The impacts of correlated supplier disruptions in supply networks

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Abstract of the bachelor's thesis

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Abstract

Supply networks are the networks through which the companies get and deliver goods and services. Disruptions in the companies' supply networks can have serious financial impacts on their performance. The correlation of these disruptions is a complex area, and there is not always a clear intuition how the correlation impacts the overall performance of the company.

In this thesis, we examine how the correlated disruptions of two suppliers impact the disruption probability of a company. We use Probabilistic Risk Assessment (PRA) -approach and model supply networks as Bayesian networks. We also implement an approach to examine correlation. By Monte Carlo simulations, we examine the disruption probabilities and the correlations with different network parameters.

The results suggest that when there are high probabilities that suppliers facing correlated disruptions propagate a possible disruption to the next supplier, the disruption probability of the company decreases as the correlation of these disruptions increases. Furthermore, the higher the disruption probabilities of the suppliers facing correlated disruptions are, and the higher the probabilities that these suppliers cause a propagation of a possible disruption to the next supplier are, the higher the disruption probability of the company is.

Keywords Supply networks, correlation, disruption, risk, Probabilistic Risk Assessment, Bayesian network, Monte Carlo simulation

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Tiivistelmä

Toimitusverkkojen kautta yritykset hankkivat toimittajilta palveluita ja tavaroita, joita ne tarvitsevat toimintaansa. Yritysten toimitusverkoissa tapahtuvat häiriöt voivat johtaa vakaviin seurauksiin yritysten kannalta. Yhdessä toimitusverkossa ilmenevät häiriöt voivat myös korreloida keskenään, mikä on kompleksinen ilmiö.

Tämä tutkimus keskittyy arvioimaan kahden toimittajan kohtaamien korreloituneiden häiriöiden vaikutusta toimitusverkossa olevan yksittäisen yrityksen todennäköisyyteen kohdata häiriö. Lähestymme ongelmaa todennäköisyyspohjaisen riskinarvioinnin näkökulmasta ja kehitämme bayesilaisiin verkkoihin perustuvan mallin, jolla tutkimme häiriöitä toimitusverkossa. Lisäämme malliin tavan tarkastella ja muokata korrelaatiota, ja teemme Monte Carlo -simulaatioita eri verkon parametrien arvoilla. Simulaatioiden perusteella arvioimme todennäköisyyttä, jolla yritys kohtaa häiriön.

Työn tulosten perusteella kahden toimittajan välisten häiriöiden korrelaatio vaikuttaa siihen todennäköisyyteen, jolla yritys kohtaa häiriön. Mikäli on todennäköistä, että kahden toimittajan mahdolliset korreloituneet häiriöt siirtyvät eteenpäin seuraavalle toimittajalle, vaikutus voidaan todentaa. Mitä suurempi toimittajien kohtaamien häiriöiden korrelaatio tällöin on, sitä pienempi on todennäköisyys yrityksen kohtaamalle häiriölle. Työssä kävi myös ilmi, että mitä suurempia ovat todennäköisyydet, että toimittajat, jotka kohtaavat korreloituneita häiriöitä, kohtaavat häiriön tai mitä suurempia ovat todennäköisyydet, että mahdolliset häiriöt siirtyvät näistä toimittajista seuraavaan toimittajaan, sitä suurempi on yrityksen todennäköisyys kohdata häiriö.

Avainsanat Toimitusverkot, korrelaatio, häiriö, riski, Todennäköisyyspohjainen riskinarviointi, bayesilainen verkko, Monte Carlo -simulointi

Contents

1 Introduction

A fire in a factory in Albuquerque on 17.3.2000 lead to a crisis in the cell phone industry. This event caused a disruption in the supply of important chips for Nokia and Ericsson, two competitors in the cell phone market. Nokia was more prepared for supply disruptions and managed to meet its production target whereas Ericsson was slow to react and suffered a big loss in potential revenue (Latour, 2001). Furthermore, when a ship got stuck in the Suez Canal in 2021, the global supply chain went into disorder (Lee and Wong, 2021).

Events, such as earthquakes (Käki et al., 2015) and floods (Kim et al., 2015), can have a disruptive impact on companies and other organisations through supply chains. A supply chain, also referred to as a supply network in this thesis, is a network through which a company gets and delivers goods and services. If one part of a company's supply network suddenly stops working, the company's ability to serve its customers and fulfill its obligations may deteriorate, leading to a decline in profits. Furthermore, analyzing supply chain is associated with cost management and can be used for reducing costs and enhancing performance (Horngren et al., 2015). By considering the disruptions while planning business operations, the risk for catastrophic failures can be mitigated.

The impacts of correlated supplier disruptions on a company are not always clear. For example, let us consider a company with many suppliers two of which face correlated disruptions. If the correlation is positive, it is more likely that these suppliers fail at the same time. Then it is also more likely that these suppliers operate at the same time. If the correlation is negative, it is more likely that these suppliers fail at different times. Then it is also more likely that these suppliers operate at different times. Depending on the company's reliance on these suppliers, these situations can impact the company in different ways. This complexity leads to a situation that there is not always a clear intuition into how the correlation of supplier disruptions impacts the company. To assess the risk in supply networks, the impact of correlation needs to be studied. In this way, the risk arising from the correlation of the supplier disruptions can be identified and prepared for.

In this thesis, we are interested in analysing supply network disruptions. The objective is to assess the impact of correlated disruptions of two suppliers to the disruption probability of the company in the network. We start by assuming that if two suppliers are correlated, then the disruptions of these suppliers are also correlated. This allows us to model the correlated disruptions as if the suppliers are correlated. We utilize Probabilistic Risk Assessment (PRA) -approach and model supply networks as Bayesian networks. We incorporate correlation into the model and use Monte Carlo simulations to obtain results for a range of parameters.

This thesis is structured as follows. In Section 2, we go through the literature to clarify concepts of supply chains and networks, disruptions, and correlations in supply networks. In Section 3, we introduce the model, network representation, simulation approach, and correlation in the model. In Section 4, we provide results and limitations. Lastly in Section 5, we summarize this thesis and main results and discuss about possible improvements of the model.

2 Literature review

In the previous section, we highlighted why it is important to study supply network disruptions and their correlation. In this section, we go through the concepts of supply chains and networks, disruptions in supply networks, modelling the disruptions in supply networks, and correlations in supply networks.

2.1 Supply chains and networks

A supply chain is a group of entities, which are linked together by flow of resources (information, services or products) from the source of the resource to a customer (Mentzer et al., 2001; Horngren et al., 2015). For example, the supply chain of a factory contains its customers and all the entities that are involved with the supplies and services that the factory uses to produce its goods. Many studies have considered a network perspective of a supply chain (Lazzarini et al., 2001; Kim et al., 2011; Käki et al., 2015). This means that the supply chain is not only a linear chain rather a multidimensional network. In this thesis, we use a supply network as a hypernym for a supply chain, meaning that supply chains are modelled as supply networks.

2.2 Disruptions in supply networks

A disruption in a supply chain is an event that blocks the normal flow of resources (Craighead et al., 2007). Many events can lead to a disruption. Examples of events that can lead to a disruption are floods (Kim et al., 2015) and earthquakes (Käki et al., 2015). We define disruption as an event, which causes a major failure in a supplier or a company. This means that the supplier or the company cannot operate.

Kim et al. (2015) divided disruptions in supply networks into two categories: node/arc-level and network-level disruptions. When a node/arc-level disruption occurs, the node/arc disrupts, but resources can still flow from sources of resources to the customer by some route. When a network-level disruption occurs, arc(s)/node(s) disrupt, and resources cannot flow from sources of resources to the customer. This division of disruptions into categories highlights that not every disruption in the network necessarily leads to a disruption of the customer.

2.3 Modelling disruptions in supply networks

Disruptions in a supply network can be modelled in different ways. Bayesian networks are one quantitative method for assessing risk in supply networks. Käki et al. (2015) used Bayesian networks to evaluate supply network disruptions and the risk related to them. They used PRA-methodology, constructed node-level metrics, and performed simulations to assess the most critical parts of networks. They also provided managerial insights from the results.

Besides Bayesian networks, there are also other quantitative models. Schmitt and Singh (2009) constructed a model based on Monte Carlo simulation and discrete-event simulation to assess risk propagation in a supply chain. They found out that the inventory level at the beginning of the disruption has significant impact to the customer service. Basole and Bellamy (2014) used a computational approach to study risk diffusion in supply networks by constructing AB-model. They found that the health of the supply network and risk diffusion are associated with the structure of the network. Li et al. (2021) constructed an agent-based computational model to analyze disruption propagation in supply networks. They considered both forward and backward propagation of disruptions and concluded that the two different directions of disruption propagation have different impacts on the network. Tang et al. (2016) used cascading failure model to evaluate robustness of an assembly supply network by simulation approach. They found out that when the node's dependency on other nodes decreases or when the node's threshold for risk propagation increases the network is more robust.

Ghadge et al. (2011) approached the disruptions qualitatively. They utilized a case study of a tsunami in Japan in 2011 and used a systems thinking approach to study risk propagation in the Japan supply network. They concluded that the risk can propagate in any direction in the supply network.

2.4 Correlation in supply networks

Today's world is interlinked (Easley et al., 2010). This means that many phenomena, entities, and markets are dependent on each other. In the context of supply networks, this means that one entity's performance can be dependent on another entity's performance or some other factor that impacts both of these entities. As argued in Section 1, correlation in supply networks is important. Among other reasons, our interlinked world can also be perceived as one of the reasons behind the importance of understanding correlation in supply networks.

Tomlin and Wang (2011) wrote about managing the risk of disruption in supply chains. They mentioned that an event, for example a natural disaster, might lead to a disruption of two suppliers that are located in the same geographical area. This means that there can occur correlation between the disruptions of suppliers. They also mentioned that the same geographical location is not the only attribute that can lead to the correlated supplier disruptions, as any shared attribute between the suppliers might lead to the correlated supplier disruptions.

While there are many approaches to modelling the disruptions in supply chains and networks in the literature, not many studies have been made to examine the correlation in supply networks. Pariazar et al. (2017) examined the correlation and inspections at suppliers in supply networks in a study of supply network design. They constructed a two-stage stochastic model to see how the risk and costs behave with the network design. They found out that when there is no inspection of goods available, the correlation between failures at the first-tier suppliers leads to an increase in costs. Raghunathan (2003) studied the value of information sharing, when there may occur correlation of demands in the supply chain. They found that the correlation in the demand impacted the surplus of the companies and thus had an impact on the incentives to form partnerships.

We contribute to the literature by studying the correlation's impact on the disruption probability of a company. Our objective is to examine how the correlated disruptions in two suppliers impact the disruption probability of the company.

3 Research methods

We first present the model used in this thesis. Then the simulation approach is reviewed and lastly, the implementation of correlation is described.

We assume that the correlation of two suppliers means that the disruptions of these suppliers are correlated. In this way, the correlation can be modelled as a correlation between suppliers in the network. The model used in this thesis is based on the model constructed by Käki et al. (2015), which is an application of the PRA-approach. Their model is based on Bayesian networks, and they use simulation approach to obtain results. Even though their model is similar to the model used in this thesis, the objectives of these studies differ. They aimed to examine disruptions in the network to define the most critical suppliers and to provide managerial standpoints. For example, they did not measure the correlation of suppliers, and they used further derived metrics. On the other hand, we examine the correlation more closely and use simple metrics to achieve the objective of this thesis.

3.1 Probabilistic Risk Assessment

Risk assessment can be divided into two categories - qualitative risk assessment and quantitative risk assessment - from which the latter is referred to as PRA. In both categories of risk assessment, the risk is measured by the likelihood and the severity of the event. In qualitative risk assessment, the likelihood and the severity are described qualitatively, for example, as words like "high" or "low". In quantitative risk assessment, the likelihood is measured by a probability or a frequency, and the severity is measured by a number, for example, the number of humans potentially being killed or injured (Stamatelatos, 2000).

The model used in this thesis is a quantitative model. The likelihood of a failure in a supply network is measured as a disruption probability. The severity of a disruption is measured as an occurrence of a disruption, which is modelled as a random variable indicating whether or not the disruption occurs.

3.2 Basics of supply network model

In this thesis, a supply network consists of nodes and arcs, which are also called elements. In this supply network, nodes represent the suppliers (e.g. companies), and arcs represent the connections between the nodes. In our model, every arc has a direction, and the source of the arc is a parent node, and the sink of the arc is a child node. In Figure 1, node j is a parent node and node i is a child node. The focal node is the node in the supply network that we are interested in and whose disruption probability we are examining.

Each node in the network can be disrupted in two distinct ways. First, a node can be disrupted as a result of a propagation of disruption from its parent nodes through arcs. This movement of disruption is called the risk propagation. In this way, a disruption can propagate through the network. We assume that a disruption in the network can only propagate from a parent node to the child node (Käki et al., 2015). Second, a node can be disrupted by some other reason, which is not dependent on the network or its structure. This is called the node's internal risk. Each arc can only be disrupted by a reason that is not dependent on the network.

A disruption propagates from a parent node to the child node if the parent node and the arc from the parent node to the child node are disrupted. This means that only a disruption of the arc doesn't lead to the disruption of the child node.

3.3 Network representation

Let $S = (V, E)$, where S is the network, V is the set containing all the nodes in the network, and $E \subseteq \{(i,j)|i,j \in V\}$ is the set containing all the arcs in the network.

In our supply network model, every element i is represented with a binary state X_i. In the state $X_i = 1$, the element i is disrupted, and in the state $X_i = 0$, the element i is not disrupted. If the element is not disrupted, it means that the element is operational and works normally. The disruption of the element means that the element is not operational and thus cannot work at all. The state of the network is constructed by the states of all the elements in the network.

There are in total x^n network states, where x is the number of possible states of an element and n is the number of elements in the network. We assume that nodes and arcs have only two possible states (x = 2), operational and disrupted. The states of the elements in the network could be modelled with more than two possible states, but this would make calculations heavier. Snyder et al. (2016) also mention that the binary model is the most usual way for modelling the supply chain system state.

Let S be a network with two nodes i and j. S = (V,E), where $V = \{i, j\}$ are the nodes of the network. Let node j be connected to node i, so arcs E of the network S are $E = \{(j, i)\}$. This network is presented in Figure 1. The probability that a disruption occurs in node i without impact of other nodes is α_i. This is called the internal risk parameter. $\beta_{i|j}$ is the probability that a possible disruption in node j propagates to node i. In other words, $\beta_{i|j}$ is the probability that the arc from node j to node i disrupts.

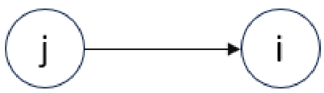

Figure 1: Simple network with two nodes and one arc (Käki et al., 2015).

There are eight states of the network, since there are two possible states for nodes i and j and two possible states for arc from node j to node i. These states and the probability that these states occur are presented in Table 1. From these network states there are five states, which lead to a disruption in node i.

Network state	X_j	$X_{i\|j}$	X_i	**P(Network state)**
1	0	0	0	$(1-\alpha_j)(1-\beta_{i\|j})(1-\alpha_i)$
2	0	0	1	$(1-\alpha_j)(1-\beta_{i\|j})\alpha_i$
3	0	1	0	$(1-\alpha_j)\beta_{i\|j}(1-\alpha_i)$
4	1	0	0	$\alpha_j(1-\beta_{i\|j})(1-\alpha_i)$
5	0	1	1	$(1-\alpha_j)\beta_{i\|j}\alpha_i$
6	1	1	0	$\alpha_j\beta_{i\|j}(1-\alpha_i)$
7	1	0	1	$\alpha_j(1-\beta_{i\|j})\alpha_i$
8	1	1	1	$\alpha_j\beta_{i\|j}\alpha_i$

Table 1: Possible states of a simple network with two nodes and one arc.

These states are in rows 2, 5, 6, 7 and 8. State 6 also leads to a disruption of node i, since propagated disruption from a parent node automatically causes a disruption in the child node. When we add these five probabilities together, we can calculate the probability of disruption of node i.

$$F_i = (1-\alpha_j)(1-\beta_{i|j})\alpha_i + (1-\alpha_j)\beta_{i|j}\alpha_i + \alpha_j\beta_{i|j}(1-\alpha_i) + \alpha_j(1-\beta_{i|j})\alpha_i + \alpha_j\beta_{i|j}\alpha_i \quad (1)$$

$$= (1-\beta_{i|j})\alpha_i + \beta_{i|j}\alpha_i + \alpha_j\beta_{i|j}(1-\alpha_i) \quad (2)$$

Thus the probability F_i that node i is disrupted can be calculated by

$$F_i = \alpha_i + \alpha_j\beta_{i|j}(1-\alpha_i) \quad (3)$$

The probability of disruption of the focal node is the key metric in this thesis. It describes the probability that the focal node in the network disrupts and thus cannot operate. Note the distinction between α_i and F_i. The α_i is internal risk probability, which refers to the probability that node i gets disrupted by some other reason than the propagation from node j. The F_i is the overall probability that node i disrupts, which includes the probabilities of every situation in which node i disrupts.

If a node in a network has more than one parent node, one parent node causing a propagation of disruption to the child node is sufficient enough to disrupt the child node. This means that if a node with many parent nodes faces a disruption from any of its parent nodes, it cannot operate anymore. In other words, this means that every parent node is vital for the node, and the node does not have two or more parent nodes that deliver the same goods or services for the node. This assumption is important, as it describes the way disruptions propagate in the network. This assumption is discussed in Section 4.

Based on (3), the probability that focal node i disrupts can be calculated exactly. We first define each possible state of the network. Then we take the states in which the focal node is disrupted and define the probabilities of these states occurring. By adding up these probabilities, we can calculate the disruption probability of the focal node. When we consider larger networks than in Figure 1, the exact formula for the disruption probability of the focal node is more challenging to derive. This is due to the fact that the number of states of nodes and arcs, and thus the number of parameters, increases exponentially. Thus, we need to find a different approach to assess the probability of disruption of the focal node in larger networks.

Simulation is a suitable tool for assessing network metrics for our purpose. Instead of deriving exact expressions, we can construct the results by using probability distributions. By sampling states for each node and arc from different probability distributions and by combining these states while considering rules for propagation of disruptions, we can construct a state for the focal node. This is the same as generating a sample from the focal node's probability distribution. By the Law of Large Numbers, if we take enough samples from this probability distribution and inspect the states of the focal node, the sum of the states of the focal node divided by the number of simulations should converge to the expected value of the state of the focal node. This expected value is the disruption probability of the focal node.

3.4 Monte Carlo simulations

Since the derivation of the exact formula for the disruption probability of the focal node in larger networks can be challenging, we use simulations for calculating this metric.

This kind of problem is well approachable by Monte Carlo simulation. In many Monte Carlo simulations, the outcome of a complex system is assessed by modelling the system by certain probability density functions. Then the simulation is performed by sampling numbers from these functions. Based on these results, different statistical metrics are calculated (Harrison, 2010).

We use Monte Carlo simulations to obtain information about the behaviour of the network. Simulations are performed with MATLAB-software. We construct the network state by generating a state for each node and arc in the network. The states are generated from binomial distributions. After all the network states are generated, the probability of disruption of the focal node is calculated.

Let the $S = \{V, E\}$ be the network with nodes $V = \{1, 2, 3\}$ and arcs $E = \{(2, 1), (3, 1)\}$. The network is in Figure 2.

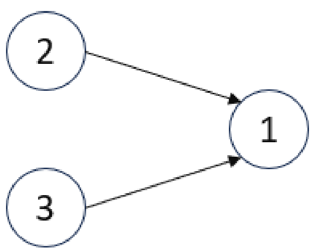

Figure 2: Network with three nodes and two arcs (Käki et al., 2015).

The state of node 1 (X_1) is generated in the following way. First, X_2 and X_3 are generated. Because neither node has parent nodes, these states are simply generated by sampling random numbers from binomial distributions. X_2 is generated from a binomial distribution with parameter α_2, which gives us α_2 of the time 1 and $(1-\alpha_2)$ of the time 0, when generating many numbers from this distribution. X_3 is generated the same way using the corresponding parameter.

Second, the impacts of these two nodes to focal node 1 are generated. States of the arcs are generated first, and then states of the arcs are multiplied by states of the corresponding nodes. $X_{1|2}$ is generated from a binomial distribution with parameter $\beta_{1|2}$. This state is then multiplied by X_2. For example, if $X_2 = 0$ and $X_{1|2} = 1$, the overall impact of node 2 to node 1 is $X_2 \cdot X_{1|2} = 0 \cdot 1 = 0$. The impact of node 3 is generated the same way with $X_{1|3}$ and X_3.

Last, X_1 is generated from three different states. The first two states are the ones generated from nodes 2 and 3, and the third one is generated from the internal risk parameter of node 1. This internal state of node 1 is generated from a binomial distribution with parameter α_1.

$$X_1 = \max(X_2 \cdot X_{1|2}, X_3 \cdot X_{1|3}, \text{Internal state of node 1}) \tag{4}$$

Based on (4), X_1 is the maximum of the internal state of node 1 and the states from nodes 2 and 3. For example, if the states from nodes 2 and 3 are 0, but the internal state of node 1 is 1, $X_1 = 1$.

The state of the focal node is collected after each simulation. After all the simulations have been performed, the number of simulations where the focal node is disrupted is counted and divided by the total number of simulations to get the probability of disruption of the focal node. For example, if 33 out of the 100 simulations turn out to disrupt the focal node, the probability that the focal node disrupts is 0.33.

3.5 Implementation of correlation

In order to modify and examine the correlation between two nodes, we create an auxiliary node into the network. This node is connected to both correlated nodes with auxiliary arcs. We want both correlated nodes to be fully dependent on the probabilities relating to the auxiliary node, and thus we use conditional probabilities to create the probabilities relating to the correlated nodes.

Let us update our simple network in Figure 2. Node s is added to the network, and it is connected to nodes 2 and 3 via conditional probabilities. Node s is auxiliary and does not represent any real object. The probabilities of disruption F_2 and F_3 are calculated with conditional probabilities.

$$F_2 = P(2) = P(2|s)P(s) + P(2|\bar{s})P(\bar{s}) \tag{5}$$

$$F_3 = P(3) = P(3|s)P(s) + P(3|\bar{s})P(\bar{s}) \tag{6}$$

From (5) and (6) we can see that the probabilities of disruption of nodes 2 and 3 are fully dependent on the probabilities relating to node s. By implementing probabilities with this approach, we can modify the correlation between nodes 2 and 3 through node s. The original network does not conceptually change much, since parameters α_2 and α_3 are just replaced by parameters F_2 and F_3. This modified network is in Figure 3.

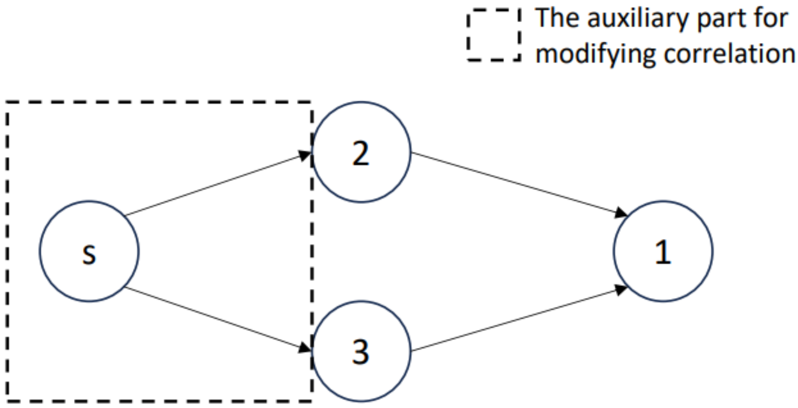

Figure 3: Network, correlation added (Käki et al., 2015).

Implementing (5) and (6) into the model creates new parameters to be considered. Because node s is auxiliary, parameters $P(2|s)$, $P(2|\bar{s})$, $P(3|s)$, $P(3|\bar{s})$, $P(s)$ and $P(\bar{s})$ do not represent any objects or qualities in reality. This allows us to choose the parameters so that the correlation varies between nodes 2 and 3. In this way, we are able to examine how the correlation between two nodes impacts the probability of disruption of the focal node.

We quantify correlation as

$$\rho(A, B) = \frac{1}{N-1} \sum_{i=l}^{N} (\frac{A_i - \mu_A}{\sigma_A})(\frac{B_i - \mu_B}{\sigma_B}), \tag{7}$$

where A and B are two different vectors, σ_A and σ_B are the standard deviations of A and B, μ_A and μ_B are the means of A and B, N is the number of data points and i is the number of a data point. This is called the Pearson correlation coefficient.

In the simulation phase, the generation logic of states in artificial part differs from the generation logic of states in other parts of the network. The state of node s (X_s) is generated, but $X_{2|s}$ and $X_{3|s}$ are not generated. If $X_s = 1$, X_2 and X_3 are generated from binomial distributions with corresponding parameters $P(2|s)$ and $P(3|s)$. If $X_s = 0$, X_2 and X_3 are generated from binomial distributions with corresponding parameters $P(2|\bar{s})$ and $P(3|\bar{s})$. The states of these two correlated nodes are collected into corresponding vectors, and the Pearson correlation coefficient between these vectors is calculated. This way correlation can be measured to examine its impact on the probability of disruption of the focal node.

Let us consider two nodes A and B with binary states. Let's inspect the correlation of these nodes. Graphs presenting the states of nodes A and B with correlation values of 1 and -1 are presented in Figures 4 and 5. Circles represent the states of these nodes, and the percentage in the middle of the circle represents the state's theoretical portion of all the states. In both figures, the disruption probabilities of node A and node B are 0.5.

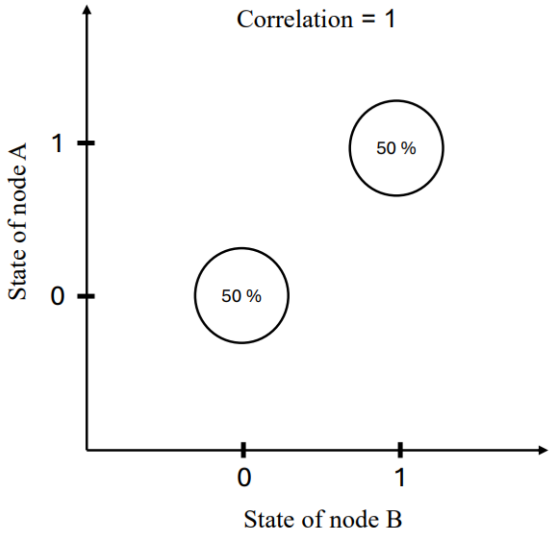

Figure 4: States of nodes A and B with correlation value of 1 between nodes A and B.

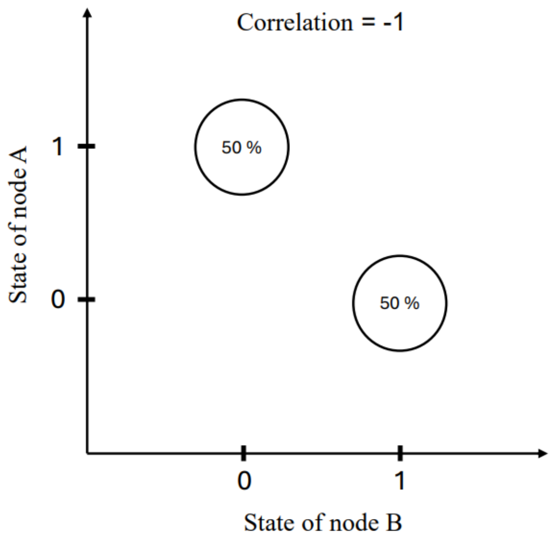

Figure 5: States of nodes A and B with correlation value of -1 between nodes A and B.

From Figures 4 and 5 we can see that the states of nodes A and B are always the same, when the correlation between these nodes is 1. Whereas when the correlation is -1, these states are always the opposite.

3.6 The impact of parameters of the correlated nodes on results

While we are interested in the correlation between two nodes, the other parameters of the correlated nodes may impact the severity of the impact of correlation on the disruption probability of the focal node. For example, let us consider the situation in Figure 3. If we have $F_2 = F_3 = 0.1$, there occurs only few disruptions from nodes 2 and 3, even though they are correlated. Same applies to $\beta_{1|2}$ and $\beta_{1|3}$. If these parameters are set to 0.1, the impact of nodes 2 and 3 on node 1 occurs more rarely. Furthermore, if there are more nodes in the network impacting node 1 than nodes 2 and 3, the impact of correlated nodes might get lost to the impact of other nodes in the network. This might happen, if we have low values for F_2, F_3, $\beta_{1|2}$ and $\beta_{1|3}$.

For these reasons, we simulate through the network with different parameters of the correlated nodes. In our example above, these parameters are F_2, F_3, $\beta_{1|2}$ and $\beta_{1|3}$.

4 Results

4.1 Network used in the study

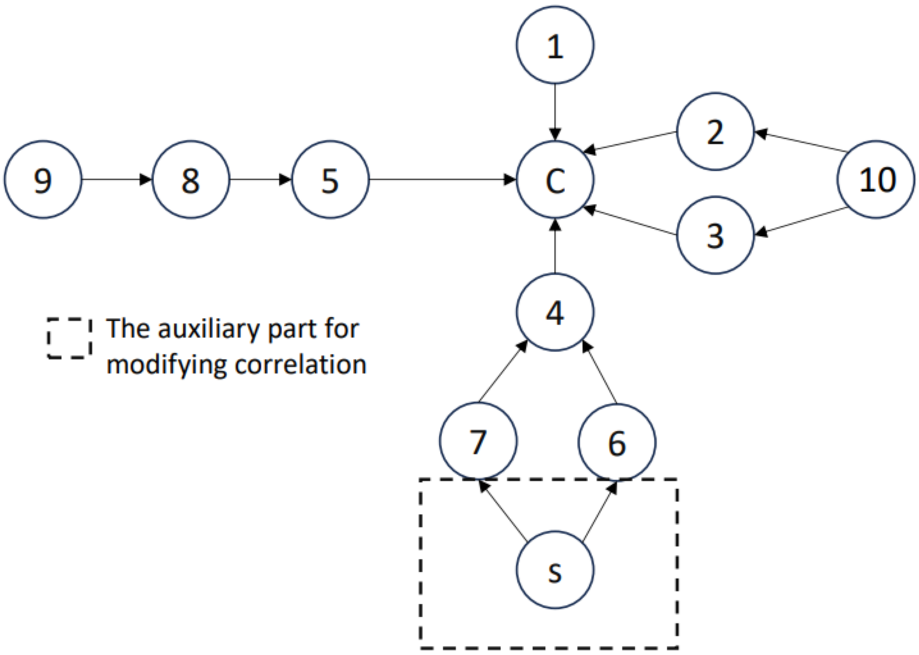

Figure 6: Modification of a network introduced by Käki et al. (2015).

We consider a network $S = (V, E)$ in Figure 6. This network contains 12 nodes and 13 arcs. The network contains auxiliary arcs from node s to nodes 6 and 7 and an auxiliary node s, which generates the correlation between nodes 6 and 7 via conditional probabilities, as presented in (5) and (6). In this way, we can examine how different levels of correlation impact the probability of disruption of focal node C.

4.2 Parameter values used in the study

Internal risk probabilities are $\alpha_i = \{0.1 | i \in V \setminus \{6, 7, C\}\}$. The value of the internal risk probability of focal node C is determined to be 0. The values of α_6 and α_7 are replaced by F_6 and F_7, which are fully dependent on the probabilities relating to node s. The values between F_6 and F_7 are the same through simulations, and thus we mark the value of them simply by α. The value of α is changed during the simulations to examine the impact of correlation to F_C with different levels of α. The values of F_6 and F_7 are produced by parameters $P(6|s)$, $P(6|\bar{s})$, $P(7|s)$, $P(7|\bar{s})$, $P(s)$ and $P(\bar{s})$ by (5) and (6). These parameter values are constructed by simulation approach.

We construct a distinct simulation where the objective is to produce the set of parameters that produces different correlations between two nodes. In this simulation, we have nodes a and b that we want to make correlated. We also have the auxiliary node s which sets up the correlation via conditional probabilities by (5) and (6). We first scope the range of parameters F_a and F_b to $\{0.2, 0.5, 0.8\}$. The values between parameters F_a and F_b are same, so we mark them by α. After this, we set $P(s)$, $P(a|s)$ and $P(b|s)$ go through the range from 0 to 1 with interval of 0.1. With each combination of these parameters, we use (5) and (6) to generate $P(a|\bar{s})$ and $P(b|\bar{s})$. If these values are greater or equal to 0 and smaller or equal to 1, we construct a 10 000 sample simulation for the states of nodes a and b. If this obligation is not fulfilled, we move on to the next combination of parameters. In the simulation, we first generate a state for node s. If $X_s = 1$, X_a and X_b are generated from the binomial distributions with corresponding parameters $P(a|s)$ and $P(b|s)$. If $X_s = 0$, X_a and X_b are generated from the binomial distributions with corresponding parameters $P(a|\bar{s})$ and $P(b|\bar{s})$. These states are collected to corresponding vectors, and after 10 000 simulations the correlation between these vectors is calculated. After all the simulations, we pick 10 different correlation values and corresponding values of $P(s)$, $P(a|s)$ and $P(b|s)$ for each value of α. These parameters can be used to construct correlation between any two nodes that do not have parent nodes. Now we use these parameters for the network in Figure 6 where $a = 6$, $b = 7$, $\alpha = F_6 = F_7$ and node s is the auxiliary node. From these parameters, we only provide the values of α in this thesis.

The probabilities that a disruption passes from node j to node i are $\beta_{i|j} = \{0.5 | (j, i) \in E \setminus \{(7, 4), (6, 4), (s, 7), (s, 6)\}\}$. The values between $\beta_{4|6}$ and $\beta_{4|7}$ are the same through the simulations, and thus we mark the values of $\beta_{4|6}$ and $\beta_{4|7}$ by β. The value of β is changed during the simulations to examine the impact of correlation to F_C with different levels of β.

Simulations with each value of α and β are performed with 10 different correlations. With each correlation, we simulate the network 100 000 times. In total, we simulate 1 000 000 times with each combination of parameters α and β. Let's call this sample of 1 000 000 simulations a set. The range of parameters α and β is $\{0.2, 0.5, 0.8\}$.

Since the simulations are performed with all combinations of parameters α and β, we simulate 9 sets in total. We select this range of α and β to be able to inspect the impact of these parameters, while still having as little different values as possible. This restriction is done, since every new value of α or β requires new sets of simulations, which take time.

4.3 Simulation results

The disruption probabilities of focal node C in Figure 6 with different values of parameters α and β are presented as a function of the correlation between nodes 6 and 7 in Figure 7. Simulations through the network are performed with different values of parameters to examine for which values of parameters α and β the correlation between nodes 6 and 7 impacts F_C.

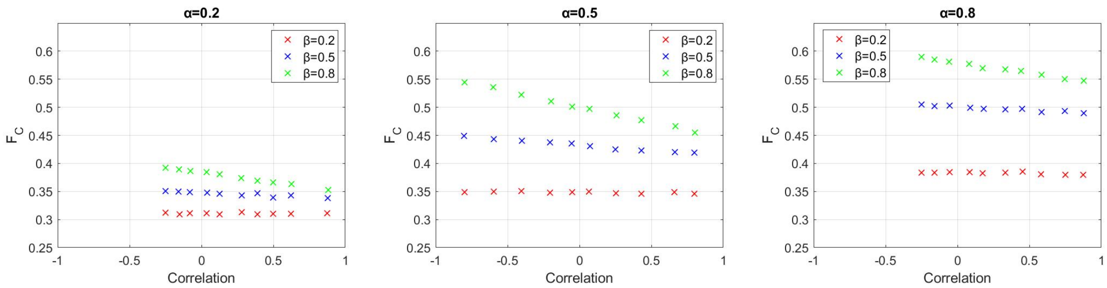

Figure 7: F_C as a function of the correlation between nodes 6 and 7 with different parameters α and β.

As we can see from Figure 7, the higher the β is, the higher the F_C is. This tells us that when $\beta_{4|6}$ and $\beta_{4|7}$ increase, F_C increases. We can also see that the higher the α is, the higher the F_C is. This means that when F_6 and F_7 increase, F_C increases.

The most interesting result occurs in the impact of correlation between nodes 6 and 7 to F_C, when β is as high as 0.8. In a situation, where the correlation in the network has no impact on F_C, a constant line could be fitted through the data points of each combination of α and β. But from Figure 7 we can clearly see that as correlation between nodes 6 and 7 increases and $\beta = 0.8$, F_C decreases with every value of α. On the other hand, when $\beta = 0.2$, the correlation has no impact on F_C.

The highest and the lowest simulated values of correlation and F_C related to these correlations with different values of α and β are presented in Table 2. On the top of Table 2, different values of α and β are presented. On the left, different values of F_C and correlation between nodes 6 and 7 are presented. The differences between values of F_C are calculated in the last two rows.

There are four combinations of α and β where the absolute value of relative difference of F_C is more than 5%. This difference is significant. These combinations are the ones where $\beta = 0.8$ and the one where $\alpha = 0.5$ and $\beta = 0.5$. In all of these combinations, the relative difference of F_C is negative. This supports the observation

	$\alpha = 0.2$			$\alpha = 0.5$			$\alpha = 0.8$		
β	0.2	0.5	0.8	0.2	0.5	0.8	0.2	0.5	0.8
The highest correlation in Figure 7	0.875	0.877	0.880	0.800	0.799	0.802	0.874	0.876	0.877
The lowest correlation in Figure 7	-0.250	-0.249	-0.250	-0.800	-0.802	-0.799	-0.250	-0.251	-0.250
F_C with the highest correlation in Figure 7	0.311	0.338	0.353	0.346	0.419	0.455	0.380	0.489	0.547
F_C with the lowest correlation in Figure 7	0.312	0.351	0.392	0.349	0.449	0.545	0.383	0.505	0.590
The difference of F_C with the highest and the lowest correlations in Figure 7	-0.001	-0.013	-0.039	-0.004	-0.031	-0.090	-0.004	-0.015	-0.042
The relative difference of F_C with the highest and the lowest correlations in Figure 7	-0.36 %	-3.65 %	-9.95 %	-1.01 %	-6.80 %	-16.52 %	-0.94 %	-3.02 %	-7.18 %

Table 2: The highest and the lowest simulated values of the correlation and F_C related to these correlations with different values of α and β.

made earlier regarding Figure 7 that when β is high, F_C decreases as the correlation between nodes 6 and 7 increases.

We can also see that when the $\beta = 0.2$, the absolute value of relative difference of F_C is less than 1.1% with every value of α. This means that the correlation has no significant impact on F_C when β is low.

4.4 Assessment of the results

According to our model, an increase in the value of α or β leads to an increase of F_C. This means that when we have higher probabilities that correlated nodes disrupt or propagate a possible disruption to the next node, it is more probable that the focal node disrupts. When we have high probabilities that correlated nodes propagate possible disruptions to their child node (β), the correlation has an impact to the probability of disruption of the focal node. In this situation, as the correlation increases between the correlated nodes, the probability of disruption of the focal node decreases. When β is low, the correlation has no impact on the disruption probability of the focal node.

The results regarding parameters α and β are intuitive. F_C is a sum of multiplications of probabilities, and if the value of some parameter increases, the value of F_C increases.

The results regarding the correlation may seem counterintuitive at the first look, as one could think that the correlation of nodes in the network must cause higher probability of disruption of the focal node. The results can be explained by examining a simple network with correlation presented in Figure 3. Let us consider two extreme examples. In the first example, the correlation between nodes 2 and 3 is 1, and $\beta_{1|2}$ and $\beta_{1|3}$ are 1. In the second example, the correlation between nodes 2 and 3 is -1, and $\beta_{1|2}$ and $\beta_{1|3}$ are 1.

In the second example, states of nodes 2 and 3 (X_2 and X_3) are always different, since the correlation is -1. Because $\beta_{1|2} = 1$ and $\beta_{1|3} = 1$, node 1 certainly disrupts from either one of the nodes, meaning that $F_1 = 1$ no matter what the α_1 is. In the first example, X_2 and X_3 are always the same, since correlation is 1. There are possibilities that $X_2 = X_3 = 0$, meaning that disruption is not propagated from nodes 2 and 3 to node 1. Therefore, node 1 might not be disrupted, and F_1 can be lower than 1, depending on α_1.

As we can see from Figure 7, there is no data for the high negative correlation, when α is high or low. If we wish to get a correlation level close to -1, there has to occur many network states, where nodes 6 and 7 have the opposite states. It is unlikely that many opposite states could be generated from the binomial distributions with the same parameter value, when the value of parameter α is high or low.

4.5 Assessment of the model

There are many assumptions related to our model, which restrict its usability. Still, this model provides important insights about the impact of correlation of two nodes to the disruption probability of the focal node.

The range of parameters α and β is $\{0.2, 0.5, 0.8\}$, which is quite small but enough to provide insights about these parameters' effect on the impact of correlation to F_C. If more detailed results are needed, the range of these parameters could be broadened. When applying a broader range, one must keep in mind that for low and high values of α, the correlated nodes are unlikely to have high negative values of correlation.

Ghadge et al. (2011) and Li et al. (2021) stated that the disruptions can propagate in more than one direction. We restrict this point of view to comply with Käki et al. (2015) and assume that disruption can propagate only from parent nodes to child nodes. They noted that this direction of disruption propagation is normal in usual supply networks in which the flow of materials is from the suppliers (the parents) to the company or to the next supplier (the child). This means that even though we make this assumption regarding the propagation of disruptions, the results of this thesis are applicable to many supply chains. Because we do not consider propagation of disruptions from child nodes to parent nodes, our results on assessing the probability of disruption of the focal node might be lower compared to the results that could be obtained with a model that considers both directions of disruptions. This happens due to the fact that adding more sources of disruption to nodes makes it more probable that nodes fail.

The propagation of disruptions in the model needs to be assessed in another way too. We assume that if any of the parent nodes causes the disruption to propagate to its child node, the child node disrupts. This is basically a situation, where every supplier is a key supplier, and a company or a supplier cannot operate without every one of its suppliers. This is not the only way to model the propagation of disruptions. A disruption could move from parent nodes to the child node, for example, if 50% or all of the parent nodes cause a propagation of disruption to the child node.

The approach to disruption propagation used in this thesis is not applicable to all situations. For example, let us consider a grocery store which has a single supplier of milk that does not provide other products for the store. If the supplier of milk cannot provide milk for the grocery store, the store can still provide other products for the customer and thus is operational. With our approach, if this store cannot get milk from its supplier of milk, it cannot operate.

Our approach might provide more precise results when assessing supply networks, where all the suppliers' products are vital for the next supplier or the company. For example, this could be a supply network of a tea shop, which sells only one type of tea. There is a supplier of tea and a supplier of cups in the shop's supply network. If one of these suppliers cannot deliver products to the shop and causes a propagation of disruption for the shop, the shop cannot sell cups of tea to its customers.

As these two examples illustrate, the propagation of disruptions is dependent on the company's nature. In the future, the impact of correlation between suppliers to F_C should be studied with different structures of disruption propagation.

The sizes of supply networks are usually large (Käki et al., 2015). We have a small and artificial network in our model, and the correlation is measured between two nodes. In some supply networks, there may be correlations between more than just two nodes. In future, the impacts of correlations between more than two nodes could be studied.

For these earlier presented reasons, our conclusions are not applicable in all situations, and one should be careful when applying them into the practice.

5 Conclusions

This thesis concentrates on the supply networks and the correlation of disruptions in them. The main objective is to understand how the correlation of disruptions between two suppliers impacts the disruption probability of the company. We assume that correlation between suppliers implies the correlation of their disruptions. We apply Probabilistic Risk Assessment -approach, and construct a Bayesian network model, which is based on the model studied by Käki et al. (2015). The correlation is implemented to the model via conditional probabilities. After constructing the network model, Monte Carlo simulations are performed with different parameters to obtain results.

The results imply that correlated disruptions of two suppliers have an impact on the probability of disruption of the company with the following condition. When there are high probabilities that a possible disruption propagates from suppliers, which face correlated disruptions, to the next supplier in the network (parameter β), the higher the correlation is, the lower the disruption probability of the company is. If the parameter β is low, the correlation doesn't impact the disruption probability of the company.

This thesis can be extended in the future. The model's network structure can be improved by modifying the propagation of disruptions. For example, in a situation of many suppliers, the propagation of disruptions can be changed so that the propagation from only one of the suppliers does not disrupt the next supplier or the company. In this way, more realistic results could be gained in situations in which the company uses many suppliers to acquire certain goods or services. Another step in improving this model would be to allow the disruptions to propagate also from companies to suppliers. This change would better reflect the propagation of disruptions because, for example, a sudden drop in the demand for a supplier's produced goods or services (disruption in company or next supplier) can disrupt the supplier. In the future, the impact of the position and the number of correlated nodes in the supply network can also be studied to examine if these qualities impact the disruption probability of the company.

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