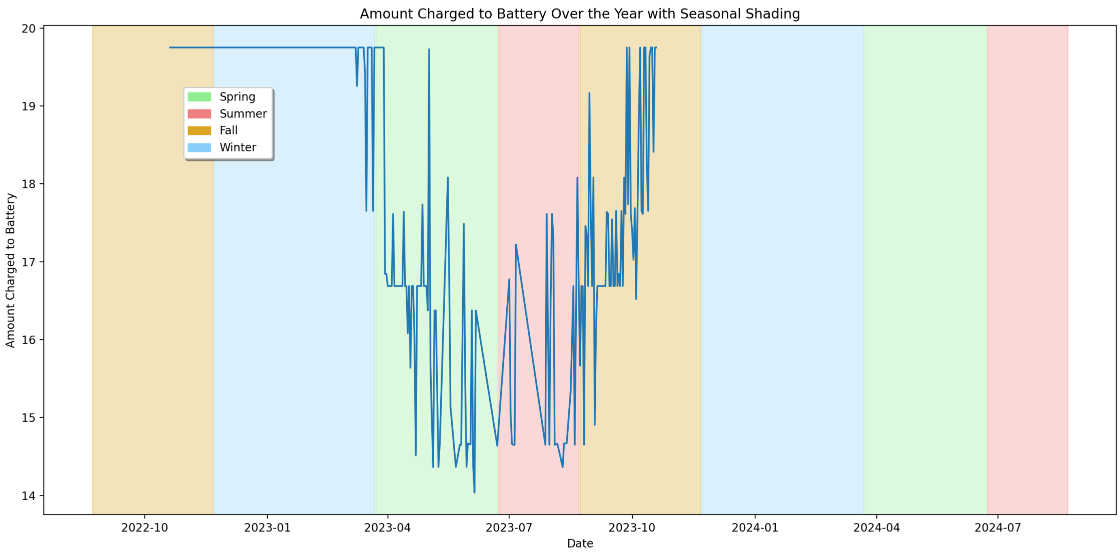

Figure 6: Amount Charged to Battery throughout the year.

In Fig. 6, we observe the amount of energy stored in batteries over the year. The seasonal shading helps to understand how different seasons affect the ability to charge the batteries. For instance, you might see more energy storage in winter seasons due to low solar production and need for storing the limited solar energy. Conversely, less energy may be stored during seasons with more sunlight due to more consistent and higher solar production. This information is crucial for planning energy storage strategies and ensuring a stable energy supply throughout the year. The revenue and battery charging graphs can be compared to understand how energy storage impacts financial returns. For example, during times when the battery charging is high, revenue might not necessarily increase if the market price is low. Conversely, even if less energy is stored in batteries, revenue might be high if the energy is sold when market prices peak.

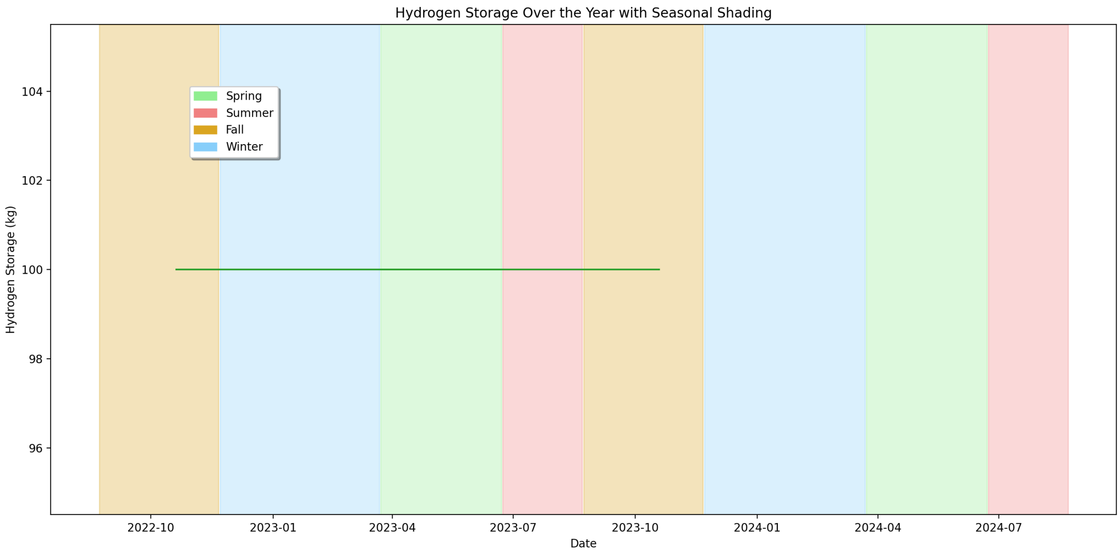

Figure 7: Hydrogen Storage at the end of the day in kg throughout the year.

Fig. 7 depicts the level of hydrogen storage at the end of the day, measured in kilograms, across the year. The stability of the line around the 100 kg mark suggests that the model consistently maintains a certain level of hydrogen storage at the end of the day, likely to ensure a steady supply of hydrogen in case of an emergency.

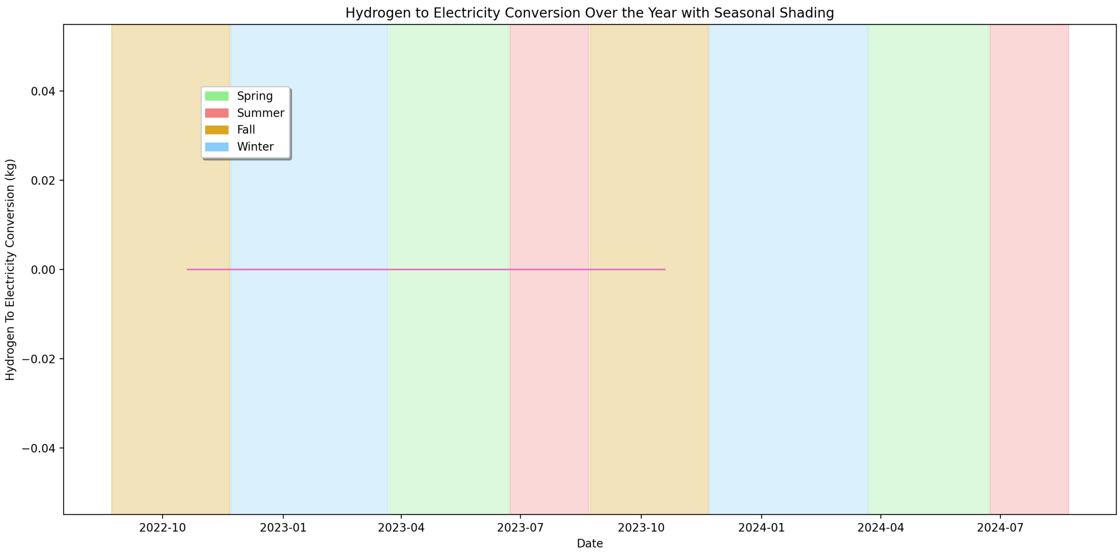

Figure 8: Hydrogen to Electricity Conversion throughout the year.

Fig. 8 shows the conversion of stored hydrogen back into electricity. The flat line at zero suggests that hydrogen is not converted back to electricity within the model. This could mean that hydrogen is either sold directly or stored for longer-term use rather than being used for electricity generation. This helps us understand that the investment in a Fuel Cell might not be worth if only focusing on it's use case to convert electricity from hydrogen.

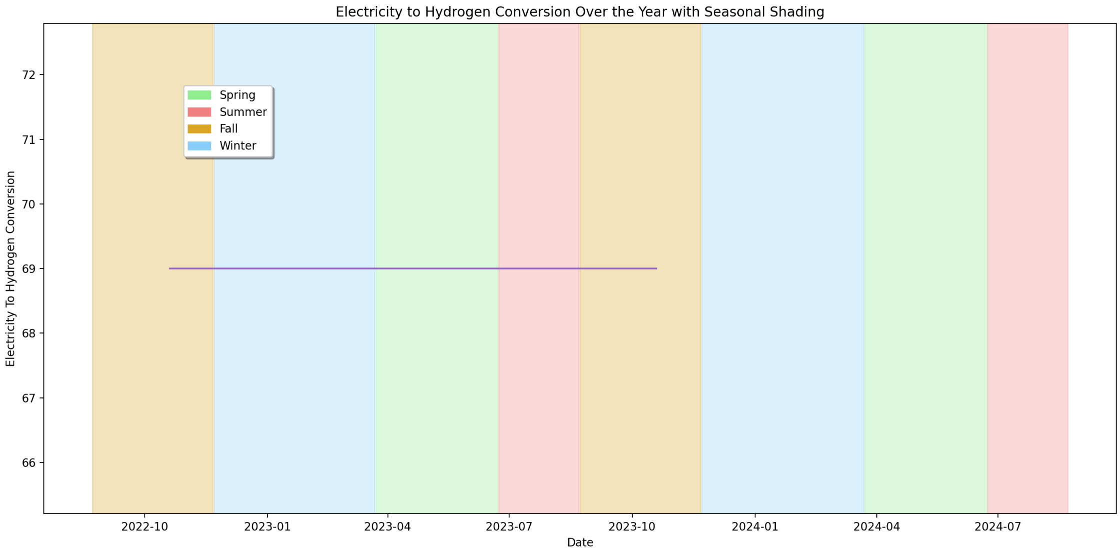

Figure 9: Electricity to Hydrogen Conversion throughout the year.

Fig. 9 indicates the amount of electricity converted to hydrogen. The consistent value of 69 MWh suggests a steady use of excess electricity to produce hydrogen, which may then be stored or sold. The model regularly utilizes available electricity for hydrogen production, which can act as a form of energy storage or as a product for the energy market.

These graphs collectively demonstrate how the optimization model strategically manages energy assets to maximize revenue, ensure energy supply, and leverage seasonal variations. The utilization of batteries for storage, solar production for generating energy, and hydrogen as a flexible energy carrier are all part of the model's integrated approach to stabilizing revenue throughout the year despite the inherent fluctuations in market prices and energy availability. We can look at how the model stabilises inherent fluctuations in more detail by exploring the variability in market prices.

4.2 Market Price Variability

As discussed earlier, The prices in the energy market are extremely volatile. This volatility can have significant impacts on the revenue streams for energy producers, particularly those that operate with renewable sources like solar energy, where production can be highly variable.
Using the optimization model presented above in Equation 16, the effect of volatile market prices can be seen on the generated revenue. The model can also be modified to help visualise the effect of the market prices on the revenue when certain elements

of the model aren't present, to better judge the necessity and importance of those elements.

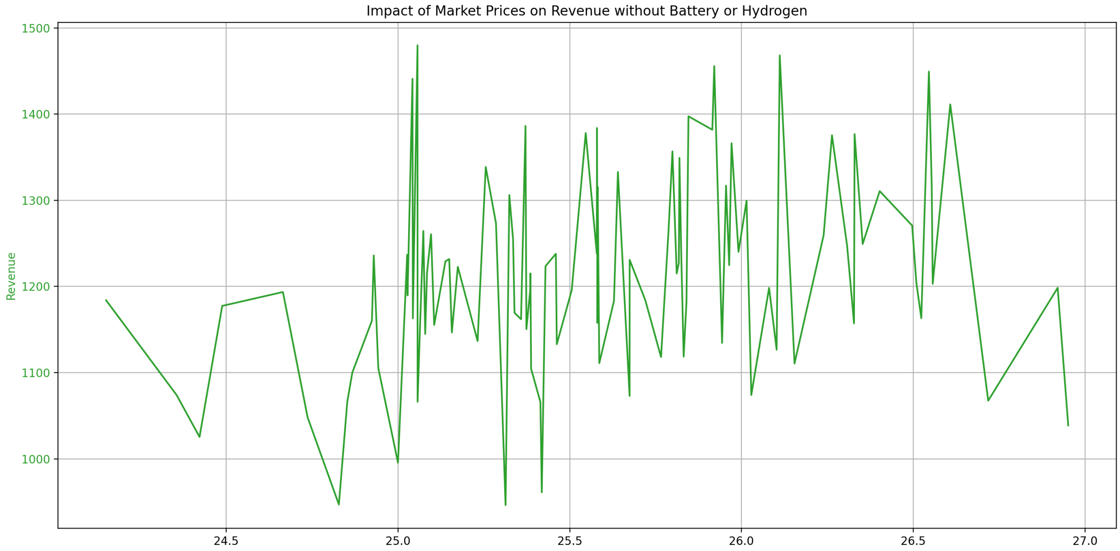

Figure 10: Impact of Market Prices on Revenue without Battery and Hydrogen.

Fig. 10 shows revenue fluctuating widely with market prices, indicating a direct dependence on selling prices without any means to store or defer the sale of produced energy. The jagged pattern suggests that revenue is highly sensitive to market conditions, which can lead to financial instability for the producer.

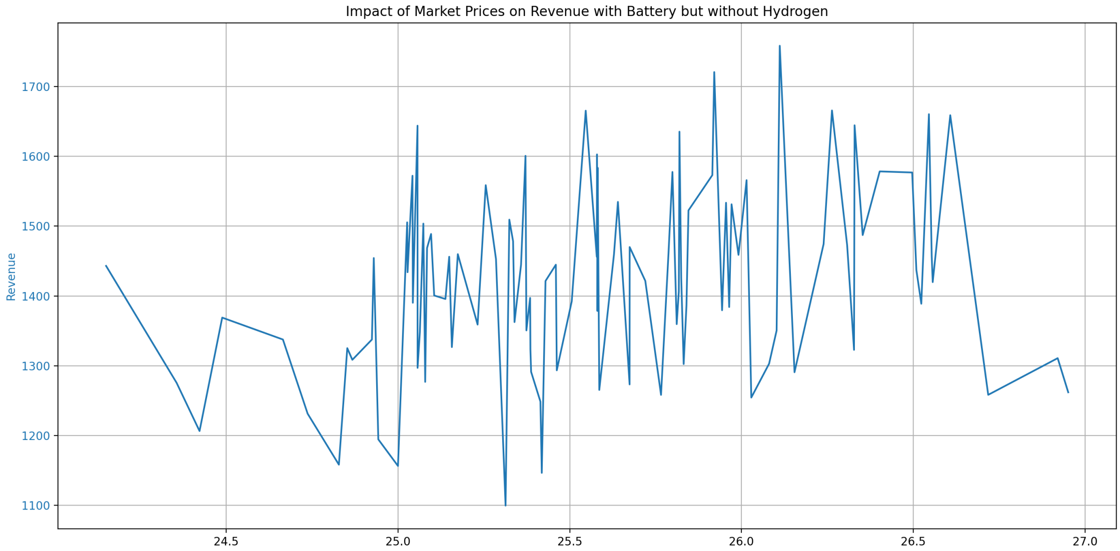

Figure 11: Impact of Market Prices on Revenue with Battery but without Hydrogen.

In Fig. 11, With the introduction of battery storage, there is a noticeable stabilization in revenue, as energy can be stored when prices are low and sold when they are higher. The battery acts like a buffer. However, the peaks and troughs indicate that the capacity or strategy of the battery storage does not entirely mitigate the volatility. This could be because the battery has a limited capacity, and it can only mitigate the impact of market prices to a certain extent.

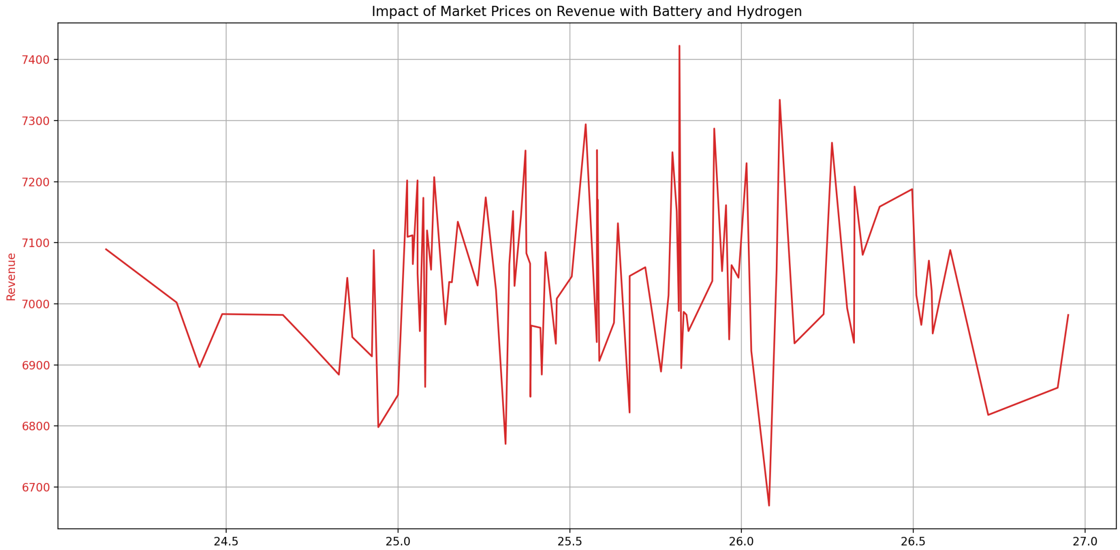

Figure 12: Impact of Market Prices on Revenue with Battery and Hydrogen

Fig. 12 demonstrates the most stable revenue profile, suggesting that the combination of battery and hydrogen storage provides the best buffer against market price volatility. The use of hydrogen storage possibly allows for long-term energy retention, offering more flexibility to wait for favorable market conditions. The smoother trend line indicates that the integrated approach to energy storage can significantly reduce the risk associated with price fluctuations.

Without any storage, the producer's revenue is highly unstable, leading to financial risk. Adding a battery storage system helps stabilize revenue somewhat, reducing but not eliminating volatility. Introducing both battery and hydrogen storage systems offers the most stable revenue, demonstrating the value of diversified energy storage solutions to mitigate the risks of fluctuating market prices.

4.3 Solar production variability

In our model, probabilistic forecasting is implemented by fitting a gamma distribution to historical solar production data. The gamma distribution is particularly suited for this purpose due to its flexibility in modeling skewed, non-negative data, a common characteristic of solar energy production. By fitting this distribution, the model captures the typical range and variability in historical solar output, providing a basis for generating realistic future scenarios. As the model employs Monte Carlo simulation to obtain numerical results through repeated random samples, it enables the exploration of a wide range of scenarios, particularly in systems with complex interactions and uncertainty, like solar energy production. By drawing random samples from the fitted gamma distribution for each time period, the model is able to simulate various plausible solar production levels for different hours of the day.

The curve on the graph below Fig. 13 represents the gamma probability density function (PDF) that has been fitted to the solar production data. This represents the likelihood of different solar production values occurring. The gamma distribution is commonly used to model data that are skewed to the right and where the values are positive, which is typical for the solar production data. The PDF starts high at the lower end of the x-axis (near zero), indicating a higher probability of lower production values, and gradually tapers off as the production value increases. This reflects that lower solar production values are more common than higher ones in your dataset. This is because there are multiple hours with no sun and hence no production, and with higher values of solar production being observed only for an hour or two maximum if the weather conditions are not cloudy. In the context of Monte Carlo simulation, this PDF can be used to generate random solar production values that are consistent with historical patterns. When one simulates numerous scenarios (each represented by a day with 24 hours of production), you draw random values from this distribution, mirroring the range of possible real-world outcomes.

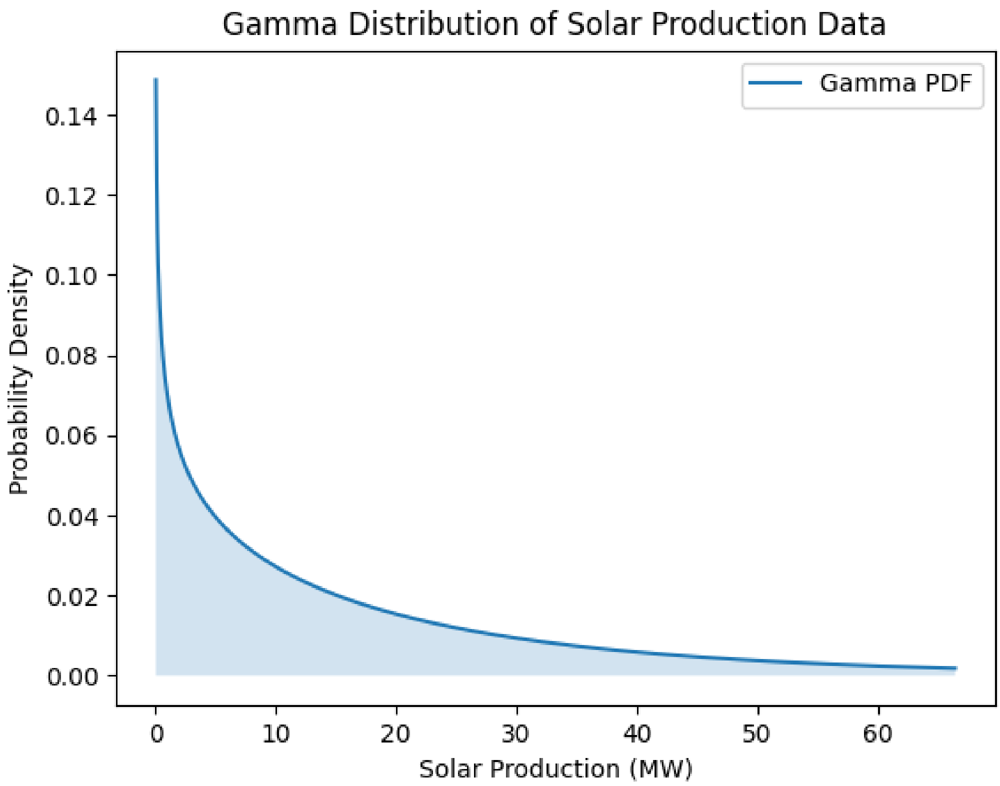

Figure 13: A visual representation of the PDF for the used solar production data

The process of using Monte Carlo Simulation and performing statistical analysis on the simulated data can be summarised by the steps below,

1. **Gamma Distribution Fitting:**
   Let X be the random variable representing solar production. The historical solar production data $D = \{d_1, d_2, \ldots, d_n\}$ where $d_i > 0$, is used to estimate the parameters of a gamma distribution. This is given by the PDF:

   $$f(x; k, \theta) = \frac{x^{k-1} e^{-\frac{x}{\theta}}}{\theta^k \Gamma(k)} \tag{32}$$

   where k is the shape parameter, θ is the scale parameter, and $\Gamma(k)$ is the gamma function evaluated at k. The parameters k and θ are estimated from the data D.

2. **Simulated Solar Production Generation:**
   For each day t, simulate m scenarios of solar production $S_{t,1}, S_{t,2}, \ldots, S_{t,m}$, where each scenario is a vector of 24 hourly values denoted by h. Each value is generated from the gamma distribution with parameters k and θ:

   $$S_{t,j} = (s_{t,j,1}, s_{t,j,2}, \ldots, s_{t,j,24}), \quad s_{t,j,h} \sim \text{Gamma}(k, \theta), \tag{33}$$

   We can have a look at the difference between the actual solar production data and the simulated data when we increase the number of scenarios Fig. 14. When running the simulation only once, the outcome is just one instance of what could happen given the input parameters. This single instance can be quite far from the actual or expected value because it represents only one possible outcome out of many. When the simulation is ran multiple times, the simulation starts to generate a range of possible outcomes. The average of these outcomes tends to converge towards the expected value, which in this case is the expected solar power production.

3. **Sunrise/Sunset Mask:**
   Define a binary mask $M_t = (m_{t,1}, m_{t,2}, \ldots, m_{t,24})$ based on the sunrise and sunset times for day t, such that:

   $$m_{t,h} = \begin{cases} 1 & \text{if } h \text{ is between sunrise and sunset} \\ 0 & \text{otherwise} \end{cases} \tag{34}$$

   The simulated production is then adjusted by the mask:

   $$S'_{t,j} = S_{t,j} \cdot M_t \tag{35}$$

4. **Revenue Calculation:**
   For each scenario j on day t, calculate the revenue $R_{t,j}$ using the objective function in equation 29.

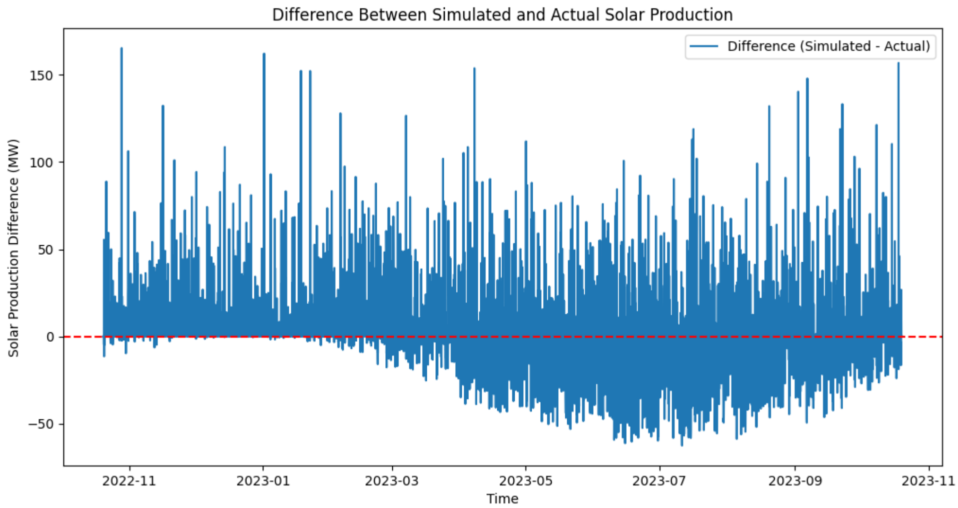

Figure 14: Difference between actual and simulated data when there is only 1 scenario

5. **Statistical Analysis:**
Perform statistical analysis on the set of revenues $\{R_{t,1}, R_{t,2}, \ldots, R_{t,m}\}$ for each day t. Calculate the expected revenue μ, the standard deviation σ, and the 5th and 95th percentiles P_5 and P_{95}:

$$\mu = \frac{1}{m} \sum_{j=1}^{m} R_{t,j} \tag{36}$$

$$\sigma = \sqrt{\frac{1}{m-1} \sum_{j=1}^{m} (R_{t,j} - \mu)^2} \tag{37}$$

$$P_5 = \text{5th percentile of } \{R_{t,1}, R_{t,2}, \ldots, R_{t,m}\} \tag{38}$$

$$P_{95} = \text{95th percentile of } \{R_{t,1}, R_{t,2}, \ldots, R_{t,m}\} \tag{39}$$

From the graphs (Fig. 14 - Fig. 17) it can be observed that as the number of scenarios increases, the variability in the difference tends to reduce, indicating a convergence to an expected value. This behavior is consistent with the Law of Large Numbers in probability theory, which states that as the number of trials increases, the average of the outcomes will tend to converge towards the expected value. The purpose of using Monte Carlo simulation in this context is to take into account the randomness and uncertainty in solar production forecasts. By generating multiple scenarios, we can understand the variability and risk associated with solar production. After running the model on all the scenarios, we can calculate the:

- **Expected Revenue** : This is the mean or average of the revenues from all the simulated scenarios. A negative value indicates that, on average, the model predicts a loss. This could happen if the costs associated with imbalances, buying energy, or other factors outweigh the revenue from selling energy.

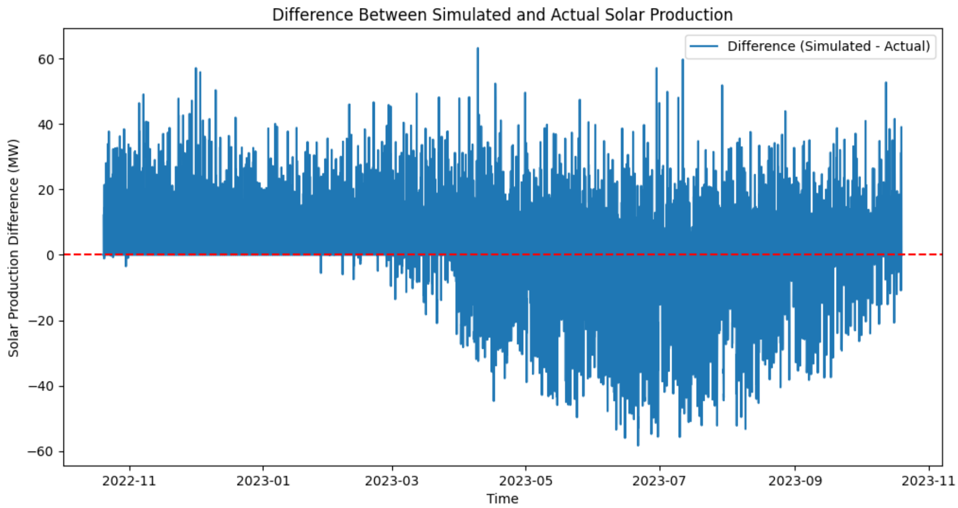

Figure 15: Difference between actual and simulated data when there are 5 scenarios

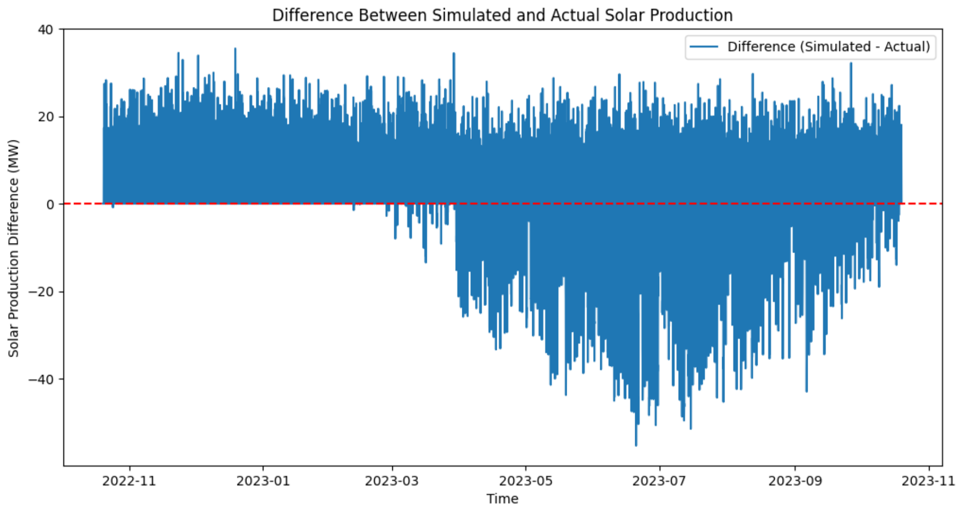

Figure 16: Difference between actual and simulated data when there are 20 scenarios

- **Revenue Standard Deviation** : This measures the amount of variability or dispersion from the average revenue. A higher standard deviation indicates a greater spread of outcomes, meaning there is higher uncertainty about the revenue outcome.

- **Percentile 5** : This is a measure of the lower boundary of revenue outcomes. Only 5 per cent of the outcomes are below this value. It is a way to understand the worst-case scenarios and risks involved.

- **Percentile 95** : This represents the upper boundary, with 95 per cent of the

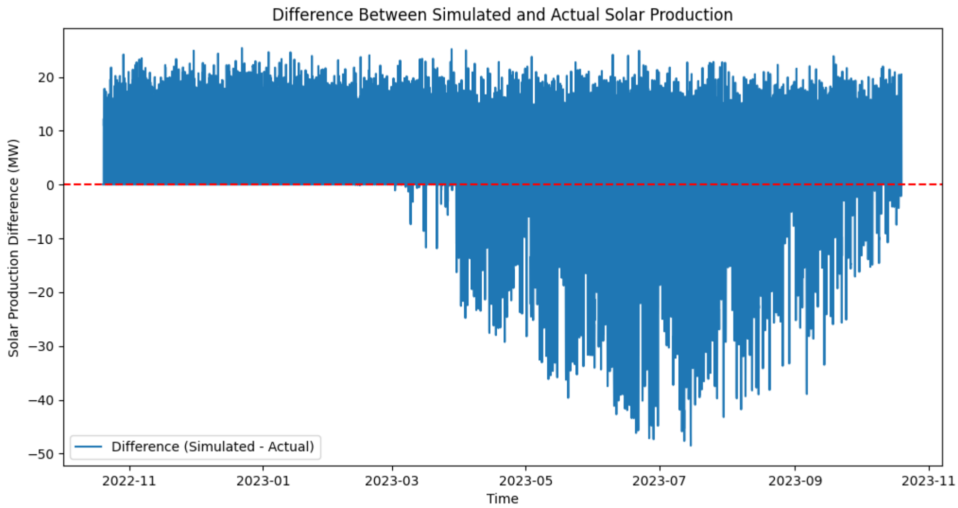

Figure 17: Difference between actual and simulated data when there are 100 scenarios

revenue outcomes falling below this value. It gives a sense of the best-case scenarios.

Fig. 18 illustrates the variability of revenue outcomes based on different percentiles. The green box represents the middle 90 per cent of all revenue outcomes. The central line (median) suggests that the median outcome is slightly negative, indicating that typically, the revenue is slightly less than the break-even point. The red box represents the top 5 per cent of all revenue outcomes, which are the best-case scenarios, where solar production was likely more accurate or favorable, resulting in positive revenue outcomes. The blue box represents the worst-case scenarios with significant negative revenue, likely due to substantial discrepancies between forecasted and actual solar production, leading to high costs in the imbalance market.
Based on the figures (Fig. 18,Fig. 19), we can see that if the actual solar production significantly deviates from what was forecasted and sold on the market, the energy provider must cover the imbalance by buying or selling the difference at potentially unfavorable prices, leading to financial losses. Therefore, investment in improving the accuracy of solar forecasts is of paramount importance to reduce exposure to imbalance markets and stabilize revenue.

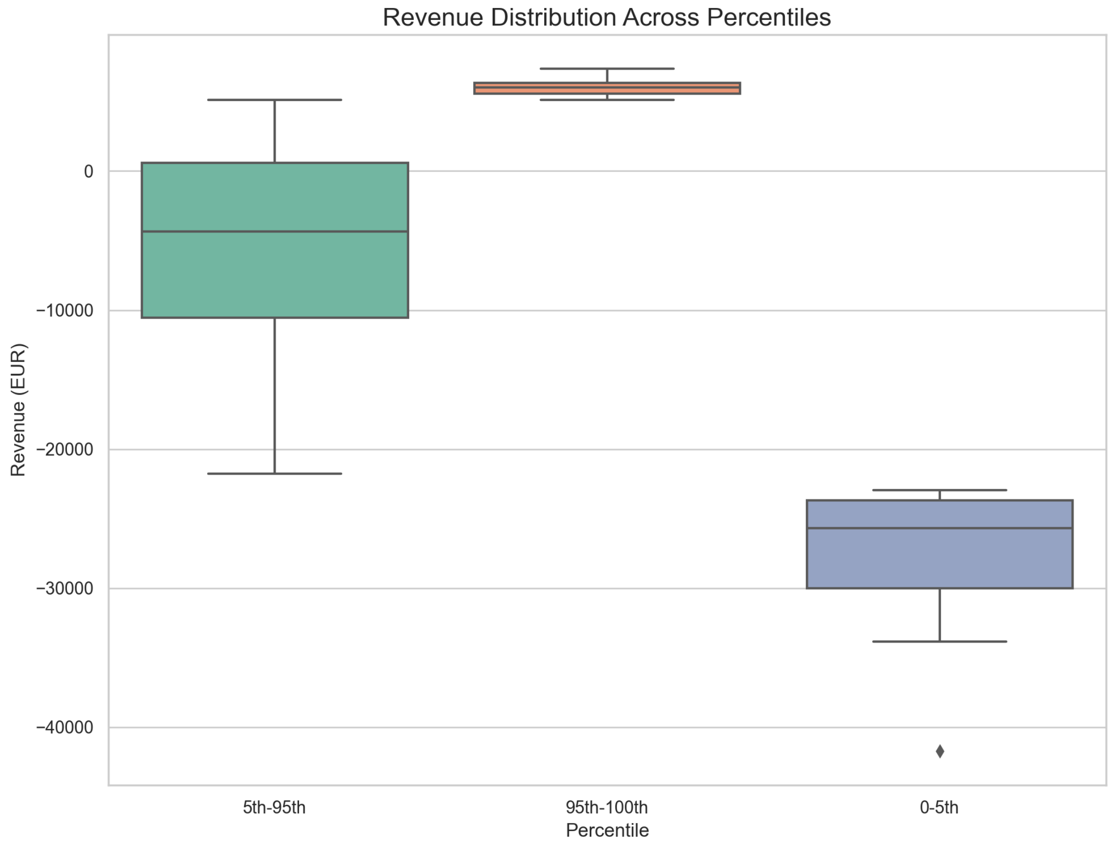

Figure 18: Visualising the spread of revenue outcomes, highlighting the 0th, 5th, 95th, and 100th percentiles on 100 simulations.

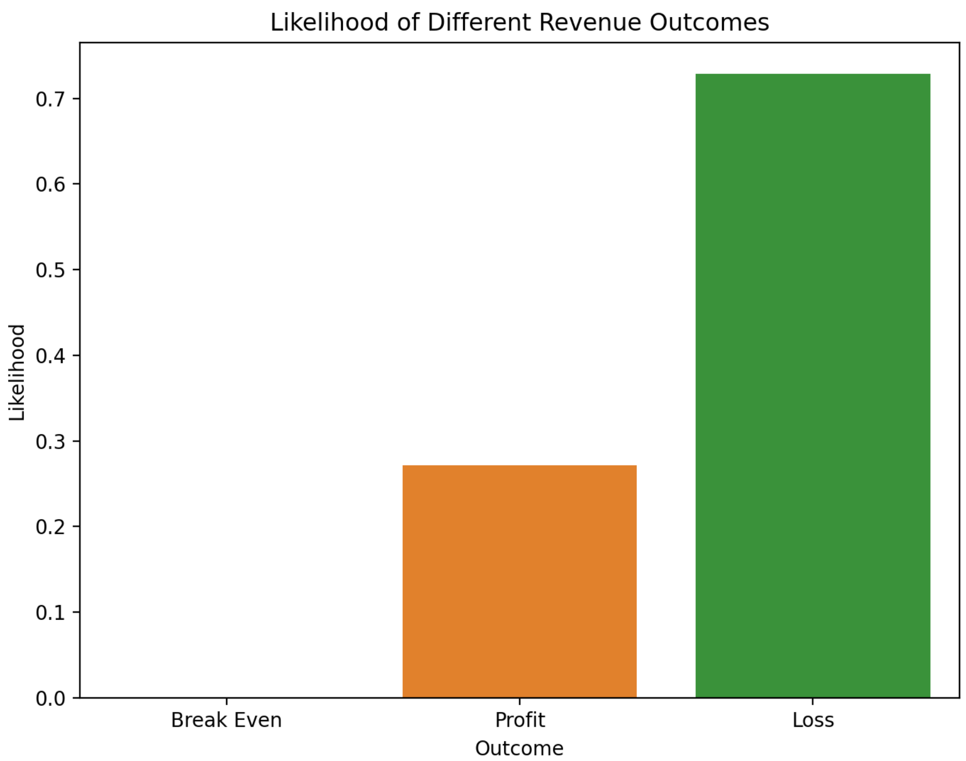

Figure 19: Likelihood of the returns on investment for the simulated solar production values

5 Conclusions and Future Directions

In conclusion, this thesis has successfully established a linear optimization framework for analyzing the viability and optimization of HES within the Nordic energy markets, focusing particularly on Finnish data. The thesis also dove into the foundational economic concepts governing energy markets and the inherent uncertainties in these markets. It also introduced Monte Carlo Simulations which was utilised to simulate various market scenarios, assessing the impact of these uncertainties on the performance and profitability of a hybrid energy system. The results obtained highlighted the effectiveness of the developed optimization model in managing uncertainties and maximizing revenue.

During the iterative model development process it was observed that on a dataset of 24 hours for a random day, The model shows a 20% increase in revenue on the addition of a 10 MW battery storage, a 24% increase in revenue on the allowing the ability to buy electricity from the grid and a 485% increase in revenue with hydrogen conversion and storage facilities implemented along with the other assets. The results indicated the importance of having a HES for the purposes of maximising revenue. The results also showed increased resilience towards the volatility of market prices, further strengthening the case for investing in a HES.

While an optimization model can enhance decision-making by proposing the most efficient use of resources based on predictions, it cannot fully compensate for inaccuracies in solar production forecasts. An optimization model is only as good as the data it is based on. If the solar production forecasts are inaccurate, even the most sophisticated optimization models may lead to suboptimal decisions and negative financial outcomes. This highlights a major limitation in a HES, which is the sources of energy being used and the accuracy of the respective source of energy being used. However, this does not weaken the case for a HES as inaccuracies in forecast will have a worse effect if there is no flexibility available that comes with the establishment of a HES.

A major area of focus to further make the case for a HES and improve decision making through the optimization framework will be the development of more accurate energy forecasting models. There can also be the inclusion of more than one source of energy, incorporation of intraday and flexibility markets in the model to further stabilise the revenue by providing additional maneuvering against the volatile market prices.

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A Python Code for Revenue Generation Model

```python
import pandas as pd
import numpy as np
import pulp as pl

from solarproductionforecast import *
from fetchprices import *

forecast_df, actual_df, df = solar_energy_production_forecast()

solar_production_forecast = forecast_df["Amount(MW)"].values
solar_production_actual = actual_df["Amount(MW)"].values
day_ahead_spot_prices = fetch_spot_prices()["Price(EUR/MW)"].values

positive_imbalance_price, negative_imbalance_price = extract_imbalance_prices()

tolerance = 1e-5  # You may adjust the tolerance level as necessary
big_M = 1000
data_points = 24  # The number of hours in the planning horizon, typically 24 for a single day.
time_frame = range(1,data_points)
grid_capacity_limit_MW = 50


# Battery Parameters
battery_capacity_MW = 10  # The maximum amount of energy that the battery can store, measured in Megawatts (MW).
battery_c_value = 0.5  #  Represents the battery's discharge rate. A C-value of 0.5 means the battery can be dis/charged at a rate that would deplete its capacity in 2 hours.
battery_efficiency = 0.9  # The efficiency of the battery in storing and discharging energy. A value of 0.9 (or 90%) means that 10% of energy is lost during charging and discharging cycles.
battery_discharge_limit = 0.8 * battery_capacity_MW # The maximum amount of energy that can be discharged from the battery at any given time, calculated as a percentage of the battery's capacity.
battery_lifetime_cycles = 5000  # The total number of charge-discharge cycles the battery can undergo before its capacity significantly degrades.
battery_SOC = 0 # Initial state of charge


# Hydrogen Parameters
hydrogen_storage_capacity_kg = 1250 # The total amount of hydrogen (in kilograms) that can be stored.
initial_hydrogen_storage = 0 # Initial amount of hydrogen in storage

electrolyser_size_MW = 6 # The capacity of the electrolyser, measured in Megawatts.
```

```
electrolyser_conversion_rate = 0.03333 # Conversion rate as a
    separate constant measured in MWh/kg.
electrolyser_efficiency = 0.6 # The efficiency of the electrolyser
    as a variable input.
electrolyser_conversion_efficiency = electrolyser_efficiency/
    electrolyser_conversion_rate # The formula for calculating
    electrolyser conversion efficiency in kg/MWh.

hydrogen_fuel_cell_size_MW = 10  # The capacity of the hydrogen
    fuel cell, measured in Megawatts.
fuel_cell_conversion_efficiency = 0.017 # The amount of electrical
    energy produced from 1 kg of hydrogen, measured in MWh/kg.

hydrogen_demand_flat = 100  # The constant amount of hydrogen
    demanded
hydrogen_market_price_EUR_per_kg = 6 # The selling price of
    hydrogen per kilogram
hydrogen_cost_price_EUR_per_kg = 6 # The cost price of hydrogen per
     kilogram



def revenue_generated():

    # Create the LP problem
    problem = pl.LpProblem("Revenue_Maximization", pl.LpMaximize)

    # Create decision variables
    day_ahead_vars = pl.LpVariable.dicts("Day_Ahead", (range(
        data_points)), lowBound=0, cat="Continuous") # The energy (
        MWh) sold in the day ahead market. Each element t denotes
        energy sold for delivery at hour t. All the deals are made
        for the next day.
    grid_buy_vars = pl.LpVariable.dicts("Grid_Buy", range(
        data_points), lowBound=0, cat="Continuous") # The energy (
        MWh) bought from the grid at each hour t.

    battery_charge_vars = pl.LpVariable.dicts("Battery_Charge",
        range(data_points), lowBound=0, cat="Continuous") # This
        variable represents the amount of energy (in MW) that is
        charged into the battery at each hour t within the planning
        horizon (defined by num_hours).
    battery_discharge_vars = pl.LpVariable.dicts("Battery_Discharge
        ", range(data_points), lowBound=0, cat="Continuous") #
        Represents the amount of energy (in MW) discharged from the
        battery at each hour t within the planning horizon.
    battery_action = pl.LpVariable.dicts("Battery_Action", range(
        data_points), cat="Binary") # Prevent simultaneous charging
        and discharging of the battery within the same hour using a
        Binary variable.
    battery_SOC_vars = pl.LpVariable.dicts("Battery_SOC", range(
        data_points), lowBound=0, upBound=battery_capacity_MW, cat="
        Continuous") # Decision variable for state of charge of the
        battery
```

```python

    # Decision variables for hydrogen
    hydrogen_storage_vars = pl.LpVariable.dicts("Hydrogen_Storage", range(data_points), lowBound=0, cat="Continuous") # Represents the amount of hydrogen (in kg) stored at each hour t within the planning horizon.
    electricity_to_hydrogen_vars = pl.LpVariable.dicts("Electricity_To_Hydrogen", range(data_points), lowBound=0, cat="Continuous") #  Total energy converted to hydrogen at each hour t in (MW)
    allow_hydrogen_sale_vars = pl.LpVariable.dicts("Hydrogen_Sale_Limit", range(data_points), cat="Binary")
    hydrogen_sold_vars = pl.LpVariable.dicts("Hydrogen_Sold", range(data_points), lowBound=0, cat="Continuous") #The amount of hydrogen sold (in kg)
    hydrogen_to_electricity_vars = pl.LpVariable.dicts("Hydrogen_To_Electricity", range(data_points), lowBound=0, cat="Continuous") # The hydrogen (in kg) converted to energy at each hour t.
    hydrogen_buy_vars = pl.LpVariable.dicts("Hydrogen_Bought", range(data_points), lowBound=0, cat="Continuous") # Amount of Hydrogen bought from the Hydrogen Market in kg.

    # Set the objective function
    problem += (
        pl.lpSum([day_ahead_vars[t] * day_ahead_spot_prices[t] for t in time_frame]) - pl.lpSum([grid_buy_vars[t] * day_ahead_spot_prices[t]] for t in range(data_points)) + pl.lpSum(hydrogen_sold_vars[t] * hydrogen_market_price_EUR_per_kg - hydrogen_buy_vars[t] * hydrogen_cost_price_EUR_per_kg for t in range(data_points))
        + pl.lpSum([max(0,solar_production_actual[t] - solar_production_forecast[t]) * positive_imbalance_price[t] + min(0,solar_production_actual[t] - solar_production_forecast[t]) * negative_imbalance_price[t] for t in range(data_points)])
    )

    # Add 50 MW grid buy/sell constraint
    for t in time_frame:
        problem += (
            day_ahead_vars[t] + grid_buy_vars[t] + abs(solar_production_actual[t] - solar_production_forecast[t])
        ) <= grid_capacity_limit_MW


    # Add the energy balance constraints
    for t in time_frame:
        problem += (
            day_ahead_vars[t] + battery_charge_vars[t] + electricity_to_hydrogen_vars[t] + solar_production_actual[t-1] -
```

```
                solar_production_forecast[t-1]
            <=  battery_discharge_vars[t]*battery_efficiency +
                grid_buy_vars[t] + hydrogen_to_electricity_vars[t]*
                fuel_cell_conversion_efficiency
        )

    # Battery operational constraints
    for t in range(data_points):
        problem += battery_charge_vars[t] <= battery_capacity_MW *
            battery_c_value

        problem += battery_discharge_vars[t] <=
            battery_discharge_limit * battery_c_value

        problem += battery_SOC_vars[t] - battery_discharge_vars[t]
            >= 0
        problem += battery_SOC_vars[t] + battery_charge_vars[t] <=
            battery_capacity_MW

        problem += battery_charge_vars[t] <= big_M * (1 -
            battery_action[t])
        problem += battery_discharge_vars[t] <= big_M *
            battery_action[t]

    # Set the initial conditions outside the loop
    problem += battery_charge_vars[0] == 0
    problem += battery_discharge_vars[0] == 0

    # Constraints for state of charge
    problem += battery_SOC_vars[0] == battery_SOC +
        battery_charge_vars[0] - battery_discharge_vars[0]
    for t in time_frame:
        problem += battery_SOC_vars[t] == battery_SOC_vars[t-1] +
            battery_charge_vars[t] - battery_discharge_vars[t]

    # Constraints to ensure state of charge is within bounds
    for t in range(data_points):
        problem += battery_SOC_vars[t] <= battery_capacity_MW
        problem += battery_SOC_vars[t] >= 0


    # Hydrogen operational constraints
    for t in range(data_points):
        problem += electricity_to_hydrogen_vars[t] <=
            electrolyser_size_MW # Amount of electricity we convert
            to hydrogen should always be less than the size of the
            electrolyser
        problem += hydrogen_to_electricity_vars[t] *
            fuel_cell_conversion_efficiency <=
            hydrogen_fuel_cell_size_MW  # Amount of electricity we
            get from hydrogen should always be less than the size of
             the Fuel cell

    # Constraints for hydrogen storage dynamics
```

```python
    problem += hydrogen_storage_vars[0] == initial_hydrogen_storage - hydrogen_to_electricity_vars[0] + electricity_to_hydrogen_vars[0] * electrolyser_conversion_efficiency - hydrogen_sold_vars[0] + hydrogen_buy_vars[0]
    for t in time_frame:
        problem += hydrogen_storage_vars[t] == hydrogen_storage_vars[t-1] - hydrogen_to_electricity_vars[t] + electricity_to_hydrogen_vars[t] * electrolyser_conversion_efficiency - hydrogen_sold_vars[t] + hydrogen_buy_vars[t]

    # Constraints to ensure hydrogen storage is within bounds
    for t in range(data_points):
        problem += hydrogen_storage_vars[t] <= hydrogen_storage_capacity_kg
        problem += hydrogen_storage_vars[t] >= 0

    # Ensure that the sum of hydrogen sold and hydrogen used to meet demand does not exceed the hydrogen stored
    for t in range(data_points):
        problem += hydrogen_storage_vars[t] >= hydrogen_demand_flat
        problem += hydrogen_sold_vars[t] <= 1500 * allow_hydrogen_sale_vars[t]
        problem += hydrogen_to_electricity_vars[t] <= 1500 * allow_hydrogen_sale_vars[t]
        problem += electricity_to_hydrogen_vars[t] <= big_M*(1-allow_hydrogen_sale_vars[t])
        problem += hydrogen_buy_vars[t] <= big_M*(1-allow_hydrogen_sale_vars[t])


    # Limit the number of times hydrogen can be sold in a day
    problem += pl.lpSum(allow_hydrogen_sale_vars[t] for t in range(data_points)) <= 1


    # Solve the problem and print the results
    problem.solve()
    print("Status: ", pl.LpStatus[problem.status])
    print("Optimal revenue: ", pl.value(problem.objective))

revenue_generated()
```

Listing 1: Python code for revenue generation optimisation model

The GitHub repository for the optimization model can be found at this link.