Department of Mathematics and Systems Analysis

Modelling and solution methods for renewables-driven energy markets

Nikita Belyak

Aalto University

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Abstract

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Abstract

Responding to the alarming climate change consequences, many countries are paying significant attention to the energy systems' transition towards environmental sustainability. As an example, European Union established an ambitious goal to become climate-neutral by 2050 compared to 10 levels, and South Korea aims to reduce greenhouse gas emissions by 37% below business-as-usual by 2030. Considering the essential role of energy markets in modern economies such targets pose a fundamental challenge to finding a potential solution that would ensure furthering human-kind well-being and decarbonisation. One of the commonly exploited crucial tools for planning energy systems transition and understanding its effect on the economy and social welfare is energy systems modelling. However, modelling techniques undergo criticism regarding the insufficient level of precision provided for police makers. In particular, two of the main challenges are associated with i) a limited number of attempts to integrate multiple energy-sector stakeholders into a single-model formulation and ii) a trade-off between the model complexity and its numerical tractability.

This dissertation addresses both of these challenges. First, it formulates a modelling framework allowing one to represent energy systems operations involving multiple generation companies and transmission system planning. Additionally, the energy system models formulated in this dissertation allow for the consideration of renewable supporting policies such as carbon tax and investment subsidies. These models provide insights into how a welfare-maximising unit may impact the increase of renewable share in the generation mix without harming the total welfare. Such a study was conducted for Nordic and Baltic countries. Second, this dissertation provides a solution method formulation that can be applied to solving proposed mathematical models. The solution algorithm was developed in two stages: i) combining Lagrangian decomposition with mixed-integer relaxation allowing one to obtain an arbitrary precise solution in case of duality gap absence and ii) embedding these techniques within a duality-based branching strategy that would ensure solving to optimality even in the presence of duality gap.

The models in this dissertation serve as a support for decision-makers trying to understand how and to which extent they can exploit the influence of the transmission system operator on the energy system with or without other supportive policies in the context of decarbonising the energy system. The solution algorithms proposed in this dissertation are generally applicable to a wide range of two-stage stochastic mixed integer problems appearing in such sectors as, for example, the design of water networks, modelling refinery processes and transportation systems.

Keywords Energy systems modelling, Non-linear optimisation, Mixed-integer based relaxation, Lagrangian relaxation, Branch-and-bound

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Preface

This dissertation is the final outcome of challenging and fruitful 4+ years of my work. While I am grateful for the experience and the expertise I have now, bringing my doctoral education path to its finale has required a lot of courage and self-will. Unfortunately, my academic journey happened to be during exceptional times of the global pandemic and the war directly affecting me and my family. These circumstances combined together with specifics of the department work environment not necessarily focused on a people-centred perspective forced me to exceed my limits in both bad and good ways, bringing up thoughts of doubt. However, luckily I had reliable and supportive people standing with me throughout my academic journey. And I would like to dedicate this dissertation to each and every one of them.

First, I would like to thank my supervisor and academic advisor Professor Fabricio Oliveira for his wide expertise and support during my whole doctoral journey. Professor Oliveira vastly impacted both my professional and personal development and his positive and friendly guidance helped me to keep going even at times of low spirit. I would also like to thank my other supervisor with whom I shared about a third of my doctoral journey, Professor Steven Gabriel. Professor Gabriel has taught me how an efficient and yet people-centred collaboration is conducted and working with him without exaggeration was one of the most exciting parts of my research. I would also like to thank International Institute for Applied Systems Analysis (IIASA) for the opportunity to broaden my knowledge while temporarily working in a multi-disciplinary department and, personally, my supervisor at IIASA Professor Nikolay Khabarov. Working with Professor Khabarov largely contributed to my knowledge of energy markets and taught me how to apply my knowledge in the context I was yet poorly familiar with. I would also like to express my gratitude to the Research Council for Natural Sciences and Engineering of the Academy of Finland for the financial support of my doctoral studies at IIASA.

I wish to also thank my colleagues throughout the years, especially Paula, Olli, Ellie, Edoardo, Alessandro, Lucas and Vilma. I was lucky to be surrounded by extraordinary people, who are great professionals and certainly cornerstones for the positive and friendly work environment. With many of them, I shared my experience inside and outside the office growing into friends. I particularly want to thank Ellie for all the empathy and support that helped me to keep going even at the hardest times. And I would also like separately thank Lucas for sharing with me most of my first-time experiences in academia during the first years.

And of course, last but not least, I want to express my gratitude to the people that were supporting me outside of academia. I want to thank my parents and my family, in particular, my mother, Ekaterina, and my grandmother, Margarita, for believing in me and always being there for me. I would also like to thank my sister, Anastasia, for her frequent mental support during times of doubt. And I would also like to thank my beloved partner Aleksei and my dear friends Daryna, Leonid, Jerome and Margarita who were also sharing with me a fair share of my doctoral journey. Without all of you, it would be so much harder (if not impossible) to accomplish what I have today.

Espoo, May 22, 2023,

Nikita Belyak

Contents

List of Publications

This thesis consists of an overview and of the following publications which are referred to in the text by their Roman numerals.

I Tiago Andrade, Nikita Belyak, Andrew Eberhard, Silvio Hamacher, Fabricio Oliveira. The p-Lagrangian relaxation for separable nonconvex MIQCQP problems. *Journal of Global Optimization*, 84, 1, 43–76, February 2022.

II Nikita Belyak, Fabricio Oliveira. A novel dual-decomposition method based on p-Lagrangian relaxation. *Submitted manuscript*, 31 pages, February 2023.

III Nikita Belyak, Steven A. Gabriel, Nikolay Khabarov, Fabricio Oliveira. Optimal transmission expansion planning in the context of renewable energy integration policies. *Submitted manuscript*, 30 pages, February 2023.

Author's Contribution

Publication I: "The p-Lagrangian relaxation for separable nonconvex MIQCQP problems"

Belyak is the primary author. Andrade proposed the original idea for the solution method. Oliveira conceptualised the idea. Under the supervision of Oliveira, Belyak implemented the algorithms, carried out the numerical experiments and wrote the manuscript. Eberhard developed and wrote theoretical contributions. Andrade, Eberhard, Hamacher and Oliveira carried out the revision of the manuscript

Publication II: "A novel dual-decomposition method based on p-Lagrangian relaxation"

Belyak is the primary author. Oliveira proposed the research topic. Under the supervision of Oliveira, Belyak implemented the algorithms, developed theoretical contributions, carried out the numerical experiments and wrote the manuscript. Oliveira carried out the revision of the manuscript.

Publication III: "Optimal transmission expansion planning in the context of renewable energy integration policies"

Belyak is the primary author. Belyak and Oliveira proposed the topic. Under the supervision of Gabriel, Khabarov and Oliveira, Belyak implemented the models, gathered the input data, carried out the numerical experiments, analysed the experimental data and wrote the manuscript. Gabriel, Khabarov and Oliveira carried out the revision of the manuscript.

List of Tables

Abbreviations

EU European Union

GDP Gross Domestic Product

GHG GreenHouse Gas

KKT Karush-Kuhn-Tucker

MIQCQP Mixed-Integer Quadratically Constrained Quadratic Programming

NP Non-deterministic Polynomial-time

RNMDT Reformulated Normalised Multiparametric Disaggregation Technique

TSO Transmission System Operator

UK United Kingdom

VRE Variable Renewable Energy

Symbols

$\mathbf{CO}_2$ Carbon dioxide

$\mathbb{R}$ Real coordinate space

1. Introduction

1.1 Background

Global energy systems transition to fully sustainable and carbon neutral is one of the present fundamental challenges from the economic and social perspectives [54]. Energy markets play an essential role in modern economies due to the sector's output share in the GDP as well as an impact on business investments and research. The transition process of energy systems firmly affects the evolution of the energy market sector. While ensuring and furthering humankind's well-being remain the key factors for the energy transition process, nowadays, substantial attention is paid to sustainability and decarbonisation [39]. Pressed by climate change, the need to minimise anthropogenic greenhouse gas (GHG) emissions has motivated many countries to search for efficient GHG emissions-free solutions for energy sectors. As an example, South Korea aims to reduce greenhouse gas emissions by 37% below business-as-usual emissions by 2030 across all economic sectors [65]. Likewise, the European Union (EU) established an ambitious target to be climate-neutral by 2050 relative to 1990 levels [15].

The decarbonisation of the energy sector inspired ongoing investigation for potential solutions, ranging from advancing the existing generation technologies and proposing new ones, such as carbon capture and storage technologies [43]. Moreover, the solutions for emissions mitigation should also take into account the possibility of decoupling the economic growth from emissions to ease the economic burden of decarbonisation. Chang et al. [14] highlight that a crucial tool for energy systems transition planning, and understating its impact on economic and social aspects of life, is energy systems modelling. The authors mention the existence of numerous modelling frameworks and computational software allowing one to account for various technical and methodological considerations when modelling energy systems.

Nevertheless, modelling tools have been criticised for providing insufficient precision of the information for the policymakers [55]. Keirstead et al. [31] name four groups of challenges associated with energy modelling techniques. Among those, they highlight the complexity of energy systems and model integration. Addressing these two issues lays the basis of this dissertation.

1.2 Energy systems complexity

While numerous techniques exist to incorporate a detailed representation of energy systems features (e.g., multi-objective modelling, bi-level programming and etc. [52, 61]), there exists a trade-off between the model complexity and computational tractability. Therefore, the architecture of such models usually involves a number of simplifying assumptions. As an example, in [57], the authors used the segment substitution method to simplify the distribution system model to ease the computational burden. In [53], the authors highlight the computational intractability of the decentralised energy generation model and examine the efficiency of different methods to reduce solution times. Virasjoki et al. [67] simplified the decision variable space by explicitly enumerating upper-level decisions due to computational difficulties associated with the complexity of the model and the size of the case study instance.

Consequently, it is crucial to develop efficient solution techniques allowing one to consider a higher level of detail in the energy model's structure without significantly increasing the computational burden. However, the solution approach is generally tailored to the model features. Energy system models are designed as mathematical programming models that involve four types of elements: i) a set of data, ii) a set of variables with their domain, iii) a set of constraints defining the feasible region for a solution, and iv) an objective function to be maximised or minimised [33, 11]. Therefore, when considering the features of energy system models, one actually refers to the nature of decision variables (e.g. discrete or integer), types of constraints (e.g., linear or non-linear), number of objectives and sub-problems that could be involved at different levels.

One of the challenging classes of mathematical programming problems is grouped by two-stage stochastic programming problems whose deterministic equivalent are non-convex mixed-integer quadratically constrained quadratic programming problems. The two-stage nature implies that there are two sets of decisions that are made in different stages. And stochasticity implies that a random event occurs in between the stages [21]. For example, Vahedipour-Dahraie et al. [66] proposed a two-stage stochastic programming model to determine the optimal scheduling with considering risk aversion and system frequency security to maximise the expected profit of the micro-gird operator. Zhou et al. [73] suggested that a two-stage stochastic programming model is an optimal design of a distributed energy system under energy demand and supply uncertainty. In [5], the authors proposed a two-stage stochastic integer programming model to optimize power production and trading for energy producers operating in competitive markets.

Solving two-stage stochastic programming problems usually involves the approximation of the random distribution with the discrete set in case these distributions are not finite [21]. This scheme allows for deriving a deterministic equivalent for the two-stage stochastic programming problem that is computationally tractable. However, the computational tractability of the resulting deterministic equivalent model may be jeopardised due to the presence of mixed-integer variables and non-linear constraints. Both mixed-integer linear programming and nonlinear programming problems are known to be NP-hard [23, 48] from a theoretical standpoint and challenging to solution methods from a computational standpoint [59]. Hence, the case where the deterministic equivalent model is represented by a mixed-integer quadratically constrained quadratic programming (MIQCQP) model appears to be particularly challenging as the problem spans challenges of both mixed-integer and non-linear natures. Moreover, in case the quadratic constraints are non-convex, one would have to rely on global optimisation methods as, even if the mixed-integer constraints are relaxed, finding a local solution would not guarantee its global optimality [29].

Nevertheless, the range of MIQCQP applications is vast. In particular, the class of models named pooling problems are MIQCQP under the assumption of linearly blending qualities [45]. Pooling problems are widely applicable in the areas of engineering that include supply–chain operations, and communications [44]. Examples include petroleum refining [32], and wastewater treatment [22], to name only a few. Another important class of MIQCQP problems applications are formed by single-level reformulation models of some bi-level optimisation problems [20, 67].

1.3 Energy models integration

Another conceptual problem related to energy modelling techniques is a very limited number of attempts to integrate models covering multiple energy sectors [31]. In particular, in the context of decarbonisation strategies, it is crucial to take into consideration the conflicting interests that the various participants pursue. While decreasing the amount of CO_2 emissions might be the primary goal of state policy-making organisations, private companies are often majorly aimed at increasing their own profits. However, for private investors, the expansion of variable renewable energy

(VRE) capacity is not necessarily profitable due to insufficient revenues occurring in liberalised energy markets (e.g., those found in Europe, the UK, and North America) [28]. Therefore, faithfully modelling a complex energy system involving players with disagreeable objectives can provide insights into how to achieve an equilibrium state.

Nevertheless, building an energy modelling tool capable of accounting for and mimicking the behaviour of all the systems' participants at once is an infeasible task. Therefore, attention is usually paid to investigating the interconnections between a subset of the energy system players. As an example, in [37], the authors propose a multi-objective model allowing to find an optimal solution that satisfies both the residential consumers willing to reduce their expenses and utilities aiming at reducing their system peak load demand. Shafiekhani et al. [62] exploited a bi-level programming approach to find a bidding equilibrium for profit-maximising producers and an entity maximising social welfare.

Two key players in the energy market system are generation companies and transmission system operators (TSO). While generators compete to serve the load demand, transmission is crucial in the sense that it enhances the competition in the energy market, which leads to better performance and efficiency in electricity production [51]. Additionally, an expanded transmission infrastructure allows for optimal use of surplus generation, efficient congestion management and reduced requirement for backup generation capacity [49]. Moreover, in the context of decarbonisation, the role of TSOs becomes even more significant as it facilitates the capability to cope with temporal availability and geographically nonuniform spread of VRE availability [1].

For example, a long-term perspective analysis for France indicated that reaching 100% VRE share in the generation mix by 2050 would require, among others, efficient interconnections to ensure power reliability [60]. Mazzanti [40] highlights that an efficient high-voltage direct current transmission system is the key environmentally-compatible solution for integrating renewable power in the grid in case of remotely located VRE plants such as offshore wind or desert photovoltaic generation. Nevertheless, the author concludes that further research activities are required to improve the performance and reliability of such transmission systems and, hence, to further increase their role in decarbonising strategies. A similar conclusion is withdrawn in [46], where the authors highlight that the design of existing transmission systems does not allow one to introduce sufficient levels of renewable energy into the grid. Moreover, managing the interaction between TSO's and generators' investments is a challenging task, making it hard to motivate renewables-driven generation infrastructure expansion [18]. Therefore, developing an efficient strategy to facilitate the increase of VRE in the generation mix requires designing new approaches for transmission network planning.

The literature groups the strategies to model renewables-driven transmission and generation infrastructure planning into two categories. The first one involves designing the system where TSOs make investment decisions anticipating all the possible developments in generation infrastructure. For example, the authors exploit such an approach in [63, 47]. However, this method has two shortcomings: i) formulating an uncertainty set comprising the possible generators' investment decisions is a challenging task, and ii) the generators are passive players incapable of reacting to the TSO's decisions. These shortcomings are addressed in modelling strategies forming the second category, where the decisions of the TSO and generation companies are made in a coordinated manner, commonly involving a model with several-level sub-problems. For example, such a strategy was explored in [46, 64, 72].

1.4 Objectives and scope

This dissertation aims to answer several research questions. Firstly, it aims to fulfil the research gap regarding the analysis of TSO impact in the context of renewables-driven energy system planning. However, this analysis requires efficient solution tools that allow us to solve large-scale several-level mathematical optimisation problems. Therefore, the development of an efficient solution approach became the second direction of this dissertation. However, while the aforementioned solution approach was supposed to be utilised in the context of solving renewable-driven energy system planning, ultimately, these two research directions did not converge. The reason behind this is the complexity of the developed energy modelling assessment that prevented completing the research objective to the extent that was originally set within the time constraints of the dissertation work. In particular, the decision spaces for TSO and generation companies were supposed to be discrete and conclusively requiring solving large-scale MIQCQP problems. However, preliminary experiments with illustrative instances identified unanticipated computational issues which required further analysis considering a larger set of instances. Our way to circumvent this issue was to relax the integrality constraints on decision space for TSO and generation companies and assume as fixed the TSO's decisions in an enumerative approach. Thus, eliminating the mixed-integer part from the MIQCQP problems, allowed us to utilise existing solution software for QCQP problems.

At the same time, the development of a solution method to efficiently solve MIQCQP problems has been conducted as an independent project. In principle, it can be applied for solving problems optimising, in a coordinated manner, discrete TSO and generators' investment decisions. The development of this efficient solution method took place in two parts. The first one involved the development of an efficient solution method that outperformed commercial solver Gurobi [27]. However, the efficiency of the proposed approach would still be jeopardised due to shortcomings associated with the mixed-integer nature of the instances considered. Hence, the second part of this research direction was dedicated to addressing that issue.

Therefore, the threefold contribution of this dissertation spans:

1. Formulation and implementation of energy market model in a coordinated manner, optimising TSO and generation companies' decisions and applying the proposed modelling approach to model the Nordic and Baltic energy system.

2. The employment of the aforementioned energy market model to study the role and extent of expanding the transmission infrastructure and altering carbon tax value and VRE subsidies distinctively or in composition in the context of renewables-driven strategies. In particular, the analysis concentrated on the changes in optimal values of total welfare, VRE share in generation mixture and total generation amount.

3. Development of an efficient solution method for non-convex MIQCQP problems relying on duality theory, mixed-integer relaxation techniques and a branch-and-bound algorithm.

Table 1.1 summarises the scope of Publications I-III in terms of their research objectives.

	Research objective
Paper I	Decomposition- and relaxation-based solution method for non-convex MIQCQP problems
Paper II	Enhancement of the solution method by employing branch-and-bound approach
Paper III	Modelling of renewables-driven energy system involving transmission infrastructure expansion

Table 1.1. Research objectives of Publications I-III

1.5 Research methods and dissertation structure

The mathematical model presented in Paper III accounts for the economic and technical features of the energy market. In particular, the model considers the transmission, generation levels and supply- and demand-based price determination. Additionally, due to the diversity of interests between the market players, i.e., TSO and generation companies, the model has a bi-level structure. Further, the equilibrium state is found by transforming the bi-level model into the mathematical program with equilibrium constraints [24] and solving it to optimality.

The solution method developed in Paper I combines a Lagrangian decomposition [26] approach with a mixed-integer-based relaxation technique [4]. Nevertheless, the proposed approach, named p-Lagrangian decomposition, would not allow for overcoming the duality gap arising from the mixed-integer nature of the MIQCQP problems. Paper II tackles that issue, enhancing the p-Lagrangian decomposition by combining it with a duality-based branch-and-bound technique [10]. Both methods proposed in Papers I and II were assessed on randomly generated MIQCQP instances.

The rest of the introductory chapter is structured as follows. Section 2 presents the methodological background of this dissertation. Section 3 discusses the results and contributions of Papers I-III. Section 4 concludes the dissertation by summarising the contributions and outlining the extensions for future research.

2. Methodological background

This chapter contains a summary of the methodological foundations explored in this dissertation. In particular, Sections 2.1.1 - 2.1.3 describe the methodological background being the basis for solution methods for MIQCQP problems. Sections 2.2 - 2.3 describe the modelling approach employed for the renewable-driven energy market assessment.

The formulation of a general two-stage stochastic mixed-integer problem, such as that discussed in Section 1.2, is

$$z^{\text{SMIP}} := \min_{x}. \left\{ c^{\top} x + \mathcal{Q}(x) : x \in X \right\}, \tag{2.1}$$

where the vector $c \in \mathbb{R}^{n_x}$ is known and X is a mixed-integer set that contains linear constraints and integrality restrictions on some components of x. The recourse function $\mathcal{Q} : \mathbb{R}^{n_x} \mapsto \mathbb{R}$ is the expected recourse value

$$\mathcal{Q}(x) := \mathbb{E} \left[\min_{y}. \left\{ f(y, \xi) : g(x, y, \xi) = 0, y \in Y(\xi) \right\} \right], \tag{2.2}$$

where, for any realisation of the random variable ξ, $f : \mathbb{R}^{n_y} \mapsto \mathbb{R}$ is defined as

$$f(y, \xi) := q(\xi)^{\top} y + \sum_{(i,j) \in B_Q} Q(\xi)_{i,j} y_i y_j,$$

$g := [g_1, \dots, g_{|M|}]^{\top}$ where $g_m : \mathbb{R}^{n_x \times n_y \times n_\xi} \mapsto \mathbb{R}, \forall m \in \{1, \dots, |M|\} := M$, is defined as

$$g_m(x, y, \xi) := T(\xi)_m x + W(\xi)_m y + \sum_{(i,j) \in B_U} U(\xi)_{m,i,j} y_i y_j - h(\xi)_m,$$

and B_Q (B_U) is an index set containing the pairs (i, j) for which the entry $Q_{i,j} > 0$ ($U_{i,j} > 0$), implying the presence of the bi-linear terms $y_i y_j$; $Y(\xi)$ is a mixed-integer set containing both linear constraints and integrality requirements on some of the variables $y(\xi)$; and $\mathbb{E}_{\xi}[\,\cdot\,]$ denotes the expectation of $\cdot$ in terms of the random variable ξ.

Following the standard procedure in the stochastic programming literature, we approximate the random distribution of variable ξ with a finite set S of realisations $\xi_1, \dots, \xi_{|S|}$, each with associated probability value $p_1, \dots, p_{|S|}$. In particular, each realisation ξ_s of ξ encodes the realisation observed for each of the random elements $(q(\xi_s), Q(\xi_s))$ and $(T(\xi_s)_m, W(\xi_s)_m, U(\xi_s)_m, h(\xi_s)_m)$, $\forall m \in M$. For the sake of notation compactness, we refer to these collections as (q^s, Q^s) and $(T^s_m, W^s_m, U^s_m, h^s_m)$, $\forall m \in M$, respectively.

Problem (2.1) can be then posed as the deterministic equivalent MIQCQP problem

$$z^{\text{SMIP}} := \min_{x,y}. \; c^\top x + \sum_{s \in S} p^s ({q^s}^\top y^s + \sum_{(i,j) \in B} Q^s_{i,j} y^s_i y^s_j) \tag{2.3}$$

$$\text{s.t.: } x \in X \tag{2.4}$$

$$T^s_m x + W^s_m y^s + \sum_{(i,j) \in B} U^s_{m,i,j} y^s_i y^s_j = h^s_m, \; \forall m \in M, \forall s \in S \tag{2.5}$$

$$y^s \in Y^s, \; \forall s \in S. \tag{2.6}$$

2.1 Solution method

2.1.1 Lagrangian decomposition

One of the possible approaches for solving the MIQCQP problems is the decomposition method allowing one to split the MIQCQP problem into several smaller sub-problems that are more tractable and can be solved independently, possibly in parallel. One of the most common decomposition frameworks is Lagrangian decomposition. The authors in [10] identified that by means of exploring the block-angular structure of the problem, Lagrangian decomposition yields stronger relation than, for example, linear programming relaxations. The first step of employing Lagrangian decomposition to MIQCQP problem (2.3) - (2.6) involves creating copies of the complicating variables — i.e., the first stage variables x - for each scenario $s \in S$ and introducing non-anticipativity conditions to prevent the first-stage decisions from being scenario dependent. The reformulated equivalent of Problem (2.3) - (2.6) can be represented as

$$z^{\text{SMIP}} := \min_{x,\overline{x},y}. \; \sum_{s \in S} p^s (c^\top x^s + {q^s}^\top y^s + \sum_{(i,j) \in B_Q} Q^s_{i,j} y^s_i y^s_j) \tag{2.7}$$

$$\text{s.t.: (2.6)} \tag{2.8}$$

$$x^s \in X, \forall s \in S \tag{2.9}$$

$$T^s_m x^s + W^s_m y^s + \sum_{(i,j)\in B_U} U^s_{m,i,j} y^s_i y^s_j = h^s_m,\ \forall m \in M, \forall s \in S \tag{2.10}$$

$$x^s - \overline{x} = 0,\ \forall s \in S, \tag{2.11}$$

where the constraint (2.11) enforces non-anticipativity for the first-stage decisions. Problem (2.7) -(2.11) has a nearly decomposable structure implying that if one was to remove Constraint (2.11), the Problem (2.7) -(2.11) could be fully decomposed into $s \in \mathcal{S}$ MIQCQP problems. Therefore, the next step of the Lagrangian decomposition is the implementation of Lagrangian relaxation to remove Constraint (2.11)

Lagrangian relaxation is a powerful tool allowing one to approximate a constrained optimisation problem with a more easily tractable version of it. Solving the relaxed problem allows for obtaining an upper (lower) bound for the solution of the primal maximisation (minimisation) problem [36]. The approximation of the original primal problem, named the Lagrangian dual problem, is done by relaxing one of the constraints and adding a penalty term associated with it, called the Lagrangian multiplier, to the objective function. In this case, the Lagrangian multiplier can be interpreted as the cost of violating the relaxed constraint.

Relaxing Constraint (2.11) in Problem (2.7) -(2.11) using Lagrangian relaxation results in the following Lagrangian dual problem

$$L_p(\mu) = \left\{ \begin{array}{l} \min\limits_{x,\overline{x},y} \sum\limits_{s\in\mathcal{S}} p^s \big(c^\top x^s + q^{s\top} y^s + \sum\limits_{(i,j)\in B} Q^s_{i,j} y^s_i y^s_j + \mu^{s\top}(x^s - \overline{x})\big) \\ \qquad\qquad\qquad\qquad\qquad\qquad : (x^s, y^s) \in (2.8) - (2.10), \end{array} \right\}. \tag{2.12}$$

2.1.2 Reformulated normalised multi-parametric desegregation technique

A conceptually different approach to solving the MIQCQP problems is approximating it with mixed-integer relaxation. An efficient technique to relax quadratic terms named normalized multiparametric disaggregation has been proposed by Castro [13]. The method relies on discretising the domain of one variable in each of the bi-linear terms. The method is closely related to the piecewise McCormick envelopes approach[41], as the discretisation of the variable domain is equivalent to splitting it into a number of uniform partitions. The size of these partitions is related to the accuracy of the approximation and can, thus, be made arbitrarily precise. However, one should bear in mind the trade-off between the accuracy of the approximation and its tractability, as improving the accuracy of the approximation increases the number of variables in this approximation, including binary variables.

More formally, the discretisation of the domain of continuous variable y_i^s in Problem (2.7) -(2.11) can be represented by the set of constraints (2.13)-(2.15)

$$y_i^s = (Y_i^{s,U} - Y_i^{s,L})(\sum_{k \in \{0,\ldots,9\}, l \in \{-p,\ldots,-1\}} k \times 10^l \times z_{i,k,l}^s + \Delta\lambda_i^s) + Y_i^{s,L}, \quad (2.13)$$

$$0 \leq \Delta\lambda^s \leq 10^p, \quad (2.14)$$

$$z_{i,k,l}^s \in \{0,1\}, \; \forall k \in \{0,\ldots,9\}, l \in \{p,\ldots,-1\}, \quad (2.15)$$

where $Y_i^{s,U}$,($Y_i^{s,L}$) corresponds to the upper (lower) bound of variable y_i^s.

In the set (2.13)-(2.15), the integer parameter $p < 0$ corresponds to the precision of approximation, as it defines the number of digits used in normalised mixed-integer representation of variable y_i^s. The term $\Delta\lambda_i^s$ ensures that we can achieve any value in the set $[0,1]$. Further, when considering the variable y_i^s in the context of the bi-linear term, i.e., a product $y_i^s \times y_j^s$, the product of binary and continuous variables $z_{i,k,l}$ and y_j^s is linearised exactly. The product of two continuous variables $\Delta\lambda_i^s$ and y_j^s is relaxed using McCormick envelopes [12]

Hence, the normalized multiparametric disaggregation of the $y_i^s \times y_j^s$ implies the discretisation of the domain of y_i^s. Assuming the lowest possible accuracy of the approximation, i.e., when precision parameter $p = -1$ and the size of partitions is 0.1, the representation of variable y_i^s is

$$y_i^s = 0 \times 10^{-1} \times z_{i,0}^s + 1 \times 10^{-1} \times z_{i,1}^s + \cdots + 9 \times 10^{-1} \times z_{i,9}^s + \Delta\lambda_i^s,$$

where $\Delta\lambda_i^s \in [0, 0.1]$ and $z_{i,0}^s, \ldots, z_{i,9}^s$ is binary. This approach has been further enhanced by Andrade et al. [4] and named reformulated normalized multiparametric disaggregation technique (RNMDT). The authors suggested the consideration of a base-two (or binary) representation, instead of a decimal, for the discretised domain. Therefore, the smallest possible interval would be 0.5 instead of 0.1 when the precision parameter is set to $p = -1$. Additionally, the authors suggested a series of reformulations and simplifications in the definition of the relaxation. These improvements allowed for significant reductions in the number of variables and constraints in the formulation, and hence, the approximation became more easily tractable. As an example, the discretisation of the y_i^s domain considering binary representation and precision parameter $p = -1$ would be

$$y_i^s = 0 \times 2^{-1} \times z_{i,0}^s + 1 \times 2^{-1} \times z_{i,1}^s + \Delta\lambda_i^s,$$

where $\Delta\lambda_i^s \in [0, 2^{-1}]$ and $z_{i,0}^s, z_{i,1}^s$ are binary.

Moreover, the authors suggested an improvement related to the process of increasing the accuracy of the approximation. The original idea in the normalized multiparametric disaggregation suggests considering smaller partition sizes for all the variables being discretised to obtain a more accurate approximation of the primal problem. The authors in [4] developed an assessment method allowing one to identify the variables whose discretisation partitions tightening would lead to a significant improvement in the whole approximation accuracy. Hence, one would only consider small partitions in the discretisation of those variables and, ultimately, require fewer additional binary variables to be added for a tighter approximation.

2.1.3 Duality-based branch-and-bound method

The composition of the RNMDT method and Lagrangian decomposition allows one to solve large-scale non-convex MIQCQP problems efficiently [3]. However, the approach demonstrated shortcomings associated with the mixed-integer nature of the instances under study. While one can obtain an arbitrary precise objective function value for the dual problem, it is not guaranteed to coincide with the objective function value of the primal problem due to the lack of convexity.

A solution to circumvent this issue was proposed by Carøe and Schultz [10]. The authors suggested considering the Lagrangian relaxation of the primal problem within the branch-and-bound framework. The idea of the branch-and-bound algorithm is to repeatedly partition the space of all feasible solutions into smaller subsets and calculate the value of the objective function over each of the subsets [35]. The objective function value, in this case, provides a lower (upper) bound for the primal minimisation (maximisation) problem. Then the subsets for which the value of the objective function value exceeds (is less than) the objective function value of a known feasible solution for the primal minimisation (maximisation) problem are excluded from further partitioning. The process of partitioning continues until a feasible solution for the primal minimisation (maximisation) problem is found for which the value of the objective function is no greater (not less) than the objective function value for any other subset.

The procedure of dividing the set of all feasible solutions into disjoint subsets differs depending on what type of primal constraints is violated by the optimal solution. In case the optimal solution $\tilde{x}^s$ from the set of all possible solutions $\tilde{X}^s$ violates the integrality constraint at the coordinate i, two disjoint subsets $\tilde{X}_1^s$ and $\tilde{X}_2^s$ such that $\tilde{X}_1^s \cup \tilde{X}_2^s = \tilde{X}^s$ formed as $\tilde{X}_1^s = \tilde{X}^s \cap \tilde{x}_i^s \leq \lfloor \tilde{x}_i^s \rfloor$, $\tilde{X}_2^s = \tilde{X}^s \cap \tilde{x}_i^s \geq \lfloor \tilde{x}_i^s \rfloor + 1$, where $\lfloor \tilde{x}_i^s \rfloor$ denotes greatest integer less than or equal to $\tilde{x}_i^s$.

Another strategy for splitting the set $\tilde{X}$ into disjoint subsets occurs if optimal solution $\tilde{x}^s$ violates the constraint that is being relaxed under the Lagrangian relaxation scheme. Carøe and Schultz [10] considered the deterministic equivalent of a two-stage mixed-integer stochastic programming problem, i.e., Problem (2.7) - (2.8). The Lagrangian relaxation was applied to relax the non-anticipativity constraints (2.11) resulting in Lagrangian dual problem (2.12).

Constraints (2.11) ensure that, for decision variable $x \in \mathbb{R}^N$, $x_1 = \cdots = x_N$. Thus, they are used to ensure that the copies of the first-stage decision variables match for all the independent sub-problems $i \in \{1, \ldots N\}$. The stochastic nature of the primal problem implies that each separate sub-problem i has a predefined probability p_i associated with it. Hence, the primal objective function takes into account the expected value of all the objective function values of separate sub-problems.

Therefore, the authors in [10] proposed the branch-and-bound algorithm that partitions the set of all feasible solutions of the Lagrangian dual problem based on the violation of both integrality and non-anticipativity conditions.

Paper I of this dissertation combines the methodologies presented in Sections 2.1.1 and 2.1.2 to develop an efficient solution method for non-convex MIQCQP problems. This algorithm was further developed in Paper II by means of exploiting the branching strategies presented in Section 2.1.3 to allow convergence to an optimal solution even in the presence of a duality gap caused by the mixed-integer nature of the MIQCQP problems.

2.2 Concepts of centralised and decentralised energy markets

In the context of defining bi-level problems representing the energy market operations, Paper III refers to concepts such as centralised and decentralised energy markets. These terms identify whether different segments of the energy system are controlled by one or multiple agents. The term "energy decentralisation" might have different interpretations depending on what is being considered as decentralised: energy hardware, ownership, knowledge, socio-political power, decision-making authority, economic market share and etc. [30]. A centralised energy system has an advantage in ensuring secure and efficient resource allocation, especially in the context of the electricity supply chains [8]. However, a centralised energy system might not be capable of efficiently handling the disturbances within the supply chain. As a result, addressing climate change issues and the need for energy security makes independent, decentralised energy systems more appealing [68] and pinpoint them as a core part of future energy development strategies around the world [69].

Paper III explores the concepts of energy centralisation and decentralisation in the context of power plans and transmission infrastructure ownership. Therefore, in the case of the centralised energy market, both power plants and the transmission system are owned by a single agent. This agent then decides on generation and transmission infrastructure expansion investments and generation levels. An example of such a system can be found in [42], where the authors evaluated the possibility of transforming a small Canary island's energy system into a 100% renewable one. In the case of decentralised energy systems, Paper III assumes power plants and transmission infrastructure to be owned by two distinct agents. Moreover, the ownership of various power plants can also be distributed between multiple generation companies.

The common approach in day-ahead energy markets is to not only consider energy supply suggested by generation companies' but also the demand provided by end-use customers as it allows for the reduction of the peak load of the system [69]. Higher prices motivate the producers to produce more and buyers to buy less. Such dependencies can be represented through a demand function $f_d(p)$ and a supply function $f_s(p)$ that for a given value of energy price p return the corresponding demand and supply quantities (q_d and q_s). There is only one price value p^* at which the demand and production quantity values coincide, i.e., $q_p = q_s = q^*$ and this point is named the economic equilibrium [24].

Let us consider the social welfare function $SW(q)$ for a single commodity market defined as

$$SW(q) = \int_0^q f_d^{-1}(q')dq' - \int_0^q f_s^{-1}(q')dq',$$

where q is a quantity of energy, $f_d^{-1}(q)$ and $f_s^{-1}(q)$ are inverse demand and supply functions, respectively. It is worth highlighting that SW, which represents the quantity of energy produced, is assumed to coincide with the demand. Let us assume the inverse demand and supply functions have negative and positive slopes, respectively. It then follows that $SW(q)$ is strictly concave. Hence, if $SW(q)$ has a maximum point $q^* = \max_q SW(q)$ it is unique and defined by the condition

$$\frac{dSW(q)}{dq} = 0. \tag{2.16}$$

Equation (2.16), in turn, implies that $f_d^{-1}(q) - f_s^{-1}(q) = 0$ or $f_d^{-1}(q) = f_s^{-1}(q)$. Hence, $q^* = \max_q SW(q)$ would be the equilibrium point, i.e., $f_d^{-1}(q^*) = f_s^{-1}(q^*)$.

The construction of equilibrium models involving multiple generation companies usually requires an assumption about producers' and consumers' behaviour regarding market price. If any of the producers or consumers can exploit the impact of their generation decisions on the market price, the energy market is said to experience imperfect competition [24]. Otherwise, if all the producers and consumers act as price takers, such a case is named perfect competition. The latter setting is considered in Paper III as it more closely reflects the Northern European energy market setting considered in the case study of interest.

Therefore, the equilibrium for a perfect competition can be calculated by maximising social welfare

$$\text{Max.} \int_0^q f_d^{-1}(q')dq' - \sum_i VC_i(q_i) \tag{2.17}$$

$$\text{s.t.: } q - \sum_i q_i = 0, \tag{2.18}$$

where q_i and $VC_i(q_i)$ are individual output and variable cost functions, respectively, for generation company i.

2.3 Bi-level programming problems

Bi-level programming problems stem from the modelling of hierarchical relationships among multiple decision makers that can be grouped into two categories based on the nature of their interaction with other decision makers: leader (upper) level and follower (lower) level [34]. As examples, bi-level models are employed to find the optimal design for bus lane networks [71], to define the optimal location for renewable power plants [38] and optimise frequencies of an urban transportation network [16]. Paper III takes into account the bi-level structure in the interactions between a transmission system operator at the upper level in the hierarchy and power market operations at the lower level, formulating it as a bi-level programming problem.

Assuming the transmission system operator acts as a welfare maximiser, the bi-level problem can be formulated as follows.

$$\text{Max. } SW(q) - C_{TSO} \tag{2.19}$$

$$\text{s.t.: } q \in \text{argmax } (2.17) - (2.18), \tag{2.20}$$

where C_{TSO} is the sum of the total and variable costs of the transmission system operator.

To tackle problem (2.19) - (2.20), a common approach is to transform it into a single-level alternative representation. Gabriel and Leuthold [25] proposed replacing the lower-level problem (2.17)-(2.18) with correspondent Karush–Kuhn–Tucker (KKT) conditions [24] in case the lower-level problem is a convex quadratic program rendering single-level mathematical program with equilibrium constraints.

For the convex problems, KKT conditions are necessary and sufficient optimality conditions if the constraint qualification holds (see, for example, [70]). For the problem (2.17)-(2.18) the KKT conditions are given by

$$-f^{-1}(q) + \lambda = 0, \tag{2.21}$$

$$\frac{dVC_i(q_i)}{dq_i} - \lambda = 0, \ \forall i \tag{2.22}$$

$$q - \sum_i q_i = 0, \tag{2.23}$$

$$\lambda \in \mathbb{R}, \tag{2.24}$$

where λ is the dual variable associated with (2.20) (see Section 2.1.1).

Hence, problem (2.19)-(2.20) can be equivalently reformulated by replacing equation (2.20) with (2.21)-(2.24), assuming that the KKT conditions are indeed optimality conditions for (2.17)-(2.18).

2.4 Relationship between the methodological developments in Papers I, II and III

Papers I and II consider two-stage stochastic mixed integer programming problems whose deterministic equivalents are MIQCQP problems. Section 1.4 highlights that under particular assumptions that render KKT conditions valid for attesting optimality, the bi-level model presented in Paper III could be transformed into a MIQCQP problem. This is precisely the connection between the general structure of the mathematical programming problems studied in Papers I, II and III.

The summary of the structure of the problem proposed in Paper III more explicitly explains the connection between its single-level representation and the MIQCQP problem. Firstly, as mentioned in Section 1.4 for the sake of easing computational tractability, the authors of Paper III relaxed integrality constraints for variables related to the capacity expansion levels of the transmission lines and generation sources. Hence, if this relaxation was not adopted the original bi-level problem would have a mixed-integer nature. Additionally, in contrast with the simplified version of the bi-level problem presented in Section 2.3, the bi-level problem proposed in Paper III involves a large number of inequality constraints at the lower level. Hence, the KKT conditions for the lower-level problem would involve complementary slackness conditions that, in turn, would contain bi-linear terms, i.e., the products of primal and dual variables. Lastly, one should take into account that the objective function of the upper level of the problem proposed in Paper III is the social welfare function $SW(q)$ defined in Section 2.2 that is quadratic, Therefore, taking into account the aforementioned details regarding the structure of the bi-level problem in Paper III one can conclude that, under the absence of relaxation of integrality constraints, the transformation of the bi-level problem into as single level alternative following the procedure described in Section 2.3 results in equivalent MIQCQP model.

3. Contributions of the dissertation

Table 3.1 summarises the research objectives, methodologies and key takeaways of Papers I-III. The research objectives of Papers I and II intersect, as they aim at formulating efficient algorithms to solve two-stage stochastic programming problems whose deterministic equivalents are non-convex MIQCQP problems presenting a separable structure. Paper I proposes an efficient method that, nevertheless, does not guarantee convergence to the primal optimal solution in cases where duality gaps are present. Paper II advances the method proposed in Paper I to address this issue. The research objective of Paper III is to model simultaneous transmission and generation infrastructure expansion in the decentralised energy market. The model is then used to analyse the generators' investment decisions in the context of VRE-supporting policies.

Methodologically, Paper I relies on the combination of classic Lagrangian decomposition and mixed-integer RNMDT relaxation [4]. Additionally, Paper I also exploits the dynamic-precision algorithm for efficiently choosing the bi-linear terms to relax, as originally proposed in [4]. Paper II relies on the dual-decomposition method proposed in [10], in which a branch-and-bound strategy takes place whenever either integrality or constraint relaxed using Lagrangian relaxation is violated. Paper III is based on a bi-level optimisation model, which is then transformed into a single-level mathematical program with equilibrium constraints [24]. In the presence of discrete capacity expansion decisions, the resulting single-level equivalent is an MIQCQP problem that, in principle, could be solved by using algorithms proposed in Papers I and II.

Papers I and II present experimental results obtained from a set of randomly generated MIQCQP instances. The modelling approach proposed in Paper III has been applied to two case studies: i) illustrative three-node instance and ii) the energy system representing the simplified version of combined Nordic (Finland, Norway, Sweden and Denmark) and Baltic (Estonia, Latvia and Lithuania) energy markets. The models in Paper III, as well as the algorithms in Papers I and II, were implemented using Julia [7] language and Gurobi [27] solver.

The following sections provide a summary of the contribution and results of Papers I-III.

Publication	Research objectives	Methodologies	Main results
Paper I: "The p-Lagrangian relaxation for separable non-convex MIQCQP problems"	Development of efficient solution method for MIQCQP problems. Evaluation of the proposed method efficiency on a range of randomly generated instances	Lagrangian relaxation, RN-MDT, Dynamic-precision-based algorithm	Novel optimisation algorithm outperforming commercial solver Gurobi in terms of computational time required to solve MIQCQP instances. We also highlight the limitations of the proposed method related to its incapability of overcoming duality gaps.
Paper II: "A novel dual-decomposition method based on p-Lagrangian relaxation"	Combining the p-Lagrangian relaxation and branch-and-bound strategies to ensure convergence to optimal solutions that are both primal and dual feasible even in case of duality gaps. Evaluation of the new method efficiency on a range of randomly generated instances.	Branch-and-bound algorithm, Frank-Wolfe Progressive hedging, proximal bundle method	Novel optimisation algorithm allowing for efficiently solving to optimality mixed-integer relaxations of MIQCQP problems that can be made arbitrarily precise. The proposed algorithm outperforms the commercial solver Gurobi in terms of computational time.
Paper III: "Optimal transmission expansion planning in the context of renewable energy integration policies"	Formulation of the mathematical model to study optimal investments in the transmission and generation infrastructure under different VRE-supporting policies. Investigation of the optimal investment strategies in the Nordic and Baltic energy markets.	Bi-level modelling, complementarity modelling, representative days via hierarchical clustering	Novel bi-level model representing the behaviour of TSO and generators under different VRE-supporting policies. Sensitivity analysis on the input parameters conducted for small-scale instances. Case study for Nordic and Baltic energy systems.

Table 3.1. Research objectives, methods and key results from the papers in the dissertation

3.1 Paper I

The paper focuses on non-convex MIQCQP problems presenting a block-angular structure allowing for decomposition as those presented by the deterministic equivalent of two-stage mixed-integer stochastic programming problems. In principle, such problems can be solved by the available open source and commercial solvers, e.g., Couenne [19] or Gurobi [27]. However, off-the-shelf solvers are not consistently capable of solving the MIQCQP problem to optimality in case of large-scale instances [67].

Therefore, Paper I proposes a novel technique to efficiently compute a dual bound for non-convex MIQCQP problems named p-Lagrangian decomposition. The p-Lagrangian decomposition combines the classic Lagrangian decomposition approach and mixed-integer relaxation. The relaxation is constructed by means of employing the RNMDT technique and its precision can be manually adjusted via a precision parameter p. Combining these two methods allows one to exploit the classic Lagrangian decomposition strategy but consider the mixed-integer relaxations instead of the actual sub-problems. Hence, one could exploit more robust mixed-integer linear programming solution methods to solve sub-problems' relaxations to optimality.

Paper I also proposes a dynamic-precision algorithm allowing one to individually adjust the RNMDT precision parameter p only for those variables whose precision tightening would provide a significant improvement in the precision of the whole problem RNMDT relaxation. To control the quality of the dual bound, as a subroutine, Paper I exploited the bundle method [6] for updating Lagrangian multipliers.

The p-Lagrangian decomposition method efficiency was tested on a set of randomly generated MIQCQP instances. The numerical experiments suggested superior performance of the p-Lagrangian decomposition algorithm over Gurobi in terms of the capability of obtaining dual bound within the predefined time limit. Additionally, the numerical experiments demonstrated significant savings in computational time if one exploits parallel computing.

Nevertheless, despite the suggested computational efficiency p-Lagrangian decomposition method demonstrated a few shortcomings. The first one is related to the exploitation of the heuristics approach as a subroutine to generate the primal feasible solution. The inaccuracy of the heuristic method might jeopardise the capacity of the p-Lagrangian decomposition method to attain desired optimality tolerance. The second shortcoming is associated with the incapability of the p-Lagrangian decomposition method to handle the duality gap arising from the mixed-integer nature of the primal problem.

3.2 Publication II

Analogously to Paper I, Paper II also addresses the development of the solution approach for non-convex MIQCQP problems presenting a block-angular structure stemming from the deterministic equivalent of two-stage mixed-integer stochastic programming problems. However, Paper II proposes a solution method that allows one to attain desirable optimality tolerance even in cases when a duality gap arises due to the mixed-integer nature of the primal problem.

The proposed solution method named p-branch-and-bound incorporates the p-Lagrangian decomposition presented in Paper I within the branch-and-bound framework proposed in [10]. The proposed p-branch-and-bound method generates the primal bound for the RNMDT relaxation of the primal problem with predefined tolerance for any precision factor p. Then, the RNMDT relaxation can be made arbitrarily precise by means of changing the precision parameter value.

The main idea of the p-brand-and-bound relies on i) solving each node sub-problem using p-Lagrangian decomposition and ii) replicating branch-and-bound procedure whenever the solution generated by p-Lagrangian decomposition violates integrality or non-anticipativity conditions. Moreover, Paper II conducts an analysis of the p-branch-and-bound convergence behaviour by comparing two different methods for solving mixed-integer relaxations of dual problems in p-Lagrangian decomposition: i) proximal bundle method and ii) Frank-Wolfe progressive hedging.

The p-branch-and-bound efficiency has been verified on two sets of random MIQCQP instances. Set 1 is inspired by [2] and considers large-scale problems involving quadratic matrices with 1% density. The numerical results demonstrated that the proposed method presents significant reductions in computational time when compared with the direct employment of a commercial solver Gurobi. Depending on the solution method exploited to solve nodes subproblems, p-branch-and-bound allowed on average to reduce the time required by Gurobi to solve RNMDT relaxation by about 32 % and 84% in case proximal bundle method and Frank-Wolfe progressive hedging has been used, respectively.

Nevertheless, when applied to the instances from Set 1 p-branch-and-bound required considering only one (root) node as already at the root node, it managed to generate the solution that satisfied both integrality and non-anticipativity primal conditions. Hence, we have also applied to p-branch-and-bound method to another set of MIQCQP instances (Set 2) that contained problems involving quadratic matrices with 90% densities. However, in order to ensure the convergence of the p-branch-and-bound within a reasonable time, we considered small-scale problems in Set 2. The numerical results demonstrated that even in cases the solution for the root node violates integrality or non-anticipativity conditions, the employment of p-branch-and-bound ensures that, ultimately, the proposed method attains zero-gap optimal solutions.

3.3 Publication III

Earlier research has emphasised the significant role of efficient transmission infrastructure in the context of mitigating the challenges following the increase of VRE generation share and its availability uncertainty [46]. Nevertheless, a research gap still exists regarding the extent to which one can consider the transmission infrastructure development as a part of the efficient renewable-driven strategy. Therefore, Paper III investigates the impact of TSO investment decisions along with other widely applicable VRE-supporting policies as a carbon tax and renewable-driven investment incentives on the optimal generation mix.

In the first part of Paper III, we propose a modelling approach for planning renewables-driven transmission infrastructure expansion in a perfectly competitive market that also accounts for the carbon taxes and the subsidies for VRE-related costs. The mathematical optimisation model relies on a bi-level formulation. The upper-level problem depicts a welfare-maximising transmission system operator that makes decisions regarding the capacity expansion of the transmission lines. The lower level of the problem represents power market operations and considers the generation companies making the decisions about generation capacity expansion and generation levels to maximise their profit. The bi-level model assumes a linear inverse demand function used to derive equilibrium demand and price. The notion of perfect competition refers to the fact that the generation companies at the lower level act as price takers, implying a lack of complete knowledge regarding the linear dependence between generation levels and price.

The lower-level problem is convex and only contains linear constraints. Hence, the KKT conditions for it are necessary and sufficient. Exploiting this fact, Paper III transforms the resulting bi-level optimization problem into a single-level mathematical problem with equilibrium constraints by replacing the lower-level problems with corresponding KKT constraints rendering quadratically constrained quadratic programming problems.

The second part of Paper III is devoted to the analysis of how the composition of alternative levels for input parameters such as transmission infrastructure expansion budget, a carbon emission tax, and monetary incentives for renewable generation capacity expansion affect the total generation mix. This analysis is executed by applying the proposed modelling approach to an illustrative three-node instance and a case study considering a simplified representation of the energy system of the Nordic (Finland, Sweden, Norway and Denmark) and Baltic (Estonia, Latvia and

Lithuania) countries. In Paper III, we considered the optimal total welfare, the share of VRE in the optimal generation mix and the total amount of energy generated as the output factors in the analysis.

Applying the proposed modelling assessment to the illustrative three-node case study allowed us to analyse the sensitivity of the model to different levels of the input parameters applied individually. The numerical results demonstrated the limited efficiency of each input parameter if not applied in composition with others. For the Nordic and Baltic countries case study, the authors considered all possible compositions of two values ("low" and "high") for the input parameters. However, trying to solve the model for a more realistic Nordics and Baltics energy system revealed too computationally demanding for state-of-the-art solvers such as CPLEX [17] and Gurobi [27]. Therefore, the authors of Paper III discretised and enumerated possible upper-level decisions and afterwards, among all the possible alternatives, determined the TSO's investment portfolio that led to the optimal solution. The numerical results indicated that one has to carefully decide on the values of the input parameters depending on what output factor they want to impact. Additionally, the conclusions regarding how input parameter values impact all the output factors slightly differ depending on whether the generation companies possess a "high" or "low" budget for the generation infrastructure expansion. One of the main findings was that regardless of the generation expansion budget value, employing the "low" value for the carbon tax is efficient enough as its further increase does not influence any of the output factors. Supposedly, this is due to the high share of VRE in Nordic and Baltic countries (more than 50%) reached with a "low" tax, which in turn dampens the impact of the conventional energy-related costs on generation companies' profits. Nevertheless, the numerical results suggest that the highest VRE share in the generation mix, total welfare and generation amount values in the energy market where generation companies have a "low" generation expansion budget are obtained when simultaneously considering "high" values for the incentive and transmission infrastructure expansion budget. Similarly, for the case when the generation companies possess a "high" generation capacity expansion budget, considering "high" values for the incentive and transmission expansion budget while keeping the carbon tax "low" led to the highest values in all the output factors.

4. Discussion

4.1 Theoretical and practical implications

The developments in this dissertation provide insights into (i) the modelling of the renewable-driven transmission expansion planning in the competitive market, (ii) the solution method for the resulting model and generalised two-stage mixed-integer stochastic programming problems and (iii) analysis of the impact of alternative levels of transmission expansion budget, a carbon tax and VRE-supporting incentive on the energy system. The algorithms developed in Papers I and II of this dissertation provide a valuable contribution to both academia and industry. In particular, they enlarge the existing literature utilising MIQCQP models that can not be solved to optimality without introducing a series of simplifications and assumptions. Additionally, the methodologies suggested in Papers I and II can be introduced into commercial and open-source available solvers due to, as demonstrated by publications, significant improvement in computation performance.

Paper III of this dissertation contributes to the literature on energy systems models and provides insights into the questions related to the exploitation of transmission infrastructure to support VRE integration into the energy system. The models and results developed in this paper also contribute to the industry. Specifically, practical implications are suggested, for instance, by (i) understanding and analysing the impact of VRE-supporting instruments, (ii) analysing the interaction between TSO and generation companies and (iii) identifying optimal investment decisions following maximising the welfare. Additionally, the modelling approach developed in the paper may serve as a basis for further development to address new research questions. Moreover, it highlights the limitations of state-of-the-art solution software and motivates the development of efficient solution approaches. Additionally, Paper III conveys a solution method for the proposed bi-level problem that, due to computational challenges, requires considerable simplifications. Nevertheless, one should bear in mind that the solution methods proposed in Papers I and II can potentially be employed for solving to optimality the bi-level problem proposed in Paper III. Due to computational challenges related to the implementation of the methods proposed in Papers I and II and the limited time frame of doctoral research, this dissertation does not address this research question directly. On the other hand, it does provide the pathway for future research developments aiming to narrow the gap between the proposed method and applications.

The application of the modelling assessment to the illustrative case study allows one to individually address the effect of each VRE-supporting policy that would be unforeseen without the use of such a simple case study structure. Moreover, it allows one to analyse the effect of exploring extreme values for VRE supporting policies, e.g. 100% subsidies for VRE expenses. As numerical experiments suggest, for a perfectly competitive market, such policy implications might lead to unexpected results, e.g., a decline in VRE share in total generation mixture considering high levels of VRE subsidies. The identification of these insights would be extremely challenging without the use of mathematical modelling. The analysis of the impact of VRE-supporting policies on the Nordic and Baltic countries case study provides valuable results for welfare-maximising decision-makers, aimed at addressing the global climate change challenges while ensuring the sustainability and security of energy systems. In particular, these results might be fruitful for the EU or national policy-makers to support the development of new financial or technical strategies.

4.2 Avenues for future research

The algorithms and modelling approach in this dissertation demonstrated some shortcomings and limitations. Hence, in this section, I suggest possible further research directions. The first, and probably the most self-evident, is to actually introduce the possibility of discrete investment alternatives. While the authors in Paper III assume continuous values for the transmission/generation expansion, it is a common practice in the literature to introduce the discrete investment strategies instead (see, for example, [9, 56]). In this case, if one was to reformulate a bi-level problem into a single-level alternative, this would require the introduction of linear relaxations for mixed-integer terms in order to ensure the convexity of the lower-level problem. Further, one could exploit the primal-and-dual constraints approach [24], resulting in the MIQCQP problem that could be solved using the solution method proposed in Paper II.

Another possible research direction concerning the modelling approach proposed in Paper III could be considering the imperfectly competitive market. i.e., Cournot oligopoly [58] instead of perfect competition. The imperfect nature of the competition, in this case, would imply that the generation companies would not act as price takers, but have knowledge about the impact of their and other market players' actions on the energy price. As a result, the generation companies would take into account this information during the decision-making process, which more closely represents reality [50]. However, such a setup brings up challenges related to modelling the lower-lower of the mathematical optimisation problem as it has to be represented by a set of sub-problems, each related to a distinct generation company.

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Errata

Publication I

I. Page 49: In Problems (12) and (13) the bounds for the variables x_1 and x_2 are supposed to be [0, 1] instead of [0, 1.75]
II. Page 60: In Problem (36) the bounds for the variables x_1 and x_2 are supposed to be [0, 1] instead of [0, 1.75]

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