Aalto University
School of Science
Degree programme in Engineering Physics and Mathematics

Planning the positions of air surveillance radars using computational spatial decision analysis

Bachelor's thesis
12.12.2019

Tommi Anttila

Aalto University
School of Science

Aalto University, P.O. BOX 11000, 00076 AALTO
www.aalto.fi

Abstract of bachelor's thesis

Author Tommi Anttila

Title of thesis Planning the positions of air surveillance radars using computational spatial decision analysis

Degree programme Engineering Physics and Mathematics

Major Mathematics and Systems Analysis **Code of major** SCI3029

Supervisor Prof. Kai Virtanen

Thesis advisor(s) M.Sc. (Tech.) Mikko Harju

Date 12.12.2019 **Number of pages** 4+29 **Language** English

Abstract

This thesis presents two decision support concepts, dominance and potential optimality, and focuses on their computation. They are used in spatial decision analysis to compare decisions that associate consequences to locations in a spatial region. The objective of the comparison is to find the best alternative(s) for the decision maker.

The comparison between decision alternatives is affected by the decision maker's spatial preferences which are represented by weights describing the importance of the locations. Preference information is considered incomplete if the information available is not enough to express the preferences exactly. In this thesis, pairwise dominance and potential optimality of the alternatives are determined by solving linear optimization problems. The decision maker's preferences are represented as the constraints of the problems. The solutions provide information of the alternatives' ranking according to the decision maker's preferences that is used in decision support.

This thesis demonstrates the decision support concepts with an air surveillance planning example in which the decision maker chooses the positions for multiple radars. The surveillance performance for each alternative, i.e., each possible combination of position candidates, is simulated and measured by detection rate and tracking accuracy across the spatial region. Dominance and potential optimality are used eliminate worse alternatives according to the provided incomplete preference information.

The number of position alternatives is successfully reduced in the example using dominance. Potential optimality provided no information. The results of the comparisons between alternatives are highly dependent on the preference information and additional preferences would contribute to more detailed results. The computational implementation of the decision support concepts are generic and can be applied to other spatial decision problems.

Keywords spatial decision analysis, incomplete preference information, dominance, potential optimality, air surveillance

Aalto-yliopisto
Perustieteiden korkeakoulu

Aalto-yliopisto, PL 11000, 00076 AALTO
www.aalto.fi

Tekniikan kandidaatintyön tiivistelmä

Tekijä Tommi Anttila

Työn nimi Päätösvaihtoehtojen spatiaalinen vertailu ja sen sovellus ilmavalvonnan suunnittelussa

Koulutusohjelma Teknillinen fysiikka ja matematiikka

Pääaine Matematiikka ja systeemitieteet **Pääaineen koodi** SCI3029

Vastuuopettaja Prof. Kai Virtanen

Työn ohjaaja(t) DI Mikko Harju

Päivämäärä 12.12.2019 **Sivumäärä** 4+29 **Kieli** englanti

Tiivistelmä

Tässä kandidaatintyössä esitellään kahden päätösmenetelmän, dominanssin ja potentiaalisen optimaalisuuden, laskenta tietokoneella. Päätösmenetelmiä hyödynnetään spatiaalisessa päätösanalyysissa, jossa vertaillaan päätösvaihtoehtoja ja tavoitteena on parhaan vaihtoehdon löytäminen päätöksentekijän tavoitteet huomioiden. Päätösvaihtoehtojen vertailuun vaikuttavat päätöksentekijän spatiaaliset preferenssit, jotka kuvaavat alueen sijaintien tärkeyttä painokertoimien avulla. Preferenssi-informaatiota kutsutaan epätäydelliseksi, jos saatavilla oleva informaatio ei riitä painokertoimien täsmälliseen ilmaisuun.

Vaihtoehtojen vertailu pareittaisella dominanssilla ja potentiaalisella optimaalisuudella perustuu tässä työssä lineaaristen optimointitehtävien ratkaisemiseen. Päätöksentekijän preferenssi-informaatio esitetään optimointitehtävän rajoituksina, ja ratkaisut kertovat vaihtoehtojen keskinäisestä paremmuusjärjestyksestä. Saatua tietoa voidaan käyttää pääätöksenteon tukena.

Tämä kandidaantintyö demonstroi menetelmien käyttöä ilmavalvontatutkien suunnitteluesimerkillä, jossa päätöksentekijä valitsee sijainteja useille ilmavalvontatutkille. Päätösvaihehdot kuvaavat simuloitujen tutkien yhteisvaikutuksen kykyä havaita ja seurata lentäviä kohteita. Menetelmien avulla vaihtoehtoja rajataan eliminoimalla huonompia päätöksentekijän epätäydellisten preferenssien perusteella.

Ilmavalvontaesimerkissä dominanssin avulla päätösvaihtoehtojen määrää onnistutaan vähentämään, mutta potentiaalinen optimaalisuus ei rajaa vaihtoehtojen määrää enempää. Tulokset ovat vahvasti sidoksissa preferenssi-informaatioon, ja lisäinformaation avulla voitaisiin rajata vaihtoehtoja entisestään. Toteutetut vertailumenetelmät ovat sovellettavissa myös muihin spatiaalisiin päätöksenteko-ongelmiin sellaisinaan.

Avainsanat spatiaalinen päätösanalyysi, epätäydellinen preferenssi-informaatio, dominanssi, potentiaalinen optimaalisuus, ilmavalvonta

Contents

1 Introduction

Decisions sometimes have consequences associated with spatial or geographical regions. For instance, in air surveillance planning, deciding where to position radars affects the performance of the surveillance. Surveillance in general has several measures, such as the ability to detect and identify targets and to follow their movement. The decision maker has to consider the radar characteristics, such as the range and operational modes, in addition to the positioning of other radars in order to evaluate the performance of the surveillance. However, overall performance is not always enough, if the objective areas are not covered well enough. When considering the spatial preferences, the decision between possible radar position alternatives quickly becomes too complex to consider thoroughly and more analytical tools are required.

Similar multi-attribute decision problems appear in many other fields which have lead to different mathematical approaches to provide decision support (see, e.g., Keeney and Raiffa [1976]). Consequence value functions that value the decision's consequences are used to compare the decision alternatives with the objective of finding the best alternative(s). More recent studies have presented the decision analysis concepts in a spatial sense (see, e.g., Simon et al. [2013], Harju et al. [2019]).

The preference information in spatial decision analysis, provided by a decision maker (DM), is represented by weights associated with each attribute and spatial locations. The DM's preferences are elicited and converted into weights (see, e.g., Öztürké et al. [2005]). However, the DM is not always able to meaningfully provide complete information that could be represented as exact weights (see, e.g., Ferretti and Montibeller [2016]). This possibility of incomplete preference information is taken into account in some of the approaches (see, e.g., Kirkwood and Sarin [1985], Salo and Hämäläinen [2010]). By presenting the DM's preferences as weight constraints rather than exact weights, the comparison between alternatives becomes a matter of optimization (see, e.g., Punkka and Salo [2013]).

Harju et al. [2019] extend the spatial decision models by a thorough axiomatization and combine them with incomplete preference information. Their preference model provides the necessary tools to compare alternatives through pairwise dominance.

This thesis follows the Harju et al. [2019] preference model and applies it to regions that contain discrete locations. This thesis provides a computational implementation of dominance and potential optimality which are demonstrated using an air surveillance planning example in which the DM has to decide between radar configuration alternatives. Different radar attributes and surveillance objectives, such as detection and tracking, are considered with the DM's location and attribute specific spatial preferences to provide decision support by reducing the alternatives.

This thesis covers the basic principles of spatial decision analysis in section 2 and presents two key decision support concepts, namely dominance and potential optimality. In section 3, the DM's preference information is formalized as constraints and computational solutions to dominance and potential optimality are formulated. The concepts are demonstrated using an air surveillance planning example in section 4 in which the results and the computational properties of the methods used are discussed. Finally, section 5 concludes the thesis.

2 Introduction to spatial decision analysis

2.1 Prerequisites

In this thesis, decision alternatives are considered with respect to a spatial region denoted by S, which consists of the individual locations $s \in S$. The subsets $S' \in S$ of the region are referred to as subregions. The region can represent for example a two-dimensional geographical map or a three-dimensional airspace. Decision alternatives, denoted by $Z : S \to C$, assign a consequence $c \in C$, where C is the set of possible consequences, to each location $s \in S$.

For instance, consider deciding the positions of fire stations in a city, as presented in Honkasaari [2016]. The decision alternatives z of the fire station positions would assign each location s in the city a time value corresponding to the response time of the fire fighters. Suppose that a possible alternative z corresponds to building the fire station at coordinates (x, y). The response time would then be $z(s) = c\sqrt{(x - s_x)^2 + (y - s_y)^2} + b$, where c is a constant describing the average driving speed of the fire truck and b represents the constant alert time it takes to get moving. In this example, $z(s) \in C = [0, \infty[$ represents the response time but the consequences could as well be discrete descriptions of the damages caused by the fire, e.g., $C = \{$ "No damage", "Some damage", "Severe damage" $\}$.

The DM's preference between decision alternatives is represented by the preference relation

$$z \succeq z'. \tag{1}$$

The notation indicates weak preference which means that the alternative z is at least as preferable as z' to the DM (see, e.g., French [1986]). Other useful relations are defined as follows:

$$z \sim z' \Leftrightarrow z \succeq z' \text{ and } z' \succeq z, \tag{2}$$

$$z \succ z' \Leftrightarrow z \succeq z' \text{ and not } z' \succeq z. \tag{3}$$

The relation $\sim$ describes that the DM is indifferent between alternatives z and z', that is, she would be equally satisfied with either of the alternatives. Strict preference $\succ$ implies that the DM would always choose z over z'.

2.2 Spatial value function

The preference relation $\succeq$ provides a pairwise comparison between alternatives, which may be sufficient when the number of alternatives is small. However, if one were to solve a complete ranking for n alternatives, an order of n^2 comparisons would be needed. Value functions that assign scalar values to decision alternatives (see, e.g. Keeney and Raiffa [1976], French [1986]) provide a more convenient way of comparison. They also contribute to the credibility and transparency of the decision making process giving a mathematical form of comparison.

In this thesis, the value function is assumed to be

$$V(z) = \int_S v(z(s))d\alpha(s), \tag{4}$$

consisting of $v : C \to \mathbb{R}$, a consequence value function that assigns a value $v(c)$ to each consequence $c \in C$, and $\alpha : 2^S \to [0, \infty[$ where 2^S is the set of all possible subsets of S, a spatial weighting function that gives the spatial weight $\alpha(S')$ for each subregion $S' \subseteq S$ [Harju et al., 2019]. The spatial weights are interpreted to describe the importance of the subregions to the DM. However, the value function is only used for ordering the alternatives, rather than interpreting the exact values.

The weights are assumed to be non-negative and specified to be between 0 and 1 such that $\alpha(S) = 1$, i.e., the integral over the weights is equal to one. The spatial weighting function α is finitely additive, that is, the weight of the union of two non-overlapping areas S' and S'' is equal to the sum of their weights,

$$\alpha(S' \cup S'') = \alpha(S') + \alpha(S''), \text{ for all } S', S'' \in S, \text{ s.t. } S' \cap S'' = \emptyset. \quad (5)$$

The consequence function values $v(z)$ are scaled similarly such that $v(c) \in [0,1]$ for all $c \in C$. Harju et al. [2019] provides an axiomatization for the spatial value function, but require that S consists of an infinite number of locations.

In this thesis, the scope is limited to finite S ignoring the underlying preference assumptions. With finite S consisting of n locations, the locations are referred to using s_i where $i \in I = \{1, 2, ..., n\}$. Denote $a_i = \alpha(\{s_i\})$, the weight of the location s_i. The weight of a subregion S' is $\alpha(S') = \sum_{s \in S'} \alpha(\{s\})$. Similarly to (4), $a_i \in [0,1]$ for all $i \in I$ and $\sum_i^n a_i = 1$. The spatial value function is

$$V(z) = \sum_{i=1}^{n} a_i v(z(s_i)), \quad (6)$$

similarly as in Simon et al. [2013]. This is extended to multiple attributes by defining a spatial value function for each attribute $j \in J = \{1, 2, ..., m\}$ (see, e.g., Keisler and Sundell [1997]). The multi-attribute value function is defined as a weighted sum over the attribute-specific values, given by

$$V(z) = \sum_{j=1}^{m} b_j V_j(z_j) = \sum_{j=1}^{m} b_j \sum_{i=1}^{n} a_{ij} v_j(z_j(s_i)), \quad (7)$$

where b_j is the attribute weight, defined similarly to a_i in (6), such that $b_j \in [0,1]$ for all $j \in J$ and $\sum_j^m b_j = 1$. The spatial weight a_{ij} is dependent on the attribute j in addition to the location i and thus the spatial weights can be defined for each attribute separately. From now on, $V(z)$ is referred to as the multi-attribute version.

2.3 Incomplete preference information

Comparing alternatives using the spatial value function is convenient when the spatial weights are known exactly. However, this is often not the case. Describing one's spatial preferences in an exact manner is a difficult task especially when the number of locations is high. For instance, the DM may not be completely aware of every detail of the application, she may not be confident enough to weight the locations exactly, or there may be too many locations for her to consider.

Preference information is said to be incomplete when the information available is not sufficient to express the weights explicitly. Instead, decision alternatives are compared based on a set of feasible weights [Salo and Hämäläinen, 1992]. The true weights that represent the DM's exact preferences are an element of this set. The comparability of the decision alternatives is more limited. A complete order is not always obtained, but the preference information can narrow the set of interesting alternatives through pairwise dominance and potential optimality.

The weights are divided into two sets, ones considering the relation between attributes and others considering the spatial region. Attribute weights b_j are collected into a vector $b \in [0,1]^m$ and are considered separately from the spatial weights. The set of all possible attribute weights is

$$B^0 = \{\, b \in [0,1]^m \mid \sum_{j=1}^{m} b_j = 1\,\}. \tag{8}$$

The set B^0 serves as the base set for the attribute weights. The set of feasible attribute weights that satisfy the preference information available is a subset of B^0, denoted by $B \subseteq B^0$.

Similar definitions are made for the spatial weights a_{ij} which form a matrix a. For any given attribute j, the base set that contains all of the possible spatial weighting vectors $a_{\cdot j}$, i.e., the j:th column of a is

$$A^0 = \{\, a_{\cdot j} \in [0,1]^n \mid \sum_{i=1}^{n} a_{ij} = 1\}. \tag{9}$$

The set of feasible spatial weights that satisfy the preference information available with respect to attribute j is denoted by $A_j \subseteq A^0$. It is worth noting that the base set A^0 is the same for all attributes j.

The value of each alternative $V(z)$ is not uniquely defined since it depends on the weights a_{ij} and b. Thus, a different approach in comparing alternatives is required. The alternatives can be analysed using the concept of dominance (see e.g., Keeney and Raiffa [1976]) and classified as dominated and non-dominated alternatives. The previous are then excluded from further analysis as the latter are considered better in light of the preference information.

Decision alternative z dominates z', denoted by zDz', if

$$\begin{cases} V(z) \geq V(z'), & \text{for all} \quad a_{\cdot 1} \in A_1, ..., a_{\cdot m} \in A_m, b \in B \\ V(z) > V(z'), & \text{for some } a_{\cdot 1} \in A_1, ..., a_{\cdot m} \in A_m, b \in B \end{cases} . \tag{10}$$

Intuitively, dominance means that the dominating alternative obtains at least equally good values for all feasible weights but strictly better value(s) for at least some feasible weights. Due to the transitivity of the preference relation in (1), dominance in (10) is transitive as well. Obviously, the requirement of strict inequality for some weights implies asymmetry, that is, zDz' simultaneously with $z'Dz$ is not possible.

However, depending on the alternatives, comparing the alternatives using pairwise dominance may not be sufficient to provide reasonable decision support. The set of non-dominated alternatives can be too large or even include every alternative. Potential optimality can be used to narrow down the alternatives further. An alternative z' is considered potentially optimal in the spatial weights given by the set A, if

$$V(z') \geq V(z), \text{ for all } z \in Z \text{ for some } a_1 \in A_1, ..., a_m \in A_m, b \in B. \tag{11}$$

Potential optimal alternatives, i.e., those satisfying (11), achieve the highest value for some feasible weights. Potential optimality is a stronger condition than dominance but it is computationally more challenging for larger numbers of locations. However, potential optimal alternatives are normally also non-dominated and as a result, computing potential optimality for already non-dominated alternatives may save resources.

Using the definitions in 10 and 11, the decision alternatives can be compared without the requirement of exact preference information. The DM can state her spatial preferences as rankings rather than precise numbers. She can for instance rank the subregions based on their priority considering a given attribute. Such preference statements are more intuitive and credible to express, even for a DM that is not an expert in mathematics. There are different methods of preference elicitation (see, e.g., Salo and Hämäläinen [1992]) that transform the preferences in a mathematically usable form. The weights satisfying the DM's statements form the feasible sets A and B.

3 Computation of dominance and potential optimality

This section presents a computational approach to analyze the decision alternatives using dominance and potential optimality. The computations are formulated as linear programming problems (see, e.g., Athanassopoulos and Podinovski [1997]) that can be solved using various algorithms and software. Appendix A provides the problems in standard form that can be solved as is.

3.1 Set of feasible weights

The feasible weights are defined by preference statements given by the DM separately for attribute and spatial weights. The statements are formalized as linear constraints related to the weights which specify the feasible set of weights (see, e.g., Salo and Hämäläinen [1992]). Finally, the constraints are combined together into a more convenient form considering the computation of the optimization problems.

3.1.1 Attribute weights

When the DM has not given any preference information and there are no weight restrictions, the set of feasible attribute weights B is equal to the base set B^0. The preference information can be elicited from the DM using different methods. The preference statements of the form "attribute j_1 is at least k times more important than attribute j_2" is transformed into a linear constraint of the form

$$\sum_{j=1}^{m} q_j b_j \leq 0, \tag{12}$$

where q_j are the corresponding real-valued coefficients. The coefficients form a vector q, for instance, $q = \begin{bmatrix} -2 & 1 & 0 & \dots & 0 \end{bmatrix}$ corresponds to the inequality $2b_1 \geq b_2$, indicating that attribute 2 is at most twice as important as attribute 1 to the DM.

Multiple preference statements are collected into a $t \times j$ matrix, where t is the number of the statements, such that

$$Q = \begin{bmatrix} q^1 \\ \vdots \\ q^t \end{bmatrix}, \tag{13}$$

where $q^1, ..., q^t$ are coefficient vectors respective to each statement. The feasible set of attribute weights B is now defined explicitly. It is the set of weights that satisfy all the preference statements in (13), i.e.,

$$B = \{\, b \in B^0 \mid Qb \leq \bar{0} \,\}. \tag{14}$$

3.1.2 Spatial weights

The DM's spatial preference statements correspond to spatial weight inequalities for a given attribute $j \in J$. The spatial weight of a subregion is the sum of the location weights,

$$\alpha_j(S') = \sum_{i \in I \,|\, s_i \in S'} a_{ij}. \tag{15}$$

Consider that the DM states that she finds subregion S' at least ℓ times as important as S'' regarding the j:th attribute. The statement corresponds to the inequality $\alpha_j(S') \geq \ell \alpha_j(S'')$, i.e., $\sum_{i \in I \,|\, s_i \in S'} a_{ij} \geq \ell \sum_{i \in I \,|\, s_i \in S''} a_{ij}$ The spatial constraints are defined similarly to (12), by

$$\sum_{i=1}^{n} p_i^j a_{ij} \leq 0, \tag{16}$$

where p_i^j are the real-valued spatial coefficients concerning attribute j.

The spatial preference statements and the corresponding coefficients are collected into a $r_j \times n$ matrix P^j for attribute j, where r_j is the number of statements regarding the attribute, similarly to the attribute coefficient matrix Q.

As in (14), the feasible set of spatial weights with respect to the j:th attribute is given by

$$A_j = \{\, a \in A^0 \mid P^j a \leq \bar{0} \,\}. \tag{17}$$

3.1.3 Combining the weights

In order to greatly simplify the upcoming computation of dominance and potential optimality, an auxiliary weight combining the attribute and spatial weights, such that $w_{ij} = b_j a_{ij}$, is introduced. Notation for the weights w_{ij} is analogous to a_{ij}. The matrix containing w_{ij} is referred to as w. The multi-attribute spatial value function (7) in terms of w_{ij} is

$$\begin{aligned} V(z) &= \sum_{j=1}^{m} b_j \sum_{i=1}^{n} a_{ij} v_j(z_j(s_i)) \\ &= \sum_{j=1}^{m} \sum_{i=1}^{n} b_j a_{ij} v_j(z_j(s_i)) \\ &= \sum_{j=1}^{m} \sum_{i=1}^{n} w_{ij} v_j(z_j(s_i)). \end{aligned} \tag{18}$$

As seen in (18), the value function $V(z)$ is now linear in terms of the weights w_{ij}. Since the sum of the spatial weights is equal to 1 for each attribute j, the attribute weights b_j are obtained as the sum of the auxiliary weights over the region

$$\sum_{i=1}^{n} w_{ij} = \sum_{i=1}^{n} b_j a_{ij} = b_j \sum_{i=1}^{n} a_{ij} = b_j. \tag{19}$$

Using this definition, the spatial weights are calculated by normalizing with b_j, such that $a_{ij} = w_{ij}/b_j$ when $b_j \neq 0$. The case $b_j = 0$ is trivial, as the total weight $\sum_{i \in I} w_{ij} = 0$. It follows that the auxiliary weights w_{ij} sum to 1. The set of possible weights W^0 is defined similarly to A^0 and B^0, such that

$$W^0 = \{\, w \in [0,1]^{n \times m} \mid \sum_{i=0}^{n} \sum_{j=0}^{m} w_{ij} = 1 \,\} \tag{20}$$

The preference statements considering both attributes and locations are presented in terms of w_{ij}. The attribute preference statements $\phi \in \{1, 2, ..., t\}$ in (12) are now of form

$$\sum_{j=1}^{m} q_{\phi j} b_j = \sum_{j=1}^{m} q_{\phi j} \sum_{i=1}^{n} w_{ij} = \sum_{i=1}^{n} \sum_{j=1}^{m} q_{\phi j} w_{ij} \leq 0 \tag{21}$$

Similarly, the spatial preference statements $\pi \in \{1, 2, ..., r_j\}$ considering attribute j are of form

$$\sum_{i=1}^{n} P_{\pi i}^{j} a_{ij} = \sum_{i=1}^{n} P_{\pi i}^{j} \frac{w_{ij}}{b_j} = \sum_{i=1}^{n} P_{\pi i}^{j} w_{ij} \leq 0. \tag{22}$$

Since the original constraints derived from the spatial and attribute specific statements are unchanged, the feasible weights satisfy every constraint. Considering that the auxiliary weights sum to 1 it follows that the spatial weights a_{ij} and attribute weights b_j obtained from the weights w_{ij} also sum to 1, respectively.

3.2 Dominance

In this thesis, the dominance between alternatives is determined by posing the dominance condition in (10) as an optimization problem with the preference information applied as constraints.

Specifically, the dominance between alternatives z and z' is defined using the following definitions (23)-(28).

$$\min_{w \in \mathbb{R}^{n \times m}} \sum_{i=1}^{n} \sum_{j=1}^{m} w_{ij}(v_j(z_j(s_i)) - v_j(z_j'(s_i))) \tag{23}$$

$$\max_{w \in \mathbb{R}^{n \times m}} \sum_{i=1}^{n} \sum_{j=1}^{m} w_{ij}(v_j(z_j(s_i)) - v_j(z_j'(s_i))) \tag{24}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} \sum_{j=1}^{m} q_{\phi i} w_{ij} \leq 0, \text{ for all } \phi \in \{\, 1, 2, ..., t \,\} \tag{25}$$

$$\sum_{i=1}^{n} P_{\pi i}^{j} w_{ij} \leq 0, \text{ for all } \pi \in \{\, 1, 2, ..., r_j \,\}, \text{ for all } j \in J \tag{26}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} w_{ij} = 1 \tag{27}$$

$$w_{ij} \geq 0 \tag{28}$$

If a non-negative solution is found to minimization in (23) and a positive solution is found to maximization in (24), z dominates z'. The constraints in

(25)-(28) are used in both problems. Constraints (25) and (26) correspond to the preference statements, (27) provides that the weights w sum to 1 and (28) assures that the weights are positive as defined. An important note is that both of the objective functions and the constraints are linear with respect to the weights w_{ij}. Without the auxiliary weights the problem would be non-linear and the solution would be considerably more difficult to find.

The number of variables grows fast if more locations are presented resulting in a potentially challenging problem to solve computationally. However, if the DM's spatial preferences are stated in terms of subregions S' consisting of several locations, the variables respective to these locations are restricted identically. The minimum and maximum values are obtained when the spatial weight of the subregion is concentrated in a single location [Harju et al., 2019]. For example for a single attribute j, assume that the sum of the weights w_{ij} in a subregion S' is a constant $\sum_{i\in I^*} w_{ij} = c$, where $I^* = \{\, i \in I \mid s_i \in S' \,\}$. Denoting $W^* = \{\, w \in W \mid \sum_{i\in I^*} w_{ij} = c \,\}$ and using the linearity of the problem, it follows that

$$\begin{aligned} &\min_{w\in W^*} \sum_{i\in I^*} w_{ij} V_j(z_j(s_i)) - V_j(z_j(s_i)) \\ =&c \min_{i\in I^*} V_j(z_j(s_i)) - V_j(z_j(s_i)). \end{aligned} \tag{29}$$

As a result, the variables w_{ij} representing the locations in S' reduced to a single variable.

Provided the locations can be divided into a smaller amount of subregions, the optimization problem reduces drastically in complexity making the computation significantly faster. As a result, increasing the amount of locations in a subregion, e.g., increasing the resolution of a map, has little effect on the computational complexity of determining the non-dominated alternatives. However, the complexity is highly dependent on the number of the alternatives as the amount of pairwise comparisons grows quadratically with respect to the number of alternatives.

3.3 Potential optimality

Potential optimality requires an alternative to achieve a value at least as good as that of all of the other alternatives. As a result, the condition can be formulated as constraints as the difference between the alternative in question and every other alternative.

The alternative z' is potentially optimal, if a solution to

$$\min_{w \in \mathbb{R}^{n \times m}} k \tag{30}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} \sum_{j=1}^{m} w_{ij}(v_j(z_j(s_i)) - v_j(z'_j(s_i))) \leq 0, \text{ for all } z \in Z \tag{31}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} q_{\phi i} w_{ij} \leq 0, \text{ for all } \phi \in \{1, 2, ..., t\} \tag{32}$$

$$\sum_{i=1}^{n} P_{\pi i}^{j} w_{ij} \leq 0, \text{ for all } \pi \in \{1, 2, ..., r_j\}, \text{ for all } j \in J \tag{33}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} w_{ij} = 1 \tag{34}$$

$$w_{ij} \geq 0 \tag{35}$$

exists. The objective function k is a constant as any feasible solution suffices. The condition in (31) states that the alternative z' achieves at least as good values $V(z)$ as every other alternative. Conditions (32)-(35) contain the DM's preference information and the weight requirements, defined similarly as in the case of dominance in (23)-(28).

Provided that the alternative's concequences are defined continuously and are not constant within a subregion, the linear programming problem consists of nm variables that cannot be concentrated. This is often the case if the concequences depend, say, on the distance of a location from a fixed point, e.g., a radar. In this case, the complexity of the problem is highly dependent on the number of locations and attributes. However, if there are equal concequences within a subregion, e.g., concequences are defined discretely, similar simplifications can be used as in the case of dominance.

4 Air surveillance application

This section demonstrates the use of dominance and potential optimality in decision support. The section presents an air surveillance example with the goal of providing recommendations for radar positions from a group of possible locations. Planning the positions plays a key role, e.g., in civilian air traffic control and military applications. The decisions are faced, for instance, in planning air surveillance networks that consist of the complete system of radars, tracking software and human operators with the goal of providing accurate information to aid the DMs in traffic control and defence.

The radar characteristics are simulated using a specifically designed tool that outputs the consequences, that is, the radar performance, in each location. The values and the previously posed optimization problems are solved in MATLAB.

4.1 Scenario

The air force of a fictional country is planning the positions of their air surveillance equipment in order to maximize coverage and surveillance quality in their airspace. The planner in charge (DM) has preselected a group of positions that fulfill the necessary geographical requirements for a radar.

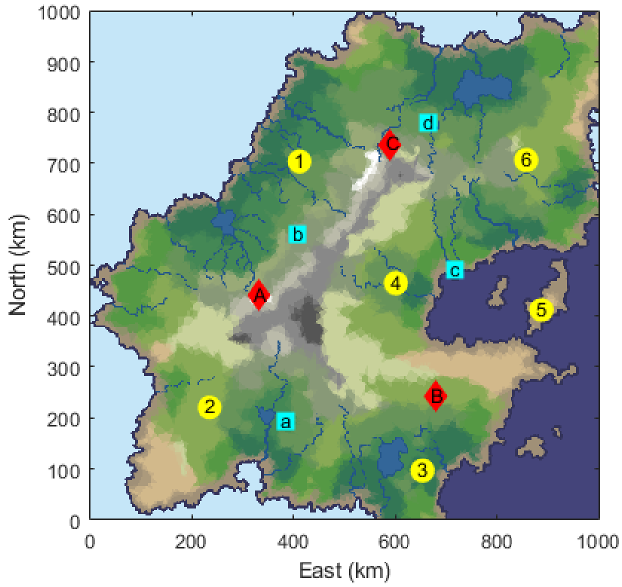

Figure 1: The map of the country. Distances are in kilometers. Position candidates for short range radars are represented with circles, medium range radar with squares and long range radar with diamonds.

The country has three different radar types in use and each of them has their own positional requirements and thus different position candidates presented in Fig. 1. The country has three short range radars available and the DM has selected six position candidates for the short range radars. There are four position candidates and two units available for the medium range radars. The country has only one long range radar and the DM has pointed out three different position candidates for it.

4.2 Decision alternatives

The performance of the radars is simulated using a computational tool that represents realistic radar models. The tool takes into account the radar properties, operational modes and stochastic elements encountered in reality. The main radar properties in this thesis are the effective range and the rotation time which describe the distance and the frequency of the observations the radar provides. The properties are presented in Table 1.

Table 1: Radar properties

Radar range	Effective range (km)	Rotation time (s)	Units available
Short	100	4	3
Medium	300	8	2
Long	500	8	1

Short range radars turn faster and thus detect targets more frequently. The medium and long range radars have greater effective ranges but rotate slower. The different operational modes contribute to the differences between radars, but are not discussed here. Since the units available and the position candidates are different for each radar type, the decision alternatives correspond to the combinations of positions. The number of alternatives in the scenario is $\binom{6}{3} \cdot \binom{4}{2} \cdot \binom{3}{1} = 360$. The radar configurations, that is, alternatives, are referred to according to the labels used in Fig. 1. For example, configuration Aab123 has a long range radar at A, medium range radars at a and b, and short range radars at 1, 2 and 3.

4.3 Attributes

The radar performance is measured in two attributes, detection rate $z_d(s)$ and tracking capability $z_t(s)$ at a given location. The performance is considered in three dimensions, but in this thesis the scope is limited to a specific altitude reducing the region to two dimensions.

The visibility of a target is dependent of its flight altitude as the ground curvature limits the minimum visible altitude. In this example, each of the radar types is installed to 15m height and the surrounding areas provide clear line of sight to every direction which corresponds to a radar horizon of approximately 14km in ground distance. The minimum altitude beyond the radar horizon grows quickly in terms of distance. Potential enemy aircraft are assumed to fly at an altitude of 4km (13100ft) to which the line of sight is clear up a distance of 240km. The computational tool takes diffraction into account resulting in slight improvements in detection beyond the direct visibility, but decreases rapidly to zero.

Each radar assigns a probability of detection to each location $s \in S$ that is obtained using the computational tool. The detection rate $z_d(s)$ for a single radar is obtained by dividing the probability of detection by the time it takes to rotate. The detection rate $z_d(s)$ describes the number of detections per second and is independent of the other radars, that is, the detection rate cumulates when more radars are added.

Tracking accuracy $z_t(s)$ is represented by three-dimensional accuracy of the observations consisting of horizontal and vertical accuracy obtained for each $s \in S$ using the computational tool. The three-dimensional accuracy is calculated as a weighted geometrical mean of the two components. The $z_t(s)$ values of the tool range from 0 to around 10000 with smaller values representing better tracking ability. In this thesis, the highest value is limited to 2000, as anything beyond that is considered as no tracking capability at all.

The attributes of alternative Aab123 are represented in Figures 2 and 3.

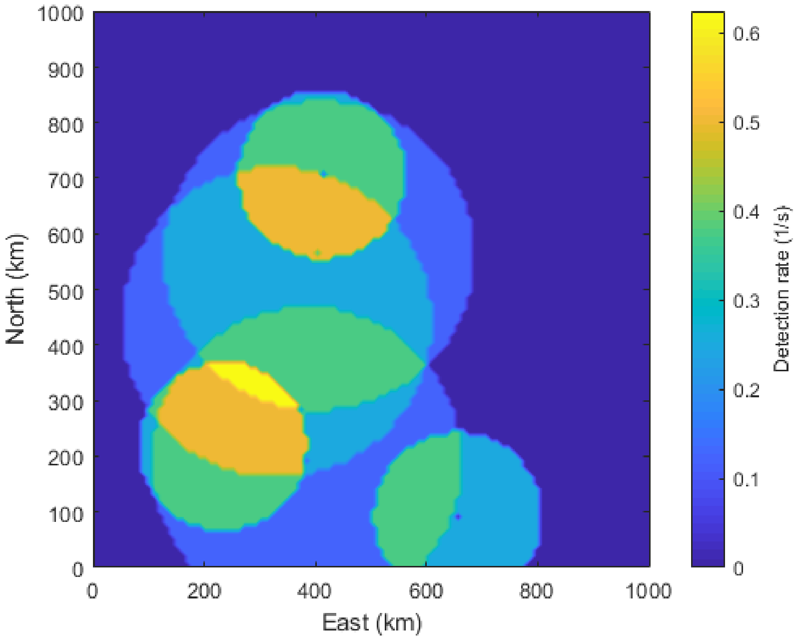

Figure 2: Detection rate of Aab123.

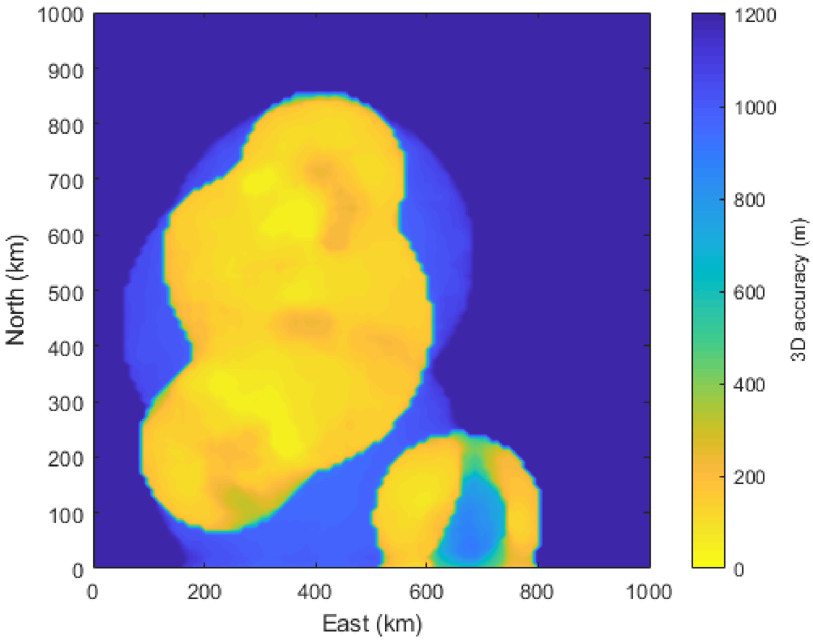

Figure 3: Tracking accuracy of Aab123.

4.4 Consequence value function

Both attributes, detection rate $z_d(s)$ and tracking $z_t(s)$, are valued using additive value functions $v_d(z)$ and $v_t(z)$, respectively. The change in $v_d(s)$ from higher detection rates is not linear. A small detection rate is considered better than no detections at all in terms of coverage. Similarly, a high detection rate is not greatly better than a little lower rate. The consequence value function for detections is given by $v_d(z) = 1 - e^{-4z_d(s)}$. Improvements in detection rate beyond 1 per second are considered to be marginal and thus the exponential coefficient 4. The function $v_d(z)$ is represented in Fig. 4.

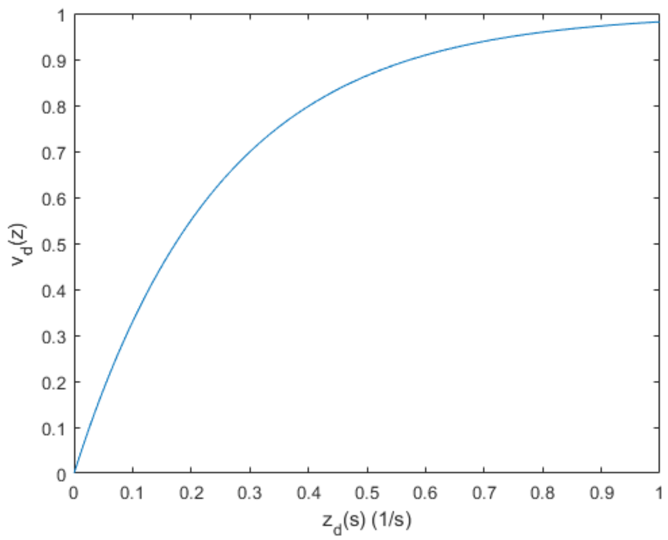

Figure 4: $v_d(z)$ in terms of detection rate $z_d(s)$.

Tracking accuracy is valued using a piecewise linear function that represent the DM's assessment on tracking importance. The slope of $v_t(s)$ is steeper the lower the $z_t(s)$ values are. Opposite to $v_d(s)$, the gains in $v_t(s)$ grow as the tracking accuracy improves, seen in Fig. 5.

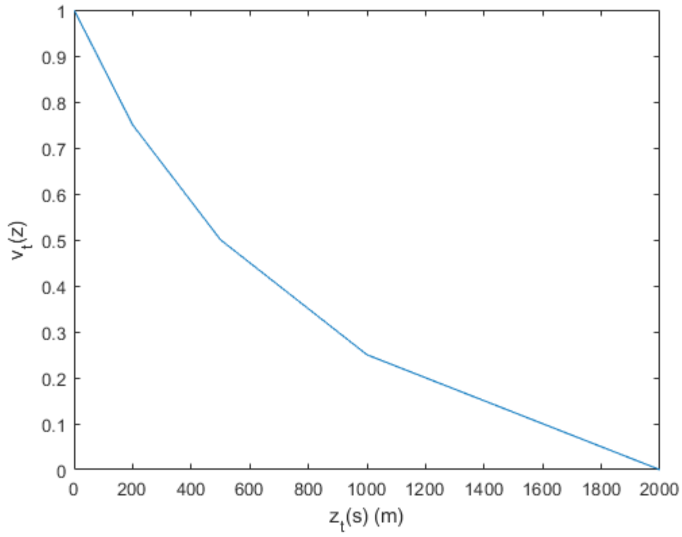

Figure 5: $v_t(z)$ in terms of tracking accuracy $z_t(s)$.

4.5 Preference information

The DM presents her spatial preferences considering geographical areas. The preference statements of form (16) rank the areas in question by their importance. The country is divided into six areas, namely Border, East, Inland, South, West and North areas. The areas numbered from 1 to 6 in the previous order are presented in Fig. 6 along with four main cities named A, B, C and D.

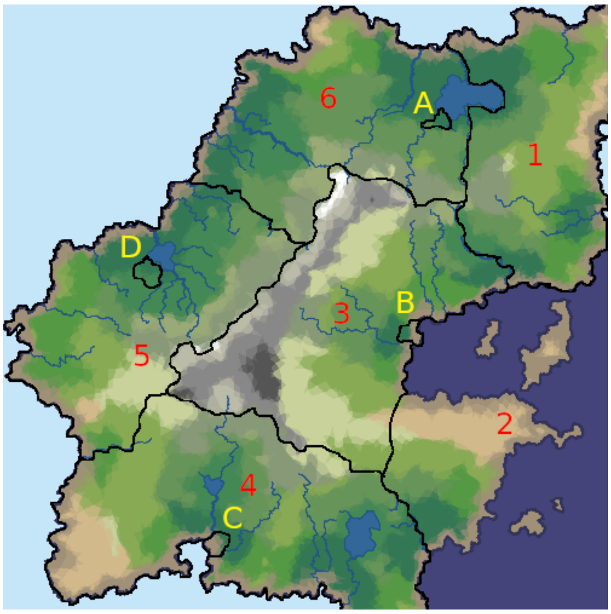

Figure 6: Map division into 6 areas with the largest cities A to D.

The country neighbored by two other countries, neighbors 1 and 2. Neighbor 1 is located in the East and it shares some land border in the region 1. Neighbor 1 is a powerful country with large land areas continuing across the sea. The relations between this country have been controversial and is considered a more likely threat. The other one, neighbor 2, is located mainly in the west, with whom the DM's country shares substantial share of the land border, is considered an ally.

The DM states that detection capability is preferred near borders, Eastern border and sea especially, and the preference gradually decreases towards inland. She ranks the areas by importance, such that $\alpha(S^1) \geq \alpha(S^2) \geq \alpha(S^6) \geq \alpha(S^4) \geq \alpha(S^5) \geq \alpha(S^3)$. Specifically,

$$x\alpha(S^{i+1}) \geq \alpha(S^i) \geq y\alpha(S^{i+1}), \tag{36}$$

where i corresponds to the indices of the previous ranking and $x = 3$ and $y = 2$. For instance, the border area is 2 to 3 times as important as the East area. The cities ABCD are considered to be a part of their respective areas.

Spatial preferences of tracking are ranked in a similar manner using (36). The DM states that the cities are considered separately, $\alpha(S^{ABCD}) = \alpha(S^A \cup S^B \cup S^C \cup S^D)$. The order is $\alpha(S^6) \geq \alpha(S^{ABCD}) \geq \alpha(S^1) \geq \alpha(S^3) \geq \alpha(S^4) \geq \alpha(S^2) \geq \alpha(S^5)$, with $x = 3$ and $y = 2$. The spatial preferences are collected in Table 2.

Table 2: DM's spatial preferences. The order of importance is presented by attribute and the exact statements are of the form (36), where i corresponds to the indices of the given order. Area ABCD represents the combined locations of the cities A, B, C and D.

Attribute	Area order
Detection	1, 2, 6, 4, 5, 3
Tracking	6, ABCD, 1, 3, 4, 2, 5

The DM divides the areas into Fig. 6 to 20 subregions to provide more detailed preference information, presented in Fig. 7.

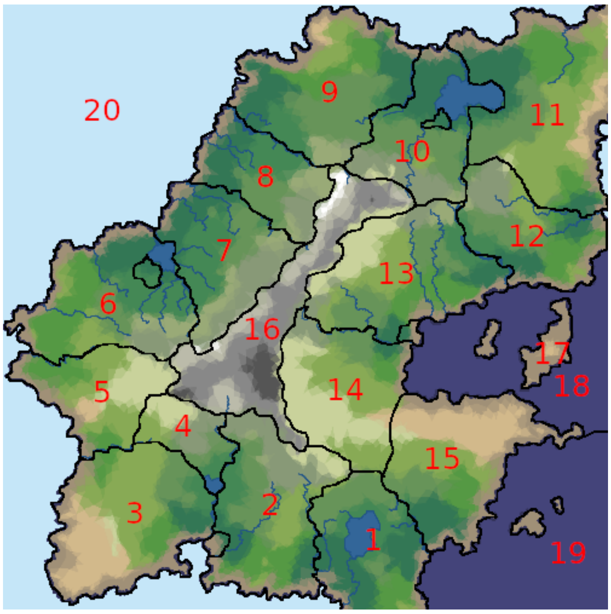

Figure 7: Regions divided into 20 subregions. Note that subregion 17 is disjoint containing all of the islands in regions 18 and 19. The subregion 20 consisting of the areas outside of the country is considered to be the least important for both attributes.

The subregions are ordered by their importance within the region they are in. The DM has assessed the subregions in a similar fashion using (36), with coefficients $x = 2$ and $y = 1.5$. The ranking is presented on Tables 3 and 4 for detection and tracking, respectively.

Table 3: DM's spatial preferences concerning detection in the subregions.

Area	Subregion ranking
1	11, 12
2	18, 19, 17, 15
3	13, 14, 16
4	1, 2, 3, 4
5	5, 7, 6
6	10, 9, 8

Table 4: DM's spatial preferences concerning tracking in the subregions.

Area	Subregion ranking
ABCD	A, B, C, D
1	11, 12
2	15, 17, 18, 19
3	13, 14, 16
4	2, 1, 3, 4
5	6, 5, 7
6	10, 9, 8

Spatial information taken into account, the DM also states that detection is considered the more important attribute, such that it has $60\% - 75\%$ of the total weight. Consequently, tracking is assessed to be between $25\% - 40\%$ of the total weight.

Since the subregions are rather coarse compared to the coverage of the radar configurations, seen for example in Fig. 4 and 5, the DM is asked to further specify the regions. She divides the subregions into two to six districts by their local importance, i.e., population and infrastructure of the area, presented in Fig. 8.

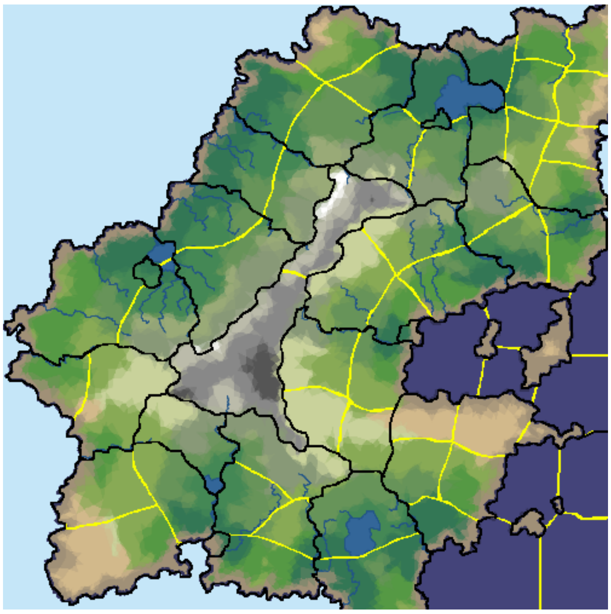

Figure 8: Subregions divided to local districts. The islands in the Southern sea area are considered together. The other two are considered separately. The total number of areas is 70.

4.6 Results

Computing the pairwise dominance for each decision alternative with the other alternatives results in 67 non-dominated (ND) alternatives, presented in Table 5 and Fig. 9.

Table 5: Occurrances of each radar position candidate in the ND alternatives.

Position	Occurrances
A	4
B	3
C	60
a	20
b	24
c	24
d	67
1	34
2	32
3	32
4	33
5	33
6	37

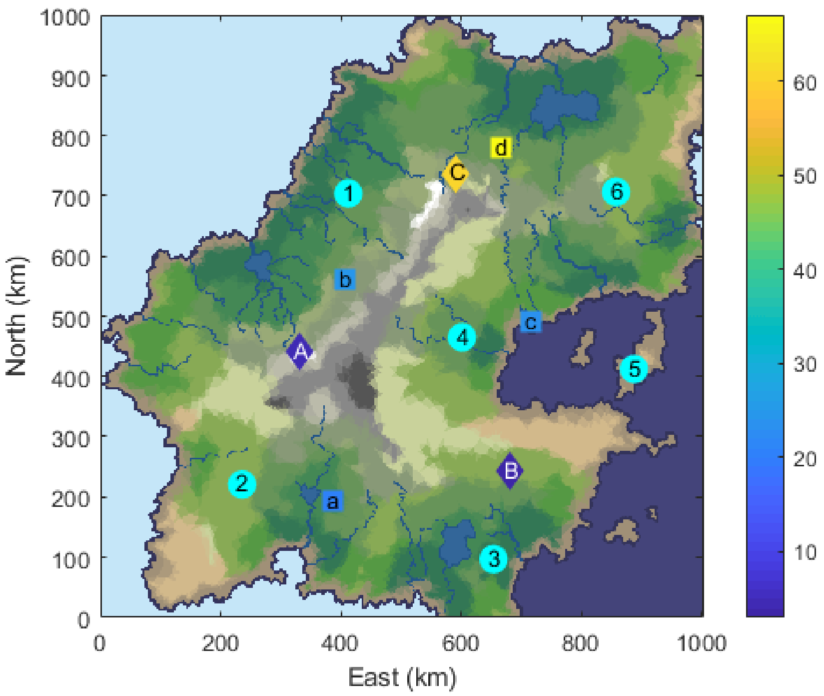

Figure 9: Occurrances of radar position candidates.

The medium-range radar location d is included in every ND alternative. As a result of the strict inequality in (10), a medium-range radar should be placed there in every case. The other locations can not be confirmed using this result even though some candidates occur more often than others. For instance, location C occurs in most ND alternatives, but since the dominance does not provide information of the mutual ordering of the ND alternatives, the alternatives with locations A and B are considered equally good.

Potential optimality calculated for the ND alternatives result in 67 potentially optimal (PO) alternatives that are exactly the same as the ND alternatives, presented in Table 5 and Fig. 9.

Using dominance to compare the possible radar position alternatives successfully eliminated 293 out of 360 alternatives. However, the remaining alternatives have to be assessed by the DM for which 67 alternatives are many to consider. The short-range radars occur in the ND alternatives more evenly distributed than the other radar types. Considering the rather coarse area division in Fig. 8 in comparison with the short-range radar effective range of 100 km, the radars cover relatively small number of regions. This suggests that differences between these radars are less likely occur. The radars are also scattered around the map while the DM's preferences focus on fewer areas, especially near the Eastern border. That said, only short-range position candidates near these more important areas are 5 and 6, the latter of which occurs the most in the ND alternatives.

The concequences of the considered attributes, detection rate and tracking accuracy, change suddenly as seen in Fig. 2 and Fig. 3. When the effective range is reached, or the minimum flight altitude becomes too high due the radar horizon, both attribute values drop drastically. The circle radii are visible in both figures. The rather coarse area division considered, the values achieved within a single area can differ from very good to very bad. Since the computation of the pairwise dominance is closely related to the value differences within the areas, quickly changing consequences may contribute to the relatively high number of ND alternatives.

The computation of the ND alternatives in this example took 560 seconds on a desktop computer. The computation depends heavily on the preference information given by the DM, since dominated alternatives are eliminated as they are found lowering the remaining pairwise comparisons. Computation of the PO alternatives took 260 seconds. However, the complexity of potential optimality depends heavily on the total number of alternatives and here it was only computed for already ND alternatives.

5 Conclusion

This thesis provided a computational implementation of dominance and potential optimality for comparing decision alternatives in spatial problems while taking the decision maker's possibly incomplete preferences into account. Comparison between alternatives is based on dominance and potential optimality which are determined using linear optimization. The feasible region is constrained by the DM's preferences which are assumed to be linear. Finally, this thesis demonstrated the computational implementation to provide decision support in an example deciding the positions of air surveillance radars.

The resulting number of non-dominated alternatives depends heavily on the preference information. The preference information used in the example considered subregions divided hierarchically in three layers. As a result, dominance narrowed down the number of alternatives to one-sixth of the original number while potential optimality provided no useful information in this case. One position turned out to occur in every non-dominated alternative suggesting it should be selected. Even though some information is provided, the DM is left with many alternatives to consider.

The implementation is generic and can be applied to various spatial decision problems with various types of preference information. In order for the optimization to be linear, the constraints were assumed to also be linear. Since dominance and potential optimality are general comparisons of the alternative's values, linearity is not strictly enforced. However, the optimization can become significantly more challenging with more complex constraints. Furthermore, as the constraints represent the DM's preferences, more complex preferences might not be natural for humans to express.

The air surveillance example considered two measures of performance, detection rate and tracking capability of aircraft in the airspace. In the example it is assumed that all aircraft to fly at a same altitude which resulted in a two-dimensional region. This assumption is rather unrealistic but simplifies the computation greatly, since otherwise different altitudes would have to be considered in every location. Multiple altitudes could be taken into account, for example, by considering attributes at two-dimensional regions for each altitude. This approach essentially doubles the number of the attributes. The DM would have to assess the mutual importance of altitudes but the spatial preferences could be different at each altitude. A few well chosen altitudes could end up with more accurate results without costing too much computationally.

References

A. Athanassopoulos and V. Podinovski. Dominance and potential optimality in multiple criteria decision analysis with imprecise information. *Journal of the Operational research Society*, 48(2):142–150, 1997.

V. Ferretti and G. Montibeller. Key challenges and meta-choices in designing and applying multi-criteria spatial decision support systems. *Decision Support Systems*, 84(1):41–52, 2016.

S. French. *Decision Theory: An Introduction to the Mathematics of Rationality.* Halsted Press, 1986.

M. Harju, J. Liesiö, and K. Virtanen. Spatial multi-attribute decision analysis: Axiomatic foundations and incomplete preference information. *European Journal of Operational Research*, 275(1):167–181, 2019.

M. Honkasaari. *Computation of Non-Dominated Alternatives in Spatial Decision Analysis.* Special assignment, Aalto University, 2016.

R.L. Keeney and H. Raiffa. *Decisions with Multiple Objectives: Preferences and Value Tradeoffs.* John Wiley and Sons, 1976.

J. Keisler and R. Sundell. Combining multi-attribute utility and geographic information for boundary decisions: an application to park planning. *Journal of Geographic Information and Decision Analysis*, pages 101–118, 1997.

C. Kirkwood and R. Sarin. Ranking with partial information: A method and an application. *Operations Research*, 33(1):38–48, 1985.

M. Öztürké, A. Tsoukiàs, and P. Vincke. Preference modelling. In *Multiple criteria decision analysis: State of the art surveys*, pages 27–59. Springer, 2005.

A. Punkka and A. Salo. Preference programming with incomplete ordinal information. *European Journal of Operational Research*, 231(1):141–150, 2013.

A. Salo and R.P. Hämäläinen. Preference assessment by imprecise ratio statements. *Operations Research*, 40(6):1053–1061, 1992.

A. Salo and R.P. Hämäläinen. Preference programming–multicriteria weighting models under incomplete information. In *Handbook of multicriteria analysis*, pages 167–187. Springer, 2010.

J. Simon, C.W. Kirkwood, and L.R. Keller. Decision analysis with geographically varying outcomes: Preference models and illustrative applications. *Operations Research*, 62(1):182–194, 2013.

A Appendix A

The matrix w is rearranged as a vector g, such that $g(i + (j-1)n) = w_{ij}$. The set of possible weights is denoted here by G^0,

$$G^0 = \{\, g \in [0,1]^{nm} \mid \sum_{j \in J,\, i \in I} g(i + (j-1)n) = 1 \,\}. \tag{37}$$

The attribute-specific spatial constraints P_j are combined diagonally to a matrix

$$P^* = \begin{bmatrix} P_1 & & 0 \\ & \ddots & \\ 0 & & P_m \end{bmatrix}. \tag{38}$$

The attribute constraints are joined into a $t \times nm$ matrix Q^*. By indexing the attribute statements using $\phi \in \{1, ..., t\}$, each matrix P_j in P^* correspond to a vector of n attribute weights given by the preference statement ϕ. Explicitly,

$$Q^*(\phi, m(j-1) + k) = Q(\phi, j), k = \{1, ..., n\}. \tag{39}$$

Both nm wide matrices are stacked together into the final constraint matrix

$$X = \begin{bmatrix} P^* \\ Q^* \end{bmatrix}. \tag{40}$$

The set of feasible weights w is given as a subset of the set of all feasible weights G^0, such that

$$G = \{\, g \in G^0 \,|\, Xg \leq \bar{0} \,\}. \tag{41}$$

Assuming the value functions are known and the consequence values can be calculated, the consequences' values are arranged into vectors

$$f_j = [v_j(s_1) \; \ldots \; v_j(s_n)] \tag{42}$$

and the vectors F_j into a vector

$$F^z = [f_1 \; \ldots \; f_m], \tag{43}$$

that represents the values of the alternative $z \in Z$.

Now, dominance between two alternatives z and z' is determined by solving

$$\min_{g \in G^0} (F^z - F^{z'})g \tag{44}$$

$$\max_{g \in G^0} (F^z - F^{z'})g \tag{45}$$

$$\text{s.t. } Xg \leq 0. \tag{46}$$

$$\sum g = 1, \tag{47}$$

where I is an identity matrix of the appropriate size.

Finally, the constraints in potential optimality are given by

$$Y = \begin{bmatrix} F^z - F^{z^1} \\ \vdots \\ F^z - F^{z^\ell} \\ X \end{bmatrix}, \tag{48}$$

where $z^1, ..., z^\ell$ represent the decision alternatives.

Potential optimality is determined by solving

$$\min_{g \in G^0} k \tag{49}$$

$$\text{s.t. } Yg \leq 0 \tag{50}$$

$$\sum g = 1. \tag{51}$$