Determining optimal condition monitoring strategies for a multi-component system

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Abstract

Maintenance is essential to ensure the reliable and safe operation of technical systems. For such systems, mathematical modeling enables optimal maintenance policy development, aiming to reduce long-term costs while still maintaining reliability. Traditionally, maintenance scheduling relies on predefined time-based intervals. However, due to advancements in sensor technology and data analysis, condition monitoring has become more effective approach. In condition-based maintenance, better maintenance decisions are made based on observations of the system state.

This thesis compares condition monitoring strategies for a multi-component system with economic and structural dependencies. The proposed condition-based maintenance model uses discrete deterioration levels to model the deterioration of components. Condition monitoring is helpful when a non-deterministic component deterioration is applied. Transition probabilities between the states are uniquely defined, and thus, the process can be modeled as a discrete-time Markov decision process. The true deterioration levels of the components are revealed only on periodic inspections. The optimal maintenance policy was determined using a modified policy iteration algorithm, and the performance of the model was evaluated using Monte Carlo simulation.

In this thesis, a three-component example system was used to test the updated deterioration behavior, and various inspection intervals and target components were compared. The additional information provided by condition monitoring was found to reduce long-term costs in most scenarios. The results indicate that inspections on the component that is the most likely to break down first are the most efficient in terms of maintenance cost reduction.

Keywords maintenance scheduling, condition-based maintenance, Markov decision process, multi-component system

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Tiivistelmä

Tekniset järjestelmät tarvitsevat kunnossapitoa luetettavan ja turvallisen käytön takaamiseksi. Matemaattisella mallintamisella pystytään muodostamaan optimaalinen huoltopolitiikka, joka pienentää pitkän aikavälin kustannuksia samalla huolehtien myös järjestelmän luotettavuudesta. Perinteisesti huollon aikataulutus perustuu etukäteen määrättyihin aikaväleihin. Mittajärjestelmien ja tiedonkäsittelyn kehittyessä kunnon valvonnasta on tullut kuitenkin tehokkaampi työkalu. Kuntoperusteisessa huollossa havaintoja järjestelmän tilasta hyödynnetään parempien huoltopäätösten tekemisessä.

Tämä kandidaatintyö vertailee erilaisia kunnonvalvontastrategioita monikomponenttijärjestelmälle, jossa on taloudellisia ja rakenteellisia riippuvuuksia. Esitetty kuntoon perustuva huoltomalli käyttää diskreettejä kuntoluokkia mallintamaan komponenttien kulumista. Jotta kunnonvalvonnalla saavutettaisiin etua, kaikkien järjestelmän komponenttien kulumiseen lisättiin epävarmuutta. Komponenttien todellinen kulumistaso paljastetaan ainoastaan jaksollisissa tarkastuksissa. Tilojen väliset siirtymätodennäköisyydet pystytään määrittämään yksiselitteisesti, joten prosessia voidaan mallintaa diskreettiaikaisella Markovin päätösprosessilla. Optimaalinen huoltopolitiikka laskettiin hyödyntäen muokattua ohjauksen iteraatio -algoritmia (engl. modified policy Iteration algorithm), ja mallin suoriutumista arvioitiin Monte Carlo -simulaation avulla.

Tässä työssä kunnonvalvontastrategioiden vertailu toteutettiin vaihtelemalla tarkastusvälejä ja tarkastettavia komponentteja. Järjestelmän kuntoa kuvaavan lisätiedon havaittiin pienentävän pitkäaikaisia kustannuksia lähes kaikissa testitilanteissa. Tulokset osoittavat, että tarkastukset kohdistettuina komponenttiin, joka todennäköisesti hajoaa ensimmäisenä, johtavat suurimpiin kustannussäästöihin.

Avainsanat huollon aikataulutus, kuntoon perustuva huolto, Markov päätösprosessi, monikomponenttijärjestelmä

Contents

1 Introduction

Most technical systems undergo degradation over time and require maintenance to ensure their continuous functioning. Typically, these systems consist of multiple components with different features, dependencies, and failure probabilities. Failures are challenging to predict due to the stochasticity in system degradation. Owners and operators want to minimize operating costs and thus avoid unnecessary maintenance and waste of resources. On the other hand, they need their machinery to be as reliable as possible, as unexpected failures can result in substantial financial losses, safety risks, and production disruptions. Therefore, proper maintenance scheduling is crucial to numerous industries, such as manufacturing, transportation, and energy.

Traditionally, maintenance scheduling is handled time-based, which means inspection and maintenance actions for the machinery in question are carried out at predetermined time windows. These maintenance windows are usually based on recommendations from the original equipment manufacturer. However, advancements in sensor technology and data analysis have enabled more accurate and cost-effective ways to monitor the condition of technical systems. Ahmad and Kamaruddin (2012) compares time-based (TBM) and condition-based maintenance (CBM) strategies in an industrial setting. They state that almost all system or component failures can be accurately predicted using measures and observations of the system state, such as vibration analysis, sound monitoring, or oil-analysis. In their paper, CBM is found to be more efficient compared to TBM in industrial applications.

This thesis aims to find an optimal solution to a CBM problem for a multi-component system with economic and structural dependencies. The work builds upon models developed by Leppinen et al. (2023), Torpo (2019), Kokkonen (2021), and Lähteenmäki (2022). Torpo (2019) presents a way to model a multi-component system using directed graphs. With suitable assumptions, the evolution of this kind of system can be modeled as a Markov decision process (MDP), and the maintenance scheduling optimization problem can be solved using a policy iteration algorithm (Leppinen et al., 2023). Kokkonen (2021) extends the model by enabling component inspections. Inspections give the system operator more accurate information on component deterioration by updating the failure distribution of the inspected component. Lähteenmäki (2022) introduces uncertainty to the deterioration of one component at a time, increasing the number of state transitions. Continuous monitoring is used to reveal the actual system state.

The model presented in this thesis applies non-deterministic deterioration to all components in the system. Different monitoring strategies are considered, and comparisons between different inspection frequencies and settings are made. The subsequent sections of this thesis are organized as follows: Section 2 gives a brief literature review and technical background. Section 3 presents the CBM model in more detail, starting with the deterioration behavior in a single-component case, then extending it to multiple components, and finally, introducing methods for solving the optimization problem using an iteration algorithm and simulations. Section 4 compares the long-term costs and number of failures associated with different monitoring strategies in a three-component example system. Finally, Section 5 concludes the thesis with a summary of the results and their limitations.

2 Background and literature review

Maintenance scheduling is a well-known optimization problem that has developed rapidly in recent years. In the past decade, the most relevant topics have included multi-component models, inspection maintenance, and prognostics and diagnostics, as concluded by Quatrini et al. (2020) in their extensive literature review on condition-based maintenance. In this thesis, the background section covers the following topics: First, Subsection 3.1 covers literature on different maintenance strategies for multi-component systems. Subsection 3.2 presents a few other CBM models, followed by the theoretical background for Markov decision processes in Subsection 3.3.

2.1 Maintenance strategies

Multi-component systems are more difficult to model than single-component systems because of different deterioration behavior and dependencies between the components. The dependencies in a multi-component system can be divided into three categories: economic, structural, and stochastic dependencies (Laggoune et al., 2010). In maintenance scheduling, an *economic dependence* between components means that the costs to repair, replace, or inspect components simultaneously differ from operations executed separately. Economic dependence is positive when simultaneous maintenance is more effective. However, it can also be negative if components are more expensive to maintain collectively. For example, shared costs in labor and equipment set economic dependencies. *Structural dependency* occurs when components are structurally connected, resulting in a situation requiring, for example, the disassembly of both structurally dependent components to reach one of them. If the deterioration or the failure of one component affects the deterioration of other components, components are said to share a *stochastic dependency*.

A maintenance strategy specifies whether components are maintained separately or simultaneously and before or after a failure. Bevilacqua and Braglia (2000) outline five different maintenance strategies:

1. *Corrective maintenance* is applied only after a system failure.

2. *Preventive maintenance* tries to schedule maintenance actions before failures to prevent breakdowns.

3. *Opportunistic maintenance* strategy utilizes maintenance opportunities, for example, by executing multiple maintenance actions simultaneously.

4. *Condition-based maintenance* uses observations from the system to better schedule maintenance actions. Maintenance actions are performed when there is evidence of a potential failure rather than following a fixed schedule.

5. *Predictive maintenance* uses condition data and modeling to predict future machine deterioration. Maintenance decisions are based on these predictions.

2.2 Condition-based maintenance

Condition-based maintenance has proven efficient in many industrial applications (e.g. Ahmad and Kamaruddin, 2012). In the literature, deterioration is often modeled as a stochastic process, and the true state of the system is revealed through inspections (e.g. Castanier et al., 2005) or by continuous monitoring (e.g. Oakley et al., 2022). Le and Tan (2013) implement both continuous monitoring and periodic inspections. They model continuous monitoring as a less precise way of observing the system state compared to inspections.

To study the relation between stochastic and economic dependencies in the context of CBM, multi-component systems with parallel components and load-sharing can be modeled following Oakley et al. (2022) and Keizer et al. (2018). Both of these studies consider systems under stochastic deterioration and perfect continuous monitoring. Andersen et al. (2022) compare the use of TBM and CBM under perfect inspections on predetermined intervals. They find that the deterioration process of the components can be discretized without a significant loss in policy performance. Castanier et al. (2005) present a CBM model where the system state can only be revealed through inspections or preventive maintenance actions. Optimal thresholds are calculated for maintenance actions in a two-component case.

2.3 Markov decision process

The deterioration of multi-component systems can be assumed to follow the Markov property, where the probability for each state transition depends solely on the current state. Discrete deterioration levels and failure states make it possible to represent the system state in a finite state space (Leppinen et al., 2023). Furthermore, the transition probabilities between these states can be uniquely defined. A process fulfilling the Markov property is called a Markov chain.

In maintenance scheduling, however, the transition process is only partially random. Maintenance decisions impact the system state and entail associated costs. This process is described as a Markov decision process (MDP) (Howard, 1960). MDP differs from a standard Markov process in that the state of the system depends on decisions made from a finite set of feasible decisions. Each decision is associated with a reward or a cost. The objective is usually to maximize the long-term reward or to minimize costs.

Partially observable Markov decision process (POMDP) models the underlying core process as a MDP but updates the true state of the system only through observations (Corotis et al., 2005). In other words, the underlying process and transition probabilities are known, but there is no certainty of the actual state at each moment. Observations can be either partial with some residual uncertainty about the system state or perfect, meaning that the actual state of the system is revealed with certainty. Corotis et al. (2005) use of POMDP to find the optimal inspection and maintenance policy for a high-way bridge.

An optimal solution to a MDP optimization problem can be found, for example, using policy iteration or value iteration algorithms. Leppinen et al. (2023) use policy iteration to find the optimal maintenance policy by minimizing long-term maintenance costs for a multi-component system with economic and structural dependencies. Puterman (2014) presents a modified policy iteration algorithm that is implemented in the model of Leppinen et al. (2023) by Parkkali (2021). The policy iteration is proven to lead to better results than traditional threshold policies in MDP-based maintenance scheduling problems (e.g. Keizer et al., 2018; Leppinen et al., 2023).

3 Model and methodology

This section presents a condition-based maintenance model incorporating non-deterministic component deterioration and inspections at periodic intervals. The system structure, component-specific deterioration and failure probabilities are considered in a general setting and can be adjusted to correspond to a real-world technical system. First, in Subsection 3.1, the deterioration process is considered in a single-component case. The multi-component system structure and failure behavior of components, as discussed in Subsections 3.2 and 3.3, are based on the model proposed by Leppinen et al. (2023). Subsection 3.4 discusses the deterioration behavior of a multi-component system and presents an approach to reveal the actual system state in periodic inspections. In Subsection 3.5, an iteration algorithm by Parkkali (2021) is used to solve the optimal maintenance policy, which is then utilized in a Monte Carlo simulation.

3.1 Component deterioration

Deterioration is first considered in a single-component case. A component can be either operative or failed. A component can be maintained in predefined and discrete maintenance windows t^k at constant intervals Δt, meaning $t^{k+1} = t^k + \Delta t$ where $k \in \mathbb{N}$ is the index of the maintenance window. If a component fails during a maintenance interval (t^k, t^{k+1}), it is replaced and restored to a "good-as-new" state in the next maintenance window t^{k+1}. Condition monitoring actions become meaningful when a non-deterministic measure of wear is introduced following Lähteenmäki (2022). Before discussing the wear measure, the failure probabilities are presented in an age-dependent case based on the model of Leppinen et al. (2023).

The last replacement time of a component is denoted as τ, allowing us to calculate the age of a component a^k at a maintenance window t^k as $a^k = t^k - \tau$. The failure probability of a component is assumed to be a function of component age $f(a^k)$. From the probability density function (PDF), we derive the probability of a component failing before age a^k as $F(a^k) = \int_{-\infty}^{a^k} f(t)\, dt$. The maximum age α of a component is the minimum value, which satisfies $\forall x > \alpha : F(x) = 1$. For simplicity, we assume that $f(a^k)$ is a linearly increasing function, and thus α can be defined.

We introduce t^f to be the failure time of a component and $R(a^k)$ the reliability of a component as the probability that the component does not fail during the next maintenance interval. The reliability can be calculated using conditional probability:

$$R(a^k) := P(t^f > t^{k+1} | t^f > t^k) = \frac{P(t^f > t^{k+1})}{P(t^f > t^k)} = \frac{1 - F(a^k + \Delta t)}{1 - F(a^k)}.$$

The failure state of a component at a maintenance window t^k is expressed as a binary value:

$$f^k = \begin{cases} 0, \text{if the component is operative} \\ 1, \text{if the component is failed.} \end{cases}$$

Now, the state of a component can be explicitly defined with the component age and failure state:

$$s_k(a^k, f^k) := \begin{bmatrix} a^k \\ f^k \end{bmatrix}, \text{where } a^k, f^k \in \mathbb{R}.$$

In some cases, time and age are not the only nor the best way to model changes in the failure probabilities. For instance, in the case of trains, the distance traveled can also be utilized to model the use of the machinery (Leppinen et al., 2023). In this thesis, we adopt a measure of wear w^k, referenced as the deterioration level, as the functional age of a component. This replaces the linearly time-dependent age a^k presented above. Using suitable instruments and monitoring, better estimates for w^k can be derived, surpassing those solely based on time or distance. In instances where no better information is available, w^k increases linearly with time, leading to a conservative prediction of deterioration.

The non-linear component deterioration in our case is modeled by allowing a component to either stay at the same deterioration level or to deteriorate by one time step Δt during a maintenance interval. To describe the probability of component deterioration staying the same, we employ a component-specific wear-dependent probability $g(w^k)$. We assume that the likelihood of the component remaining at the same deterioration level decreases when it becomes more deteriorated.

Due to the non-deterministic deterioration, the deterioration level w^k of a component can either stay the same or increase, or the component can fail during a maintenance interval (t^k, t^{k+1}). Probabilities for these three outcomes, denoted by $p_s(w^k)$, $p_g(w^k)$ and $p_f(w^k)$, are calculated in Equations 1, 2, and 3 respectively (Lähteenmäki, 2022).

$$p_s(w^k) = g(w^k)(1 - p_f(w^k)) \tag{1}$$

$$p_g(w^k) = (1 - g(w^k))(1 - p_f(w^k)) \tag{2}$$

$$p_f(w^k) = 1 - R(w^k). \tag{3}$$

In our model, the failure probabilities are assumed to follow the Weibull distribution with component-specific shape and scale parameters. Table 1 provides an example of probabilities for a component to remain at the same deterioration level and the calculated reliability of the component. Weibull shape parameter $k = 5$ and scale parameter $\lambda = 11$ are used, with a maintenance interval Δt set to 1. Table 2 summarizes the state transition probabilities presented in Equations 1, 2 and 3.

w^k	0	1	2	3	4	5	6	7	8	9	10
$g(w^k)$	0	0.4	0.3	0.2	0.15	0.125	0.1	0.1	0.1	0.1	0.1
$R(w^k)$	1.00	1.00	1.00	1.00	1.00	0.99	0.97	0.95	0.91	0.85	0.78

Table 1: Probabilities for staying at the same deterioration level and reliability for an example component

w^k	0	1	2	3	4	5	6	7	8	9	10
p_f	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.05	0.09	0.15	0.22
p_s	0.00	0.40	0.30	0.20	0.15	0.12	0.10	0.09	0.09	0.08	0.08
p_g	1.00	0.60	0.70	0.80	0.85	0.86	0.87	0.85	0.82	0.76	0.70

Table 2: State transition probabilities for an example component

Without any inspections, faster deterioration is presumed due to the conservative assumption of deterioration. However, by using condition monitoring, the assumed deterioration level of the monitored component can be updated between the maintenance intervals. Inspections or other condition monitoring actions are assumed to be perfect, meaning that the actual state of the component can always be determined.

To conclude, the assumptions for the deterioration process of a component are:

- For each component, there are three possible transitions between two maintenance windows.
  - The component can stay at the same deterioration level: $w^{k+1} = w^k$
  - The component can deteriorate: $w^{k+1} = w^k + \Delta t$
  - The component can fail: $f^{k+1} = 1$
- The failure probability p_f depends only on the wear w^k of the component.
- Components deteriorate during the first maintenance interval after their replacement.

3.2 System structure

Most technical systems consist of more than one component that can deteriorate, fail, and be maintained separately. However, the mathematical modeling of such systems is complicated due to e.g. dependencies between the components. We consider a system of $n \in \mathbb{N}$ components whose state can be expressed as the matrix

$$s^k(w^k, f^k) := \begin{bmatrix} (w^k)^T \\ (f^k)^T \end{bmatrix}, \text{where } w^k, f^k \in \mathbb{R}^{n\times 1}.$$

The vector w^k consists of the deterioration levels w_i^k and the vector f^k of the failure states f_i^k of each component $i \in \{1, 2, ..., n\}$.

The dependencies between the components are modeled using a directed graph consisting of a root node 0 and n nodes that present the maintenance actions for a component replacement. Each arc (i, j) connecting nodes has a weight c_{ij} representing the component-specific maintenance costs. The weight of an arc going from node i to j indicates the maintenance cost of j when i is also maintained at the same maintenance window. Costs for every maintenance action are calculated starting from the root node. A fixed set-up cost c_0 represents the cost of initiating a maintenance action, such as downtime costs and getting the required personnel and equipment in place.

The model incorporates a positive economic dependency between the components. How strong the dependency is can be influenced by adjusting the set-up cost and the weights of the arcs between nodes. A structural dependency in the model occurs when a node representing the replacement of a component is not directly connected to the root node. Nevertheless, as no node can be entirely isolated from the root node, the structurally dependent components can still be maintained simultaneously with some other component. The system has no stochastic dependencies, as all components are assumed to deteriorate and fail independently.

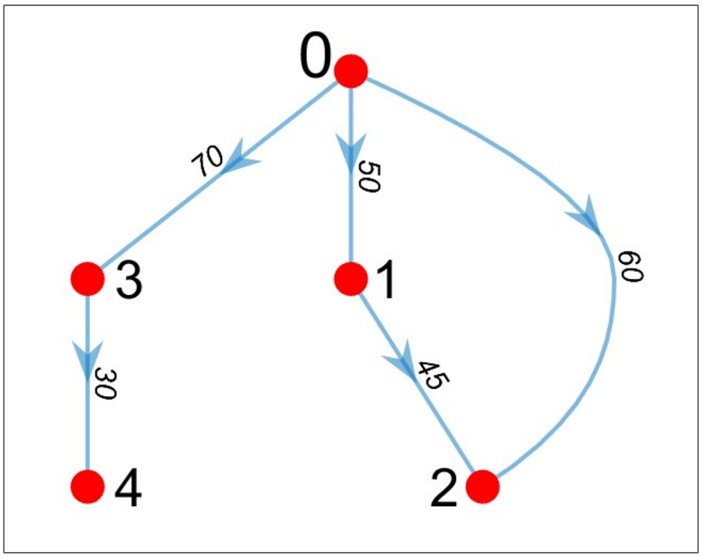

Figure 1: System of four components presented as a directed graph

Figure 1 shows an example system with four components. In this example, components 1, 2, and 3 can be replaced independently with maintenance costs of 50, 60, and 70, respectively, added to the set-up cost c_0. However, there is a structural dependency between components 3 and 4, meaning component 4 can be maintained only at the same maintenance window as component 3 with an added cost of 30. Additionally, component 2 is more cost-efficient to replace simultaneously with component 1 than independently as $c_{12} = 45 < 60 = c_{02}$.

3.3 Maintenance actions

In the model, all components in the system are considered critical, meaning that the failure of any component results in the failure of the entire system. Component failures are assumed to be independent of each other. As a result, the reliability of the system is defined as the probability that no component fails during the next maintenance interval (t^k, t^{k+1}):

$$R_{sys} := \prod_{i=1}^{n} R(w_i^k)_i, \tag{4}$$

where $R(w_i^k)_i$ is the reliability of component i at a maintenance window t^k.

To prevent system failures, a reliability threshold $\rho \in (0, 1)$ is introduced, ensuring that the reliability of the system must remain above a certain level: $R_{sys} \geq \rho$. This constraint restricts the number of feasible system states when assuming increasing failure rates for components.

The event of component i failing during a maintenance interval (t^k, t^{k+1}) is denoted by E_i^k. Since only one component can fail at a time, the probability $P(E_i^k)$ is calculated as a conditional probability

$$P(E_i^k(w^k)) = \frac{\int_0^{\Delta t} \prod_{j \neq i}^{n} [1 - F_j(w_j^k + t)] f_i(w_i^k + t) dt}{\prod_{i=1}^{n} [1 - F_i(w_i^k)]}. \tag{5}$$

To ensure that failed components are replaced and the reliability threshold is consistently met, replacement actions can be performed to restore one or more components to a "good-as-new" state. These maintenance actions are represented by a binary vector x^k, termed the maintenance portfolio. Whether or not component i is replaced in the maintenance window t^k, is represented by the binary decision variable

$$x_i^k = \begin{cases} 0, & \text{if the component is not replaced} \\ 1, & \text{if the component is replaced.} \end{cases}$$

Maintenance actions are perfect in that replacements restore the maintained components back to a "good-as-new" state. If conservative deterioration is presumed, the deterioration level of each component grows each maintenance interval by Δt. Therefore, the conservative prediction of the system state in the following maintenance window is

$$w_i^{k+1} = \begin{cases} w_i^k + \Delta t, & \text{if } x_i^k = 0 \\ \Delta t, & \text{if } x_i^k = 1. \end{cases}$$

Conservative deterioration in the following maintenance interval (t^k, t^{k+1}) is always presumed when determining reliability or failure probabilities in Equations 4 and 5. Thus, even the components which do not deteriorate can fail.

In order for a maintenance portfolio x^k to be feasible for a state s^k, it must satisfy three properties:

1. It fulfills the reliability threshold ρ with updated state s^{k+1} assuming deterministic deterioration.

2. It replaces failed components.

3. It satisfies structural dependencies, meaning that there is a path from the root node to all nodes for the selected actions.

The cost of a maintenance portfolio x^k, noted by $c(x^k)$, can be calculated from the directed graph using Edmond's algorithm (see Torpo, 2019). The cost includes a constant set-up cost c_0, which is added every time at least one component is replaced. If a component fails, additional costs arise due to, for example, unexpected system shutdown. Thus, a component-specific corrective surplus r_i is added to the maintenance cost of a failed component. Consequently, the incurred total costs for a maintenance window t^k, noted c^k, can be expressed with the component-specific corrective surpluses collected in vector r as $c^k = c(x^k) + f^k r^T$.

3.4 Monitoring in the model

Each component can either stay at the same deterioration level, deteriorate, or fail during a maintenance interval Δt, as described in Section 3.1. Therefore, when considering a multi-component system, the number of possible state transitions increases exponentially with the number of components in the system. Despite each component having three possible state transitions, only one component can fail at a time, limiting the number of possible state transitions from state s^k to s^{k+1} to be $2^n(n+1)$ where n denotes the number of components in the system.

Whether the components in the system will deteriorate or not in the maintenance interval (t^k, t^{k+1}) is given as a binary vector $q^k \in \{0,1\}^n$, where $q_i^k = 1$ if and only if $w_i^{k+1} = w_i^k$. Since each component deteriorates independently, the probabilities for each possible value y of q^k can be calculated as a joint probability

$$P(q^k = y) = \Pi_{i=1}^n [y_i g(w^k)_i + (1 - y_i)(1 - g(w^k)_i)] \quad (6)$$

The joint probability for each state transition can be calculated by multiplying Equation 6 with the probability for the corresponding failure state z, $P(f^k = z)$, given by either Equation 4 or 5.

We demonstrate this with an example on a three-component system. For this system, the probabilities of the deterioration level staying the same during a maintenance interval are in Table 3, and the parameters for the distributions of the failure times are in Table 4. The Weibull distributions are visualized in Figure 2.

w^k	1	2	3	4	5	6	7	8	9	10
$g(w^k)_1$	0.4	0.3	0.2	0.15	0.125	0.1	0.1	0.1	0.1	0.1
$g(w^k)_2$	0.3	0.25	0.25	0.2	0.175	0.175	0.15	0.15	0.125	0.1
$g(w^k)_3$	0.5	0.25	0.125	0.1	0.1	0.1	0.1	0.1	0.1	0.1

Table 3: Staying probabilities for the example system

Component	k	λ
Component 1	5	11
Component 2	2	20
Component 3	15	10

Table 4: Weibull parameters for failure PDFs

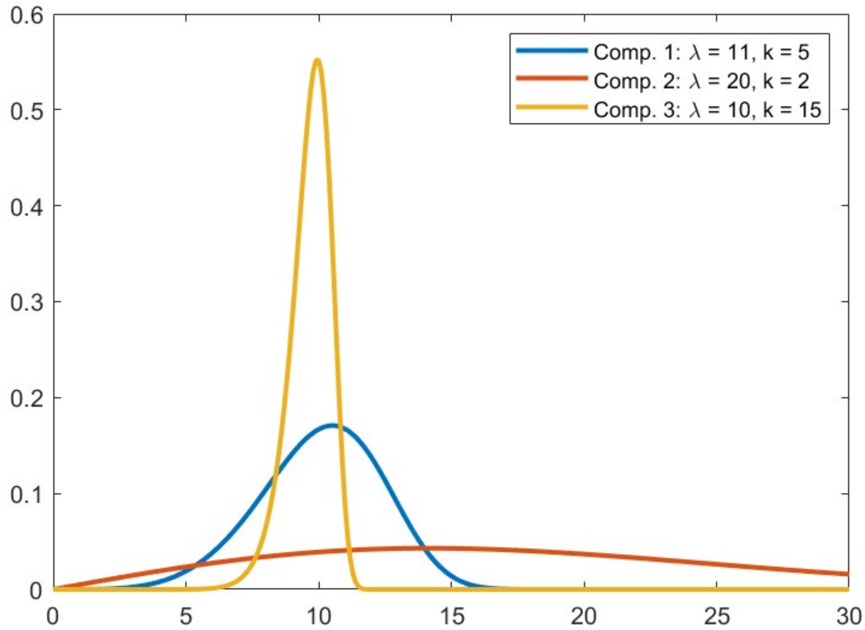

Figure 2: Distributions of failure times in an example system

A system of three components has 21 possible state transitions. The probabilities for the component deterioration can be calculated using Equation 6, and the probabilities for the possible failure states can be attained with Equations 4 and 5. To demonstrate, five of these joint probabilities are calculated in Table 5 for the example system presented above and an initial state of

$$s^k = \begin{bmatrix} 4 & 3 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

where the deterioration levels of the components are $w^k = (4, 3, 2)^T$ and the failure states are $f^k = (0, 0, 0)^T$.

s^{k+1}	$P(s^{k+1} \mid s^k)$
$\begin{bmatrix} 4 & 3 & 2 \\ 0 & 0 & 0 \end{bmatrix}$	$R_{sys}((4,3,2)^T)P(q^k = (1,1,1)^T) \approx 0,9\%$
$\begin{bmatrix} 5 & 4 & 3 \\ 0 & 0 & 0 \end{bmatrix}$	$R_{sys}((4,3,2)^T)P(q^k = (0,0,0)^T) \approx 46,4\%$
$\begin{bmatrix} 5 & 3 & 3 \\ 0 & 0 & 0 \end{bmatrix}$	$R_{sys}((4,3,2)^T)P(q^k = (0,1,0)^T)) \approx 15,5\%$
$\begin{bmatrix} 4 & 4 & 3 \\ 0 & 1 & 0 \end{bmatrix}$	$P(E_2((4,3,2)^T))P(q^k = (1,0,0)^T) \approx 0,4\%$
$\begin{bmatrix} 5 & 4 & 2 \\ 1 & 0 & 0 \end{bmatrix}$	$P(E_1((4,3,2)^T))P(q^k = (0,0,1)^T) \approx 0.1\%$

Table 5: Transition probabilities for an example system

While having uncertainty in the model, the actual state of the system at a time t^k remains unknown. However, certain situations provide clarity: when a component is replaced, it is restored to a "good-as-new" condition, and its deterioration level in the next maintenance window is set to Δt. In addition, component failures cannot go unnoticed. Apart from these situations, the maintenance decisions are based on a prediction of the deterioration of the components. This model uses conservative predictions, meaning that each component is presumed to deteriorate during each maintenance interval unless proven otherwise.

To acquire more information on the state of the system, the condition of selected components can be monitored. Monitoring can be carried out using various means, such as electronic sensors or physical inspections. The actual state of the selected components is revealed through perfect periodic inspections. The inspection intervals are predetermined, and if inspections occur in every maintenance window, the component is considered to be continuously monitored. With the updated information provided by inspections, better maintenance decisions can be made.

3.5 Optimal maintenance policy and simulation

The optimal maintenance policy $U(s)$ is calculated using a modified policy iteration algorithm with Gauss-Seidel method, as presented by Parkkali (2021). For each feasible state s, the algorithm chooses a maintenance portfolio that minimizes the long-term maintenance costs. The underlying deterioration process is known when calculating the optimal policy using the principles of a partially observable Markov decision process. This means the algorithm assumes non-deterministic deterioration behavior, as presented in Section 3.4. As a result, the number of state transitions in the algorithm is increased.

The evolution of the system and the performance of different monitoring strategies can be assessed using Monte Carlo simulation. The simulation maintains two separate versions of the system state: s^k_{real}, which keeps a record of the real deterioration of the system following the non-deterministic deterioration behavior, and $s^k_{assumed}$, which stores the assumed deterioration levels and is updated according to the chosen inspection interval and target components. The simulation runs for k_{end} time steps, starting from initial state of $w^k_{assumed} = w^k_{real} = (0, 0, 0)^T$ for a three-component system. The Monte Carlo sample size of M is used, and each simulation run is labeled as $m = 1, 2, 3, ..., M$. From each simulation, the total costs and number of failures are collected. Other information, such as average age when maintained, failure distributions, or frequency of maintenance portfolios, can be attained similarly.

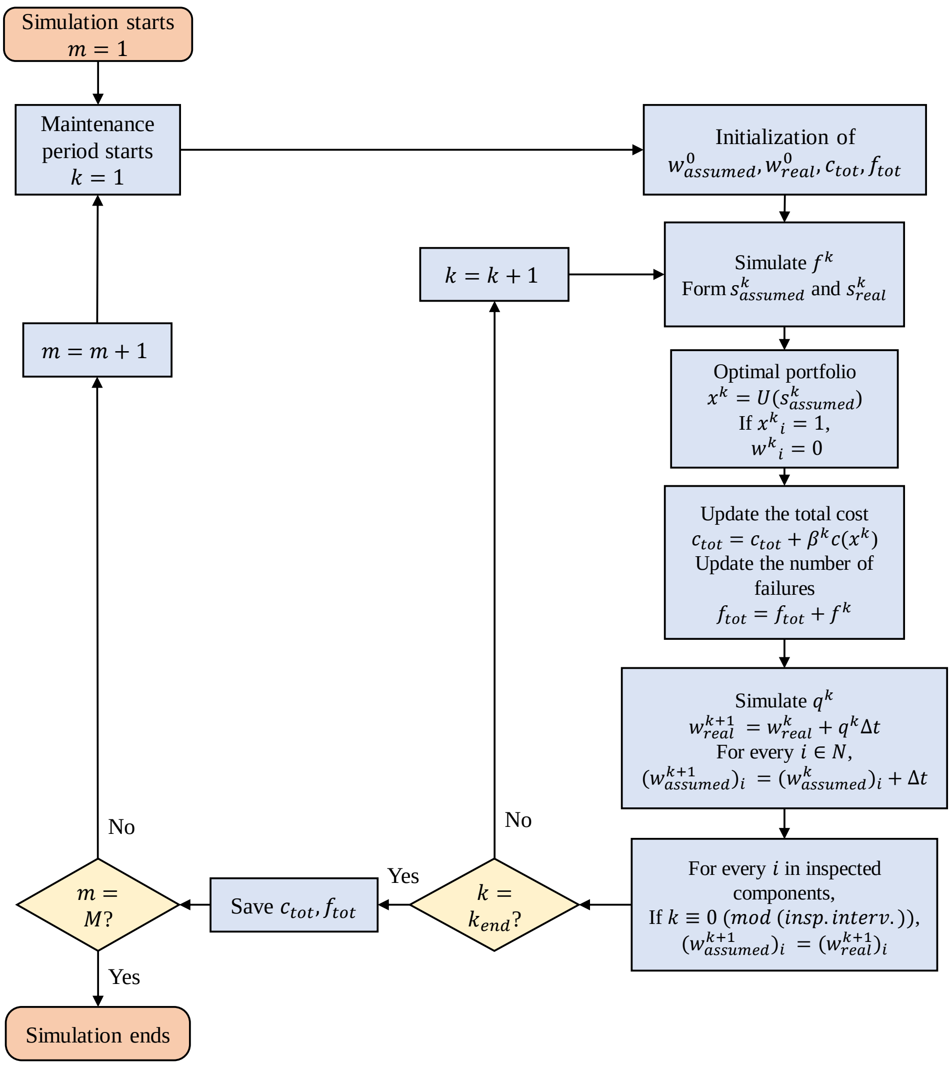

Figure 3: A flow chart of the simulation

A flow chart of the process is presented in Figure 3. First, the values of c_{tot} and f_{tot} are set to zero, and values of w^k are initialized. The failure state f^k is simulated using probabilities from Equations 4 and 5 and the optimal portfolio corresponding to the new states $s^k_{assumed}$ and s^k_{real} is chosen. A discount factor β is used to reduce future maintenance costs. The discounted cost $\beta^k c(x^k)$ of the maintenance portfolio x^k is added to the total cost. The value of q^k is simulated using probabilities presented in Equation 6, and w^k_{real} is updated accordingly. The assumed deterioration level $w^k_{assumed}$ is updated deterministically with a conservative assumption of deterioration. Components are inspected when the remainder of k and the inspection interval is zero. The simulation runs until k reaches k_{end}, and then the total costs and number of failures from the run are stored. A large enough M is used to ensure accurate results.

4 Results

The model in Section 3 was tested using a three-component example system. The example system represents a vehicle with three replaceable components: brakes (B), wheels (W), and engine (E). The directed graph of the system with maintenance costs is visualized in Figure 4. The simultaneous replacement of brakes and wheels is cost-effective due to shared dismantling, while engine replacements are the most expensive.

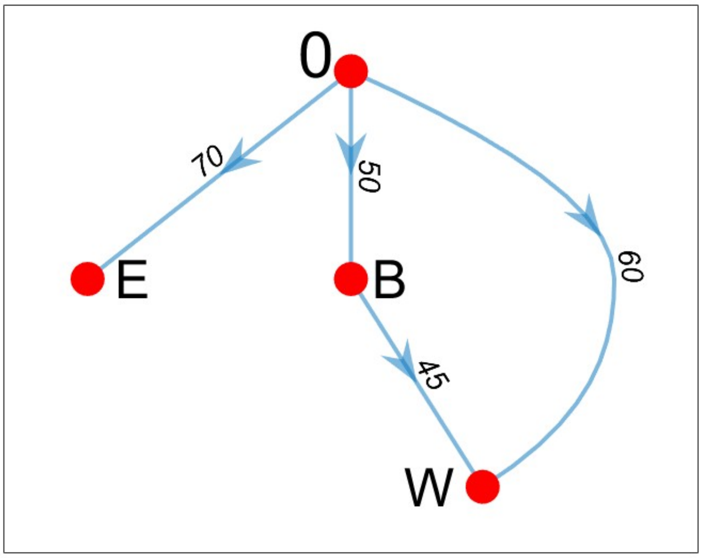

Figure 4: Three-component system used in the study

Values for the function $g(w^t)$ and failure distributions for the vehicle parts are the same as used in Section 3.4 and are found in Tables 3 and 4, respectively. Brakes correspond to component 1, wheels to component 2, and engine to component 3. In this scenario, the components can be replaced and inspected during scheduled visits once a year at a repair shop, giving us the maintenance interval of $\Delta t = 1$. Set-up cost of $c_0 = 60$ and corrective surpluses $r = (100, 120, 150)^T$ were used, along with a reliability threshold $\rho = 0.95$ and a discount factor of $\beta = 0.99$.

All components in the model deteriorate non-deterministically according to the probabilities in Equation 6. The real deterioration state of a component is revealed only during perfect inspections at predetermined intervals. In this section, the set of components subject to these inspections and the inspection interval are varied, and long-term costs and the number of failures are compared between the scenarios.

The computations were carried out using Matlab R2023a software on a Dell Latitude 7490 laptop with Quad 1.70 GHz Intel Core i5-8350U CPU and 16 GB of RAM. When allowing non-deterministic deterioration, the computations become more computationally intensive due to the increased number of state transitions in the partially observable Markov decision process. Particularly, the constant computing of the deterioration probabilities in Equation 6 significantly extends the computing times of the policy iteration algorithm. While the deterministic model by Leppinen et al. (2023) takes only 0.3929 seconds, it takes 60.9856 seconds for the updated model to find the optimal policy.

4.1 Average cost

Table 6 presents the average costs of all eight combinations of components monitored, using an inspection interval of two. This means that the actual state of the inspected components is revealed every second maintenance interval. The third column presents the percentage change in costs compared to the scenario where no component is monitored. From the data, it can be observed that increasing the number of components monitored decreases the long-term average costs. This is because inspections update the assumed condition of the component and prevent premature maintenance actions, resulting in fewer maintenance costs.

Monitored components	Average costs	Compared to no monitoring
-	33.99	-
W	31.83	-6.35 %
E	30.77	-9.48 %
W, E	29.34	-13.68 %
B	29.09	-14,41 %
B, W	28.37	-16.54 %
B, E	27.59	-18.81 %
B, W, E	26.76	-21.26 %

Table 6: Average cost of different scenarios

Furthermore, the monitoring of brakes leads to the most substantial decrease in long-term costs. This is because the brakes are the most likely to break down first due to their failure probability distribution. Simultaneous replacements are more desirable due to the economic dependence in the system. If the component most likely to fail first can be maintained later, it also enables more efficient maintenance of the other components.

4.2 Number of failures

Another interesting change when moving to a condition-based model is the number of failures. Table 7 presents the long-term average component failures per 100 maintenance intervals for the eight scenarios with different components monitored, using an inspection interval of two. From the table, it is evident that the number of failures increases as more components are inspected. When a component is not monitored, it is assumed to be more deteriorated than in reality, leading to earlier and potentially unnecessary maintenance actions. Inspecting components reduces redundant replacements and thus increases the number of failures. Moreover, the number of failures when using a deterministic model is 2.31 failures per 100 maintenance interval, which is higher than the failure rates in any scenario using non-deterministic deterioration. Inspecting brakes seems to have the most significant effect again.

Monitored components	Failures	Compared to no monitoring
-	1.14	-
W	1.28	+ 12.26 %
E	1.31	+ 14.72 %
B	1.35	+ 18.36 %
B, E	1.42	+ 24.61 %
B, W	1.46	+ 28.15 %
W, E	1.53	+ 34.14 %
B, W, E	1.61	+ 41.86 %

Table 7: Average number of failures in 100 maintenance intervals

To conclude, our conservative assumption of deterioration leads to earlier maintenance of components when the components are not inspected. Therefore, component inspections lead to fewer required maintenance actions. This helps lower average costs, but also more failures occur.

4.3 Effect of inspection interval

The model enables the adjustment of the inspection interval relative to the maintenance interval. The simulation was run with all eight scenarios and with inspection intervals from 1 to 6. The long-term average costs and the number of failures are visualized in Figures 5 and 6.

The figures show that costs do not decrease as much, nor does the number of failures grow significantly for longer inspection intervals. It seems that increased inspection interval decreases the effect of condition monitoring as inspections are performed less frequently, causing increased uncertainty in the deterioration of the components. Another interesting observation from the data is that with an inspection interval of 5, condition monitoring appears to have no effect. The optimal policy and conservative presumption of deterioration lead to all components being replaced simultaneously at the age of 5. Consequently, after all the components have been

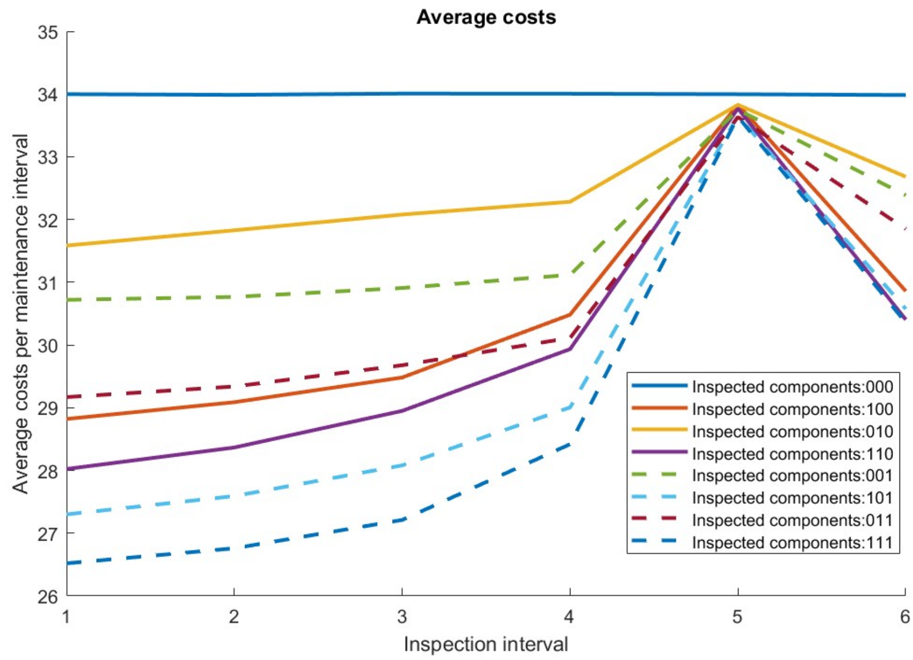

Figure 5: Average costs per maintenance interval with different inspection intervals

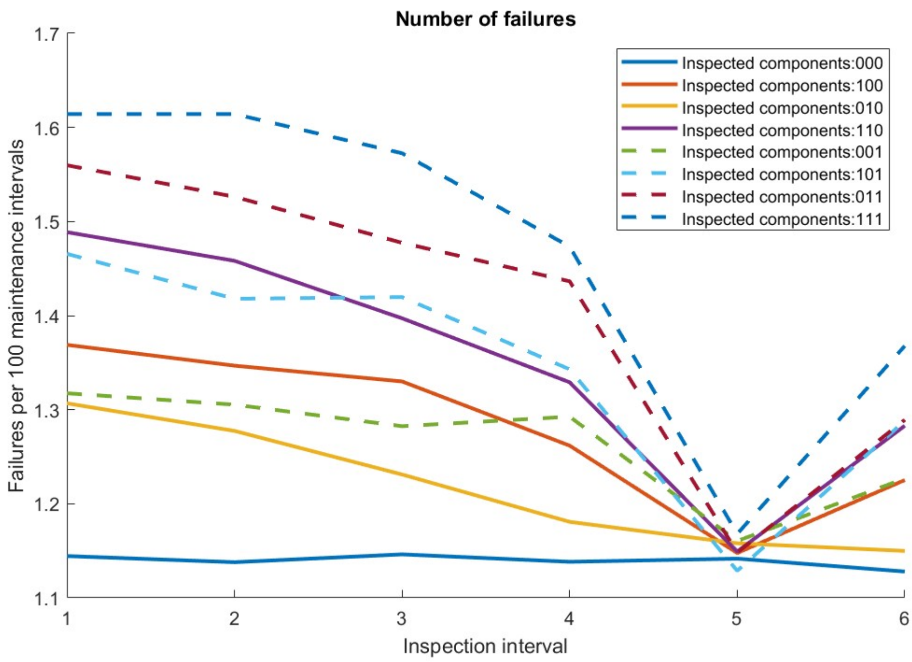

Figure 6: Number of failures per 100 maintenance intervals with different inspection intervals

replaced, inspections do not affect the results. When the inspection interval exceeds 5, there is again a notable impact on both the cost and failure data.

4.4 Effect of staying probabilities

The staying probabilities $g(w^k)$ describe how fast the components are deteriorating. The system can be under different conditions and loads and as a result, the staying probabilities can vary. The values of $g(w^k)$ presented in Table 3 were multiplied with a coefficient of 1.5 and 0.5. For instance, staying probabilities corresponding to state $w^k = (1, 1, 1)^T$ become $g(w^k) = (0.6, 0.45, 0.75)^T$ when using coefficient 1.5 and $g(w^k) = (0.2, 0.15, 0.25)^T$ with coefficient 0.5. The change in long-term costs is visualized in Figure 7. In the figure, the change is compared to the deterministic result where the uncertainty in the deterioration behavior is not considered, and the staying probabilities are set to zero.

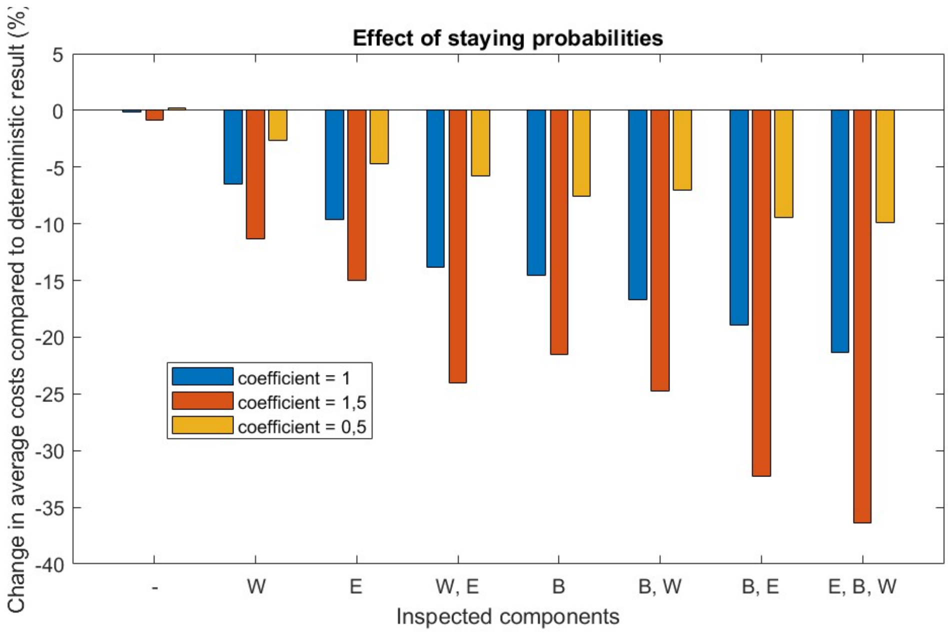

Figure 7: Average costs with different staying probabilities compared to a deterministic result

From Figure 7, it is obvious that the benefit of condition monitoring is more significant when the staying probabilities $g(w^k)$ are increased. When the staying probabilities are higher, the uncertainty in the system deterioration grows. Due to this uncertainty, also components in relatively good condition are replaced, and unnecessary costs arise. However, when no components are inspected, the values of $g(w^k)$ do not have an effect on the long-term costs. If no condition monitoring is in use, the maintenance actions usually follow a predetermined schedule, and the actual deterioration rate does not affect incurred costs.

5 Conclusion

This thesis presents a multi-component condition-based maintenance model with economic and structural dependencies. The system structure and maintenance actions are based on the model developed by Leppinen et al. (2023). The comparison between different condition monitoring strategies is meaningful only when non-deterministic deterioration with uncertainty is applied to all components in the system. In this work, a model with the updated deterioration behavior was tested by varying inspection intervals, target components, and deterioration probabilities. The additional information provided by condition monitoring was found to reduce long-term costs in most scenarios.

First, we considered the non-deterministic deterioration in a single-component case, and three possible state transitions were presented similarly as in the work of Lähteenmäki (2022). The multi-component system structure follows the work of Leppinen et al. (2023) where the system state is discretized, and the evolution of the system is modeled as a Markov decision process. However, the number of possible state transitions in a multi-component system grows exponentially with the new deterioration behavior. The optimal maintenance policy for the extended MDP was calculated using a modified policy iteration algorithm following the work of Parkkali (2021). A Monte Carlo simulation of the system evolution with partial observability through periodic inspections was implemented on Matlab.

The model was tested using an example system with three differently behaving components. Monitoring was found to reduce long-term maintenance costs by up to over 30 percent in some cases where the system deterioration was set slow. Generally, costs can be reduced by increasing the number of components inspected and the inspection frequency. This, however, also increased the number of component failures. Most effective was the monitoring of the component with the poorest failure distribution, meaning the component was the most probable to fail first.

To conclude, introducing frequent monitoring to the system, with, for example, electronic sensors can make a significant impact by reducing long-term costs. On the other hand, when reliability is prioritized, more conservative predictions of wear effectively reduce system failures. The results depend on the chosen system and policy. Therefore, these findings cannot be generalized unequivocally, and more comprehensive testing is needed.

The model is adaptive as both inspected components and inspection interval can be changed easily, as well as the system structure, together with failure and deterioration parameters. On the other hand, the challenges of the model include necessary assumptions related to deterioration and maintenance and inspection actions. Additionally, computations may become intractable with the added state transitions.

The model could be generalized even further in future research by relaxing the needed assumptions. For example, systems with a stochastic dependency could be modeled if the assumption of the criticality of each component could be relaxed or the possibility for parallel components or load sharing would be enabled (see e.g. Oakley et al., 2022). More uncertainty could be added to the model by enabling imperfect inspections or maintenance actions (see e.g. Le and Tan, 2013). Testing the theoretical model with real-life data from the industry would also be essential.

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