Alternative distance functions for change minimization in manufacturing network optimization

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Abstract

As the transportation sector attempts to decrease its emissions, the demand for renewable fuel is expected to increase. The growing demand for biofuel feedstock and end products and the global nature of the feedstock and end product markets result in increasingly complex supply chains. Renewable fuel companies aim to address the complexity by implementing mathematical models to execute their operations in the most profitable manner. Some of these models include components designed to mitigate uncertainties in the supply chains arising from several factors such as production and shipment issues or changes in forecasts.

This thesis further develops the uncertainty mitigation approach of a mid-term bi-objective optimization model of a renewable fuel manufacturing network. The two objectives of the model are maximizing gross margin and minimizing plan changes, measured in tons, with respect to a reference plan. The change minimization objective was designed to find a profitable solution that minimizes deviation from that reference plan, as changing a plan induces hidden costs not captured by the model. In this thesis, a new formulation of the change minimization objective is presented. The original model minimizes the total volume of change but does not capture the number of individual changes, each of which has some cost. Thus, this thesis presents an alternative objective function corresponding to the number of plan changes into the original model.

To minimize the number of optimization runs, the change count minimization objective was incorporated into the existing volume change minimization objective. As the volume change is often significantly larger than the change count, the combined change minimization objective consists of a normalized sum of the two objectives.

This novel model was tested using two different data sets and compared to the original model. The novel approach was successful in producing a close-to-ideal gross margin while increasing the volume change and reducing the number of plan changes in comparison to the original approach. This suggests that the novel approach offers benefits in mitigating hidden costs. However, the prolonged solving time of the novel model limits its practical use for decision makers. Reducing the computational complexity of the model offers potential for future research.

Keywords multi-objective optimization, supply chain optimization, manufacturing network

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Tiivistelmä

Uusiutuvien polttoaineiden kysynnän odotetaan kasvavan liikennealan pyrkiessä vähentämään päästöjään. Uusiutuvien polttoaineiden ja niiden raaka-aineiden kasvava kysyntä sekä globaalit markkinat johtavat yhä monimutkaisempiin toimitusketjuihin. Jalostajat pyrkivät vastaamaan monimutkaisuuteen ottamalla käyttöön matemaattisia malleja, joiden avulla jalostustoimintaa voi toteuttaa mahdollisimman kannattavasti. Osa näistä malleista on suunniteltu lieventämään toimitusketjujen epävarmuustekijöitä, jotka johtuvat esimerkiksi tuotanto- ja kuljetusongelmista tai ennusteiden muutoksista.

Tässä tutkielmassa jatkokehitettiin uusiutuvan polttoaineen tuotantoketjun optimointiin kehitettyä kaksitavoitteista optimointimallia. Alkuperäisen mallin kaksi tavoitetta olivat käyttökatteen maksimointi ja tonneissa mitattu suunnitelmamuutoksien minimointi. Muutosten minimointitavoitteen tarkoitus on löytää kannattava suunnitelma, joka eroaa mahdollisimman vähän aiemmasta suunnitelmasta, sillä suunnitelman muuttaminen aiheuttaa piilokustannuksia. Tässä tutkielmassa esitellään vaihtoehtoinen tapa muotoilla muutosten minimointitavoite. Alkuperäinen malli minimoi muutosten voluumia, mutta ei ota huomioon muutosten lukumäärää, vaikka piilokustannuksia syntyy jokaisesta yksittäisestä muutoksesta. Täten alkuperäiseen malliin lisättiin vaihtoehtoinen kohdefunktio, joka vastaa suunnitelmamuutosten lukumäärää.

Optimointikierrosten määrän minimoimiseksi muutosten lukumäärän minimointitavoite sisällytettiin aiempaan muutosten voluumia minimoivaan tavoitteeseen. Koska voluumin muutos on tyypillisesti huomattavasti suurempi kuin muutosten määrä, yhdistetty muutosten minimointitavoite koostuu näiden kahden tavoitteen normalisoidusta summasta.

Uutta mallia testattiin kahdella eri aineistolla ja verrattiin alkuperäiseen malliin. Uuden mallin avulla löydettiin ratkaisuja, joiden käyttökate oli samalla tasolla kuin verrokkiratkaisuilla, mutta muutosten lukumäärä oli pienempi. Tulosten perusteella uusi malli voi lieventää muutoksista aiheutuvia piilokustannuksia alkuperäistä mallia paremmin. Päätöksentekijöiden näkökulmasta uuden mallin käytännöllisyyttä rajoittaa kuitenkin sen pitkä ratkaisuaika. Potentiaalinen jatkotutkimuskohde on, miten mallin laskennallista vaativuutta voisi vähentää.

Avainsanat monitavoiteoptimointi, tuotantoketjun optimointi, tuotantoverkko

Contents

1 Introduction

As incentives and mandates for greenhouse gas emission reduction are imposed on the transportation sector, renewable fuel demand is expected to soar [1]. To address the growing demand, an increase in biofuel raw material supply is required, which results in more complex supply chains.

In order to address the complexity of supply chains, organizations implement network optimization models which allow them to gain decision support for ensuring profitability. In fast-paced industries, basic profit-maximizing optimization models have a clear deficiency: as uncertainties arise from several factors such as changes in prices and forecasted demand, production issues and shipment delays, the optimality of the obtained result can be susceptible to change.

Uncertainty mitigation is essential for maintaining competitiveness of the companies in the complex and fast-paced biofuel refining industry [2]. One approach for mitigating uncertainty in the supply chain of a biofuel refining company is the bi-objective model for scenario optimization of a manufacturing network proposed by Vuola [3]. The model is used in the mid-term sales and operations execution process (S&OE) of the case company. In S&OE, the planning work and the execution of the plan overlap. Due to this, each change in the S&OE plan gives rise to a hidden cost. For example, conducting a change may require both transportation of physical material and effort from the supply chain planners or logistics operatives.

To create a profitable plan while mitigating the hidden costs, the model proposed by Vuola [3] involves two objectives: profit maximization and minimization of deviation from the reference plan. The change minimization objective considers the total volume change between the reference plan and the optimized plan. The total volume change is the sum of the volume changes on each arc and each period of the multi-period manufacturing network model. The bi-objective approach aims to ensure that the newly optimized plan is profitable while remaining close to the previous plan, which is already being executed. However, it has been noticed that the change minimization objective is not ideal for the business case. As the current change minimization objective only considers the total volume change, it does not take into account the number of changes. For instance, it gives equal weight for ten changes of magnitude one and one change of magnitude ten. However, performing one large change is likely to induce less effort and therefore smaller costs than several small changes. Thus, minimizing the overall number of deviations from the reference plan is also in the interests of the decision maker (DM).

The aim of this thesis is to further develop the bi-objective model for scenario optimization proposed by Vuola [3], by adding a third objective function corresponding to the number of deviations from the reference plan. Section 2 outlines the background of the thesis, which includes the supply chain network and the existing biobjective optimization model. Section 3 formulates the novel change count minimization objective. The new model formulation was tested and its ability to reduce the number of changes was shown. However, the solving times of the model were significantly increased with the new formulation. Testing results are presented in section 4. Finally, section 5 summarises the findings of this thesis.

2 Background

This section introduces the supply chain network structure and the existing bi-objective network optimization model of the case company. Subsection 2.1 presents the network structure. Subsection 2.2 discusses multi-objective optimization generally before presenting the existing bi-objective model and its challenges this thesis intends to address. Lastly, subsection 2.3 defines the multi-objective optimization method which the current bi-objective method utilizes.

2.1 Supply chain network

A network is a system of connected entities. The entities can be represented with a set of nodes, while their connections can be represented with a set of arcs. This thesis considers the manufacturing network model suggested by Vuola [3], in which the nodes represent either supply or demand markets or one of two types of internal nodes: inventory and refinery nodes. Supply market nodes function as sources of feedstock for the supply chain, while demand market nodes receive the end products. Inventory nodes are used for storing feedstock or end products. The refining process takes place in refinery nodes, which can also function as storage for feedstock and end products. The arcs represent the opportunity to move physical material or criteria from one node to another. Criteria refer to the sustainability certificates used by the case company. As the flow of goods on a given arc happens only in one direction, the manufacturing network discussed in this thesis is a directed network. Furthermore, the directed network contains no cycles, which means it is a directed acyclic network. Other constraints related to the flow of material in the model include inventory, transportation and production capacities and a yield percentage in the refining process.

To formulate the network more precisely in terms of sets, let G denote a multi-period manufacturing network consisting of a set N of nodes, a set A of lanes (denoted $(i, j) \in A$ where i is the starting node and j is the end node) and a set T of periods. The lanes represent the possibility to move any physical materials (denoted by a set PM) and any criteria combinations (denoted by a set CC) from one node to another.

Given the case company objective to maximize gross margin (GM), the relevant network optimization problem type is minimum cost flow. The goal in minimum cost flow problems is to minimize overall cost subject to supply, demand and capacity constraints. Naturally, this corresponds to maximizing overall profit subject to the constraints. Furthermore, as the case company refines several products from a variety of feedstocks, the network model is a multi-commodity network model. Moreover, the model requires the capability to transfer goods across multiple periods.

2.2 Multi-objective optimization

Multi-objective optimization (MOO) is an area of decision-making which involves finding the best solution for problems with several, sometimes conflicting, objective functions. That is, given a set of k objective functions $f_i(\mathbf{x})$, $i = 1, \ldots, k$, the goal of MOO is to find a set of solutions $X^* \subseteq X$, where X represents the feasible solution space, such that the solutions in X^* represent the best possible trade-offs between the competing objectives [4].

In multi-objective optimization, the notion of optimality is replaced by Pareto optimality. A solution $\mathbf{x}$ to a MOO problem is Pareto-optimal if it cannot be improved within the feasible space in any objective function without worsening at least one other objective function. That is, if there does not exist another solution $\mathbf{x}'$ such that:

- For all $f_i, i = 1, \ldots, k$, where f_i represents the i^{th} objective function to be minimized, $f_i(\mathbf{x}') \leq f_i(\mathbf{x})$, and for all $f_i, i = k+1, \ldots, n$, where f_i represents the i^{th} objective function to be maximized, $f_i(\mathbf{x}') \geq f_i(\mathbf{x})$.
- There exists at least one $j \in 1, \ldots, n$ such that $f_j(\mathbf{x}') < f_j(\mathbf{x})$ for minimization objectives or $f_j(\mathbf{x}') > f_j(\mathbf{x})$ for maximization objectives.

The set of all Pareto optimal solutions is referred to as the Pareto front. Essentially, solving a multiobjective optimization problem usually corresponds to generating points belonging to the Pareto front or finding the entire set of such points [4].

The network optimization model suggested by Vuola [3] solves a biobjective optimization problem with GM and the total magnitude of deviation from the reference plan as its objectives to be maximized and minimized, respectively. These objectives are often conflicting: for example, a new sales opportunity can on the one hand increase GM and on the other hand increase deviation from the reference plan. The problem is solved using the ε-constraint method (defined in subsection 2.3), which generates a subset of the Pareto front. This thesis aims to include a third objective, minimization of the number of deviations from the reference plan, to the model. This renders the model tri-objective.

2.3 The ε-constraint method

The ε-constraint method is a well-known technique for solving multi-objective optimization problems. Like many other multi-objective methods, it converts the initial multi-objective problem into a single-objective problem which can be solved with regular single-objective optimization techniques. In the ε-constraint method, one of the objective functions is chosen as the primary function to be optimized. The values of the other objective functions are bounded by additional constraints [5]. By changing the bounds on the other objectives, one can obtain new points of the Pareto front. Assuming an original problem of the form

$$\begin{aligned} \max \quad & (f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_k(\mathbf{x})) \\ \text{s.t.} \quad & \mathbf{x} \in S, \end{aligned}$$

where $\mathbf{x}$ is the decision variable vector, $f_1(\mathbf{x}), \dots, f_k(\mathbf{x})$ are the k objective functions and S is the feasible region, the ε-constraint method converts the problem into the following form:

$$\begin{aligned} \max \quad & f_1(\mathbf{x}) \\ \text{st} \quad & f_2(\mathbf{x}) \geq \varepsilon_2, \\ & f_3(\mathbf{x}) \geq \varepsilon_3, \\ & \vdots \\ & f_k(\mathbf{x}) \geq \varepsilon_k, \\ & \mathbf{x} \in S. \end{aligned}$$

By variation of the parameters $\varepsilon_2, \dots, \varepsilon_k$ new Pareto-optimal solutions can be obtained. Each $\varepsilon_i, i \in 2, \dots, k$, can vary within the interval $[z_i^{nad}, z_i^*]$, where z_i^{nad} is the worst value of $f_i(\mathbf{x})$ over the Pareto front (referred to as the nadir values) and z_i^* is the best value of $f_i(\mathbf{x})$ over the Pareto front (referred to as the ideal values). $z_i^*, i \in 2, \dots, k$ can be found by maximizing the corresponding objective functions individually. $z_i^{nad}, i \in 1, \dots, k$ are generally difficult to obtain in problems with more than two objective functions, but are possible to estimate using a so-called payoff table. It is constructed the following way: first, we obtain k solutions, corresponding to optimizing each objective function independently. For each solution, we also calculate the values of other objective functions and create a table containing these values. The worst value in the table for objective function i is then an estimate of z_i^{nad} [4].

In the bi-objective network optimization model suggested by Vuola [3], GM is chosen as the primary objective function. The secondary objective function used as a constraint is the change minimization objective. This thesis attempts to include a third objective function corresponding to the number of changes with respect to the reference plan, to address the business use case of the model more accurately. The change induces two issues: firstly, the ε-constraint method requires more optimization runs to generate an adequate representative set of the Pareto front with three objective functions instead of two. Secondly, nadir values are difficult to obtain in problems with more than two objective functions [4]. These concerns are addressed in section 3 of this thesis.

3 Mathematical formulation

This section examines the mathematical formulation of the new objective functions. Subsection 3.1 presents the formulation of the current volume change objective and the new change count objective. Subsection 3.2 discusses the alternative techniques for modeling the binary variables in the change count objective. In subsection 3.3, we present a method for combining the volume change and change count objectives. Finally, section 3.4 addresses the normalization needed to give equal weight for both terms of the combined change minimization objective.

3.1 The new objective function

The current volume change minimization objective is of the form

$$\begin{aligned} \min_{\mu,\nu} \quad & (\sum_{t\in T} tw_t(\sum_{k\in PM}\sum_{i,j\in A}\mu^k_{ijt} + \sum_{m\in CC}\sum_{i,j\in A}\nu^m_{ijt})) \\ \text{s.t.} \quad & -\mu^k_{ijt} \leq x^k_{ijt} - y^k_{ijt} \leq \mu^k_{ijt} \\ & -\nu^m_{ijt} \leq u^m_{ijt} - w^m_{ijt} \leq \nu^m_{ijt} \\ & \mu^k_{ijt} \geq 0 \\ & \nu^m_{ijt} \geq 0, \end{aligned} \tag{1}$$

where x^k_{ijt} is the volume of physical material k on lane (i, j) at time t and y^k_{ijt} is its reference solution, and u^m_{ijt} is the volume of criteria m on lane (i, j) at time t and w^m_{ijt} is its reference solution. tw_t is the weight given to period t.

Applying the same weights tw_t for each period t, the novel change count minimization objective is of the form

$$\begin{aligned} \min_{\alpha,\beta} \quad & (\sum_{t\in T} tw_t(\sum_{k\in PM}\sum_{i,j\in A}\alpha^k_{ijt} + \sum_{m\in CC}\sum_{i,j\in A}\beta^m_{ijt})) \\ \text{s.t.} \quad & -\mu^k_{ijt} \leq x^k_{ijt} - y^k_{ijt} \leq \mu^k_{ijt} \\ & -\nu^m_{ijt} \leq u^m_{ijt} - w^m_{ijt} \leq \nu^m_{ijt} \\ & \mu^k_{ijt} \geq 0 \\ & \nu^m_{ijt} \geq 0 \\ & \mu^k_{ijt} \leq M^k_{ijt}\alpha^k_{ijt} \\ & \nu^m_{ijt} \leq N^m_{ijt}\beta^m_{ijt} \\ & \alpha^k_{ijt}, \beta^m_{ijt} \in \{0, 1\} \end{aligned} \tag{2}$$

where x^k_{ijt} is the volume of physical material k on lane (i, j) at time t and y^k_{ijt} is its reference solution, and u^m_{ijt} is the volume of criteria m on lane (i, j) at time t and w^m_{ijt} is its reference solution. tw_t is the weight given to period t. α^k_{ijt} and β^m_{ijt} are binary variables representing whether a change occurs on lane (i, j) at time t for physical material k and criteria m, respectively. M^k_{ijt} and N^m_{ijt} are chosen to be larger than any conceivable magnitude of change, thus restricting α_{ijt}^k and β_{ijt}^m to be equal to 1 if a change occurs. If no change occurs on lane (i,j) at time t for physical material k or criteria m, the variables α_{ijt}^k and β_{ijt}^m are allowed to be 0. These so-called big-M constraints are a typical method for coupling the values of continuous variables to the values of binary variables [6].

3.2 Modeling techniques for binary variables

The standard technique for coupling the value of a continuous variable x to the value of a binary variable z is a "big-M" constraint of the form $-Mz \leq x \leq Mz$ for a constant $M > 0$ which is an upper bound of x [6]. The big-M method has been used for a similar plan change count function in supply chain optimization by Jatty et al. [7]. For a nonnegative x the constraint simplifies to $x \leq Mz$. The resulting constraint enforces the implication $x > 0 \implies z = 1$. Considering our model, the technique is suitable for coupling the values of α_{ijt}^k and β_{ijt}^m to those of μ_{ijt}^k and ν_{ijt}^m, respectively. This is achieved via constraints of the form $\mu_{ijt}^k \leq M_{ijt}^k \alpha_{ijt}^k$ and $\nu_{ijt}^m \leq N_{ijt}^m \beta_{ijt}^m$ for physical material k and criteria combination m on lane (i,j) and period t. Using this technique, the variable α_{ijt}^k would assume value 1 if a change occurs in the volume of physical material k on lane (i,j) and period t, which is equivalent to the event that $\mu_{ijt}^k > 0$. Note that these constraints do not enforce that $\mu_{ijt}^k = 0 \implies \alpha_{ijt}^k = 0$ or that $\nu_{ijt}^m = 0 \implies \beta_{ijt}^m = 0$. Nevertheless, the only objective function containing the variables α_{ijt}^k and β_{ijt}^m is the new change count minimization function, which only contains positive multiples of the binary variables. It is thus reasonable to assume that the solver "prefers" to set the binary variables equal to 0 whenever possible (that is, whenever the corresponding continuous variables are equal to 0).

Naturally, implementing such "big-M" constraints necessitates selecting appropriate values for the constants M_{ijt}^k and N_{ijt}^m for $k \in PM$, $m \in CC$, $(i,j) \in A$, $t \in T$. For a constraint of the form $\mu_{ijt}^k \leq M_{ijt}^k \alpha_{ijt}^k$, the value of M_{ijt}^k must be an upper bound of μ_{ijt}^k. If not, some otherwise feasible solutions would be cut off because of the constraint. In some cases, this could even lead to the model being infeasible.

Fattahi et al. [8] discuss other challenges related to the "big-M" approach. Firstly, common mixed-integer linear program (MILP) solving methods, including cutting-plane and branch-and-bound algorithms, operate through iterative relaxations of the constraints. To some extent, the performance of the methods is dependent on the strength of these relaxations. Small values for the "big-M" constants may result in stronger relaxations whose feasible sets are smaller. Secondly, large values for the "big-M" constants could lead to numerical issues. This suggests that choosing as small big-M values as possible can improve model performance.

Khoshniyat and Törnquist Krasemann [9] studied the effect of the big-M values on model computation time. They suggest that the computation time of a MILP can substantially decrease with an appropriate choice of big-M. They note, however, that it may not be easy to significantly bound the big-M constants, as too small values might change the solution space.

In our case, bounding the big-M is difficult. Finding the maximum flow of a physical material k or criteria combination m on lane (i, j) on period t is an optimization problem in itself. Finding that quantity is obviously required to calculate the corresponding maximum deviation from the reference plan, that is, the maximum of μ_{ijt}^k for physical material k and ν_{ijt}^m for criteria combination m. Thus, there is no immediate method for bounding the big-M constants. However, the maximum flow of a physical material k or criteria combination m on lane (i, j) on period t may be possible to calculate using a recursive algorithm. This is because the parameters of the model include upper bounds for the amount of supply from any supply market on any period. The recursive graph traversal approach is possible due to the fact that the supply chain network is a directed acyclic graph. Developing such an algorithm is not part of the scope of this thesis, but offers potential for future research.

As an alternative to the big-M approach, we present a similar modeling technique for coupling binary variables to continuous variables. The validity of the approach must be assessed in the testing phase. The supposed advantage of the approach is that it does not require estimating or implementing any additional constants. It relies on a simple notion: a constraint of the form $x \leq xz$, where $x \in \mathbb{R}_0^+$ and $z \in \{0, 1\}$, enforces the implication $x > 0 \implies z = 1$. In our case, the constraints are of the form $\mu_{ijt}^k \leq \mu_{ijt}^k \alpha_{ijt}^k$ and $\nu_{ijt}^m \leq \nu_{ijt}^m \beta_{ijt}^m$. Similarly to the big-M approach, the constraint does not enforce $\mu_{ijt}^k = 0 \implies \alpha_{ijt}^k = 0$. Nevertheless, it is reasonable to assume the solver "prefers" to set the binary variables to 0 whenever possible, as their sum is a minimizable objective.

3.3 Combining the change minimization objectives

As the original model uses the ε-constraint method, it is preferable to combine the two change minimization objectives into one function before the solution process. This is due to the larger number of optimization runs and model adjustments that would have to be made if the same ε-constraint method were used for a tri-objective problem.

For reducing the number of objective functions before the optimization procedure, Koski and Silvennoinen [10] suggest dividing the objective functions into groups. Within each group, a weighted sum of the objective functions in that group forms a new objective function. This new objective function replaces the functions that constitute it. Koski and Silvennoinen [10] also state that every Pareto-optimal solution of the novel problem is a Pareto-optimal solution of the initial problem. However, the reverse result generally does not hold. This means that in practice the method may be unable to discover some Pareto-optimal solutions of the original problem. In our case, using a weighted sum $f_2' = w_2 f_2 + w_3 f_3$ of the volume change minimization objective

f_2 and the change count minimization objective f_3 is sufficient, since generating only a subset of the original three-dimensional Pareto front adequately serves the business purpose.

The weights w_2 and w_3 must be chosen so that they give equal importance to both the volume change f_2 and the change count f_3. The two functions generally do not share the same order of magnitude, as the volume change f_2 is likely to be larger than the change count f_3. On average, when a change occurs, the change in volume is larger than 1. Due to this, a normalization scheme for both functions is beneficial. The normalization essentially involves modifying weights w_2 and w_3 such that $w_2 f_2$ and $w_3 f_3$ operate on the same scale. It is possible to add constant terms c_2 and c_3 to keep the scale of $w_2 f_2 + c_2$ and $w_3 f_3 + c_3$ constant (for example, to ensure both functions are within [0,1]).

3.4 Normalization

Miettinen [4] presents several methods for normalizing objective functions in multi-objective problems. A method proposed by Osyczka [11] divides each objective function f_i by its ideal value z_i^*. However, this method does not utilize information about the nadir value z_i^{nad}. A more exact normalization which takes the entire range of the function into account uses the form

$$f_i^{normalized} = \frac{f_i - z_i^*}{z_i^{nad} - z_i^*} = \frac{1}{z_i^{nad} - z_i^*} f_i - \frac{z_i^*}{z_i^{nad} - z_i^*} \tag{3}$$

This form guarantees that $f_i^{normalized} \in [0, 1]$. We can then construct the new combined and normalized change minimization objective f_2' from the normalized forms of f_2 and f_3:

$$f_2' = f_2^{normalized} + f_3^{normalized} = \frac{f_2 - z_2^*}{z_2^{nad} - z_2^*} + \frac{f_3 - z_3^*}{z_3^{nad} - z_3^*} \tag{4}$$

In this form, f_2' gives equal importance to both f_2 and f_3. Furthermore, $f_2' \in [0, 2]$.

Implementing this normalization scheme requires estimating the nadir and ideal values of both change minimization functions f_2 and f_3. As discussed in part 2.3, a common method for estimating the nadir values is a so-called payoff table. Constructing the table requires optimizing each objective function individually. For the current implementation, the respective maximization of GM and minimization of volume change are performed. Thus, constructing a payoff table would require an additional optimization run for minimizing the change count. As the change count and volume change minimization objectives are closely related, it is reasonable to assume that the solution obtained when solely maximizing GM decently estimates both nadir values z_2^{nad} and z_3^{nad}. Note that in the bi-objective case, the payoff table method yields the true nadir value of both objective functions. However, with more than two objectives, it only yields an estimate of the nadir values [4]. Therefore, the more convenient method of estimating both z_2^{nad} and z_3^{nad} from the GM-maximizing solution will be used. Similarly, we use the volume change-minimizing result to estimate both ideal values z_2^* and z_3^*. By using these estimates, it is possible to avoid individually minimizing the change count.

4 Results

This section outlines the results obtained from the implementation of the tri-objective model, which was developed to improve upon the bi-objective model proposed by Vuola [3].

The novel tri-objective model was evaluated and compared to the original bi-objective model using two data sets. Three optimization runs were conducted with both data sets: run 1 and run 2 used the novel change minimization objective, while run 3 used the original volume change minimization objective. Runs 1 and 2 differed in the method that was used to model the binary variables representing changes in the network plan. Run 1 modeled the binary variables with an approach that does not require estimating a big-M (formulated in subsection 3.2). Run 2 used the big-M method for modeling the binary variables. As discussed in section 3.2, the big-M parameter must be set to some upper bound of the individual volume change variable to prevent cutting off otherwise feasible solutions. The nadir value of the second (minimizable) objective function corresponding to the sum of volume changes is a logical upper bound for any individual volume change. Thus, the estimate of the nadir value of the volume change minimization objective, z_2^{nad}, was used as the big-M parameter for run 2. Both binary variable approaches were included in the testing to assess whether they produced similar results and whether they differed in solving time. Similarly, the objective function values and the solving time from run 3 were compared to those produced by runs 1 and 2 to assess differences in performance between the original and the novel model.

Two solutions from the middle of the Pareto front per optimization run and per data set were obtained. In accordance with the ε-constraint method, these middle solutions were obtained by variation of the ε parameter which bounds the secondary objective function. As the novel model uses $f_2' \in [0, 2]$ as the secondary objective function, $\frac{2}{3}$ (for middle solution 1) and $\frac{4}{3}$ (for middle solution 2) were used as ε values during runs 1 and 2. For run 3, the ε values were obtained the following way: the interval $[z_2^*, z_2^{nad}]$ was segmented into three equal portions, and ε was set to the first endpoint and the second endpoint of the central segment for middle solution 1 and middle solution 2, respectively.

In order to discuss the performance of the alternatives in maximizing GM without disclosing the actual GM numbers, the GM values were scaled. For both data sets, GM was maximized and given a scaled value of 1. Other GM values were scaled by dividing them by the maximized GM value. As the model included soft constraints and corresponding penalty terms in the GM maximization objective, the true GM value was extracted from the objective value by adding the values of the penalty terms to the initial objective value. Due to the effect of the penalty terms, the true GM value extracted from the solely GM-maximizing solution was lower than the true GM value of some middle solutions. Nevertheless, scaling was conducted by dividing the middle solution GM values by the GM value from the GM-maximizing solution in all cases.

4.1 Data set 1

Table 1 presents the nadir and ideal values of the volume change and change count objectives for each optimization run with data set 1. The similarity of the nadir and ideal values for each optimization run suggests that the approximation approach used for z_2^{nad}, z_3^{nad} and z_3^* is justified.

Run	z_2^{nad}	z_3^{nad}	z_2^*	z_3^*
Run 1	25533.99987	3017	13924.04469	1862
Run 2 (big-M)	25717.08738	3008	13924.04469	1857
Run 3 (control)	25603.58072	3030	13924.04469	1898

Table 1: Ideal and nadir values for data set 1.

Table 2 presents the objective values of the first middle solutions obtained using data set 1. The GM value of each middle solution from runs 1 and 2 was compared to the GM value of the corresponding middle solution from run 3. This percentage difference in GM is shown in the '% improved' columns of tables 2, 3, 6 and 7. Table 3 presents the objective values of the second middle solutions obtained with data set 1. The big-M method (run 2) and the alternative binary variable modeling method (run 1) performed equally well. Based on the objective values, the methods successfully reduced the number of changes at the expense of the volume change objective. The results demonstrate that the new model is able to significantly reduce the number of plan changes while retaining a close-to-ideal GM.

Run	**Scaled GM**	**% improved**	**Volume change**	**Change count**
Run 1	0.9871885143	-1.462%	20065.76124	1998
Run 2 (big-M)	0.9998685762	-0.196%	19716.39928	2057
Run 3 (control)	1.001831908	0%	17817.22336	2304

Table 2: Objective function values for data set 1, middle solution 1.

Run	**Scaled GM**	**% improved**	**Volume change**	**Change count**
Run 1	1.000018277	0.001%	22850.1401	2493
Run 2 (big-M)	0.9999894573	-0.002%	22824.32539	2510
Run 3 (control)	1.000004776	0%	21710.40204	2705

Table 3: Objective function values for data set 1, middle solution 2.

Table 4 presents the optimization completion times for the first data set. They represent the time taken to complete the entire optimization procedure, with two middle solutions. While the obtained results are promising in terms of objective values, a caveat exists: the computational effort required by the new model is high. While testing the new model, it was noticed that in some cases the optimization failed to complete even after 15 minutes. Therefore, a time limit of 15 minutes was applied to each middle solution run. Nevertheless, in all test cases 15 minutes per middle solution was enough to yield sensible results. Reducing the computation time of the new model offers potential for future research, as it is a major factor in the usability of the model from the perspective of the decision maker.

Run	**Time to complete optimization**
Run 1	30 minutes and 22.8 seconds
Run 2 (big-M)	31 minutes and 15.33 seconds
Run 3 (control)	9 minutes and 18.91 seconds

Table 4: Optimization completion times for data set 1.

To demonstrate the differences between the old and the new model, the distribution of the plan changes was analyzed further. Figure 1 shows histograms of the plan changes for runs 1 and 3. The histograms demonstrate that the new model significantly reduces the number of the smallest changes.

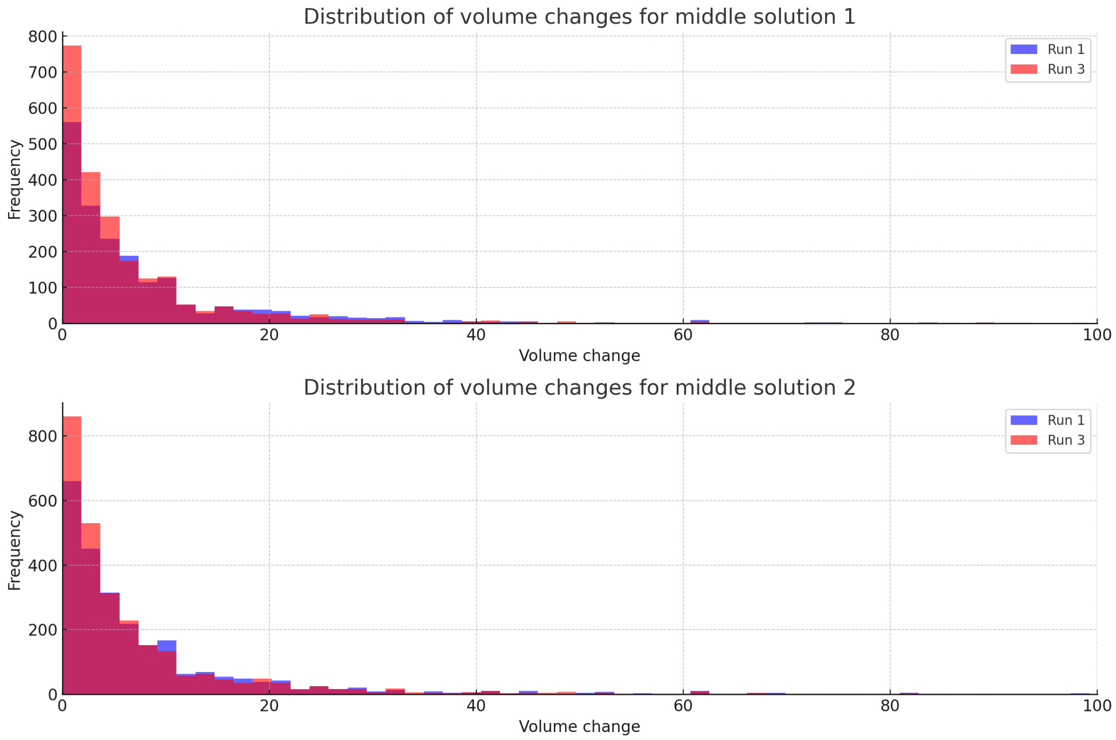

Figure 1: Distribution of the volume changes of middle solutions 1 and 2 for runs 1 and 3, using data set 1.

4.2 Data set 2

Table 5 presents the nadir and ideal values of the volume change and change count objectives for data set 2. As was the case for data set 1, the similarity of the ideal and nadir values for each run suggests that the approximation approach for z_2^{nad}, z_3^{nad} and z_3^* is justified.

Run	z_2^{nad}	z_3^{nad}	z_2^*	z_3^*
Run 1	25150.33988	2788	11173.52346	1387
Run 2 (big-M)	24905.44682	2781	11173.52346	1380
Run 3 (control)	25078.64381	2783	11173.52346	1394

Table 5: Ideal and nadir values for data set 1.

Table 6 presents the objective values for the first middle solution with the second data set, while table 7 shows the objective values for the second middle solution with the second data set. The novel model (runs 1 and 2) was able to achieve a comparable GM with a significant reduction in the change count.

Run	**Scaled GM**	**% improved**	**Volume change**	**Change count**
Run 1	0.9988476067	-0.076%	17209.19365	1706
Run 2 (big-M)	0.9994112497	-0.020%	17034.82979	1705
Run 3 (control)	0.9996087231	0%	15808.56358	2070

Table 6: Objective function values for data set 2, middle solution 1.

Run	**Scaled GM**	**% improved**	**Volume change**	**Change count**
Run 1	1.00356319	0.360%	21668.60617	2193
Run 2 (big-M)	0.9990158299	-0.094%	21171.06938	2186
Run 3 (control)	0.9999595394	0%	20443.6037	2479

Table 7: Objective function values for data set 2, middle solution 2.

Table 8 presents the optimization completion times for the second data set. Similarly to data set 1, runs 1 and 2 which used the novel model were significantly slower than the control run.

Run	**Time to complete optimization**
Run 1	40 minutes and 18.55 seconds
Run 2 (big-M)	39 minutes and 29.09 seconds
Run 3 (control)	9 minutes and 25.85 seconds

Table 8: Optimization completion times for data set 2.

To demonstrate the differences between the old and the new model, the distribution of the plan changes was analyzed further. Figure 2 shows histograms of the plan changes for runs 1 and 3. The histograms demonstrate that the new model significantly reduces the number of the smallest changes.

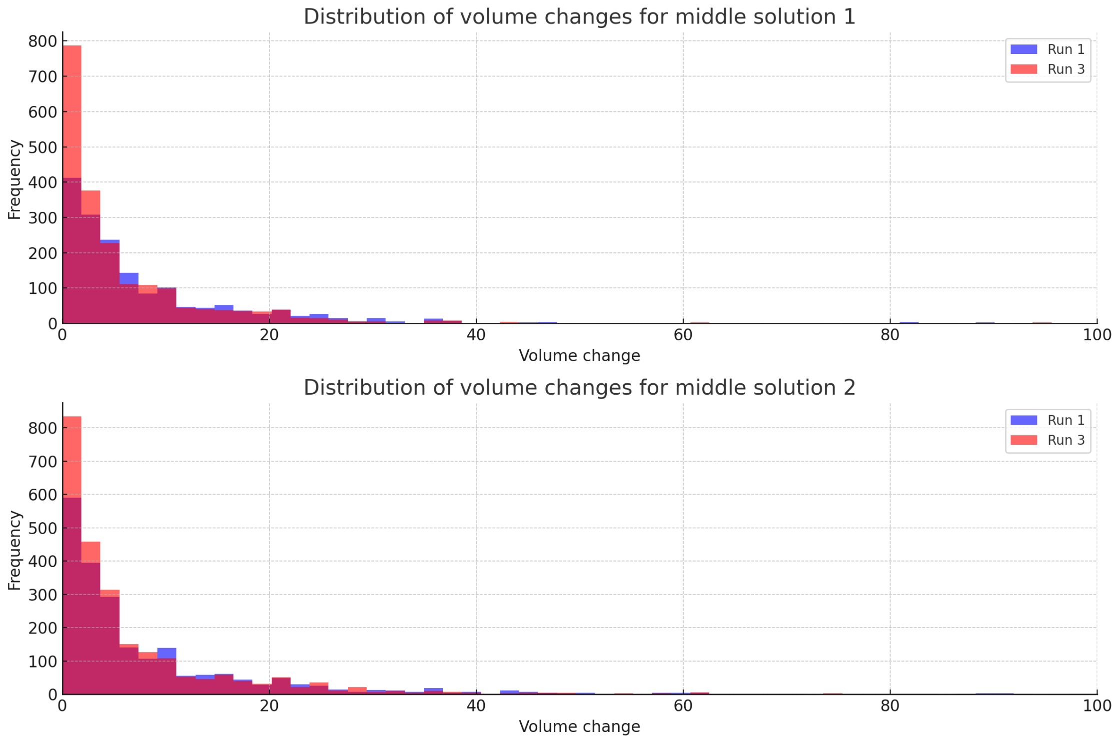

Figure 2: Distribution of the volume changes of middle solutions 1 and 2 for runs 1 and 3, using data set 2.

4.3 Conclusions

The results from the tests in subsections 4.1 and 4.2 seem promising as the novel model significantly reduced the number of the smallest changes. The novel model was able to find solutions with a close-to-ideal GM while significantly reducing the number of changes and slightly increasing the volume change. This is relevant considering the business use case of the model, as each individual change induces hidden costs.

However, the prolonged solving time of the model is a major caveat. In the test cases, the solving time of the novel model was approximately 20–30 minutes longer than when using the original model. As the model is intended for daily use and analysis of different scenarios, the solving time is a significant factor from the perspective of the decision maker.

Altogether, the model has potential for future development. Reducing the solving time of the model is crucial in order to increase usability for the decision maker. For example, an algorithm for obtaining tighter bounds for the big-M values (discussed in subsection 3.2) may be a viable method for reducing the computation time of the model. Furthermore, examining the asymptotic computational complexity of both the original and the novel model is of interest, as the complexity of the supply chains may increase in the future.

5 Summary

As a result of the transportation sector's attempts to decrease its emissions, the demand for renewable fuel is estimated to increase. The growing demand for renewable fuels and their feedstocks increases the complexity of the supply chains of the industry. This is exacerbated by the global nature of their markets. Thus, companies develop mathematical models to execute operations profitably and mitigate uncertainties in their global supply chains.

The current bi-objective model used for S&OE optimization of a biofuel refining company was further developed in this thesis. The current model involves two objectives: GM maximization and minimization of deviation from a reference plan. The latter is measured in tons. However, as each plan change induces a hidden cost, it is in the interest of the case company to not only minimize the volume change in tons, but to also minimize the overall number of deviations from the reference plan. The primary goal of this thesis was to examine the possibility to enhance the existing model by integrating a third objective function that accounts for the number of deviations.

The third objective function accounting for the number of deviations was formulated as a sum of binary variables. For each continuous decision variable, a corresponding binary variable was added to the model. Each binary variable was designed to assume value 0 when the corresponding continuous variable is equal to 0, and value 1 otherwise. Essentially, the third objective function counts the number of changes between the optimizable plan and a reference plan. The binary variables indicating changes were implemented with two alternative methods: a standard big-M method and an alternative method. The former used the nadir value of the volume change minimization objective as the big-M parameter which is chosen to be an upper bound of any individual volume change in the multi-period, multi-commodity network. The latter method did not require estimating any upper bound for the volume changes. Both binary variable approaches performed equally well.

To minimize the number of model adjustments and required optimization runs, the third objective function was incorporated into the existing change minimization function. To accurately capture the trade-offs of the two change minimization objectives, their normalized sum was chosen as the new change minimization objective. The normalization was conducted using approximated ideal and nadir values of both change minimization objectives.

The novel model was able to find solutions with close-to-ideal GM and significantly fewer plan changes compared to the old model. However, the practicality of the novel model as decision support for the supply chain planners is severely limited by the prolonged solving time. Reducing the solving time offers potential for future research.

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