Optimal forest rotation under carbon pricing and multiple age-dependent risks

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Abstract

This thesis is about optimal forest rotation under carbon pricing and risk of a forest damage. The expected net revenue gained from a forest is calculated taking into account the price of timber, carbon sequestration subsidies, and taxes on the release of carbon. Risk of a forest damage, which would lead to a loss of income, is considered in the model. The risk is modeled to be dependent on the age of the forest, using a non-homogeneous Poisson process. Four age-dependent risk profiles are considered. Both the price of carbon and magnitude of the risk of a damage are varied. Optimal time to harvest the forest, which maximizes the net revenue gained from the forest, is calculated. The results show that when the risk of a damage is elevated for younger forests, longer rotation times are obtained with a lower price of carbon, than in the case of other risks. However, the net revenue at the optimal point is the lowest when the risk is elevated at younger forests, and greatest when the risk is elevated for older forests. With all the risk profiles, raising the price of carbon also raises the expected net revenue significantly.

Keywords Optimal forest rotation, carbon pricing, forest damage, non-homogeneous Poisson process, climate change

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Tiivistelmä

Tämä kandidaatintyö käsittelee optimaalista metsänkiertoa hiilidioksidin hinnoittelun ja metsätuhoriskin vallitessa. Metsästä saatava odotettu nettotulo lasketaan ottaen huomioon puun hinta, hiilensidontatuki ja hiilidioksidin vapauttamisesta maksettavat verot. Mallissa otetaan huomioon metsätuhoriski, joka realisoituessaan johtaa tulonmenetyksiin. Mallissa riski riippuu metsän iästä epähomogeenisen Poisson-prosessin mukaisesti. Laskenta tehdään neljälle eri riskiprofiilille. Sekä hiilen hintaa että vahinkoriskin suuruutta vaihdellaan, ja lasketaan optimaalinen hakkuuajankohta, joka maksimoi metsästä saadut nettotulot. Tulokset osoittavat, että kun vahinkoriski on korkeampi nuoremmille metsille, pidempiin kiertoaikoihin päästään alhaisemmalla hiilen hinnalla kuin jos riski on korkeampi vanhoille metsille. Nettotulot optimaalisessa pisteessä ovat kuitenkin pienimmät, kun metsätuhon riski on suuri nuorilla metsillä, ja suurimmat, kun riski on korkea vanhoille metsille. Kaikilla riskiprofiileilla hiilidioksidin hinnan nostaminen nostaa myös odotettuja nettotuloja merkittävästi.

Avainsanat Optimaalinen metsänkierto, hiilen hinnoittelu, metsätuho, epähomogeeninen Poisson prosessi, ilmastonmuutos

Contents

1 Introduction

As concerns over climate change have grown, many countries have agreed to set targets for carbon neutrality. Finland has a goal of being carbon neutral by 2035, and the carbon sinks play an important role on the road to carbon neutrality. In addition to the current carbon sinks, the target is to increase the annual carbon sequestration by 3 million tonnes of CO_2 equivalent by 2035. (Huttunen et al., 2022, p. 27)

However, in 2021 research results showed that the land use sector of Finland has become a source of emissions (Official Statistics of Finland, 2021). One reason for this is that the carbon sinks have collapsed due to increased amounts of logging the forests. Around the same time, interest in the fight against biodiversity loss has increased. In Finland, a major cause of extinction of species is changes in forest habitats, especially the decrease of old forests, large trees and deadwood (Rassi et al., 2010, p. 36).

Letting the forests grow older before logging them would help with both of these problems. By delaying the logging of the forests, the carbon they have sequestered would stay stored and out of the atmosphere for a longer time. From a biodiversity perspective this could also have positive results, since the amount of old forests would increase.

However, a big part of forests are privately owned commercial forests, and the forest owner's goal is usually to maximize the income from the forest. From the forest owner's perspective, keeping the forests growing can be risky, as there is a risk of a forest damage present. A damage, for example a fire, could destroy a significant part of the forest and lead to a loss of income.

The forest owner could be motivated to keep the forest growing by giving the stored carbon a price. If the forest owner earns money for storing carbon, and has to pay taxes for releasing carbon, it may become more economically viable to delay logging the trees. However, this can be very expensive for the party responsible for carbon pricing, if a lot of carbon is stored and the price of carbon is high.

In this thesis, the effects of carbon pricing and risk of a damage on the logging time of a forest are examined. The economically optimal time of harvest is calculated when both of these factors are present. The risk of a damage is dependent on the age of the forest. Four different risk profiles are presented. Both the price of carbon and magnitude of the risk are varied.

2 Background

Traditionally, the Faustmann formula has been used to calculate the expected income from a bare land, assuming that trees will be planted on the land. If the rate at which the forest grows is known, as well as the discount rate, replanting cost and the price of timber, the expected land value at time t can be calculated using Faustmann's formula.

Besides producing timber, forests can be used for storing carbon. This is one way to decrease the amount of carbon in the atmosphere and lessen the magnitude of climate change, but the forest owner doesn't directly benefit from the stored carbon. However, the owner can be encouraged to store carbon in their forests by paying income for storing carbon, or, charging taxes for releasing the carbon that has been stored. Carbon pricing will thus affect the total income that the forest owner gains from growing the forest.

However, there are risks present when a forest is grown older. Damages such as fire, storm or pests can affect the results of forest growing negatively by destroying trees from the site. The risk of a forest damage can be a significant factor for the forest owner to consider as they are making the decision of weather to cut down the forest or keep it growing.

Some of these risks are dependent on the age of the forest. For example, mammal damage and many fungal diseases are types of damage that threaten especially young forests (Yli-Kojola, 2005, p. 97). Older forests on the other hand are more exposed to damages caused by wind (Yli-Kojola, 2005, p. 97) and some insects, such as European spruce bark beetle (Overbeck and Schmidt, 2012).

Various studies have examined the problem of maximizing the net revenue from a forest stand under carbon pricing or the risk of a forest damage. In Reed (1984) the standard Faustmann formula is extended with a risk of a forest damage, which follows a Poisson process. Reed's study presents both a model with a constant risk of damage and another one where the risk depends on the age of forest. The constant risk is modelled as a homogeneous Poisson process, while the age-dependent risk follows a non-homogenous Poisson process. In their study, Van Kooten et al. (1995) include carbon uptake benefits in the Faustmann formula. Ekholm (2020) combines these two methods and presents a model where both the risk of a damage and carbon uptake benefits are considered, however, only a constant risk of a damage is considered.

The aim of this thesis is to expand the model in Ekholm (2020) by allowing the risk of damage to be dependent of the age of the forest. This can be done following the methods presented in Reed (1984); replacing the homogeneous Poisson process with a non-homogeneous Poisson process.

3 Methods

3.1 Forest damages and carbon pricing

The model in Ekholm (2020) is as follows. The symbols used in the model can also be found in Table 1.

When a forest is harvested, the forest owner gains income. The yield of the harvest is defined by the price of timber $P_f(T)$ (unit €$/m^3$) and stem volume $\nu(t)$ (m^3/ha). Time is denoted by t and T is the rotation length, both in years. The year the forest is planted is $t = 0$, meaning that t also denotes the age of the forest. The forest is only harvested if no damage occurs during the rotation; when $t > T$.

The forest owner gains subsidies for storing carbon each year the forest is not harvested or no damage occurs. The subsidies are paid according to the amount of

Table 1: Symbols and their meanings

Symbol	Signification	Unit
α	Carbon content per stem volume	t_{CO2}/m^3
β	Fraction of carbon that remains stored after harvest	%
γ	Fraction of carbon that remains stored after a forest damage	%
T	Rotation length	years
r	Discount rate	%
R	Regeneration cost	€/ha
P_c	Price of carbon	€/t_{CO2}
$P_f(T)$	Price of wood	€/m^3
$\nu(t)$	Stem volume at age t	m^3/ha
$p(t)$	Probability of a damage at age t	%
$D(t)$	Net revenues if forest damage occurs at time t	€/ha
$H(T)$	Net revenues if no damage occurs, and forest is harvested at time T	€/ha
$V(T)$	Expected net present value with rotation length T	€/ha

carbon that is stored the given year. This is defined by α, the total carbon content per stem volume (t_{CO2}/m^3), and $\nu(t)$. The price of a ton of carbon is defined by P_c, which is in €/t_{CO_2}.

If the forest is harvested or a forest damage occurs, the forest owner has to pay a tax for releasing carbon. The amount of taxes is defined similarly as the subsidies: the price of carbon is the same, P_c. After harvesting, the fraction of carbon that remains stored is β. The owner needs to pay the tax for fraction $1-\beta$, since this is the part of carbon that will be released in the atmosphere. Similarly, if a damage occurs, the fraction of carbon remaining is γ, and the owner needs to pay for $1-\gamma$ of the total carbon content in the forest.

In the Ekholm model, the probability of a damage is constant for each year. This means that the damages follow a homogeneous Poisson process. Times between damages are exponentially distributed with λ as the average rate of damages per year:

$$p(t) = \lambda e^{-\lambda t}, \tag{1}$$

where *p(t)* denotes the probability that a damage occurs at the age t.

A single rotation will either end in harvest or forest damage. Function *D(t)* describes the net revenue of a single rotation that ends in forest damage at time t. Function *H(T)* describes the net revenue of a single rotation that ends in harvest, at time T as that is the rotation length. In addition to income from timber, carbon taxes and carbon subsidies, regeneration costs, denoted with R, affect the calculation. As the aim is to calculate the present value, the expected future incomes are discounted to the present using r as the rate. The different sources of income and costs are itemized in table 3.1.

Table 2: Net revenue gained from a single rotation, and its components

	Revenue or cost after a damage, $D(t)$	Revenue or cost after harvest, $H(T)$
Carbon subsidies	$\alpha P_c \int_0^T e^{-r\tau}\nu'(\tau)d\tau$	$\alpha P_c \int_0^T e^{-r\tau}\nu'(\tau)d\tau$
Carbon taxes	$e^{-rt}(1-\gamma)\alpha P_c \nu(t)$	$e^{-rT}(1-\beta)\alpha P_c \nu(T)$
Timber profit	0	$e^{-rT}P_f(T)\nu(T)$
Regeneration cost	$e^{-rt}R$	$e^{-rT}R$

Therefore, the functions for the present value of a single rotation are

$$D(t) = \alpha P_c \int_0^t e^{-r\tau}\nu'(\tau)d\tau - e^{-rt}((1-\gamma)\alpha P_c \nu(t) + R), \tag{2}$$

in case of a damage, and

$$H(T) = \alpha P_c \int_0^T e^{-r\tau}\nu'(\tau)d\tau + e^{-rT}((P_f(T) - (1-\beta)\alpha P_c)\nu(T) - R) \tag{3}$$

if no damage occurs and the forest is harvested at time T.

Using all the information described above, the model calculates *V(T)*, the expected net present value of infinite rotations of length T.

$$V(T) = \int_0^T p(t)(D(t) + e^{-rt}V(T))dt + \int_T^\infty p(t)(H(T) + e^{-rT}V(T))dt \tag{4}$$

This can then be rearranged so that *V(T)* is on the left side. Finally, the following expression for the expected net present value is achieved:

$$V(T) = \frac{\int_0^T p(t)D(t)dt + \int_T^\infty p(t)H(T)dt}{1 - \int_0^T p(t)e^{-rt}dt - e^{-rT}\int_T^\infty p(t)dt}. \tag{5}$$

3.2 Age-dependent risk

Up to equation (5), the methods above follow the model of Ekholm (2020) directly. However, the model in this thesis differs from Ekholm's, as an age-dependent risk is presented. As the probability function in equation (1) follows a homogeneous Poisson process, the probability of a damage is constant each year. To form an age-dependent model, the probability function can be replaced with a probability function that follows a non-homogeneous Poisson process. In such a process, the average rate of damages is a time-dependent function $\lambda(t)$, instead of a constant λ. The probability function is now of the form

$$p(t) = \lambda(t)e^{\int_0^t -\lambda(s)ds}, \tag{6}$$

where $\lambda(t)$ is a function that represents the magnitude of the risk over time. For example, a forest damage whose probability increases as the forest ages, can be described by choosing a function that increases with an increasing t.

3.3 Risk profiles

This thesis looks at four risk profiles. The aim is to represent that at a certain age, a forest might be at higher risk of suffering a damage. However, no easy-to-use data on the age-dependency of forest damages is available. Therefore, the risk profiles are assumptions of how the risk could behave. Their intent is to describe that there are different types of damages, and their age-dependencies differ from each other.

In the first case, the risk of a forest damage is constant. This equals the case of Ekholm (2020), and is used for comparing and validating the results. In the second one, the risk is greater for younger forests. As young forests are especially exposed to mammal damages (Yli-Kojola, 2005), this could describe, for example, a situation where there have been a lot of cervids in the forest area. In the third case the risk is greater for older forests. The risk could behave like this if, for example, the forest were located in an area where there have been many sightings of the European spruce bark beetle (Overbeck and Schmidt, 2012). Finally, in the fourth case, the risk is elevated for both young and old forests, but lower for mid-aged forests. This could represent a situation where there are many types of risk present, but their combination threatens the young and old forests the most.

Based on these risk profiles, four functions are formed. The functions are of following form:

$$\lambda(t) = k(\delta - \theta \cdot Gamma(\alpha, \beta)), \tag{7}$$

where $Gamma(\alpha, \beta)$ is a gamma distribution, with α as shape and β as rate. The gamma distribution is used here because it allows the function $\lambda(t)$ to achieve the shape of each risk profile. However, its values are quite small, therefore θ is used for magnifying the gamma distribution.

In order to the cases being comparable with each other, the sums of the functions over time are equal at 150 years. In other words, a forest of age 150 would have faced an equal volume of risk during its life, regardless of which risk profile is present. To achieve this, δ is used for leveling the probability of a damage between different risk profiles, so that they sum up to the same value at time $t = 150$. And finally, k is used for scaling the functions to a scope where the probability of a damage could move. Each risk profile has their own values for α, β, θ and δ. The functions can be seen in Figure 1.

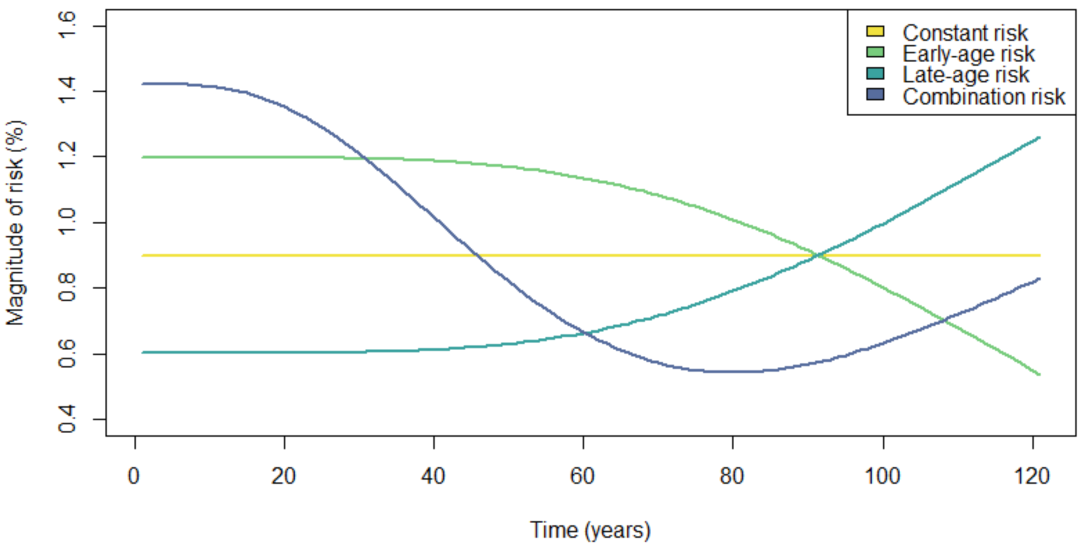

Figure 1: The functions that describe the four risk profiles

The purpose of these functions is to imitate the previously described four risk profiles. The purpose is not to give exact values of what a risk of a damage at certain time is. They are formed so that their shapes act the same way as the risk profiles do, and their magnitude is the same as the assumed probability of a damage.

3.4 Varying risk levels and price of carbon

In addition to the four different cases, the intensity of the risk varies within each case. The purpose is to examine how the magnitude of the risk affects the optimal time. This is done with the parameter k in equation (7): it receives values from 0.0 to 0.01, with 0.001 as the step size. As a result, there are 11 different functions of each risk profile. The functions can be seen in Figure 2. Each of these functions will be set as $\lambda(t)$ in (6).

Likewise, the price of carbon will be varied, with values from 0 to 100 €/t_{CO_2} every 10 €/t_{CO_2}. This allows the effect of carbon pricing on the calculation to be examined.

There are now 11 functions of the risk of each case, and 11 prices for carbon. The optimal time of harvest is calculated with each function, combined with each value of price of carbon. As a result, the optimal time of harvest is calculated 121 times with different inputs, for each case. The R function `optimize` is used for the optimization. The output of the optimization algorithm is the optimal time of harvest and the expected net revenue at the optimal point.

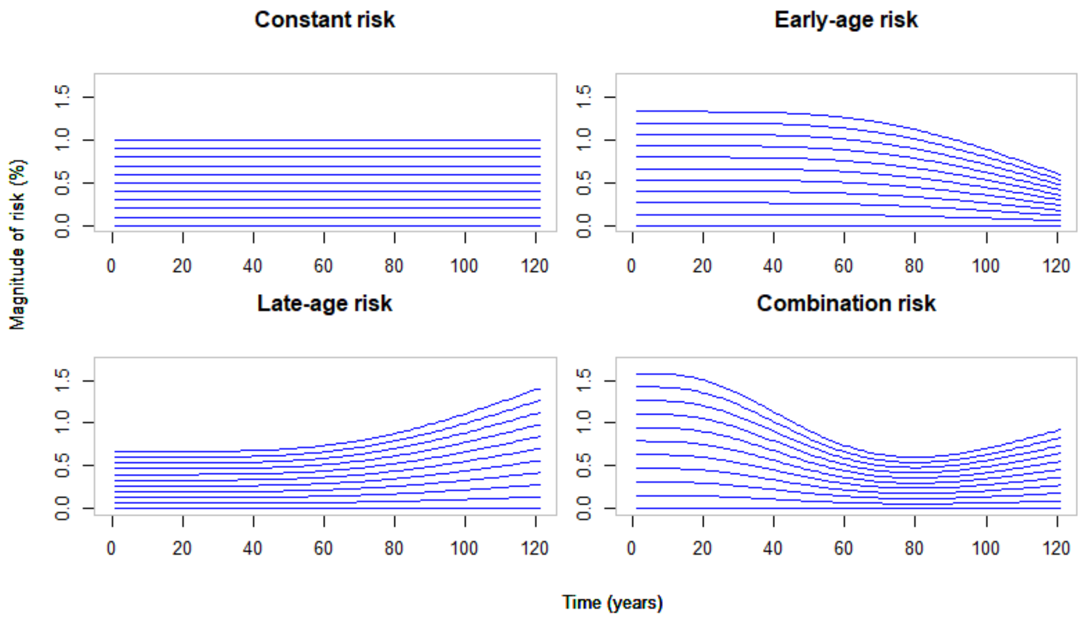

Figure 2: The risk functions with different values of k.

4 Results

4.1 Optimal time of harvest

The optimal times of harvest calculated by the optimization are shown in contour plots in Figure 3. They can be analyzed by comparing the graphs of risk profiles to each other.

When the risk of a damage is elevated for younger forests, in early-age risk, long harvesting periods can be achieved with fairly low price of carbon. Also, when the price of carbon is high, an increase in the value of k has little effect on the optimal time of harvest. And, when the price is small, the optimal times act very similarly as with the constant risk profile.

The observations seem logical: Once the forest has survived its youth without suffering a damage, the risk of a damage becomes so low that it no longer has much of an effect. Even small additions to the price of carbon increase the optimal time significantly, since keeping the forest growing and sequestering carbon allows the forest owner to gain more income, and has a very low risk of a damage.

With the case of a late-age risk, where old forests are more exposed to damages, optimal time of harvest reaches age 60 with a smaller price of carbon, than when the risk level is constant. This is expected, because at that age, the risk is still lower in late-age risk than with a constant risk. Contours of greater ages, 100-150 years, are only reached when the carbon pricing is high, and they lean to the right side. This is due to the high risk at older ages: The forest is at an elevated risk of suffering a damage, so there is a great interest of harvesting the forest before that

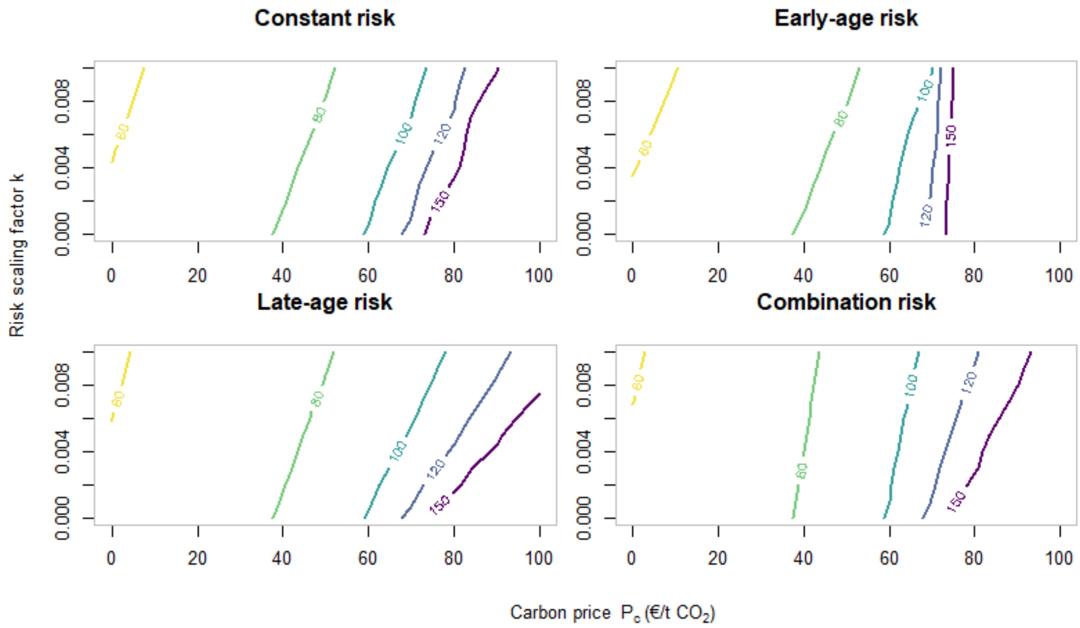

Figure 3: Contour plot of optimal rotation length T.

happens. To increase the optimal age, lots of subsidies for storing carbon are needed to compensate the elevated risk.

The contours of the combination risk resemble more the contours of constant and late-age risk than the contours of early-age risk. This is because the graph shows contours of years 60-150, where the combination risk is more similar to constant and late-age risk. The optimal time of harvest reaches age 60 with a smaller price of carbon than with any other risk profile. The contours of 60, 80 and 100 years also seem to be more vertical than with the other cases, which implies that at those ages, the increase of k has less effect on the calculation. This could happen because between years 60 and 110, the risk is at its lowest with the combination risk, so there is less interest of cutting down the trees before a damage happens.

4.2 Expected land value with optimal time of harvest

As Figure 4 shows, the expected land value is highest with the late-age risk profile, second highest with the constant risk profile, second lowest with the combination risk profile and lowest with the early-age risk profile. This can also be seen in Figure 5: the contours of expected land value in the early-age risk profile are leaning towards the right side the most, meaning that higher carbon pricing is needed to compensate the rise of k.

The results show that lower revenues are gained when risk is elevated for younger forests. Damages at young age are especially harmful, since all the forests need go through those ages, and they cannot be avoided by harvesting the forest a few years

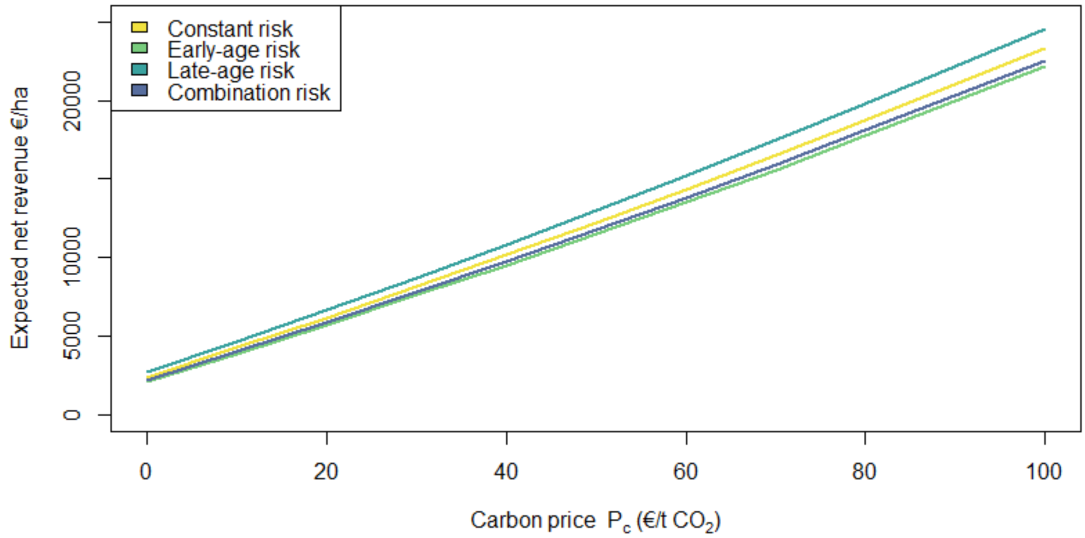

Figure 4: Expected land values with respect to price of carbon, when the value of k is 0.010.

beforehand. Additionally, no great subsidies can be gained for storing carbon, since at young ages the forests sequestrate very little carbon.

The relation between the cases with lowest profits, early-age risk, and highest profits, late-age risk, is shown in Figure 6. The graph shows that the higher the price of carbon, the lower the relative difference between the incomes. In a way, carbon subsidies and taxes smooth out the differences between different cases. However, the absolute difference between the two increases, as can be seen from Figure 4.

The Figures 4 and 5 also show that carbon pricing raises the total income gained by the forest owner significantly: the income multiplies as the prices increases from 0 to 100 €/t_{CO_2}.

5 Conclusions

This thesis adds the age-dependency of a risk of a forest damage using methods presented in Reed (1984), to the model of Ekholm (2020), in which the expected land value of a forest is calculated taking into account both the risk of a forest damage age as an age-independent risk, and the pricing of sequestered and released carbon. Four risk profiles are used for representing how the risk might behave. With these factors present, the economically optimal time of harvesting the forest is calculated.

With all the risk profiles, a high price of carbon leads to a high income, and an increase on the growth time of the forest. The results show that the greatest income is achieved when the risk is elevated for older forests. However, with such type of

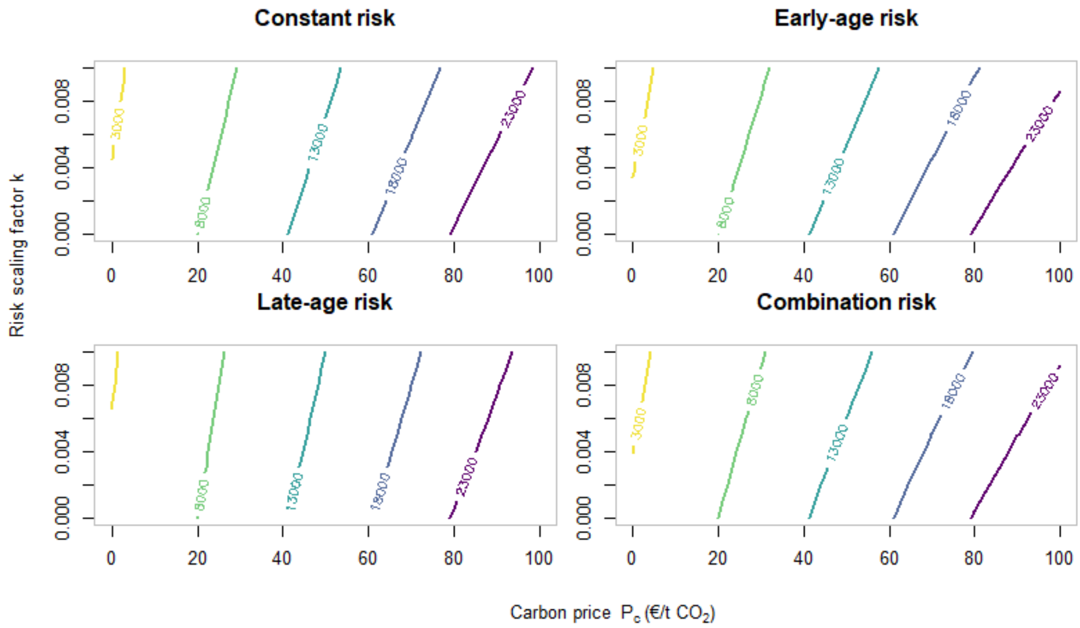

Figure 5: Contour plot of expected land values in €/ha, when the optimal time of harvest is used.

a risk, it is harder to reach very long growing times, and a high price for carbon is required in order to delay the optimal time of harvest. Longest growing times are reached when the risk is elevated for younger forests, but the income is at its lowest with this risk profile.

It is to be noted that no real data is used in this thesis: both the magnitude of the risk as well as the shapes of the risk profile are assumptions of how the risk could behave. Similarly, the prices of carbon are decided according to what it could be. However, the methods presented in this thesis could be used with real data to produce results that can be considered to reflect the real world.

The methods presented in this thesis could be extended to model different types of risks and forest management practices. In this model, the only type of harvesting is clear-cutting, and the risk is only considered to be dependent of the age of the forest. It could be complemented for example with methods of Patto and Rosa (2022), where also commercial harvesting is considered, both as a source of income and a way of preventing a forest fire. Other than age, also other dependencies for the risk could be considered, such as location or biotope of the forest,

More research could be done on the relationship between carbon pricing and age-dependent risk as well. In this thesis, increasing the price of carbon also increases the expected net revenue. It could be interesting to see at what level the risk of a damage would have to be for carbon taxes to outweigh carbon subsidies.

Carbon pricing itself could also be further explored. Now it leads to a multiplication of the expected net revenue, which can be problematic if the payer of the

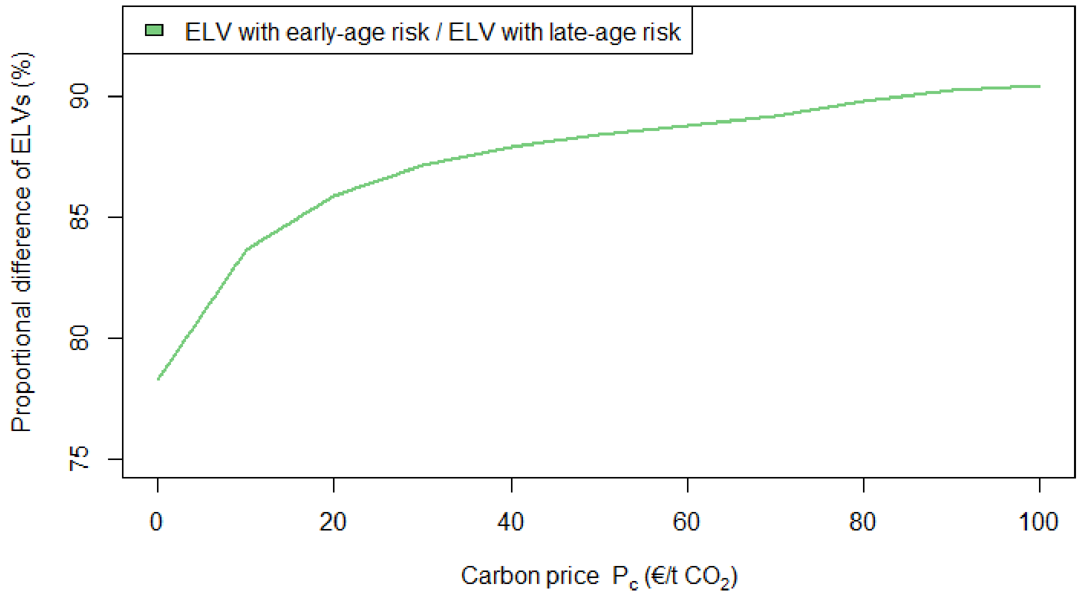

Figure 6: Proportional difference of expected land value (ELV) with early-age risk profile and late-age risk profile , when the value of k is 0.010.

subsidies is, for example, the state. Specifically, one could examine whether an increase in the carbon tax without an increase in carbon subsidies would lead to higher growth rates without a significant increase in the expected land value. If the state has a certain amount of money available for carbon subsidies, and the price of carbon determines how much forest area is covered by the subsidies, one could also explore how high the price should be in order to maximise the amount of carbon sequestered.

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