

Aalto University
School of Science
Master's Programme in Mathematics and Operations Research

Olli Herrala

An efficient strategy for solving stochastic programming problems under endogenous and exogenous uncertainties

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Supervisor: Prof. Fabricio Oliveira
Advisor: Research Prof. Tommi Ekelholm

Author:	Olli Herrala	
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Advisor:	Research Prof. Tommi Ekhholm	
	<p>Despite multi-stage decision problems being common in production planning, there is a class of such problems for which a general solution framework does not exist, namely problems with endogenous uncertainty. Methods from decision analysis and stochastic programming can be used, but both require significantly constraining assumptions. In order to overcome the current challenges, Decision Programming combines approaches from these two fields, making it possible to acquire optimal strategies for different decision problems.</p> <p>Decision Programming is strictly limited to problems in which uncertainty events and decisions are taken from a finite discrete set, reducing its applicability to problems with continuous decision spaces. Discretizing a continuous decision space increases the problem size and can lead to computational intractability.</p> <p>This thesis presents a problem decomposition approach extending the Decision Programming framework. The decomposition approach allows for considering continuous decision and uncertainty spaces in problems with a suitable structure. The proposed framework is applied to three different problems, including a large-scale production planning problem from literature. The main example in this thesis is a novel approach on climate change mitigation cost-benefit analysis, where R&D is carried out simultaneously with the emissions abatement decisions. The R&D projects provide information on the climate damage severity and decrease the price of abatement. Problems with similar structure have not been discussed in the literature, and the extended Decision Programming framework is able to solve the problem to optimality.</p>	
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	<p>Vaikka monivaiheiset päätöksenteko-ongelmat ovat yleisiä tuotannon suunnittelussa, erääseen ryhmään näitä ongelmia ei ole yleistä ratkaisumenetelmää. Tämä johtuu niinsanotusta endogeenisestä epävarmuudesta. Näihin ongelmiin voidaan soveltaa stokastisen optimoinnin ja päätösanalyysin menetelmiä, mutta kummatkin vaativat merkittäviä rajoittavia oletuksia. Uusi menetelmä, Decision Programming, yhdistää stokastisen optimoinnin ja päätösanalyysin menetelmiä mahdollistaen optimistrategioiden löytämisen erilaisissa päätösongelmissa.</p> <p>Decision Programming rajoittuu ongelmiin joissa satunnaistapahtumat ja päätökset valitaan äärellisistä diskreeteistä joukoista. Tämä rajoittaa sen soveltuvuutta ongelmiin joissa päätösjoukot ovat jatkuvia, sillä tällaisen päätösjoukon diskretointi kasvattaa ongelman kokoa ja saattaa johtaa laskennallisiin haasteisiin.</p> <p>Tässä työssä esitellään Decision Programming -viitekehystä laajentava hajotusmenetelmä jonka avulla voidaan ratkaista ongelmia, jotka sisältävät jatkuvia päätös- ja epävarmuusjoukkoja. Menetelmän soveltaminen vaatii kuitenkin ongelmalta sopivan rakenteen. Työssä esitettyä menetelmää sovelletaan kolmeen esimerkkiongelmaan, joista yksi on suuren mittakaavan tuotannosuunnitteluongelma kirjallisuudesta. Työn pääesimerkki on uudenlainen lähestymistapa ilmastonmuutoksen hillinnän kustannus-hyötyanalyysiin, jossa tutkimustyötä tehdään samanaikaisesti päästövähennysten kanssa. Tutkimusprojekteilla saadaan lisätietoa ilmastovaikutusten vakavuudesta ja lasketaan päästövähennysten hintaa. Vastavanlaisia ongelmia ei ole ennen käsitelty kirjallisuudessa ja laajennettu Decision Programming -viitekehys mahdollistaa optimiratkaisun löytämisen tässä esimerkissä.</p>		
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Chapter 1

Introduction

In order to make good decisions, individuals and institutions must consider the consequences of their choices as well as possible uncertainties. A single, independent decision is often easy to make, but when it has an uncertain impact on the future, thus affecting later decisions, the problems become more complicated. These *multi-stage decision problems* are common in e.g. production planning.

1.1 Problem statement

Despite the prevalence of multi-stage decision problems, there is no general framework for solving them. Decision analysis is often used, since influence diagrams and corresponding decision trees are relatively easy to build, and their generality allows for representation of several types of interaction between decisions and uncertainties. However, influence diagrams are hard to solve, as they are often solved using recursion-based solution methods. Additionally, forming a decision tree requires all decisions and uncertainties to be discrete, and the earlier decisions and uncertainty realizations must be known at the time of any given decision, which is known as the no-forgetting assumption.

Continuous decisions can be handled by stochastic programming, and the deterministic equivalents of stochastic problems are often relatively easy to solve. However, endogenous uncertainty, where decisions have an effect on future uncertainties, leads to larger problems in which computational tractability can become an issue without exploiting problem-specific knowledge. Different types of uncertainty are discussed in Chapter 2.

A novel approach called Decision Programming attempts to combine the strengths of influence diagrams and stochastic programming. Currently, it is

capable of combining the expressiveness of influence diagrams with the deterministic equivalent formulations from stochastic programming and efficiently solving problems with both exogenous and endogenous uncertainties as linear programming problems. Decision Programming is explained in more detail in Chapter 3.

1.2 Scope of the thesis

Despite its efficiency, Decision Programming still inherits the requirement of discrete decision and uncertainty spaces from influence diagrams. This could be dealt with by discretizing any continuous variables, but this often leads to problems that are too large to be computationally tractable. In this thesis, we extend the existing Decision Programming framework to be able to handle partly continuous problems with an exploitable structure, which is further explained and experimented in Chapter 4. The extended framework is tested on a large scale real-world problem related to climate change in Chapter 5. This problem was our original motivation for the presented developments. The overall results are summarized and concluded in Chapter 6.

Chapter 2

Literature review

2.1 Stochastic programming

In 1955, Dantzig [13] described how linear programming could be extended to include uncertainty. From this paper, a new field called stochastic programming quickly emerged. Stochastic programming is a field of mathematical programming that aims at incorporating uncertainty of the input data into an originally deterministic setting. This is achieved using scenarios, that is, combinations of the realizations of uncertain parameters. The example from Dantzig's paper concerns shipping items from a factory to an outlet, where the demand is uncertain. Another example of stochastic programming is recourse problems [42], where decisions are made in at least two stages, and the uncertainty is resolved between stages. The decision processes can incorporate more complicated decision settings, for example in climate change mitigation discussed in Chapter 5, where the uncertainties in climate sensitivity and damages are gradually resolved over time in a scenario tree [16].

In stochastic programming, uncertainties are divided in two main categories. The first and more widely researched type is called *exogenous uncertainty*. In exogenous uncertainty, the decisions have no effect on the probability distribution or the observed outcome of the future chance events. A simple example of this is a basic lottery, where the decision is to buy a ticket with a given probability of winning, and the uncertainty is resolved after buying the ticket. The decision has no effect on the winning probability of a ticket or the time at which the uncertainty is revealed.

The approach for solving problems with uncertainty is based on representing the problem as a scenario tree, from which a deterministic equivalent of the problem is constructed. A scenario is a combination of uncertainty realizations, and the objective function usually consists of an expected value

over all scenarios. Ruszczyński [37] explains this process in detail, defining the so-called non-anticipativity constraints (NACs). However, deterministic equivalent problems are often large-scale optimization models [8]. This results from the scenario representation requiring a potentially large number of NACs and independent decision variables for each scenario, which can in turn lead to computational intractability.

Stochastic programming with exogenous uncertainty has been researched widely, and as exogenous uncertainty is not the main focus of this thesis, we refer to [38] for a comprehensive review on exogenous uncertainty modelling.

The other type of uncertainty is called *endogenous uncertainty*, where the decisions can affect the uncertainty. This is a considerably more difficult class of problems, mostly because of the dependence of uncertainties increasing the problem size, and the lack of comprehensive and efficient frameworks [12]. Efficient, problem-specific solution methods based on heuristics and problem structure have been developed for, e.g., the capacity expansion problem in [18], [20] and [3], further explored in Chapter 4.

Table 2.1 presents selected references from the literature, specifying the types of uncertainty handled in each paper. It can be seen that the first paper considering endogenous uncertainties was published more than four decades after the first paper on stochastic optimization in general [13]. In the paper, Jonsbråten et al. [26] consider a subcontracting problem, where the production costs of different products are uncertain, but are learned when production is started.

Table 2.1: The classification of uncertainties covered in selected papers

Reference	Exog.	Type 1 Endog.	Type 2 Endog.
Dantzig [13] (1955)	x		
Walkup and Wets [42] (1967)	x		
Ruszczyński [37] (1997)	x		
Jonsbråten et al. [26] (1998)	x		x
Lauritzen and Nilsson [27] (2001)	x	x	
Sahinidis [38] (2004)	x		
Goel and Grossmann [18] (2006)	x		x
Colvin and Maravelias [11] (2008)			x
Peeta et al. [35] (2010)		x	
Gupta and Grossmann [19] (2011)	x		x
Calfa [10] (2014)	x		
Hellemo [23] (2016)	x	x	
Apap and Grossmann [3] (2017)	x		x
Salo et al. [39] (2019)	x	x	
Parmentier et al. [34] (2019)	x	x	
This thesis (2020)	x	x	x

Goel and Grossmann [18] further specify that there exist at least two types of endogenous uncertainty, to which Apap and Grossmann [3] and Hellemo [23] refer as Type 1 and Type 2. In Type 1 uncertainty, the decision maker can influence the underlying probability distributions, e.g., investing on a marketing campaign to potentially increase demand. In problems with Type 2 uncertainty, the decision maker is unable to influence the underlying probabilities, and the uncertainties are thus exogenous. However, the resolution of the uncertainty is conditioned to the timing of the decision, as in the subcontracting problem in [26], where the production costs are learned instantly when production is started (instantaneous revelation) or the oil-field infrastructure planning in [21], where one year of production is required before the size of the field can be determined (delay in revelation). While decisions in Type 2 uncertainty do not change the underlying probability distribution, there is often a positive value of (im)perfect information. Value of information is the increase in utility resulting from (im)perfect information regarding the decision process.

Hereinafter, we refer to problems with Type i uncertainty as Type i problems. Of the two types, Type 2 problems are the more widely researched. In the literature review by Apap and Grossmann [3], only eight publications considering Type 1 problems are listed, while Type 2 problems researched in the same period include gas-field development [18], scheduling of pharmaceu-

tical clinical trials [11] and process networks with uncertain yields [19]. As stated by Apap and Grossmann [3], the two types of endogenous uncertainty are not mutually exclusive, and the combination of Type 1 and 2 uncertainties is referred to as Type 3 by Hellemo [23]. Hellemo et al. [24] mention that Type 3 problems have not been discussed in the literature so far.

2.2 Influence diagrams

In decision analysis, multi-stage decision problems are often represented as influence diagrams [25] instead of mathematical programming formulations as in stochastic programming. Influence diagrams are acyclic graphs consisting of decision, chance and value nodes connected by arcs. An example of an influence diagram is found later in Fig. 4.1. The square nodes represent decisions, circular nodes represent uncertainty, and diamond shaped nodes are value nodes. The meaning of the arcs depends on the target node. For a chance node, the incoming arcs determine the nodes that affect the uncertainty realization, while for a decision node, the incoming arcs show which states are known when making the decision. For a value node, the arcs show the nodes that affect the utility value calculation.

If the so-called no-forgetting assumption holds, meaning that earlier decisions are known when making later ones, an influence diagram can be solved by forming an equivalent decision tree and employing dynamic programming or other well-established techniques. However, in many settings, the no-forgetting assumption does not hold. For such problems, Lauritzen and Nilsson [27] present Limited Memory Influence Diagrams (LIMID) with a solution method termed Single Policy Updating (SPU). SPU is guaranteed to find an optimal policy only if the diagram is soluble, as defined in [27]. However, Mauá et al. [28] show that finding an optimal strategy for such problems is NP-hard.

In addition to the problems of the no-forgetting assumption, influence diagrams suffer from the inability to include chance, logical or resource constraints, because the decisions made in a given node cannot depend on the other branches of a decision tree representation. In order to include such constraints, a stochastic programming formulation of the problem is required, as in the portfolio problem by Gustafsson and Salo [22].

2.3 Conclusions

Despite the advances in both stochastic programming and decision analysis, there are still many problems that are difficult to solve with the current methods. First, the no-forgetting assumption restricts both frameworks, despite SPU and other approaches in decision analysis. As seen in Table 2.1, the research on endogenous uncertainty, especially Type 1, is relatively scarce because there are few general methods in stochastic programming for dealing with such problems. With influence diagrams, endogenous uncertainty can be easily modelled, but the decisions need to be discrete for one to be able to form the equivalent decision tree and thus solve the problem. Influence diagrams also suffer from the inability to include constraints that depend on multiple branches of the decision tree, such as risk constraints.

The next section presents Decision Programming [39], a novel framework combining the expressiveness of LIMIDs and the efficient solution methods of stochastic programming, thus overcoming both the limitations of the no-forgetting assumption and the decision tree representations of the problem, however still retaining the requirement of discrete nodes in the influence diagram. Parmentier et al. [34] also present a solution method based on an influence diagram formulation of the decision problem. Instead of directly converting the influence diagram to a mixed integer linear programming (MILP) problem as in Decision Programming, the influence diagram is first modified into a rooted junction tree (a thorough explanation can be found in [34]) and the problem is then represented as a MILP program.

This thesis extends Decision Programming towards allowing continuous nodes in some parts of the influence diagram representation. This is achieved by exploiting problem structure and enables us to solve challenging problems without discretizing the continuous nodes, avoiding explosions in the number of scenarios leading to computational intractability. Additionally, we apply Decision Programming to problems with Type 2 endogenous uncertainty, something that was not presented in the original paper on Decision Programming [39].

Chapter 3

Framework

To address the limitations of no-forgetting and endogenous uncertainty in decision analysis and stochastic programming, Salo et al. [39] developed a novel approach called Decision Programming. Decision programming formulates a decision problem as a mixed integer linear programming (MILP) problem, which can be efficiently solved using off-the-shelf solvers. In addition to solving the issues discussed above, the approach also enables considering multiple objectives and chance constraints.

The approach is based on an influence diagram representation of the problem, where each decision and chance node has a finite number of possible states, and scenario paths consisting of a single state for every node in the diagram. Let us define the information set $I(j)$ as the set of nodes from which there is an arc to node j and information states $s_{I(j)} \in S_{I(j)} = \prod_{i \in I(j)} S_i$ as the combination of states s_i for nodes $i \in I(j)$. Because the influence diagram is acyclic, we can order the nodes such that $i < j$. A scenario path is defined as $s = (s_1, \dots, s_n) \in S = \prod_{i \in C \cup D} S_i$, where C and D are the sets of chance and decision nodes, respectively. We use the notation $z(s_j | s_{I(j)}) \in \{0, 1\}$ for decisions conditional on their respective information state, and define a local decision strategy $Z_j : S_{I(j)} \mapsto S_j$ for decision node j as a function that maps all information states to corresponding decisions. This way, we can define a decision strategy to be compatible with path s if $Z(s) = \{Z_j \in Z \mid z(s_j | s_{I(j)}) = 1, j \in D\}$. Compatibility means in practice that the decision strategy maps the information states of the decision nodes to the path s . The basic formulation of Decision Programming is shown in (3.1)-(3.6).

$$\max_{Z \in \mathbb{Z}} \sum_{s \in S} \pi(s) \mathcal{U}(s) \quad (3.1)$$

$$\text{s.t.} \quad \sum_{s_j \in S_j} z(s_j | s_{I(j)}) = 1, \quad \forall j \in D, s_{I(j)} \in S_{I(j)} \quad (3.2)$$

$$0 \leq \pi(s) \leq p(s), \quad \forall s \in S \quad (3.3)$$

$$\pi(s) \leq z(s_j | s_{I(j)}), \quad \forall s \in S \quad (3.4)$$

$$\pi(s) \geq p(s) + \sum_{j \in D} z(s_j | s_{I(j)}) - |D|, \quad \forall s \in S \quad (3.5)$$

$$z(s_j | s_{I(j)}) \in \{0, 1\}, \quad \forall j \in D, s_j \in S_j, s_{I(j)} \in S_{I(j)}. \quad (3.6)$$

The objective function (3.1) is the expected utility where s denotes a scenario path, while $\pi(s)$ and $\mathcal{U}(s)$ are the corresponding path probabilities and utility values. A decision strategy $Z \in \mathbb{Z}$ consists of decisions $z(s_j | s_{I(j)}) \in \{0, 1\}$ for all $j \in D$. The variable $z(s_j | s_{I(j)})$ attaining value 1 corresponds to a strategy in which at the decision node $j \in D$, the information state $s_{I(j)}$ is mapped to the decision $s_j \in S_j$. Constraint (3.2) states that in each decision node, given an information state, only one alternative can be chosen. The next three constraints define the path probabilities $\pi(s)$. Inequality (3.3) states the non-negativity of the path probabilities, as well as an upper limit $p(s) = \prod_{j \in C} P(X_j = s_j | X_{I(j)} = s_{I(j)})$, which is given by the conditional probabilities calculated along the path s . This implies that the actual path probability cannot exceed an upper limit given by the conditional chance event probabilities. The next constraint (3.4) sets $\pi(s)$ to zero if the decision strategy is incompatible with the path s . With these two constraints, assuming positive utilities $\mathcal{U}(s)$, $\pi(s)$ is set to the upper bound $p(s)$ if the decision strategy is compatible with path s and 0 otherwise. However, utilities can be negative and thus (3.5), where $|D|$ is the number of decision nodes in the problem, is needed to keep the probabilities of paths in the chosen decision strategy equal to $p(s)$.

As shown in [39], Decision Programming allows for solving a spatially distributed decision process where n decisions are made in parallel with no communication between the decision makers and thus, the no-forgetting assumption does not hold. The problem is solved in a reasonable time for up to 8 decision makers. The number of nodes in this problem is 19, including the value node, the number of paths is $2^{18} = 262144$, and the problem can be shown to not be soluble as defined in [27]. Therefore, while the single policy updating has no guarantee on convergence to optimal strategies, Decision

Programming as a MILP problem is guaranteed to converge to the global optimum.

Decision Programming is a significant contribution to multi-stage decision making and stochastic programming, enabling multi-objective decision making, relaxing the no-forgetting assumption and handling constraints involving risk factors such as value at risk (VaR). However, the approach is strictly limited to discrete, finite state spaces, greatly narrowing the set of potential applications. The following section presents an approach for relaxing this requirement in problems with a suitable structure.

Chapter 4

Novel developments

The development of the extended framework is presented in this chapter, starting with an illustrative example and the description of the decomposition approach. The decomposition is first tested on a simple problem and compared to a deterministic equivalent MILP formulation of the problem without Decision Programming. After this, the framework is used on a large-scale production planning problem from literature, and the results are compared with those from earlier solution approaches.

4.1 Decomposition

4.1.1 Illustrative example

In order to illustrate the challenges of discrete and finite decision and uncertainty spaces to which the earlier formulation of Decision Programming was limited, we introduce an example describing a product development process, presented as an influence diagram in Fig. 4.1.

First, a new product is chosen for development from a set of product concepts, the development process having multiple alternative outcomes, namely failure, partial success and full success, the probability distribution depending on the product. After the product development is finished, it is possible to launch a marketing campaign for the product. When the binary go/no-go marketing decision has been made, the price of the product is decided, and the profit is determined considering the price and the uncertain demand.

This is a relatively simple influence diagram with decision dependent probabilities, but formulating this as a Decision Programming problem requires discrete and finite decision and uncertainty spaces, i.e., defining sets of possible prices and demand values. Methods for optimal discretization of

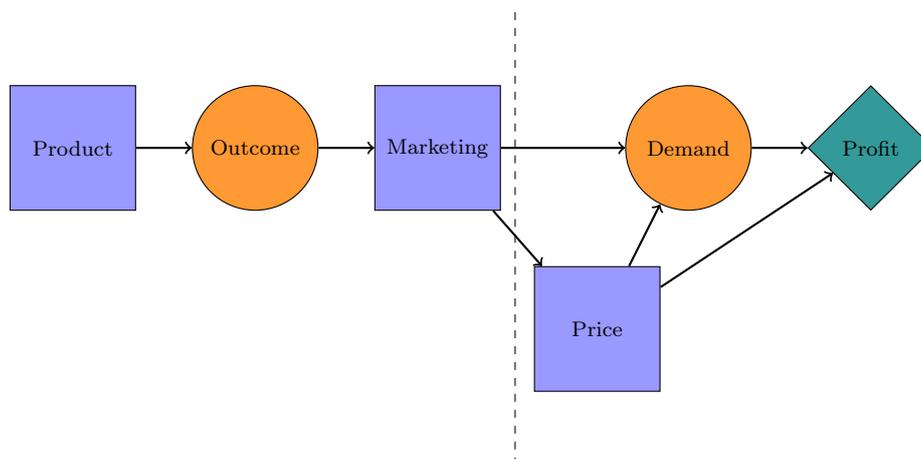


Figure 4.1: Influence diagram of the product development problem

probability distributions exist, see [36]. Despite being theoretically straightforward, this approach is limited in practice, as the total number of scenarios quickly increases when nodes are more finely discretized. This in turn leads to an increase in the number of variables and constraints in the Decision Programming formulation (3.1)-(3.6). As the size of a MILP problem increases, the running time typically increases more than linearly, ultimately leading to computational intractability.

4.1.2 Decomposition approach

If a problem has a suitable structure, one can divide it into separable parts and employ alternative solution approaches to each part. This way, one can circumvent the limitations arising from the trade-off between precision and computational tractability, resulting from the discretization. If the influence diagram can be ordered in a way where the endogenous decisions are discrete and made before any decision that requires being represented by continuous variables, the continuous portion of the problem can be replaced with an optimization problem that depends on the path s from the Decision Programming model. In Fig. 4.1, this separation is shown as a dashed line. While the demand uncertainty is endogenous, we assume that the company has conducted enough research on price elasticity, thus being able to estimate the demand distribution for different price points. This optimization problem can be seen as an extra node at the end of the diagram, and it might encompass a combination of multiple chance, decision and utility nodes. In practice, this *optimization node* is an arbitrary mathematical programming problem $\min\{f(x, s) : g(x, s) \geq 0; h(x, s) = 0\}$ dependent on the scenario

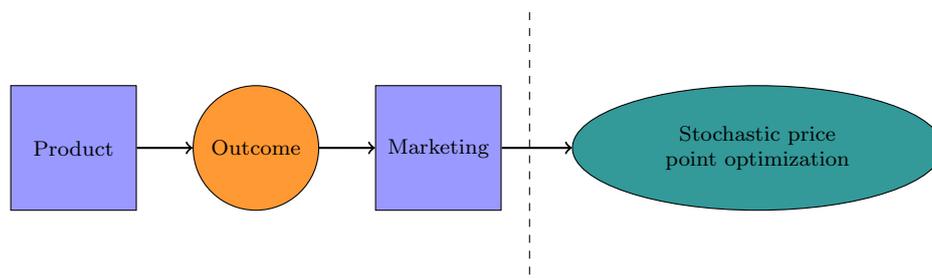


Figure 4.2: Influence diagram of the product development problem with the continuous part represented as a single node

path s in the Decision Programming influence diagram. The simplified influence diagram of the illustrative problem is shown in Fig. 4.2. In this illustrative example, the optimization node only consists of a single decision and chance node, along with the value node. In the problems presented later in this thesis, the optimization nodes are significantly more complicated.

A potential challenge with this approach is that we need to solve the optimization problem for all valid state combinations of relevant nodes, namely each scenario path $s \in S$. If the optimization node itself is complicated, possibly nonlinear and/or stochastic, this process could take considerably long. However, these problems are independent for individual paths and can thus be solved in parallel, reducing running times. Another possibility is that the utility value might be the same for different paths, e.g. in situations where the utility does not depend on a node in the discrete part. In such cases, fewer path utility calculations are needed as a single utility corresponds to multiple paths.

4.2 Motivating example

As a motivating example, we use a production planning example, modified from [41]. A company named Reddy Mikks produces interior and exterior paints, using raw materials $M1$ and $M2$. Table 4.1 presents the basic data. Additionally, the daily demand for interior paint cannot exceed that of exterior paint by more than 1 ton, and the maximum daily demand of interior paint is 2 tons. Apart from these constraints, demand is not limited. This leads to a profit maximizing linear optimization problem (4.1)-(4.6).

$$\text{maximize } z = 5x_1 + 4x_2 \quad (4.1)$$

$$\text{s.t. } 6x_1 + 4x_2 \leq 24 \quad (4.2)$$

$$x_1 + 2x_2 \leq 6 \quad (4.3)$$

$$-x_1 + x_2 \leq 1 \quad (4.4)$$

$$x_2 \leq 2 \quad (4.5)$$

$$x_1, x_2 \geq 0, \quad (4.6)$$

which has an optimal solution in $(x_1, x_2) = (3, 1.5)$ with a total profit of \$21000.

Table 4.1: The basic data for Reddy Mikks

	Tons of raw material per ton of		Maximum avail- ability (tons)
	Exterior paint	Interior paint	
Raw material, $M1$	6	4	24
Raw material, $M2$	1	2	6
Profit per ton (\$1000)	5	4	

As an extension of the original problem, we consider that the company can opt for developing a R&D project in two phases, where they can first choose to invest in one of three different R&D programs or not invest at all. After observing whether the project succeeded, they can again invest in one of the three projects. The projects, with their corresponding costs and first-phase success probabilities are presented in Table 4.2. In the first two projects, the goal is to increase the efficiency of producing the raw materials in a related process, thus increasing the availability in paint production. The third project can be seen as a marketing campaign that, if successful, increases the revenue and consequently, the profits. If a project succeeds in the first phase, it cannot be completed again, and if a project fails, it will decrease the probability of failing the same project in the second phase by 50% because of lessons learned from the failed project. Additionally, the raw material related projects are similar enough for a project in the other to decrease the probability of second phase failure in the other by 25%.

Table 4.2: The R&D projects available for Reddy Mikks Company

Project	Cost (\$1000)	Probability of success
Increase $M1$ availability by 10% (M1)	0.75	0.4
Increase $M2$ availability by 10% (M2)	1.20	0.6
Increase profits by 10% (PPT)	0.90	0.55

From this problem description, we can develop formulation (4.7)-(4.12) for each set of project decisions and uncertainty realizations. The decisions are denoted by binary variables $D_{i,j}$, where $i \in \{1, 2\}$ is the R&D phase and $j \in \{M1, M2, PPT\}$ one of the R&D projects listed in Table 4.2. The uncertain outcomes of these projects, success or failure, are denoted with binary random variables C_i . The value of the decision variables $D_{i,j}$ is 1 if project j is executed in phase i , and for the chance nodes, $C_i = 1$ if the project in phase i succeeds, 0 otherwise. The probability distribution of C_1 depends on the chosen project, while C_2 additionally depends on the first phase outcome. However, these dependencies have been omitted in this illustrative notation for clarity. The success of a project is represented with the indicator function $\mathbb{I}_j(C, D)$, depending on the decisions and uncertainty realizations as presented in Table 4.3. Only the seven feasible combinations are presented, the total number of combinations is $2^4 = 16$. The value of the indicator function is 1 if and only if the project has been performed in phase i and it is successful ($C_i = 1$), otherwise it is zero.

Table 4.3: The logic table for indicator function $\mathbb{I}_j(C, D)$

$D_{1,j}$	C_1	$D_{2,j}$	C_2	$\mathbb{I}_j(C, D)$
0	0	0	0	0
0	0	1	0	0
0	0	1	1	1
1	0	0	0	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1

$$\max z = (1+0.1(\mathbb{I}_{PPT}(C, D)))(5x_1 + 4x_2) - \text{R\&D costs} \quad (4.7)$$

$$\text{s.t. } 6x_1 + 4x_2 \leq (1 + 0.1(\mathbb{I}_{M1}(C, D)))24 \quad (4.8)$$

$$x_1 + 2x_2 \leq (1 + 0.1(\mathbb{I}_{M2}(C, D)))6 \quad (4.9)$$

$$-x_1 + x_2 \leq 1 \quad (4.10)$$

$$x_2 \leq 2 \quad (4.11)$$

$$x_1, x_2 \geq 0 \quad (4.12)$$

The modified problem (4.7)-(4.12) extends the original (4.1)-(4.6), in that if some of the projects succeed, the corresponding profits or availability constraints are correspondingly increased due to technological development. However, this formulation requires an appropriate modeling of the indicator function \mathbb{I}_j . In what follows, we present two approaches for solving this problem in a linear form. First, the deterministic equivalent of the two-stage stochastic problem is formulated. Next, the problem is represented as an influence diagram and solved using Decision Programming with the original (continuous) optimization problem as the optimization node.

4.2.1 MILP

The stochastic MILP model corresponding to the deterministic equivalent problem is presented in (4.13)-(4.31) and the notation in Table 4.4. Eq. (4.13) is the objective function, consisting of the expected values of three terms: the base profit without considering project PPT, the additional profit from project PPT, and the costs of the projects. Because of the type 1 endogenous uncertainty, the indices d_1, c_1, d_2, c_2 cannot be replaced with scenarios $s \in S$, constructing a traditional scenario tree with non-anticipativity constraints. The scenario probabilities in Type 1 problems are conditional on earlier decisions, rendering the scenario tree formulation incompatible with such problems.

Table 4.4: Notation description for (4.13)-(4.31), decision variables in **bold**

Symbol	Description
$d_1, d_2 \in \{1, 2, 3, 4\}$	The first and second phase R&D projects: 1-3 correspond to projects in Table 4.2, 4 corresponds to performing no research
$c_1, c_2 \in \{0, 1\}$	Outcome of first and second phase projects, respectively. 0: no success, 1: success
$p(c_1 d_1)$	probability of first phase outcome c_1 with project d_1
$p(c_2 d_1, c_1, d_2)$	probability of second phase outcome c_2 with project d_2 , and first phase project d_1 with outcome c_1
$\mathbf{x}_{1,d_1,c_1,d_2,c_2}$	production amount of exterior paint
$\mathbf{x}_{2,d_1,c_1,d_2,c_2}$	production amount of interior paint
\mathbf{r}_{1,d_1}	first phase project decision (1: project d_1 performed)
$\mathbf{r}_{2,d_1,c_1,d_2}$	second phase project decision (1: project d_2 performed)
y_{d_1,c_1,d_2,c_2}	base profit $5x_{1,d_1,c_1,d_2,c_2} + 4x_{2,d_1,c_1,d_2,c_2}$
ϕ_{d_1,c_1,d_2}	$r_{1,d_1}r_{2,d_1,c_1,d_2}$
γ_{d_1,c_1,d_2,c_2}	$\phi_{d_1,c_1,d_2}y_{d_1,c_1,d_2,c_2} = r_{1,d_1}r_{2,d_1,c_1,d_2}y_{d_1,c_1,d_2,c_2}$
M	a large constant used in the linearization of the product of variables

$$\text{maximize } z = \sum_s p(c_1|d_1)p(c_2|d_1, c_1, d_2)\gamma_{d_1, c_1, d_2, c_2} \quad (4.13)$$

$$+ 0.1 \left\{ \sum_{d_1, c_1, c_2} p(c_1|d_1)p(c_2|d_1, c_1, 3)c_2\gamma_{d_1, c_1, 3, c_2} \right.$$

$$\left. + \sum_{c_1, d_2, c_2} p(c_1|3)p(c_2|3, c_1, d_2)c_1\gamma_{3, c_1, d_2, c_2} \right\}$$

$$- 1.0(r_{1,1} + \sum_{d_1, c_1} p(c_1|d_1)r_{2, d_1, c_1, 1})$$

$$- 1.6(r_{1,2} + \sum_{d_1, c_1} p(c_1|d_1)r_{2, d_1, c_1, 2})$$

$$- 1.2(r_{1,3} + \sum_{d_1, c_1} p(c_1|d_1)r_{2, d_1, c_1, 3})$$

$$\text{s.t. } 6x_{1, d_1, c_1, d_2, c_2} + 4x_{2, d_1, c_1, d_2, c_2} \leq (1 + 0.1(c_1r_{1,1} + c_2r_{2, d_1, c_1, 1}))24 \quad \forall d_1, c_1, d_2, c_2 \quad (4.14)$$

$$x_{1, d_1, c_1, d_2, c_2} + 2x_{2, d_1, c_1, d_2, c_2} \leq (1 + 0.1(c_1r_{1,2} + c_2r_{2, d_1, c_1, 2}))6 \quad \forall d_1, c_1, d_2, c_2 \quad (4.15)$$

$$-x_{1, d_1, c_1, d_2, c_2} + x_{2, d_1, c_1, d_2, c_2} \leq 1 \quad \forall d_1, c_1, d_2, c_2 \quad (4.16)$$

$$x_{2, d_1, c_1, d_2, c_2} \leq 2 \quad \forall d_1, c_1, d_2, c_2 \quad (4.17)$$

$$\sum_{d_1=1}^4 r_{1, d_1} = 1 \quad (4.18)$$

$$\sum_{d_2=1}^4 r_{2, d_1, c_1, d_2} = 1 \quad \forall d_1, c_1 \quad (4.19)$$

$$r_{2, d_1, c_1, d_1} \leq 1 - c_1r_{1, d_1} \quad \forall d_1, c_1 \quad (4.20)$$

$$y_{d_1, c_1, d_2, c_2} = 5x_{1, d_1, c_1, d_2, c_2} + 4x_{2, d_1, c_1, d_2, c_2} \quad \forall d_1, c_1, d_2, c_2 \quad (4.21)$$

$$\phi_{d_1, c_1, d_2} \leq r_{1, d_1} \quad \forall d_1, c_1, d_2 \quad (4.22)$$

$$\phi_{d_1, c_1, d_2} \leq r_{2, d_1, c_1, d_2} \quad \forall d_1, c_1, d_2 \quad (4.23)$$

$$\phi_{d_1, c_1, d_2} \geq r_{1, d_1} + r_{2, d_1, c_1, d_2} - 1 \quad \forall d_1, c_1, d_2 \quad (4.24)$$

$$\gamma_{d_1, c_1, d_2, c_2} \leq M\phi_{d_1, c_1, d_2} \quad \forall d_1, c_1, d_2, c_2 \quad (4.25)$$

$$\gamma_{d_1, c_1, d_2, c_2} \leq y_{d_1, c_1, d_2, c_2} \quad \forall d_1, c_1, d_2, c_2 \quad (4.26)$$

$$\gamma_{d_1, c_1, d_2, c_2} \geq y_{d_1, c_1, d_2, c_2} - (1 - \phi_{d_1, c_1, d_2})M \quad \forall d_1, c_1, d_2, c_2 \quad (4.27)$$

$$r_{1, d_1}, r_{2, d_1, c_1, d_2} \text{ binary} \quad \forall d_1, c_1, d_2 \quad (4.28)$$

$$x_{1, d_1, c_1, d_2, c_2}, x_{2, d_1, c_1, d_2, c_2} \geq 0 \quad \forall d_1, c_1, d_2, c_2 \quad (4.29)$$

$$\phi_{d_1, c_1, d_2} \geq 0 \quad \forall d_1, c_1, d_2 \quad (4.30)$$

$$\gamma_{d_1, c_1, d_2, c_2} \geq 0 \quad \forall d_1, c_1, d_2, c_2 \quad (4.31)$$

Constraints (4.14)-(4.17) are the constraints of the original problem, where (4.14) and (4.15) are modified to take the outcomes of projects M1 and M2 into account by adding 10% to the independent term if the corresponding project is successfully completed. Constraints (4.18) and (4.19) define decision variables r as binary in both R&D phases, and limit the number of projects researched in each phase to one. Constraint (4.20) prevents a successful project from being performed again.

Furthermore, in (4.21) we define y as the profit without project PPT. Constraints (4.22)-(4.24) are a linearization of the product of two decision

variables $\phi_{d_1, c_1, d_2} = r_{1, d_1} r_{2, d_1, c_1, d_2}$ and (4.25)-(4.27) a further linearization of $\gamma_{d_1, c_1, d_2, c_2} = \phi_{d_1, c_1, d_2} y_{d_1, c_1, d_2, c_2} = r_{1, d_1} r_{2, d_1, c_1, d_2} y_{d_1, c_1, d_2, c_2}$. The model is linear with 561 constraints and 260 decision variables, of which 68 are binary. The original model (4.1)-(4.6) has two variables and six constraints. The model with R&D is thus approximately a hundred times larger than the original model, illustrating how the inclusion of uncertainty and decision stages affects the problem size.

4.2.2 Decision Programming

The structure of the Reddy Mikks problem with R&D is presented in Fig. 4.3. The approach consists of two separate parts: the first consists of the influence diagram with endogenous uncertainties and performing the R&D projects. After the two R&D phases have been completed, the second part is calculating the utility associated with each scenario path using the optimization model (4.7)-(4.12).

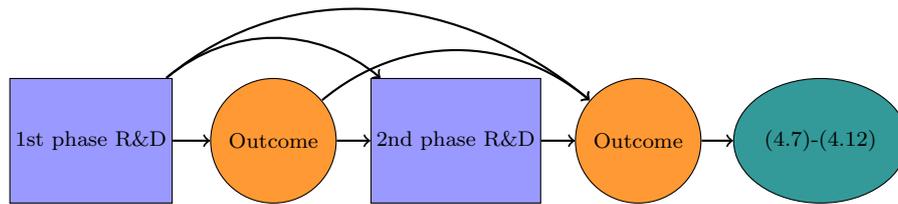


Figure 4.3: Influence diagram of the extended problem

The optimal decision tree maximizing the expected total utility is presented in Fig. 4.4. The expected utility can be easily calculated to be \$21606, which is 2.9% higher than the original optimum without any R&D. The probability of R&D being inefficient and resulting in profits lower than the solution of the original problem (\$21000) is 10.1%. The only scenario where this happens is when the project PPT fails in both stages.

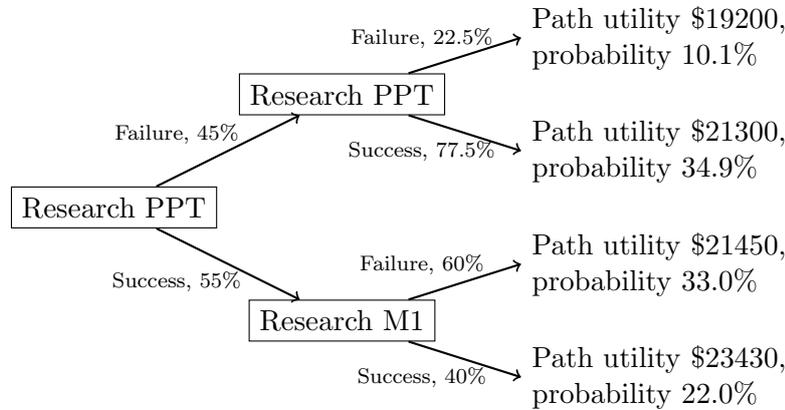


Figure 4.4: Decision tree with individual success probabilities, path utilities and their probabilities

The total number of decision variables in the deterministic equivalent MILP model is 260, of which 68 are binary. The number of constraints is 561. The Decision Programming approach leads to a problem with 100 variables, 36 of which are binary, and 201 constraints. However, a smaller problem (4.7)-(4.12) with two decision variables and six constraints needs to be solved for all 64 scenario paths. On a standard laptop¹, solving the deterministic equivalent MILP and the Decision Programming model with Gurobi 8.1.0 using all four threads took around 10 and 2.5 seconds, respectively. The path utility calculation for Decision Programming is not taken into account here because it could be parallelized, leading to significant decreases in the total solution time.

The Decision Programming approach is clearly superior in this example problem. It leads to a smaller problem that is solved four times faster than the deterministic equivalent model. Moreover, the Decision Programming framework easily generalizes to many different problems that can be represented as an influence diagram, while formulating deterministic equivalent problems such as (4.13)-(4.31) is more challenging.

While Decision Programming is clearly faster in solving the optimal decision strategy, the improved performance of Decision Programming in this case benefits from the precalculation of paths and their probabilities. This construction of the model takes nearly 10 seconds, bringing the two approaches close to each other when comparing total running times. With parallelization however, the computational time could be reduced considerably.

¹i5-4200U at 1.60GHz with Turbo Boost up to 2.60GHz, 8GB DDR3-1600 RAM

4.3 Capacity expansion problem

In order to properly test the applicability of the method, we consider the capacity expansion problem presented by Apap and Grossmann [3]. The problem consists of a simple process network where product A is produced from chemical B through an existing process. Buying B is relatively expensive, and it is possible to expand the process network to produce B from significantly cheaper raw materials C and D. The demand of A is uncertain, with two equally probable realizations in each of the eight time periods. The yields of the new processes are also uncertain with three equally probable realizations. The demand uncertainty is exogenous, while the yield uncertainty is a Type 2 endogenous uncertainty, as its exogenous realizations are learned only when the process is installed. Each process can also be expanded in order to increase its capacity. A more thorough description of the problem is presented in [3] and [18], and the process network is also presented in Fig. 4.5 for convenience.

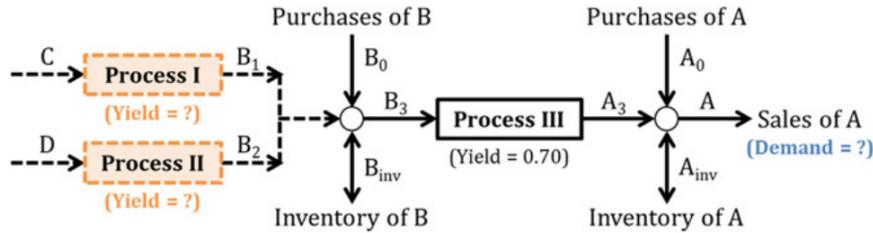


Figure 4.5: Process network for the capacity expansion problem, originally from [3].

The continuous decisions in this problem are the flow decisions corresponding to Fig. 4.5 and the process capacity expansion amounts. The decisions concerning when to install a new process are discrete, and the uncertainties are also represented by discrete probability distributions. The continuous decisions need to be moved to the optimization node, and in the resulting Decision Programming part of the problem, shown in Fig. 4.6, there are three types of nodes. Nodes Y_I and Y_{II} are the yield realizations of the two processes, nodes $D_{i,t}$ are the decisions to install process i at time t , and nodes $O_{i,t}$ are the observed yields of process i at time period t .

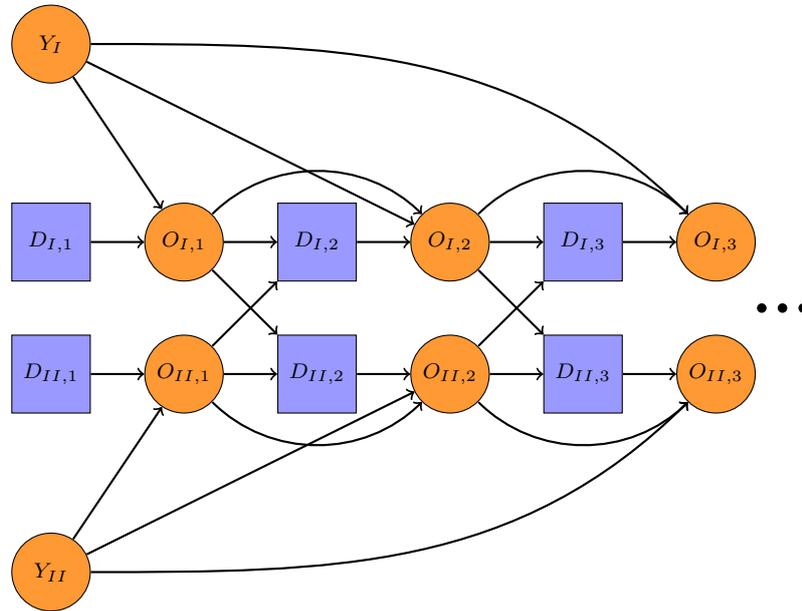


Figure 4.6: Decision programming structure of the capacity expansion problem for the first three periods

The observation nodes are presented as chance nodes, but they could also be modeled as deterministic nodes in this case, since they map the information state of the node to a single value in a deterministic way with no uncertainty. These nodes are used to model Type 2 endogenous uncertainty in an influence diagram. Representing Type 1 endogenous uncertainty in an influence diagram is trivial, as the arcs from decision nodes to chance nodes represent uncertainty depending on decisions. However, Type 2 endogenous uncertainty requires these observation nodes in order to provide correct information states to the decision nodes.

If the previous observation is non-existent, meaning that we had no information on the yield in the previous period since the process had not been installed, the current observation depends on the decision to install the process in this period. Otherwise, if the yield is observed in an earlier period, it is not forgotten and remains the same in all future periods. For each decision, with the exception of the first one, the information state of the decisions consists of the previous period yield observations for both processes.

This problem structure is a slight simplification of the original problem proposed in [3], in order to maintain tractability in the Decision Programming part of the problem. In [3], the problem was solved using a sophisticated decomposition method. We assume the effect of demand realizations on the timing of process installation to be minor. Therefore, we choose to exclude

the demand from the influence diagram and handle the demand scenarios in the optimization node. This reduces the number of scenario paths by a factor of k^8 , where k is the number of possible demand realizations in each time period. However, it also results in smaller information states for the decision nodes compared to [3], and thus possibly suboptimal solution strategies. We also “forget” the time when the processes are installed, knowing only the yield realizations of the installed processes when making decisions. Both of these are simplifications to the original problem, in which the information state of each decision node would contain all previous demand realizations and the time periods when each process has been installed, but the effect of these assumptions is shown to be small.

The number of scenario paths in this network grows exponentially as we add new stages to the problem, reaching trillions of combinations in the 8-stage model. However, the corresponding number of feasible scenario paths is only 729, since the processes can only be installed once and the deterministic chance (observation) nodes must correspond to the decision nodes. The 729 paths correspond to nine alternatives for the installation time of each of the two new processes (stages 1-8 and no installation) and three possible yields for both processes, resulting in $9^2 \times 3^2 = 729$ scenario paths. We refer to the stages of initial process installations as the timing of uncertainty revelation.

For each timing in the Decision Programming formulation, we solve a stochastic MILP problem for operating the process and obtain the expected profit for of the nine yield realizations with the given timing. Each of these problems has 331,776 variables (55,296 binary) and up to one million constraints, and the solution times can vary from minutes to hours, depending on the problem instance and the optimality tolerance. The structure of the MILP is presented in (4.32)-(4.55) and the notation explained in Table 4.5.

Table 4.5: Notation description for (4.32)-(4.55)

Symbol	Description
$s \in S$	Scenario
$p(s)$	probability of scenario s
$t \in T = \{1, 2, \dots, 8\}$	time period
d	inflation factor
$r(t, s)$	revenue in period t , scenario s
$c(t, s)$	costs in period t , scenario s
$A_0(t, s), A_1(t, s), A_3(t, s),$ $B_0(t, s), B_1(t, s), B_2(t, s), B_3(t, s),$ $C(t, s), D(t, s)$	flows corresponding to Fig. 4.5
$p \in \{P_I, P_{II}, P_{III}\}$	processes I to III
$Y(p, s)$	yield of process p in scenario s
$A_{inv}(t, s), B_{inv}(t, s)$	inventories of A and B in period t , scenario s
$V(p, t, s)$	process capacity
$\Delta(p, t, s)$	expansion amount in process p
$y(p, t, s) \in \{0, 1\}$	binary expansion decision
$c_{exp}(p, t, s), c_{op}(t, s)$	expansion and operating costs
$c_{ef}(p, t), c_{ev}(p, t)$	fixed and variable expansion costs
$c_{ov}(p, t)$	variable operating costs
$c_{inv}(t, s), c_p(t, s)$	costs of storing and purchasing materials A and B
$c_B, c_A, c_{Binv}, c_{Ainv}$	costs of B and A, and their storage
$A(t, s)$	demand of A in period t , scenario s
p_a	sale price of A
$X(t, s)$	$[y, \Delta, C, D, B_0]$
$\tau(s, s')$	the time period in which two scenarios s and s' become distinguishable

$$\max \varphi = \sum_s \sum_t p(s)(1+d)^{t-1}(r(t,s) - c(t,s)) \quad (4.32)$$

s.t.

$$B_1(t,s) = Y(P_I, s)C(t,s) \quad \forall t, s \quad (4.33)$$

$$B_2(t,s) = Y(P_{II}, s)D(t,s) \quad \forall t, s \quad (4.34)$$

$$A_3(t,s) = Y(P_{III}, s)B_3(t,s) \quad \forall t, s \quad (4.35)$$

$$B_{inv}(t,s) - B_{inv}(t-1,s) = B_1(t,s) + B_2(t,s) + B_0(t,s) - B_3(t,s) \quad \forall t, s \quad (4.36)$$

$$A_{inv}(t,s) - A_{inv}(t-1,s) = A_3(t,s) + A_0(t,s) - A(t,s) \quad \forall t, s \quad (4.37)$$

$$V(p,t,s) = V(p,t-1,s) + \Delta(p,t,s) \quad \forall p, t, s \quad (4.38)$$

$$B_{inv}(0,s) = B_{inv,0} \quad \forall s \quad (4.39)$$

$$A_{inv}(0,s) = A_{inv,0} \quad \forall s \quad (4.40)$$

$$V(p,0,s) = V_{p,0} \quad \forall p, s \quad (4.41)$$

$$C(t,s) \leq V(P_I, t, s) \quad \forall t, s \quad (4.42)$$

$$D(t,s) \leq V(P_{II}, t, s) \quad \forall t, s \quad (4.43)$$

$$B_3(t,s) \leq V(P_{III}, t, s) \quad \forall t, s \quad (4.44)$$

$$\Delta(p,t,s) \leq y(p,t,s)\Delta_{max} \quad \forall p, t, s \quad (4.45)$$

$$\Delta(p,t,s) \geq y(p,t,s)\Delta_{min} \quad \forall p, t, s \quad (4.46)$$

$$c_{exp}(p,t,s) = y(p,t,s)c_{ef}(p,t) + c_{ev}(p,t)\Delta(p,t,s) \quad \forall p, t, s \quad (4.47)$$

$$c_{op}(t,s) = c_{ov}(P_I, t)C(t,s) + c_{ov}(P_{II}, t)D(t,s) + c_{ov}(P_{III}, t)B_3(t,s) \quad \forall t, s \quad (4.48)$$

$$c_{inv}(t,s) = c_{B_{inv}}B_{inv} + c_{A_{inv}}A_{inv} \quad \forall t, s \quad (4.49)$$

$$c_p(t,s) = c_b B_0 + c_a A_0 \quad \forall t, s \quad (4.50)$$

$$c(t,s) = \sum_p c_{exp}(p,t,s) + c_{op}(t,s) + c_{inv}(t,s) + c_p(t,s) \quad \forall t, s \quad (4.51)$$

$$r(t,s) = A(t,s)p_a \quad \forall t, s \quad (4.52)$$

$$A_0(t,s) = A_0(t,s') \quad \forall t < \tau(s,s') \quad (4.53)$$

$$A(t,s) = A(t,s') \quad \forall t < \tau(s,s') \quad (4.54)$$

$$X(t,s) = X(t,s') \quad \forall t \leq \tau(s,s') \quad (4.55)$$

The objective function φ in (4.32) is the discounted expected profit of the process, where s denotes a scenario defined by the yield and demand realizations. Parameter $p(s)$ is the probability of scenario s , d is the discount rate, $r(t,s)$ and $c(t,s)$ are the revenue and total costs in scenario s at time t . Constraints (4.33)-(4.35) represent the three processes (labeled PI, PII and PIII) and their yields. Constraints (4.36) and (4.37) connect the changes in inventories to other flows in the process network presented in Fig. 4.5 and constraint (4.38) determines the capacity of process p at time t . Constraints

(4.39)-(4.41) determine the corresponding initial inventories and capacities. Constraints (4.42)-(4.44) state that the flows into the three processes can not exceed the capacity for the corresponding process.

If a process is expanded (or installed), the expansion must be between Δ_{min} and Δ_{max} , as stated by constraints (4.45) and (4.46). The decision variable y is a binary variable representing whether or not an expansion is carried out for process p in period t , and the constraints (4.45) and (4.46) also limit the actual expansion to zero when y is zero and there is thus no expansion. The next two constraints (4.47) and (4.48) define the expansion and operating costs, respectively. Expansion costs consist of a fixed part for the decision to expand and a variable part depending on the size of the expansion. Operating costs only depend on the input flow of the process. Constraints (4.49) and (4.50) define the inventory and material costs, while the constraints (4.51) and (4.52) define the total costs and revenues in period t . Variables c_i are the purchase costs of materials, while p_a is the selling price of final product A.

Finally, a large set of non-anticipativity constraints (NACs) must be defined. $\tau(s, s')$ denotes the time period in which the scenarios s and s' become distinguishable, meaning that either they have different demands $A(t, s)$ in $t = \tau(s, s')$ or that process I or II is installed in $t = \tau(s, s')$, revealing different yields for the two scenarios.

The decomposition of the problem is based on separating the timing of the initial process installation from the other decisions. The scenarios $s \in S$ in (4.32)-(4.55) comprise both the demand and yield scenarios, and as the timing of uncertainty revelation in the yields is predetermined for each MILP problem, the remaining uncertainties are purely exogenous. After solving (4.32)-(4.55) to optimality for a given timing, we get all nine path utilities corresponding to that timing. This is done by calculating the expected discounted profit over all demand realizations for each of the nine combinations of yield realizations.

In this capacity expansion problem, Apap and Grossmann [3] reached a 1.41% optimality gap in 974 seconds. On a similar computer², using Julia 1.1.0 and Gurobi 9.0.0 instead of GAMS 24.3.3 and CPLEX 12.6.0.1, the Decision Programming approach was tested with optimizing the individual timings to different optimality gap conditions. The total solution times with different stopping conditions are presented in Fig. 4.7.

²i5-4200U at 1.60GHz with Turbo Boost up to 2.60GHz, 8GB DDR3-1600 RAM

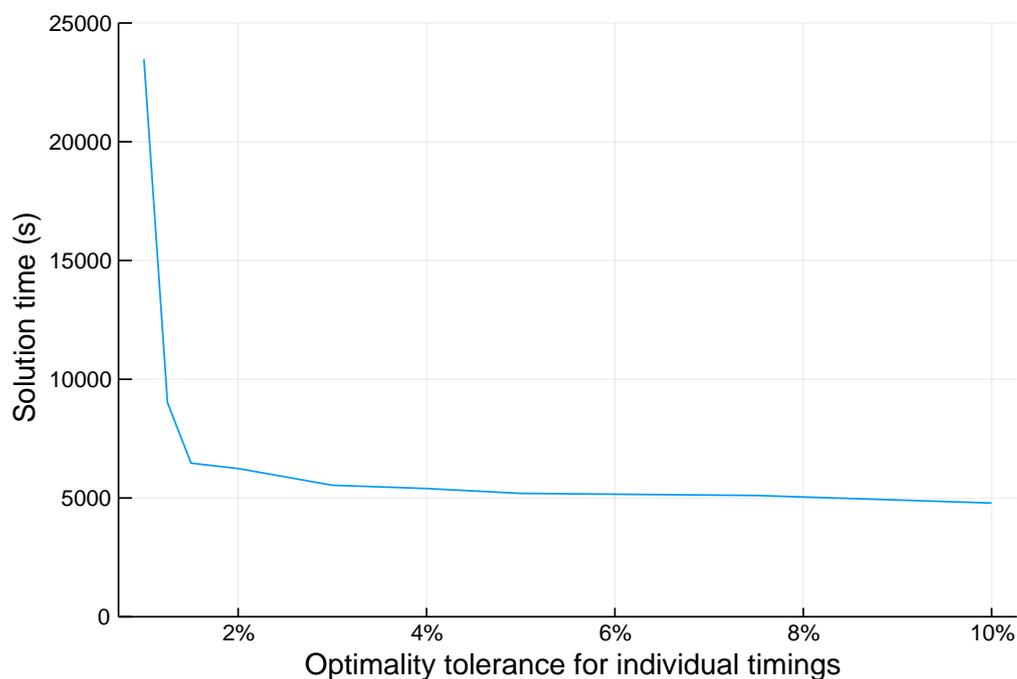


Figure 4.7: Solution times with different optimality conditions

From Fig. 4.7, we can see that the effect of the stopping condition on the solution time is minor when the gap is at or above 1.5%. With tighter optimality requirements, the solution times quickly increase because for some timings, decreasing the gap is difficult. Using the path utilities calculated at a 1.5% optimality tolerance, we get a solution 142,677. The timing strategy corresponding to this solution is to install process II in the first period, and if the yield is observed as the lowest possible realization, process I is installed in the second period.

Despite being able to reach this solution in under two hours, Decision Programming is an unfitting approach for this capacity expansion problem. The path utilities need to be solved in sets of nine because of the structure of the problem. Therefore, the value that is optimized in the path utility calculations is the expected value of the nine path utilities, in this case the average since all yield realizations are equally likely. We are able to reach a 1.5% optimality gap in all sets of utilities, but this does not mean that the gaps of the individual paths are below 1.5%.

Since knowing the optimality gap for the timing problems gives us no information on the gaps for the individual paths, nothing can be said about the gap of the solution of the full problem. If we had some guarantee on the individual path gaps, we could possibly use those gaps to obtain some

information on the optimality of the final solution. With this framework, in this problem, the utilities must be solved to optimality in order to get anything more than a feasible solution. The solution we are able to obtain, 142,677, is inside the gap reported in [3]. This at least suggests that the method works correctly.

In order to obtain some insight on how long it would take to solve this 8-stage problem to optimality, we examined shorter versions of the problem with two to six stages. These smaller problems are obtained by excluding time periods from the end of the original problem. The solution times for these problems are presented in Fig. 4.8.

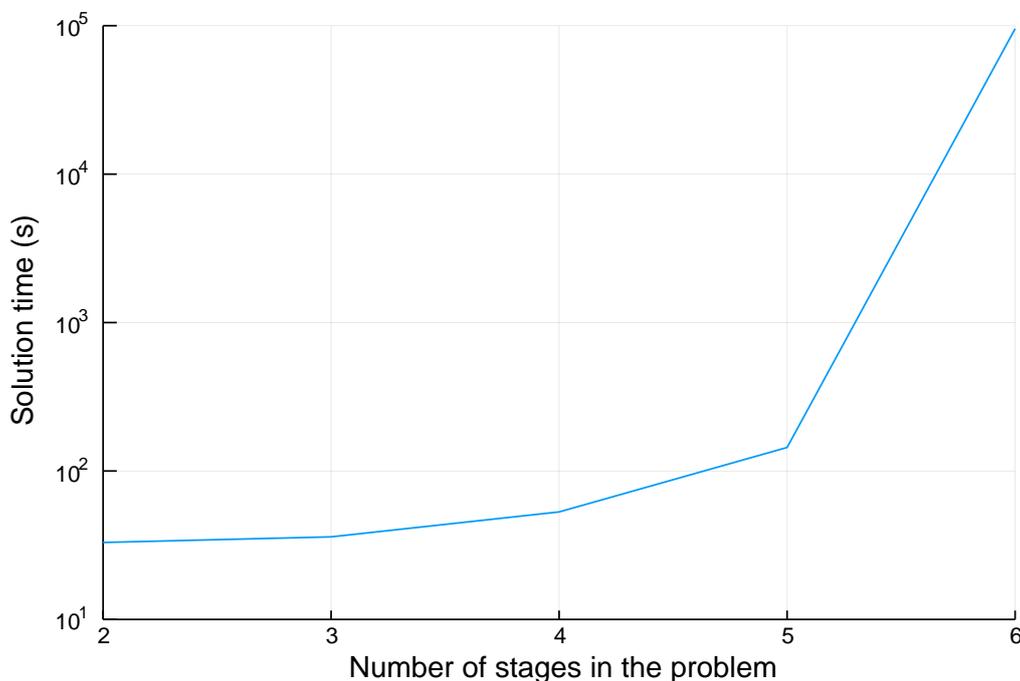


Figure 4.8: Solution times with two to six stages

In Fig. 4.8, the solution times are reasonably small, under three minutes, for up to five stages. When the sixth stage is introduced, the solution time increases to over 26 hours, indicating that solving the 8-stage problem to optimality would be practically impossible with any computer available today. This solution time can not be reduced significantly by parallelizing the utility calculations, since 74% of the solution time is spent on solving the utility of one of the 49 timings, and a further 22% on two more timings. This shows the main challenge in this problem, we end up spending a majority of solution time on a path that is not in the optimal policy. We can easily check

that even if all path utilities in the timing that we spent 74% of the solution time on increased by 10%, it would still not be in the optimal policy. The optimal policy for the 6-stage problem is the same as in our solution for the 8-stage problem, process II is installed in the first period, and process I in the second period if the yield of process II has the lowest possible realization.

Overall, the problem can be modeled as a Decision Programming problem with some simplifications. The stochastic nature of the path utility problems results in large and hard-to-solve problems, but also in that the subproblems for each timing have to be solved to optimality. Comparing the efficiency to the methods in [3] is difficult, since they solve the problem to gaps of 0.99% and 1.41% while our method is unable to produce an optimality gap for the solution. The Decision Programming framework is based on the concept of scenario paths, and solving Decision Programming problems thus requires optimal path utilities in order to be able to achieve an optimal solution for the full problem. This leads to the framework having significant challenges when the path utility calculations are as demanding as in this example, but we are nevertheless able to get some results by decreasing the problem size.

Chapter 5

Climate change mitigation with R&D

5.1 Literature review

For the past 40 years, starting from the first World Climate Conference (WCC-1)[1] in 1979, there has been increasing scientific, political and overall interest in climate change. In 1979, the specialists in WCC-1 agreed that there was a possibility of significant climate effects by the mid-21st century from the burning of fossil fuels, deforestation and changes of land use. It was also agreed that climate change should be extensively studied in order to be able to predict and react to the changes. In the next 11 years leading to WCC-2[2] in 1990, preliminary consensus had emerged on the expected warming in the 21st century, and it had become evident that if the increase of greenhouse gas (GHG) concentrations continued as it did, the socio-economic impacts would be severe.

In 1991, Nordhaus [30] criticized the “call to arms” in WCC-2 for lacking cost-benefit analysis (CBA). He presented a framework for weighing the costs and benefits of greenhouse gas abatement, acknowledging that both cost and damage estimates were highly uncertain. Additionally, the model he presented is greatly oversimplified compared to modern approaches. The abatement is done in one stage instead of being a process spanning multiple decades and involving gradual learning on the damages and climate sensitivity. Despite these uncertainties and simplifications, this paper was an important first step in the literature considering optimal GHG abatement strategies.

The next year, Nordhaus [31] presented the DICE (Dynamic Integrated Climate-Economy) model, similar to the approach in 1991, but with dy-

namics built into the model. Currently, after more than 25 years, a revised version of DICE is still used in climatic CBA. A major improvement in DICE, when compared to earlier approaches was that it allowed for determining an optimal abatement strategy for, e.g., 10 year periods, showing the optimal GHG levels for each decade. In its original version, the social cost of carbon, describing the economic cost of a ton of added CO₂ emissions, was significantly underestimated because of inaccuracy in multiple underlying variables [29]. As a result, the optimal abatement strategies were remarkably modest compared to modern estimates in [16], for example.

In the past 25 years, the essential idea of these CBA models has had relatively minor changes. The mitigation cost curves, as well as the damage costs, have been researched and iterated to more reliable estimates. However, due to extremely long time horizons in climate change assessment, there is still substantial uncertainty in modeling the dynamics of average global temperature increase. The uncertainty in two parameters, namely climate sensitivity (equilibrium temperature increase from doubling of CO₂ concentration) and the damage exponent in DICE [33], is explored in [17] with an important finding that the uncertainty in the damage severity is much more significant than that in climate sensitivity.

In the 21st century, a stronger consensus has emerged on that emissions need to be reduced in order to limit the damages from an uncontrolled temperature rise. To support this abatement process, there has been increasing focus in energy and climate R&D with the goal of reducing the costs of abatement, thus enabling policymakers to adopt a heavier abatement policy. The impact of technological R&D projects in the energy sector has been widely researched (see, for example [4–7]). These references offer data on both the success probabilities of different projects and their impact on abatement costs, based on expert elicitations. In a related context, Nordhaus [32] discussed induced innovation in climate R&D. Blanford [9] researched the relationship between investment size and success probability of a project as well as R&D portfolio optimization.

5.2 SCORE

The CBA model used in this study is a simplified version of SCORE (Stochastic Cost Optimization for Reducing Emissions), an integrated assessment model (IAM) similar to DICE, linking economical factors such as global economic output and climate impacts such as temperature change and the resulting economic damage. The two uncertain parameters in the model are climate sensitivity and the DICE damage exponent. A key element in

SCORE is the gradual revelation of these parameters. The gradual learning process is modeled as a scenario tree, with slightly varying structures in different versions of the model. In the original model in [15], hereafter referred to as SCORE-14 because it was published in 2014, the uncertainties are revealed in a binomial lattice with 50% branching probabilities. In this model, learning occurs every ten years. In the simpler model described in [17], denoted here as SCORE-20, the learning happens in two stages (between years 2050 and 2070, and after 2070). Initially, there are three possible realizations for both parameters. In each learning stage, one extreme value is removed. The decision variables in SCORE are the greenhouse gas abatement levels in different stages and scenarios.

The cost of abatement is calculated using cost curves fitted in [15]. Based on the abatement decisions, an estimate of temperature change is calculated and used in the DICE damage function. Thus we can calculate the cost of damages caused by the rise in temperature. Summing these (discounted) abatement and damage costs, we obtain the total cost of a given abatement strategy. Using this cost model, SCORE chooses the values for decision variables such that the expected total discounted cost over all scenarios is minimized.

5.3 Problem structure

We consider a setting where climate change R&D is done in two stages, first in 2020, then in 2030, after the results from the first round of projects have been observed. The goal of these R&D projects is to reduce abatement costs with new technologies or gain better information on the severity of climate damages. In the first stage, three projects are available to choose from, and two in the second stage. After the two project rounds, we run the simplified version of SCORE from year 2050 onward. The model presented here is based on SCORE-20 [17]. In this model, it is also possible to perform research in order to gain early information on the climate damage severity. Additionally, the decisions on emissions levels are made in three steps instead of two. Decision Programming is used for solving the full model. The influence diagram of the R&D SCORE problem is presented in Fig. 5.1.

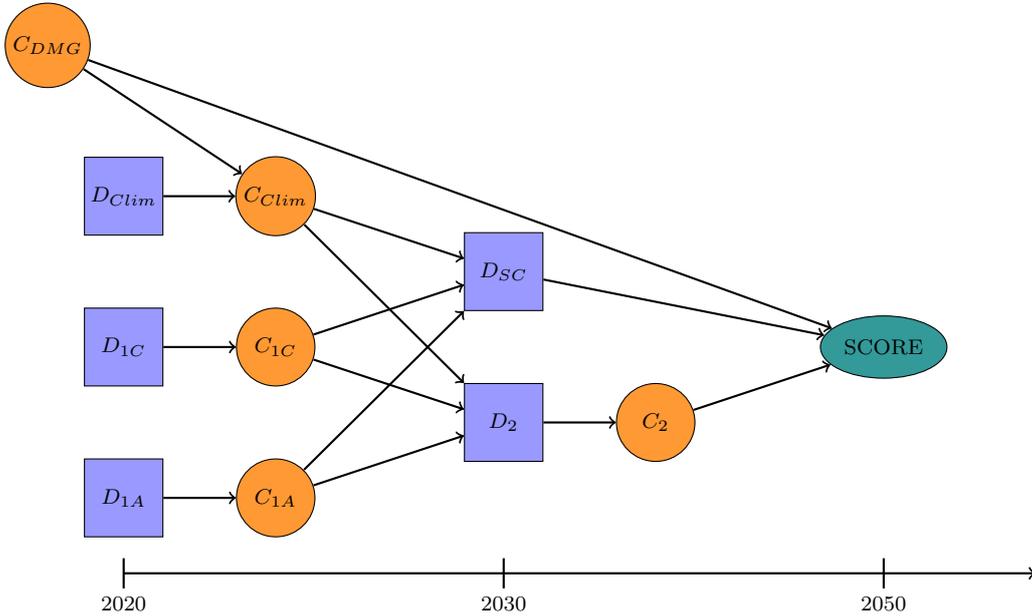


Figure 5.1: Influence diagram of R&D SCORE

The node C_{DMG} corresponds to the DICE damage exponent. The damage exponent was chosen as the random variable of interest because of the earlier results showing that the optimal mitigation strategy is highly sensitive to its distribution [17]. The decision node D_{1C} along with the corresponding chance node C_{1C} (success/failure of the project) represents a project aimed to decrease costs in the scenario where climate damages are small, while D_{1A} is the decision to perform a project aiming to decrease the costs when the damages are high. These correspond to the **C**onventional and **A**lternative technology R&D projects described by Baker and Adu-Bonnah [4]. The project D_{Clim} gives further (imperfect) information about the damage parameter value, so that the decisions made in 2030 can be made under more accurate information.

In the second phase, projects for high and low climate damages are available, but only one can be researched in D_2 . This represents a situation with insufficient resources for completing two projects, and puts pressure on researching the realization of C_{DMG} . While it is not necessarily realistic that only one of the projects can be performed in 2030 because of insufficient resources, it makes the Type 2 endogenous uncertainty more prominent in this illustrative setting. The node D_{SC} represents a discrete approximation of the first stage mitigation decision in SCORE, and after all the projects are finished in 2050, the later mitigation decisions are determined in SCORE to minimize total mitigation and damage costs.

The value of climate damages in C_{DMG} can be low, medium or high, with corresponding probabilities 30%, 40% and 30%. This value can be discovered through research denoted by D_{Clim} . The outcome C_{Clim} of the research is an example of Type 2 endogenous uncertainty, where the underlying probability distribution C_{DMG} is unaffected by the decision D_{Clim} , but the observation C_{Clim} depends on these two. If we choose to perform the research project, there is a 20% chance of failure, which means that no new information is learned. If the project succeeds, the true value of the damage parameter is learned. If the research is not performed, the observation is the same as in the case of a failed project. The cost of this project is 500 billion USD. We highlight that this is an overly simplistic representation of a complex research topic, especially in that we assume that the project can either fail or succeed, with zero probability of leading to wrong conclusions. On the other hand, including this inaccuracy in the project outcomes would be straightforward, if reliable data on projects of this nature were available.

5.4 Input data

The costs of the R&D projects as well as their impacts on the mitigation cost are based on the estimates of Baker and Adu-Bonnah [4], as described next. The impact of a specific technology on the costs is denoted by the coefficient α and the actual effect depends on the technology as presented in Fig. 5.2. Alternative technologies pivot the cost curve down, and the cost of abating a fraction μ of the emissions with achieved technological impact α is $\tilde{c}(\mu, \alpha) = [1 - \alpha]c(\mu)$, where $c(\mu)$ is the cost of abating a fraction μ with existing technologies. We use the cost curves $c(\mu)$ from [15]. In this model, the higher cost curves are used, because they can be seen as a pessimistic estimate where R&D is ineffective, which corresponds to our base scenario where the projects do not succeed.

Conventional technologies pivot the cost curve to the right, the abatement cost being $\tilde{c}(\mu, \alpha) = c(\max[\frac{\mu - \alpha}{1 - \alpha}, 0])$. The cost of researching a technology is $g(\bar{\alpha}) = \frac{\kappa \bar{\alpha}^2}{1 - \bar{\alpha}}$, where $\bar{\alpha}$ is the expected impact of the technology and κ is a scaling factor.

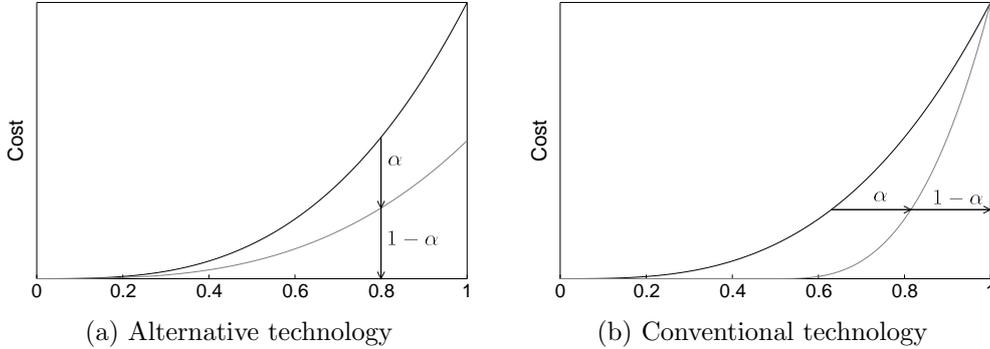


Figure 5.2: Effect of technology on the cost curve, black curve represents the cost with no technological change and the x-axis shows greenhouse gas abatement as portion of baseline emissions, figures adapted from [4].

In our example, different κ -coefficients are used for conventional and alternative technology projects, and separate impacts α along with corresponding success probabilities for each individual project. The total mitigation and damage costs are calculated in the final SCORE node, using the discrete decision D_{SC} as the first stage mitigation value and considering the success of each project. As the damage parameter has a fixed value in each scenario, the mitigation strategy is first determined with the available information, then the realized costs of this strategy are calculated, with a known level of damages. The uncertainty revelation process is based on [17]. A high or low damage level translates to the first damage branch in SCORE being certain, while medium damages results in low and high damages having the same probability in 2050. In 2070, perfect information on the value is achieved and the branch thus corresponds to the value of the chance node C_{DMG} .

5.5 Results

In the following results, values used for projects in 2020 and 2030 are $\kappa_1 = 15000$, $\kappa_2 = 25000$, respectively. The impact of a successful project is $\alpha = 0.3$ and the success probability is 70% for all projects. The mitigation range in 2030 is set to 20% to 75% decrease from baseline emissions with three uniformly spaced values (2030 emissions 25%, 52.5% or 80% of baseline).

The optimal strategy is to start by researching everything in 2020. After this, the 2030 R&D decision depends on the outcomes of the 2020 research, e.g. alternative technologies are researched if the damages are observed to be high, as expected. Medium mitigation is chosen if high damages are observed,

otherwise low mitigation is sufficient. The costs of the strongest mitigation alternative outweigh the benefits in all scenarios, since early mitigation is relatively expensive. This results from the cost curves in the model assuming some technological development and mitigation thus becoming cheaper in the future (see [15] for a more detailed explanation of the cost curves).

The expected total cost in this case is 130,972, and a cumulative distribution function (CDF) of the outcomes is shown in Fig. 5.3. We can see that the costs range roughly from 45,000 to 350,000, with a significant skewness. The expected cost of 131,000 is considerably higher than the median cost of roughly 100,000, suggesting that the high costs are more extreme. This is also evident from the CDF, since there are cost as high as 350,000 while the lowest costs are around 45,000, much closer to the expected value. The expected cost without any R&D is 183,816, and the benefit from R&D is thus 29%.

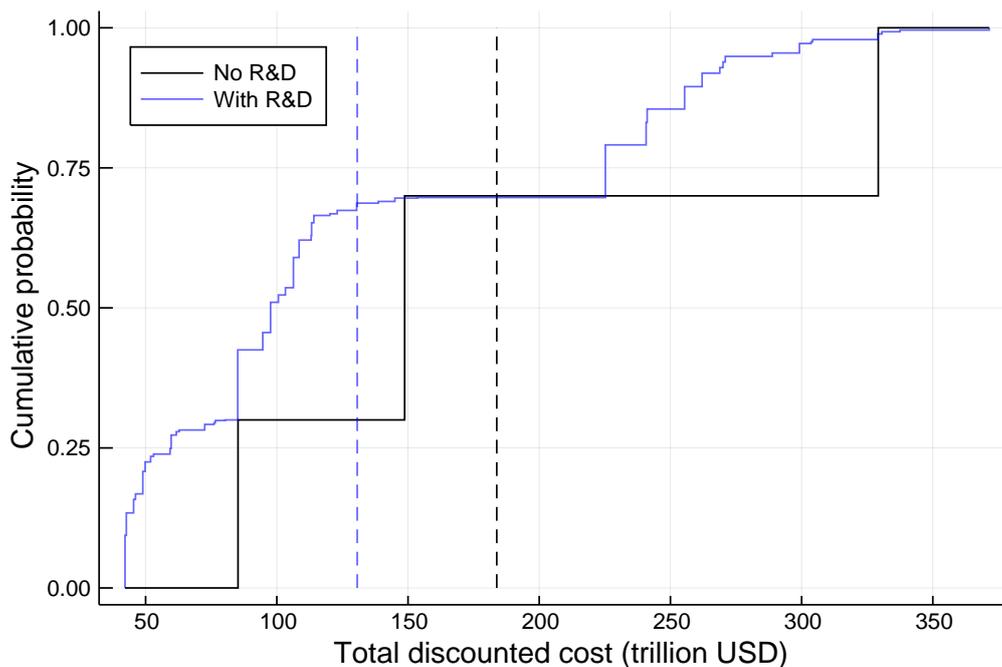


Figure 5.3: CDF of total costs with no R&D (black) and R&D (blue)

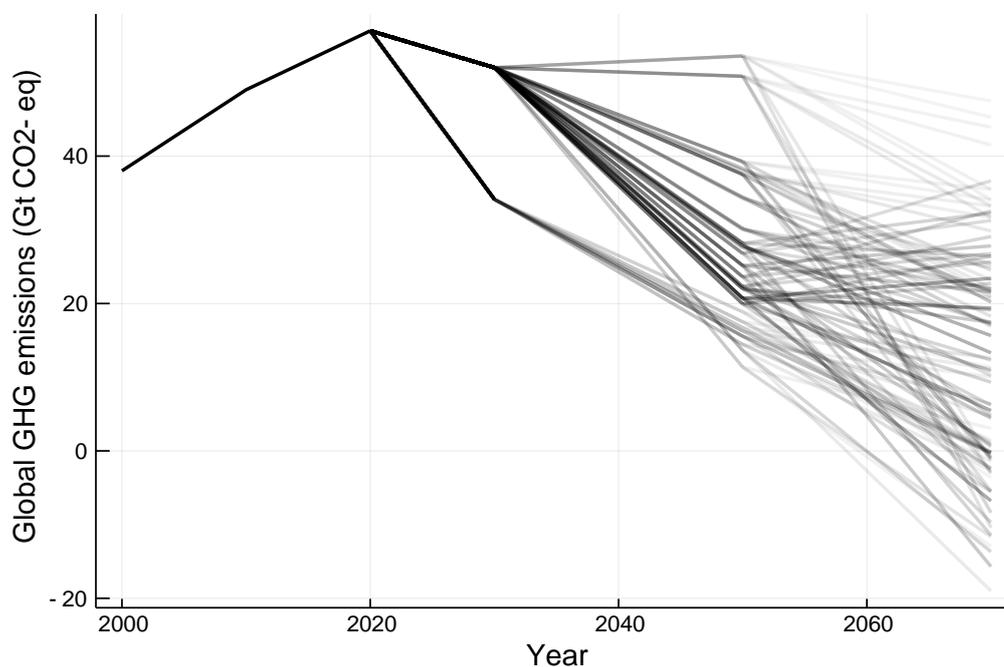


Figure 5.4: The optimal abatement strategies in different scenarios

In Fig. 5.4, the optimal abatement in different scenarios is presented for the optimal strategy without risky R&D (3 abatement options in 2030). The color of the lines represents the probability of the path, the darker the more certain. We can see that in 2030, the most aggressive abatement option (16.25Gt emissions, 25% of baseline) is not used, and the lowest abatement is more likely than the medium. The 2050 mitigation is aggressive, and the same trend continues in 2070, where a significant number of scenarios has negative emissions. The results are similar to those presented in [16]. While this is not the main output of this research, it suggests that the method is reasonable as it does not change the optimal emissions strategy significantly from earlier research.

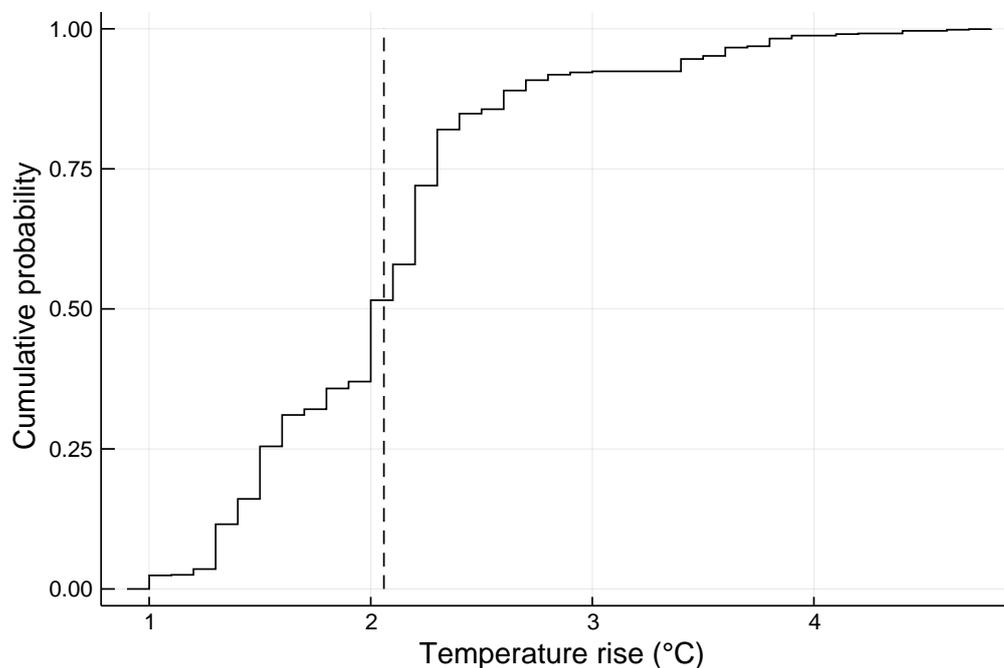


Figure 5.5: The CDF of temperature increase

The increase in global temperature corresponding to strategies in Fig. 5.4 is shown in Fig. 5.5. Approximately 90% of the probability mass lies between 1°C and 3°C and the expected increase is around 2.1°C, but the right tail is relatively fat and long. These extreme cases are likely to correspond to the highest costs in Fig. 5.3, although this was not explored further. As in the original SCORE [16], the damage function is parametrized in a way that at a 3°C rise with medium damages, the damages are 2.1% of global income. However, with temperature changes this large, the damage exponent changes the result drastically. With low damages, the damages at +3°C are only 0.7% of the global income, while with high damages, the corresponding number is 19%, a Great Depression scale loss. The severity of the high damage scenario is further illustrated in that a temperature rise of over 4.6°C with high damages would cause damages higher than the global income.

5.6 Computational considerations

The total number of scenario paths in the model with three discretization steps is 6,912, which is still reasonably small. A major challenge is the fact that the optimal mitigation strategy needs to be calculated for each

scenario. However, solving the nonlinear optimization in SCORE takes only around 0.1 seconds per scenario, using a standard laptop¹. Thus, calculating the mitigation strategies takes roughly 10 minutes with no parallelization. Determining the optimal strategy after this takes roughly four minutes, and the total running time is 15 minutes.

As discussed earlier in this thesis, discretizing a continuous decision variable is a simplification that can lead to suboptimal solutions. When the 2030 mitigation is discretized more finely than with three points, the optimal expected cost decreases because the discretization represents the decision space better. With the 2030 mitigation having five alternatives (20%, 34%, 48%, 61% and 75% decrease from baseline emissions) and adding new, more ambitious-but-riskier alternatives to first stage technology projects ($\alpha = 0.6$, success probability 50%), the optimal expected utility is 112,544. The total number of scenarios is 25,920 in this case, more than three times the number of scenarios in the previous model. As of now, this seems to be too large for efficiently solving the problem to optimum on the aforementioned laptop using Gurobi 8.1.0. The lazy constraints discussed in [39] were also tested on this problem, but they proved to be ineffective. However, running this model on a high-performance computational cluster such as Triton² in the Aalto University School of Science “Science-IT” project makes it possible to reach the optimal solution in reasonable time.

In Fig. 5.6, the effect of the number of discretization steps, as well as the effect of adding the riskier first stage R&D alternative can be seen. The total cost is calculated by summing the abatement costs until 2100 and the damage costs in years 2050-2200, with a 3% discount factor. The black line shows the CDF without any R&D and five discretization steps. The blue line corresponds to not having the riskier R&D alternative with the finest discretization, while in cases represented with the red lines, it is available (but not mandatory). The risky alternative makes the value of R&D around 25% higher, and adding more abatement options in 2030 decreases the cost, approaching some value corresponding to infinitely fine discretization or continuous abatement. Based on the results in Fig. 5.6, that value is likely to be around 112,000 with the riskier R&D included, assuming that the range of abatement values includes the optimal values. If the discretization range does not cover the optimal decision values, the solution is suboptimal with any number of discretization points.

¹i5-4200U at 1.60GHz with Turbo Boost up to 2.60GHz, 8GB DDR3-1600 RAM

²2x Xeon E5-2680 v2 CPUs at 2.80GHz, 256GB of DDR3-1667 RAM

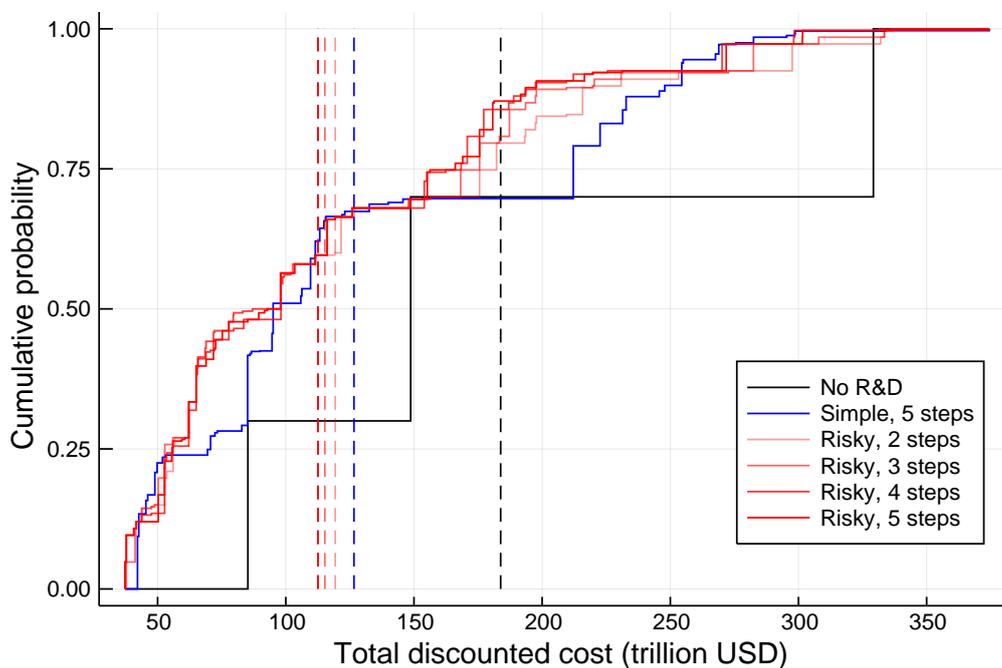


Figure 5.6: A CDF of total costs in different scenarios. “Risky” scenarios have the riskier R&D projects available while the “Simple” scenario does not.

All of the scenarios with R&D in Fig. 5.6 seem to be better than the no R&D scenario at a glance, but there is a small risk of a cost higher than the worst case without R&D, because of the possibility of all the projects failing and not producing any added value despite the costs. With all discretizations, the expected costs with the riskier projects available are lower than the best scenario without riskier R&D available, suggesting that the riskier projects have a significant impact.

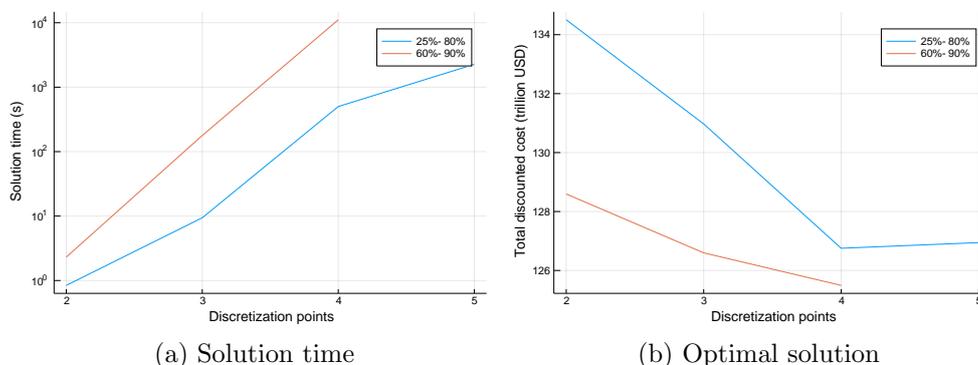


Figure 5.7: The tradeoff between solution time and accuracy. The percentages in the legend refer to the range of 2030 emissions as percentage of baseline.

In this CBA problem, there is a clear trade-off between solution time and accuracy resulting from the discretization of the 2030 mitigation decision. The solution times in Fig. 5.7a are from using 10 threads on Triton and the blue line corresponds to the discretization window used so far. The solution times grow exponentially with the number of discretization points, while the optimal solution in Fig. 5.7b clearly approaches some value with a small (≤ 5) number of points even with a very wide range of abatement values. With this window of values, the solution with two points is 6% higher than with four or five points. However, by examining the results in Fig 5.4, it can be seen that the most aggressive abatement options are not used, signaling that a tighter range of values for 2030 abatement could be used.

The performance was also analyzed with 2030 emissions being 60%-90% of the baseline (instead of 25%-80%). While the solution improvement with four points was only 1% compared to the solution with the original range, even the solution with only two points was close to the best solution obtained with the 25%-80% range. However, the solution times simultaneously increased more than tenfold. Therefore, solving this problem with five points was considered intractable even with this computational environment.

5.7 Discussion

Overall, this illustrative example on cost-benefit analysis of climate change mitigation is, despite many parameter values having little or no justification, successful in demonstrating the proposed framework in a versatile way. First, the problem can be separated into a discrete two-stage endogenous

part (R&D) and a continuous stochastic two-stage nonlinear optimization problem with little simplification. The only necessary simplification is that the first abatement decision must be moved from the continuous problem to the discrete part, because the second stage of technological R&D is performed simultaneously with the abatement decision, and a requirement of the decomposition method presented in this thesis is that the influence diagram can be arranged such that the discrete-valued nodes with endogenous uncertainties are separable from the continuous decisions in the path utility calculation as presented in Fig. 4.1. The effect of this discretization was examined, and as expected, the solution time increased fast with the addition of abatement options, although the number of scenarios increased linearly. However, the results suggest that the final solution was close to the true optimum with 4 abatement options, and we were able to solve problems of this size to optimality.

In addition to the discretization, this problem also illustrates the flexibility of the method with respect to the different types of uncertainty discussed in [24]. The uncertainty in climate sensitivity inside the simplified SCORE model is purely exogenous. The technological R&D projects shift the probability distribution of abatement costs towards lower costs (Type 1 endogenous uncertainty), while the climate R&D project affects the timing of uncertainty revelation (Type 2 endogenous uncertainty). Therefore, this problem can be classified as a Type 3, examples of which have not previously been discussed in the literature.

Chapter 6

Conclusions

The goal of this thesis was to extend the Decision Programming framework to be able to solve problems with continuous variables. The proposed decomposition approach makes it possible to solve problems with a separable structure where the final nodes in the influence diagram can be represented as an optimization node. The extended Decision Programming framework was tested on three problems. The goal was to test the applicability of the proposed approach to different problems to better understand its strengths and weaknesses.

In the motivating paint selling example (Section 4.2), the problem was formulated as both a stochastic MILP and a Decision Programming problem. Formulating multi-stage stochastic MILPs is challenging and time consuming, while the Decision Programming version was straightforward to formulate in addition to being easier to understand, mostly because of a more direct connection to an influence diagram representation of the problem. The size of the Decision Programming problem was approximately half of that of the MILP version.

After the motivating example, the method was tested on a large-scale problem from literature. The capacity expansion problem from [3] was solved with Decision Programming in order to test the applicability of the method in problems with Type 2 endogenous uncertainty, also referred to as decision dependent information structure. The yield of the new processes was uncertain, and was revealed when the processes were installed. This type of uncertainty usually leads to an optimization problem with a large number of non-anticipativity constraints (NACs) and possible intractability. In Decision Programming (in an ideal situation), all of the endogenous uncertainty can be separated into the discrete influence diagram, thus eliminating the need for NACs. However, the structure of this problem did not allow full separation, leading to some simplifications in the information available in the

decision making process, and difficult continuous optimization problems for path utilities. Despite this, the achieved solution was inside the optimality gap reached by Apap and Grossmann [3] in 2017, but we could not provide a gap for our solution.

The main motivating problem was a case study combining ideas from literature on climate change mitigation into a Type 3 endogenous problem. The key component of the example was a simplified version of the SCORE model presented in [16]. This version of SCORE is a 3-stage stochastic nonlinear problem minimizing the total costs of emissions abatement and climate damages. In the Decision Programming part, we had a two-stage R&D decision process, where one research project aimed at providing more information on the uncertain parameter in climate damages, and the other projects reduced abatement costs. The first emissions abatement decision was made in the discrete part, and the impact of this discretization was examined with different settings. In principle, discretizing a continuous variable usually leads to suboptimal solutions, and while this effect was indeed observed, the solutions seem to be close to the optimum even with relatively few discretization points. The decision strategies and the probability distribution of the objective value were also examined, and it was found that for this problem, the distribution had a fat tail in the higher costs.

The parameter values and distributions for climate change are highly uncertain, and in the scope of this thesis, they were not thoroughly researched. Therefore, the results presented in this thesis should not be taken as evidence for policy instructions for fighting climate change, but as a demonstration of the framework.

Three different problems were solved, one for each type of endogenous uncertainty, in which Type 3 is defined as a combination of Types 1 and 2. While the computational efficiency was not comparable to the specialized heuristics in the capacity expansion problem [3], the strength of the proposed framework lies in the understandability and generality of the model. Compared to large mixed-integer problems with disjunctive constraints, the Decision Programming approach has a clear connection to the influence diagram presentation of the problem. While this connection is helpful in understanding the problem, it can also pose challenges, as some problems are difficult to formulate as discrete influence diagrams.

6.1 Computational efficiency

In the motivating example, the solution times for the Decision Programming approach and the traditional MILP were similar without parallelization. In

the capacity expansion problem, the full problem could not be solved to optimality, and an optimality gap was not obtained due to the problem structure. In the climate change mitigation study, tractability was also an issue, and using a high-performance computational cluster instead of a standard laptop was necessary. As the problem size increased, the solution time seemed to increase exponentially. Overall, the computational efficiency of the method can be improved, but it is nevertheless a versatile, general framework for solving multi-stage decision problems. The capacity expansion heuristics developed by Apap and Grossmann [3] are based on years of development and are more or less optimized for the exact problem type they are dealing with, and therefore it was expected that our method would not be as fast as the reference method.

6.2 Future work

The Decision Programming framework allows for addressing Type 3 endogenous problems, a class of problems that have not been discussed in the literature because no solution methods have existed prior to Decision Programming. More Type 3 problems should be examined in order to understand how to best apply Decision Programming to such problems. In addition to the Type 3 problems, Decision Programming also allows for different risk considerations such as conditional value-at-risk (CVaR) and distributionally robust optimization.

In the scope of this thesis, we only considered problems with an exploitable structure and did not attempt to develop a more general framework that would allow any node to be continuous. With further development, progress towards such a framework would be possible.

In order to improve computational tractability in large-scale problems, it might be beneficial to create heuristics and other, possibly problem-specific solution strategies. As an example of problem-specificity, the previously developed lazy constraints did not help with the problems in the problems in this thesis, even though they enabled tractability in previous research. Parallelization of the path utility calculations was not used in this thesis, even though it could have a major impact on the total solution time when the number of paths is large.

Because Decision Programming combines stochastic programming and decision analysis, approaches from both fields can be adopted to improve tractability. First, the influence diagram can be modified with existing strategies presented in e.g. [40]. Second, different decomposition methods such as Lagrangian decomposition can be used in solving the deterministic equivalent

problem. Other novel approaches for multi-stage stochastic programming, including stochastic dual dynamic programming (SDDP) [14] could also be used to decrease solution times.

Overall, the work in this thesis extends the capabilities of Decision Programming, which, despite being a very novel framework, is a versatile and general solution method for decision problems with exogenous and endogenous uncertainty. The developed decomposition approach allowing the consideration of continuous decision spaces vastly improves the applicability and tractability of the framework in problems with such decisions.

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