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OPTIMAL PILOT DECISIONS AND FLIGHT TRAJECTORIES IN AIR COMBAT

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Abstract: The thesis concerns the analysis and synthesis of pilot decision-making and the design of optimal flight trajectories. In the synthesis framework, the methodology of influence diagrams is applied for modeling and simulating the maneuvering decision process of the pilot in one-on-one air combat. The influence diagram representations describing the maneuvering decision in a one sided optimization setting and in a game setting are constructed. The synthesis of team decision-making in a multiplayer air combat is tackled by formulating a decision theoretical information prioritization approach based on a value function and interval analysis. It gives the team optimal sequence of tactical data that is transmitted between cooperating air units for improving the situation awareness of the friendly pilots in the best possible way. In the optimal trajectory planning framework, an approach towards the interactive automated solution of deterministic aircraft trajectory optimization problems is presented. It offers design principles for a trajectory optimization software that can be operated automatically by a nonexpert user. In addition, the representation of preferences and uncertainties in trajectory optimization is considered by developing a multistage influence diagram that describes a series of the maneuvering decisions in a one-on-one air combat setting. This influence diagram representation as well as the synthesis elaborations provide seminal ways to treat uncertainties in air combat modeling. The work on influence diagrams can also be seen as the extension of the methodology to dynamically evolving decision situations involving possibly multiple actors with conflicting objectives. From the practical point of view, all the synthesis models can be utilized in decision-making systems of air combat simulators. The information prioritization approach can also be implemented in an onboard data link system.

Keywords: air combat, decision analysis, dynamic decision-making, game theory, influence diagram, nonlinear programming, optimal control, simulation, team decision theory, uncertainty

Academic dissertation

Systems Analysis Laboratory
Helsinki University of Technology

Optimal pilot decisions and flight trajectories in air combat

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Publications

The dissertation consists of the present summary article and the following papers:

[I] Virtanen, K., Raivio, T., and Hämäläinen, R.P., "Decision Theoretical Approach to Pilot Simulation," *Journal of Aircraft*, Vol. 36, No. 4, 1999.

[II] Virtanen, K., Raivio, T., and Hämäläinen, R.P., "Influence Diagram Modeling of Decision Making in a Dynamic Game Setting," *Proceedings of the 1st Bayesian Modeling Applications Workshop of the 19th Conference on Uncertainty in Artificial Intelligence*, 2003.

[III] Virtanen, K., Hämäläinen, R.P., and Mattila, V., "Team Optimal Signaling Strategies in Air Combat," *IEEE Transaction on Systems, Man, and Cybernetics - Part A: Systems and Humans*, accepted for publication.

[IV] Virtanen, K., Ehtamo, H., Raivio, T., and Hämäläinen, R.P., "VIATO – Visual Interactive Aircraft Trajectory Optimization," *IEEE Transaction on Systems, Man, and Cybernetics – Part C: Applications and Reviews*, Vol. 29, No. 3, 1999.

[V] Virtanen, K., Raivio, T., and Hämäläinen, R.P., "Modeling Pilot's Sequential Maneuvering Decisions by a Multistage Influence Diagram," *Journal of Guidance, Control, and Dynamics,* Vol. 27, No. 4, 2004.

Contributions of the author

The possibility of applying decision theory to pilot decision-making was initially presented by prof. Raimo P. Hämäläinen. Kai Virtanen was the main contributor in introducing the ideas as well as in developing the influence diagram models representing pilot decisions in air combat and trajectory optimization in Papers I, II, and V. He also developed the multiattribute value model and the interval approach in Paper III. The programming of the IPAC software in Paper III was performed by Ville Mattila. Kai Virtanen and Dr. Tuomas Raivio were the main contributors in Paper IV by introducing the approach towards the automated solution of trajectory optimization problems. Kai Virtanen also programmed the VIATO software in Paper IV. Kai Virtanen was the principal author in Papers I, II, III, and V. Paper IV was written jointly by the authors.

Preface

This thesis has been carried out in the Systems Analysis Laboratory of the Helsinki University of Technology. I am deeply grateful to Professor Raimo P. Hämäläinen, the Director of the Laboratory, for the opportunity to work in the Lab, such an uncommon innovative scientific community, as well as for familiarizing me with the scientific world in general. Without Raimo's excellent foresight and dynamic touch, the thesis would have remained only as a bad dream. I express my sincere gratitude to my instructor and friend, Dr. Tuomas Raivio. In spite of my more or less explosive mind, Tuomas has always been there with his constant and patient support and guidance. I owe You, the MAN, a book! I also wish to thank Professor Harri Ehtamo for his significant contributions during the past years. The preliminary examiners of the thesis, Professor Alain B. Haurie and Professor Prakash P. Shenoy are greatly appreciated.

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Hyvinkää, March 2005

Kai Virtanen

1. Introduction

Today's air-to-air combat is a complex conflict situation in which pilots face complicated decision-making problems. The decisions concern maneuvering, the use of weapons as well as the utilization of onboard devices, such as communication instruments, sensors, and countermeasures. The outcome of the combat depends on decisions and actions taken by the pilots as well as on aircraft performance and effectiveness of a weapon system. Analyses of air combat tactics and technologies as well as pilot training are time-consuming, expensive, and risky tasks. Hence, different mathematical models have been developed and utilized for analyzing air combat and for supporting pilot training. For a comprehensive review of air combat maneuvering and weapon employment, see Shaw (1985). For an overview of principles and practices of air combat analysis, see, e.g., Feuchter (2000).

Air combat models can be classified roughly into two major branches: optimization models providing optimal flight trajectories and simulation models applying synthesis approaches of pilot decisions. The thesis at hand contributes to both of the branches by introducing new modeling approaches for the analysis and synthesis of pilot decision-making as well as for the optimal planning of flight trajectories. The main emphasis of the thesis is on developing novel uncertainty representations in both synthesis and optimization frameworks.

In the thesis, the maneuvering and missile launching decisions of a pilot in one-on-one air combat are modeled, synthesized, and simulated by using influence diagrams (Howard and Matheson, 1984). Unlike existing synthesis approaches, the constructed influence diagram graphically describes the factors of the decision process and explicitly handles the pilot's multiple goals and preferences under conditions of uncertainty. In this model, the state of the adversary is observed but behavioral aspects of the adversary are ignored. These issues are considered in an influence diagram game formulated later in this thesis. The game model contains elements describing the rational maneuvering of the adversary as well. It produces the best myopic maneuvering strategies with respect to the pilot's preferences and available information under the assumption that the adversary behaves in the worst possible way. The new synthesis models extend the use of the formalism of influence diagrams into decision environments that evolve in time.

For combat situations with multiple actors, the thesis develops a value-focused prioritization approach motivated by needs of a data link system (e.g., Hura et al., 2000; Sturdy, 2000) that exchanges automatically tactical information between cooperating air units. The approach prioritizes available information and provides team optimal signaling strategies by solving a signaling problem that describes the optimal exchange of information between the players during a multiplayer air combat. A value function (e.g., Keeney, 1992; Keeney and Raiffa, 1993) is used for capturing preferences of the team of pilots and interval analysis (see, e.g., Salo and Hämäläinen, 2001) is utilized for representing uncertainties. Compared to existing prioritization methods used currently in the data link system, the new approach offers a novel way to prioritize tactical data and seems to improve the situation assessment capabilities of the pilots.

The thesis contributes to optimal trajectory planning by introducing an approach towards the interactive automated solution of deterministic aircraft trajectory optimization problems. The approach gives fundamental guidelines and design principles for the construction of a trajectory optimization software that can be operated without knowledge of optimization theory and mathematical modeling. Finally, the thesis applies the methodology of influence diagrams in trajectory planning by constructing a multistage influence diagram that represents the pilot's sequential maneuvering decisions in a one-on-one air combat setting. Preference optimal flight trajectories are obtained by solving the multistage model with a new nonlinear programming (e.g., Bertsekas, 1995) based solution procedure developed in the thesis. This modeling framework enables the incorporation of preference representations and uncertainty elements into optimal flight trajectory planning which is a theme elided almost totally in the current literature. On the other hand, the presented multistage influence diagram is the first elaboration in which the formalism of influence diagrams is used for modeling a dynamic multistage decision situation.

Traditionally, air combat optimization models are based on the paradigms of optimal control and the theory of dynamic games. Optimal control, also referred to as dynamic optimization, can be utilized for planning optimal trajectories in air combat settings in which a single actor optimizes his or her maneuvering. For a comprehensive review of optimal control, see Bryson and Ho (1975) or Leitmann (1981). The theory of dynamic games, or the theory of differential games referring to dynamic games where differential equations play an important role (Basar and Olsder, 1995), can be applied for modeling one-on-one air combat where both actors behave optimally. For an overview, see Basar and Olsder (1995) and for a more rigorous analysis, see Krasovskii and Subbotin (1988). The solution of the optimization models provides optimal flight paths over the total flight time of a combat setting with respect to given performance indices. Available solution methods and computational power enable off-line solution only. Nevertheless, optimal solutions offer versatile possibilities for analyzing air combat maneuvering especially when the numerical solution process of trajectory optimization problems is automated which is one of topics of this thesis. However, the analysis based on optimization or game theory is limited to one-on-one air combat only.

Aspects related to the modeling of the preferences of pilots as well as to the representation of uncertainty are typically ignored in the optimization models. These topics are taken into account in air combat simulation approaches. They can be divided into batch (e.g., Stehlin et al., 1994; Goodrich et al., 1995; Anon., 2002a; Anon., 2002b) and real-time-piloted (e.g., Goodrich et al., 1992; Bell and Crane, 1993) simulations. Batch air combat simulations allow the study of tactics and aircraft performance in a controlled and repeatable environment where multiple actors can be involved. Real-time-piloted simulations enable tactical experimentation and training of human pilots in a realistic environment. The simulation approaches of both types utilize computer guided aircraft that require a decision-making model for synthesizing the control and other combat decisions of the pilots. Existing decision-making models are based on computational techniques of artificial intelligence (McManus and Goodrich, 1989; Chappell, 1992; Stehlin et al., 1994; Glaerum, 1999), heuristic scoring methods (Bent, 1994; Lazarus, 1997), and discrete dynamic games (Austin et al., 1990; Katz and Ross, 1991; Katz, 1994). In addition, systems combining artificial intelligence and predetermined game optimal solutions have been suggested (Shinar, 1991; LeMenec and Bernhard, 1995). A shortcoming of these approaches compared to the optimization models is that they do not produce optimal controls over a long flight time because the controls are selected based only on the current information about the evolution of the combat.

One purpose of the thesis is to shed light on issues related to incorporating preferences, perceptions, and beliefs of decision makers as well as uncertainty elements into dynamic conflict settings. This is done by tackling optimal air combat maneuvering and decision-making from the perspective of decision analysis. For a comprehensive review of decision analysis, see Bunn (1984), von Winterfeldt and Edwards (1986), Keeney (1992), and Keeney and Raiffa (1993). Decision analysis is a discipline for formalizing the analysis of decisions. Its foundations are related to the roots of utility theory developed by von Neumann and Morgenstern (1947). Thus, decision analysis can be seen as the application of decision theory based on the axioms of probability and utility to real-world problems (Henrion et al., 1991). The primary aim of decision analysis is to help the decision maker think systematically about his or her objectives, preferences, as well as about the structure and uncertainty of a decision problem. In the field of decision analysis, there exists a versatile set of methods, techniques, and tools for modeling decision situations in a formal and transparent manner and for formally solving them. For a recent review of decision analysis applications, see Keefer et al. (2004).

The influence diagram introduced by Howard and Matheson (1984) (see also, e.g., Shachter, 1986; Oliver and Smith, 1990; Tatman and Shachter, 1990; Smith, 1994) is a decision theoretical tool for structuring and solving decision problems under conditions of uncertainty. On the one hand, it is a graphical paradigm for describing variables and factors affecting a decision situation as well as their relationships. On the other hand, the influence diagram allows quantitative analysis of the problem by determining the optimal solution to the problem subject to the decision maker's subjective measure of objective attainment which can be represented with a utility function (e.g., Keeney and Raiffa, 1993). Originally, the methodology of influence diagrams considers static decision problems. The thesis extends the use of influence diagrams into dynamic decision situations. It introduces influence diagram representations that contain an explicit model for the evolution of the state of a decision environment which is represented by a set of differential or difference equations.

Paper I of the thesis introduces an influence diagram for modeling and synthesizing the myopic decision-making process of a pilot under conditions of uncertainty during one-on-one air combat. The same combat setting is described as an influence diagram game in Paper II. Paper III presents a value-focused approach for prioritizing uncertain information in a multiplayer air combat. Papers IV and V of the thesis focus on optimal trajectory planning. The latter one deals with the automated solution of optimal flight paths. In Paper V, the formalism of influence diagrams is utilized for representing preferences and uncertainties in flight path optimization.

The present summary article is organized as follows. In Section 2, the inherent features of air combat decision-making are discussed and the use of air combat models is justified. In addition, current approaches for modeling air combat are surveyed. Section 3 reviews each paper of the thesis and introduces shortly methodologies used in the thesis. In addition, the relationships of the papers to the earlier literature as well as their contributions are described. Section 4 considers open research topics raised by the thesis and gives concluding remarks.

2. Current modeling approaches of military aviation

2.1 Rationale for air combat models

The actions and decisions of pilots in air combat are guided by validated and predetermined tactics and basic fighter maneuvers (e.g., Shaw, 1985). Nevertheless, the decision-making processes are not self-evident due to several inherent features of the decision environment. The decision problems can contain multiple conflicting objectives. Supersonic speed capabilities and effective maneuverability of modern aircraft create a dynamic and transient nature for the decision environment. This complicates the evaluation of different decision alternatives because the combat state is not predictable for a long time ahead.

The decision-making capability of the pilots relies on their situation awareness which is based on information acquired by visual detection, radio communications, and by using onboard avionics. The combat decisions must be made under conditions of uncertainty because the available information is incomplete due to radar measurement errors and non-real-time information received from other communication devices. The pilot is not necessarily aware of the behavior and goals of the adversaries, which increases uncertainty on the ultimate consequences of different decision alternatives and control commands. Furthermore, the performance of the adversary's aircraft and the effectiveness of his or her weapons are not known necessarily by the friendly pilots.

The above mentioned features occur in the decision-making of a single pilot that fights against a single adversary. The one-on-one combat can be seen as the building block of a multiple aircraft scenario in which a team of pilots is in combat with an enemy team. In such a setting, the decision-making becomes more complex because the cooperative nature of the decision processes within the team of the pilots must be taken into account. The team as a whole needs to assess the combat situation by inferring the mutual information and joint goals as well as by reasoning the intentions of the whole enemy team. In the selection of flight tactics, the cooperation and the coordination through information exchange via, e.g., a signaling network are required.

Analyses of air combat can be conducted with a trial and error experiment that requires several airborne test flights. Such an experiment is expensive, time-consuming and poses a risk to life and property because human errors might end in fatal accidents. Some air combat situations cannot be analyzed in a battlefield exercise. In addition, uncertainty about hostile threat systems might prevent experimentation in a realistic training environment. On the other hand, it is not rational to engage in a real combat for testing the performance of new air combat technologies or for validating new operational concepts. Because air combat occurs rarely and has large uncertainties, there is also a lack of realistic air combat data.

The application of mathematical air combat models is often the most convenient as well as the cheapest and quickest way to obtain information about system performance or the value of new ways of conducting air combat missions. A realistic air combat model should contain features describing the dynamics and uncertainties of the decision environment as well as representing multiple conflicting objectives of the pilots. In addition, aspects on team decision-making should be considered. Unfortunately, the solution of an air combat model including all these features is beyond the state of the art. Hence, one has to make a tradeoff between the realism and solvability of the model. Naturally, the purpose of use also defines a suitable model class.

In the following review of existing air combat models, first optimal trajectory planning approaches are introduced. Then, the synthesis approaches of control decisions utilizing some foundations of the first model class are presented. In addition, flight mechanical models are discussed.

2.2 Optimal trajectory planning

As stated in the introduction, quantitative analysis of air combat has its roots in the theory of optimal control. It allows the determination of the controls transforming the state of an aircraft from its initial state to another state in the best possible way with respect to a given performance index. In practice, for problems containing nonlinear equations of motion of an aircraft, only optimal open-loop controls can be solved. For a historical review of optimal control including first flight trajectory studies, see Bryson (1996) and for an overview of numerical solution techniques, see, e.g., Bryson (1999) and Betts (2001).

In air combat related optimization problems, the flight time is a commonly used performance index. The time required to achieve a given altitude and velocity is minimized in a minimum time climb problem (e.g., Kelley and Cliff, 1986; Raivio et al., 1996). A minimum time trajectory to a target is determined in a minimum time interception problem (e.g., Shinar and Spitzer, 1988; Fan et al., 1995; Glizer, 1997). On the other hand, optimal trajectories are utilized in air combat mission planning which can be based on estimated risk minimization (Vian and Moore, 1989) as well as on the avoidance of detection by the radar of a hostile force (Norsell, 2003). The theory of optimal control can also be applied for designing optimal avoidance maneuvers for an aircraft that is pursued by a missile using a known feedback guidance law (Shinar and Guelman, 1994; Ong and Pierson, 1996; Raivio and Ranta, 2003).

The theory of differential games (e.g., Basar and Olsder, 1995) can be applied for modeling air combat situations with more than one decision maker. Although theoretical elaborations on differential games involving several players have been developed (e.g., Petrosjan, 1993), team problems containing realistic aircraft models have received less attention in the open literature. However, recently stochastic optimization has shed some light to the determination of large-scale air combat tactics (Mulgund et al., 2001).

Unlike multiple actors, one-on-one air combat models have been subject to profound research effort. For instance, an encounter between a missile and an aircraft can be represented by a pursuit-evasion game, first introduced by Isaacs (1965), whose cost function, e.g., the elapsed time, is minimized by the missile and maximized by the aircraft (e.g., Breitner et al., 1993; Lachner et al., 1995; Ehtamo and Raivio, 2001; Glizer and Shinar, 2001). On the other hand, the pursuer can also be an aircraft that tries to reach the launching position of weapons (Järmark, 1985; Moritz et al., 1987; Grimm and Well, 1991). In addition to the flight time, the cost of the game involving two aircraft can be, e.g., the distance to the border of a nation (Raivio and Ehtamo, 2000b).

A pursuit-evasion game admits qualitative and quantitative solutions (e.g., Blaquiere, 1969). The former one divides the state space of the game into the set of initial states that lead to a capture or evasion of the evader. For a capturability assessment of a missile and an aircraft, see Raivio (2001). Quantitative solutions announce game optimal control strategies of the players. The solution of this type for a realistic air combat game formulation is obtained in an open-loop representation of a feedback strategy (e.g., Breitner et al., 1993; Lachner et al., 1995; Ehtamo and Raivio, 2001). Feedback solutions can be computed approximately, e.g., with a neural network approach (Pesch et al., 1995; Lachner et al., 1996) or other function approximation methods.

In pursuit-evasion games, the roles of the players must be fixed beforehand. This is usually not the case for two aircraft committing to a combat because the aircraft may switch several times from a pursuing to an evasive maneuver and vice versa. For such situations, two-target game models have been proposed (Blaquiere, 1969; see also Grimm and Well, 1991). In a two-target game, the objectives of the players are to avoid the adversary's target set while attempting to drive the state of the game to their own target sets.

The qualitative solution of a two-target game divides the state space into regions leading to different outcomes that can be capture of one player by the other, joint capture, or no capture at all. Nevertheless, computational difficulties prevent practical applications. For a comprehensive analysis with a greatly simplified vehicle models, see Davidovitz and Shinar (1989). Quantitative solutions would give game optimal strategies for both players. However, there are several modeling issues that have not been solved in a satisfactory way. An unresolved conceptual aspect is that the concept of optimality in the regions not leading to a capture is ambiguous (e.g., Ghose and Prasad, 1993). Even though the concept of game optimality is clear in regions leading to a capture of either player, the fact that the game has two target sets has to be taken into account. For instance, a combat game approach (Ardema, 1985) leads to an ill-posed pursuit-evasion game formulation containing a state constraint whose purpose is to prevent the win of the evader. For multicriteria game formulations (Ghose and Prasad, 1989; Prasad and Ghose, 1991), different solution concepts corresponding to concepts of multicriteria optimization are defined and outlined on a theoretical level, but, according to the author's knowledge, none of these elaborations has lead to practical applications. In addition to the conceptual shortcomings, the computation of optimal feedback controls for two-target games containing realistic aircraft models is intractable. An open-loop solution can be obtained by converting the original game into a single criterion differential game and solving it (Järmark, 1985; Moritz et al., 1987).

To summarize, practical applications on optimal trajectory planning are restricted to air combat settings that involve one actor or two actors with fixed roles. The corresponding optimization and differential game formulations can only be solved off-line and in an open-loop form. However, resulting optimal strategies give valuable information for different off-line analyses of air combat because the solutions are optimal over long flight times with respect to well-defined performance criteria.

2.3 Synthesis of control decisions

Optimal trajectory planning approaches originating from the theory of optimal control and differential games cannot be applied directly in the synthesis of pilot decisions due to several reasons. Optimization models cannot be solved in real-time and hence they cannot be used in online simulation. The literature on optimal trajectory planning seldom pays attention to the structure of performance criteria that should model the preferences and possibly multiple objectives of pilots. The current optimization models do not capture uncertainties about, e.g., preferences of adversaries and state measurements because perfect information structure is typically assumed. As mentioned earlier, there are conceptual difficulties on role assignment even in one-on-one air combat models (e.g., Kelley and Lefton, 1978; Merz, 1985), and multiple aircraft scenarios do not fit in differential game formulations (Shinar, 1991).

In general, in the synthesis models of control decisions, the time is discretized and the best controls are selected at discrete decision instants. A general principle is to first obtain the possible states of a combat after a short planning horizon by projecting each control alternative into the future and by predicting the state of the adversary. Then, one associates a score to each predicted combat state by using a ranking approach that should represent the preferences of the pilots. Finally, the control alternative leading to the highest score is executed until the next decision instant is reached.

The ranking approaches suggested earlier in the literature include rule-based expert systems (McManus and Goodrich, 1989; Chappell, 1992; Stehlin et al., 1994; Glaerum, 1999) and heuristic value driven models (Bent, 1994; Lazarus, 1997). In addition, fuzzy logic (Akbari et al., 2003) and neural networks (McMahon, 1990) have been applied. In a simple air combat expert system, the combat states are evaluated with combat geometry rules (Burgin and Sidor, 1988). More advanced systems (Goodrich and McManus, 1989; McManus, 1990) utilize a fixed set of questions representing different goals. In a value-driven heuristic approach (Bent, 1994; Lazarus, 1997; see also Anon., 2002c), the combat state achieved with a feasible control alternative is evaluated by using a value function. This approach is also used for modeling team decision-making.

A common feature of these scoring approaches is that the behavior of the adversary is observed but nothing is assumed. This is inconsistent with the fact that the adversary is a rational decision maker with similar goals as the actor who is selecting the best control. The rational behavior of the adversary is taken into account by synthesis frameworks utilizing game formulations. In such a framework, a pursuit-evasion or two-target game is solved approximately by discretizing time and control variables as well as by shortening the planning horizon. Austin et al. (1990) present an approach where the consequences of feasible control alternatives are evaluated using a deterministic scoring function and the best controls are obtained by solving a zero-sum matrix game. Katz and Ross (1991), and Katz (1994) introduce a zero-sum game in an extensive form with a probabilistic score. Shinar and Glizer (1995) solve a pursuit-evasion game approximately with a moving horizon control, also referred to as model predictive control, approach (e.g., Camacho and Bordons, 1998). Kelley and Cliff (1980) propose the concept of reprisal strategies that utilizes the nonoptimal behavior of the adversary in a two target game.

In an alternative synthesis approach based on a game formulation, predetermined optimal control strategies or given control laws are used as decision alternatives and the best alternative at each decision instant is selected by using a suitable technique. Neumann (1990) proposes a matrix game in which the decision alternatives of the players are nonoptimal feedback control laws. On the other hand, the ranking can be made with the help of artificial intelligence (AI) models. For instance, given solutions of optimal control problems (Powell and Maier, 1988) as well as of pursuit-evasion games (Rodin et al., 1987; Shinar et al., 1988) can be scored by using a knowledge based approach. Combining pursuit-evasion game solutions with AI techniques, it is also possible to construct a model that represents a duel between two aircraft carrying missiles (Shinar, 1991). The system of this type can also be utilized, e.g., for informing a pilot about the launching time of a missile in one-on-one combat (LeMenec and Bernhard, 1995).

It should be noted that although the reviewed approaches make it possible to cover issues like preferences and uncertainties in air combat modeling, they do not produce long-sighted optimal controls. The use of AI techniques leads to a heuristic selection of controls. On the other hand, when the control decision is synthesized with the approaches predicting the evolution of the combat state a short planning horizon ahead only, the resulting control is not an optimal long-sighted solution but rather the best myopic control command in the light of available information and the given preference representation. However, the short planning horizon allows the use of accurate aircraft models that pose difficulties in the numerical solution of optimal control and differential game formulations.

2.4 Aircraft models

In all the air combat modeling frameworks, a representation for the motion of an aircraft is needed. The modeling of both translational and rotational dynamics leads to a six degrees of freedom (6-DOF) model consisting of 12 nonlinear differential equations. The state variables of this model are the position and altitude of the aircraft in a three dimensional inertial frame, the velocity vector, the Euler angles determining the attitude of the aircraft as well as the angular rates denoting yaw, pitch, and roll motions about the center of mass of the aircraft. The control variables are associated with elevator, aileron, and rudder deflections that control the aerodynamic forces and moments affecting the aircraft. In addition, tangential acceleration is controlled with the throttle setting. The construction of the 6-DOF model is considered extensively in Stevens and Lewis (1992).

If the rotational dynamics is ignored, the 6-DOF model is reduced to a three degrees of freedom (3-DOF) point-mass model (e.g., Miele, 1962). It consists of a set of six nonlinear differential equations describing the motion of an aircraft as a point-mass in the three dimensional space. The state variables of the point-mass model represent the position, altitude, and speed of the aircraft as well the flight path angle and the heading angle. These angles determine the direction of the velocity vector. The normal acceleration of the aircraft is controlled with the loadfactor and the tangential acceleration with the throttle setting, respectively. The loadfactor is the ratio of the lift force and the gravitational force that affect the aircraft. Instead of the loadfactor, the angle of attack can also be used for commanding the normal acceleration. The lift force is directed away from the vertical plane with the bank angle. The complete point-mass model is presented, e.g., in Paper IV.

In addition to the equations of motion, aircraft models contain different constraints for state and control variables that are set by the pilot and the aircraft itself. The aircraft models also include parameters depending on an aircraft type. Such parameters are, e.g., the wing area and mass of an aircraft. The mass can also be treated as a state variable, if it is reduced considerably due to the fuel consumption during a flight. The aircraft types are also distinguished by drag coefficients characterizing the drag force, the maximal available thrust force, and the lift coefficient used in the calculation of the lift force. Many model parameters are affected by the density of the air and the Mach number. Hence, the properties of the atmosphere must also be taken into account in the aircraft models.

The use of the 3-DOF model may lead to unrealistic motion of an aircraft especially in situations in which the aircraft maneuvers excessively, because the model does not contain limits for the angular velocities. However, the numerical integration of the 3-DOF model requires less computational effort than the computation based on the 6-DOF model. There are also large uncertainties in 6-DOF modeling related to, e.g., the estimation of aerodynamic data (Hoffren and Sailaranta, 2001). For these reasons, modified aircraft models have been developed. For instance, the 6-DOF model can be simplified by omitting moment equations (e.g., Hoffren and Sailaranta, 2001). On the other hand, the realism of the 3-DOF model can be increased by adding angular rate constraints (Raivio and Ranta, 2003). Some evidence on the accuracy of the 3-DOF model is given in Hoffren and Raivio (2000). It should be noted that less detailed models than the complete 3-DOF model discussed above are often used in applications of optimal control and differential games. For instance, optimal maneuvering can be considered in the vertical or the horizontal plane only, constant speed flight vehicles can be assumed, or constant drag coefficients can be applied.

3. The thesis

The thesis consists of five individual papers. The main theme of Papers I, II, and III is related to the analysis and synthesis of pilot decision-making whereas Papers IV and V focus on optimal trajectory planning. Table 1 classifies the papers according to the aspects discussed in the previous section. The first aspect is the number of players involved in an air combat setting under consideration. Second, the length of a planning horizon, i.e., myopic or long-sighted referring to optimization over total flight time, is indicated. The third aspect implies the formalism utilized for representing uncertainties. Finally, the papers are categorized based on how the behavior of the adversary is taken into account.

Paper	Number of players	Planning horizon	Uncertainty representation	Behavior of adversary
I	one-on-one	myopic	probability	state observations
II	one-on-one	myopic	probability	rational
III	many-on-many	zero	intervals	state observations
IV	single	long-sighted	-	-
V	one-on-one	long-sighted	probability	given maneuver

Table 1. The classification of the papers.

In the following, the contributions of each paper are described and short introductions to techniques and methodologies utilized in the thesis are given. Section 3.1 discusses the synthesis models developed in Papers I, II, and III as well as decision theoretical methods applied in these papers and in Paper V. Section 3.2 presents the basics of the numerical solution of optimal control problems as well as the optimal trajectory planning frameworks developed in Papers IV and V.

3.1 Decision theoretic analysis and synthesis of pilot decisions

3.1.1 Value-focused decision-making and influence diagrams

The main idea in the utilization of decision analysis is the divide and conquer orientation (Keeney, 1992). This refers to the process in which a decision problem is decomposed into smaller subproblems which are analyzed and evaluated separately. Then, the components are composed and overall insights and recommendations on the original problem can be given. The analysis of this type is conducted by using decision theoretical methods, techniques, and tools that can be applied for modeling, structuring, and analyzing decision problems under conditions of certainty as well as under conditions of uncertainty. Such approaches also offer a possibility to transfer the perceptions, preferences, beliefs, and attitudes of a decision maker into decision models. For an overview of basic approaches, see, e.g., Bunn (1984), von Winterfeldt and Edwards (1986), and Keeney and Raiffa (1993).

The multiattribute value theory (e.g., Keeney, 1992; Keeney and Raiffa, 1993) provides a value-focused approach to decision-making problems with no uncertainty. In this approach, the state of the world is described with a set of attributes that are related to competing objectives. Each decision alternative leads to the specific levels of the attributes. A single attribute value function maps the level of an attribute into a value scale. The single values are aggregated by calculating, e.g., a weighted sum which determines the overall ranks of the decision alternatives. The multiattribute value theory is utilized in the information prioritization approach elaborated in Paper III of the thesis.

In Papers I, II, and V, the decision-making of a pilot under uncertainty is tackled by utilizing the multiattribute utility theory (von Neumann and Morgenstern, 1947; see also Keeney and Raiffa, 1993). The utility theoretical representation of a decision problem is often referred to as a Bayesian decision model. Then, the subjective probability interpretation (see, e.g., Lindley, 1994) is applied for representing uncertainty and the goodness of the decision alternatives' consequences is measured by the von Neumann-Morgenstern utility (von Neumann and Morgenstern, 1947). The subjective probability interpretation means that the probability of an event is a measure of a person's degree of belief in the event, given the information available to that person (Henrion et al., 1991). When ranking decision alternatives based on utility theory, each state of the world is evaluated with a utility function and is associated with a probability. The utility function is similar to a value function, i.e., it consists of single attribute utility functions and their weights. If the decision maker whose preferences and opinions are captured into the decision model is prepared to accept the utility theoretical definition of rationality, i.e., the axioms of the utility theory (see, e.g., Keeney and Raiffa, 1993), the rational choice is the decision alternative providing the highest expected utility.

In addition to the probability theory, other formalisms for representing uncertainty have also been introduced. In Paper III, uncertainties are incorporated into a value-focused decision framework with interval analysis (Salo and Hämäläinen, 1992; Salo and Hämäläinen, 1995; Salo and Hämäläinen, 2001). It can also be applied in group decision-making to aggregate the preferences of all group members in a single model (Hämäläinen and Pöyhönen, 1996). Interval analysis allows the variables and parameters of a decision model to vary within given ranges. Hence, the exact attribute levels associated with decision alternatives can be replaced with intervals. The interval analysis also enables the decision maker to give incomplete preference statements about the parameters of the decision model.

A primary modeling paradigm used in the thesis is the methodology of influence diagrams (Howard and Matheson, 1984; see also Shachter, 1986; Oliver and Smith, 1990; Smith, 1994) that is exploited for representing decision processes of a pilot in Papers I, II, and V. The influence diagrams allow the structuring and analysis of decision problems under conditions of uncertainty. They are directed acyclic graphs that graphically describe a decision-making situation and provide a methodological basis for the ranking of the available decision alternatives. The representation of influence diagrams can be divided into the three levels: relation, function, and number (Howard and Matheson, 1984). The relation level represents the qualitative structure of the problem that is expressed by the topology of the graph. The function level specifies the functional form of the relationships among the variables of the graph. In the number level, the quantitative details of the dependence of each variable are represented. A simple air combat related example of an influence diagram is presented in Section III C of Paper I.

The nodes of an influence diagram represent decision, random, or deterministic variables. Arcs leading into a random or deterministic node describe probabilistic or functional dependence. Informational arcs pointing to a decision node imply which quantities are known to the decision maker before the decision is made. In an influence diagram, a utility function is used for modeling the decision maker's preferences and for evaluating the consequences of the decision. The solution of an influence diagram (e.g., Shachter, 1986; Tatman and Shachter, 1990) provides probability distributions of the utility for all decision alternatives that can be ranked based on these distributions. An overview of the solution techniques of influence diagrams is given in Section II B of Paper V.

Decision trees (e.g., von Winterfeldt and Edwards, 1986; Pearl, 1987) are closely linked to influence diagrams, since any influence diagram can be converted into a decision tree. These representation schemes have different advantages in the modeling of decision situations (e.g., Covaliu, 1995). Influence diagrams provide a compact representation for decision problems by hiding many of the less interesting details whereas complex problems may lead to very large decision trees. Thus, influence diagrams are ideal for obtaining an overview of a decision problem and communicating with an expert in the application area. The connection of influence diagrams with decision trees are considered and discussed in Papers I and V. Note that influence diagrams containing only chance nodes are called Bayesian networks, or equivalently, belief networks that have received much attention in the field of artificial intelligence. The most common task solved by using these networks is probabilistic inference. For a comprehensive introduction to Bayesian networks, see, e.g., Pearl (1988) and Jensen (2001).

Contrary to game theory, decision analysis focuses on the analysis of decision situations involving a single decision maker. In addition, there are techniques dealing with group decision-making. Note that a group decision problem refers to a situation in which the preferences of group members are typically different whereas in a team decision problem (e.g., Ho and Chu, 1972; Chu, 1972; see also Basar and Olsder, 1995), such as considered in Paper III, all group members have a common goal. Despite their different perspectives, the foundations of decision theory and game theory have common roots laid down by von Neumann and Morgenstern (1947). Therefore, it might be possible to apply some foundations of decision analysis for modeling games as well. An example is the multi-agent influence diagram framework introduced recently by Koller and Milch (2001, 2003). It extends the formalism of influence diagrams into noncooperative game settings. A multi-agent influence diagram describes graphically a static game in a more compact way than traditional game formulations. Paper II of the thesis considers the utilization of influence diagrams in modeling a dynamic game setting, that is, a one-on-one air combat.

3.1.2 Influence diagram for the pilot's decision process

Paper I introduces an influence diagram for modeling the maneuvering and missile launching decisions of a pilot in one-on-one air combat, see Fig. 1. It takes into account the preferences of the pilot as well as uncertainties related to the decision-making. The inherent dynamics of the decision environment is represented with a 3-DOF point-mass aircraft model.

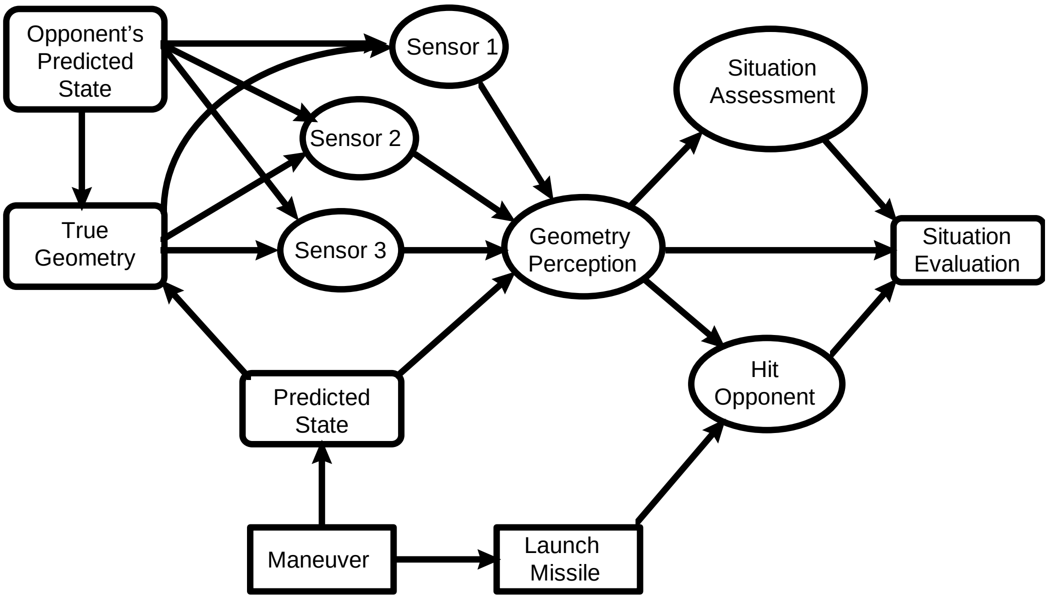

Figure 1. An influence diagram representing the decision process of the pilot in one-on-one air combat.

In the influence diagram, the continuous control variables of the point-mass model are replaced with discrete control alternatives. At the decision instant, the pilot's future states after the planning horizon are obtained by integrating the equations of motion with each control alternative. The anticipated states and the given state of the adversary define the possible combat states that can be achieved with the feasible control alternatives. Because the length of the planning horizon is relatively short, the character of the influence diagram can be considered myopic.

In Paper I, the uncertainty of the model originates from the pilot's imprecise observations about the combat state. The observation process is modeled with the chance nodes containing continuous normal probability distributions from which the uncertain values of the combat state are generated. The variances of the distributions can be considered as a measure for the accuracy of available sensors. The state observations define the pilot's belief on the combat state. The belief affects the threat situation assessment of the pilot as well as the probability of hit of a missile assessed by the pilot. The threat situation is represented with a discrete random variable whose outcomes are neutral, advantage, disadvantage, and mutual disadvantage. In practice, the probabilities of these outcomes as well as the missile's probability of hit are inferred according to Bayesian reasoning that refers to the updating of the probability distributions by using Bayes' theorem (e.g., von Winterfeldt and Edwards, 1986; Lindley, 1994).

The influence diagram associates a probability and a utility with each possible combat state. The utilities depend on the pilot's comprehension on the true combat state, the threat assessment as well as on the missile's probability of hit. In practice, the utility measure is computed with utility functions that describe the preferences and multiple objectives of the pilot. When solving the influence diagram, the probability distribution of the utility is obtained for each control alternative. The ranking of the available alternatives are made based on these distributions using, e.g., the maximum expected utility criterion. The resulting control alternative can be considered as being optimal with respect to the available information and the given preference model.

It should be noted that the expected utility is not a perfect indicator of what might happen since the risk of decision alternatives varies. Although the alternative with the highest expected utility is executed, there is a probability that the coming combat situation will be worse or better than the pilot assumes. Uncertainties and risks related to different decision alternatives can be analyzed with the probability distributions of the utility. Standard deviation is a measure of how widely the values are dispersed in a distribution and thus it is an indication of the risk. Minimal and maximal possible utilities indicate the worst and the best possible outcome. On the other hand, the risk attitude of the pilot can be taken into account with different decision criteria. The maximin criterion leads to the selection of the alternative whose worst possible utility is highest. Thus, it represents the behavior of a risk averse decision maker. A risk prone behavior can be described with the maximax criterion referring to the alternative whose best utility is highest.

The influence diagram constructed in Paper I can be used in the synthesis of the pilot's decision-making process. Due to the graphical representation of the influence diagram, it could be constructed, validated, and updated together with pilots because the representation can be understood by individuals who only have little decision theoretical background. In rule-based systems (Burgin and Sidor, 1988; McManus, 1990; Chappell, 1992; Stehlin et al., 1994), pilots can validate and analyze the decision models only by studying simulation results. These systems have also been found complex to understand and difficult to modify (Masson and Moffat, 1994).

The possibility to treat uncertainty, human preferences, and attitudes towards risk in a well-established way makes the influence diagram approach an interesting choice over expert systems. These systems typically have problems in dealing with human preferences and attitudes towards risk (Henrion et al., 1991). In the influence diagram, the utility functions describe the preferences. Tradeoffs between competing objectives are characterized by the weight parameters in the utility function, whereas in rule-based systems the tradeoffs must be expressed explicitly. On the other hand, the scoring approaches based on value functions (Bent, 1994; Lazarus, 1997) do not treat the uncertainty on control alternatives explicitly which is represented with probabilities in the influence diagram model.

In addition to the synthesis purpose, the influence diagram offers several analysis possibilities. For instance, Paper I presents how sensitivity analysis can be conducted. Furthermore, the influence diagram can be applied for identifying the effects and value of new information that could reduce the underlying uncertainty of the decision model.

3.1.3 Influence diagram game for the pilot's control decision

The myopic decision process of the pilot is represented in Paper I by assuming that the pilot receives information about the state of the combat via imprecise measurements but nothing is assumed about the behavior of the adversary. Paper II formulates an influence diagram game that contains explicit control variables as well as other components needed for modeling the control problems of both players involved in one-on-one air combat. The objective of the players in the game is to reach their own target sets before the adversary. In other words, the combat setting is similar to a two-target game formulation. In practice, the target set could represent the firing envelope of a missile.

The influence diagram game shown in Fig. 2 is constructed by combining two conventional influence diagrams describing the control decision. One of these diagrams represents the control decision from the viewpoint of a single player whose belief about the decision process of the adversary is represented by the other diagram. In the influence diagram game, the future states of the players are calculated by integrating the equations of motion with feasible controls. The anticipated states define possible combat states from which uncertain observations are received by the players. Both of them infer the threat situation assessment which is modeled using discrete random variables whose outcomes reflect the distances between the combat states and the target sets. The probabilities of the outcomes depend on the state observations and are computed with Bayes' theorem. Utility functions are used for measuring the overall preferences of the players in different combat states. In addition to the state observations, the utilities are affected by the threat situation assessments.

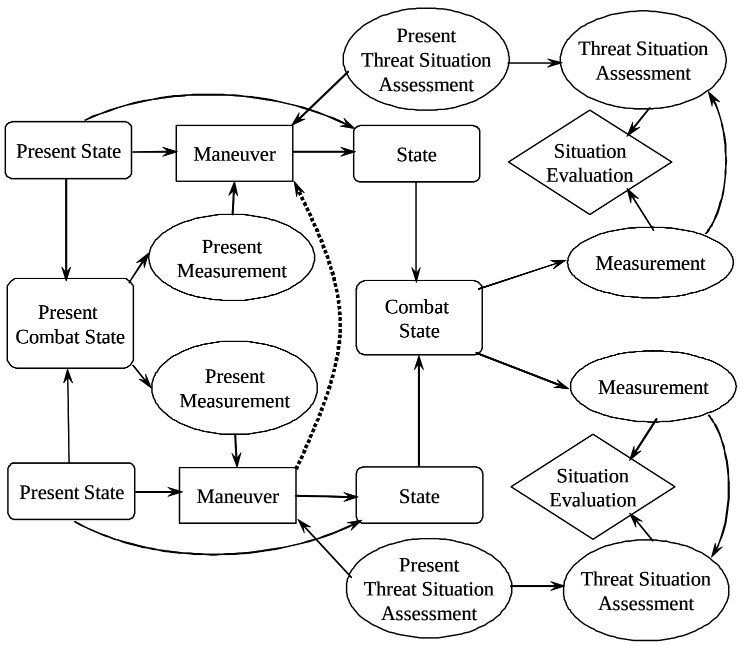

Figure 2. An influence diagram game representing the control decisions of the pilots in one-on-one air combat.

Because both the players have utility functions of their own, the game is non-zero-sum. The payoff functions of the game are associated with the expected utility. The influence diagram game allows the use of both discrete and continuous control variables. In the former case, the most desirable control alternative is obtained by converting the diagram into a game in strategic form. The continuous control variables lead to two simultaneous optimization problems that can be solved using nonlinear programming. The solution of the air combat game provides the best control for one player under the supposition that the adversary behaves in the worst possible way. The game optimal controls are obtained in a feedback form, i.e., the best controls are given as a function of all the available information at the particular decision instant but they are myopic due to the short planning horizon.

The information structure of the influence diagram game can be symmetric or asymmetric which is indicated by a dashed arc in Fig. 2. The former structure refers to a situation in which both the players are aware of all the elements of the game. Then, the game optimal solution is a Nash equilibrium (e.g., Basar and Olsder, 1995). The equilibrium of this type does not always exist and is not necessarily unique (e.g., Basar and Olsder, 1995). This problem can be avoided by assuming an asymmetric information structure. This means that one player, the leader, whose decision process and belief about the adversary's decision process are represented in the influence diagram, has knowledge of all the elements of the game whereas the adversary, the follower, makes the decision based on his or her probabilities and utilities as well as on the leader's action. Then, the influence diagram game admits a solution called a Stackelberg equilibrium (see, e.g., Basar and Olsder, 1995).

The influence diagram game in which the control variables are discrete can be solved in real-time. Hence, the game model could be used, e.g., in decision-making systems of air combat simulators for synthesizing the control decision. The graphical game representation offers similar benefits as the conventional influence diagram described in Paper I. In addition, the influence diagram game overcomes shortcomings that appear in the existing synthesis approaches utilizing game formulations (Austin et al., 1990; Katz and Ross, 1991; Katz, 1994). For instance, the payoff functions of these approaches are simple and they are constructed without a profound analysis on modeling preferences and uncertainties. The influence diagram game can also be utilized in the analysis of the pilot's control decision. All the analysis possibilities discussed in the previous subsection are available. The two-player setting also allows some additional analysis features. For instance, the effect of differences in the preferences of the players can be studied.

The underlying assumption in Paper II is that the players select the controls rationally by maximizing their payoff functions. The resulting control for one player can also be seen as the best strategy against the worst possible action of the adversary. In most cases, such strategies guarantee a secure outcome for the player. It is obvious that if the adversary behaves nonoptimally, the assumption may be overly stringent. Nevertheless, a rationality assumption is better than mere prediction based on observations from the state of the adversary. Considering the fact that the adversary is a rational decision maker with meaningful goals, the worst case assumption is likely to be correct at least for most of the time. In some case, it is even possible that the feedback nature of the solution combined with the worst case behavior results in a reprisal strategy (Kelley et al., 1980) like behavior where the nonoptimal behavior of the adversary is utilized.

The constructed influence diagram game could be used in a simulation procedure that solves game optimal control sequences of the players over the total duration of a two-target game approximately. In the simulation procedure, the control decisions are taken at discrete decision stages and the players act simultaneously. At each decision stage, the best control strategies are identified by solving the influence diagram game and they are employed until the next decision stage is reached. A similar moving horizon control approach for dynamic discrete time games is presented in Cruz et al. (2002). It should also be noted that the influence diagram game could offer a way to unify quantitative and qualitative solutions of two-target games. A prior knowledge, observations and likelihood functions used in Bayes' theorem all contribute to the threat situation probabilities that classify the states, and utilities and their weights associated with the combat states guide the selection of the controls throughout the dynamic game setting.

The graphical representation of the influence diagram game constructed in Paper II coincides with the multi-agent influence diagram framework introduced by Koller and Milch (2001, 2003). However, this modeling paradigm is unsuitable for representing the control decisions under consideration. The dynamic game formulation at hand includes a state and an explicit model, i.e., a differential equation, for the underlying dynamics of the decision environment which are not considered in the formalism of multi-agent influence diagrams.

3.1.4 Team optimal signaling strategies

In Papers I and II, the pilot's decision-making processes are analyzed in a one-on-one air combat. Paper III moves away from the two players setting and studies the prioritization of information under conditions of uncertainty in many-versus-many air combat. The prioritization study is motivated by needs of a data link system (e.g., Hura et al., 2000; Sturdy, 2000) that allows real-time exchange of state information about friendly or hostile aircraft within a cooperating team of pilots. The system automatically relays encrypted messages and provides a reliable way to transmit information under electronic countermeasures generated by the hostile force.

The data link system cannot relay all the available information because its transmission capacity is limited. On the other hand, the most crucial information must be transmitted in order to achieve the best possible situation awareness of the friendly pilots. Hence, the most important and essential information with respect to the overall goals of the friendly team should be identified by prioritizing the dynamically varying importance of the aircraft. This creates the signaling problem, shown in Fig. 3, that can be considered as a team decision problem under uncertainty. The solution of the problem gives the team optimal signaling strategy referring to the best sequence of state information included in successive data link messages. Paper III introduces a novel prioritization approach for solving such a strategy during a many-versus-many air combat.

The prioritization approach is based on a value function that captures the preferences of the pilots of the friendly team. It evaluates the individual importance of a hostile aircraft from the viewpoint of each member of the friendly team. In practice, the value function maps the momentary combat state onto an importance measure. In addition, the importance of a given aircraft is influenced by the elapsed time since the previous transmission instant of the message about this aircraft. The time factor enables the data link system to retransmit information about a particular aircraft although it would have low importance calculated based on the combat state only. The overall importance index of the hostile aircraft is determined by aggregating the individual one-on-one importances. Because the transmitting aircraft can also send its own state information, an importance index for it is needed. This index is constructed by averaging the importance indices associated with earlier transmitted messages. The final selection of the contents of the data link messages is made based on the importance indices of the hostile and transmitting aircraft.

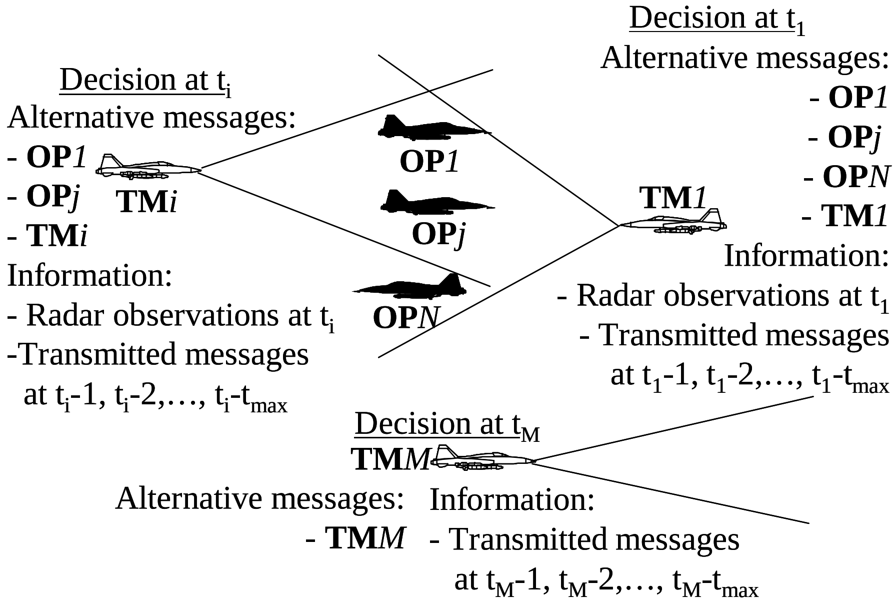

Figure 3. The signaling problem in M-versus-N air combat.

Interval analysis is used for treating the underlying uncertainty of the signaling problem. The state information of the aircraft is not known precisely and, in addition, incomplete preference statements can lead to imprecise parameters of the value function. The state variables and the parameters are now allowed to vary within given ranges. One-on-one importance intervals are calculated with nonlinear programming (e.g., Bertsekas, 1995) by maximizing and minimizing the value function subject to the feasible ranges. Then, the prioritization approach gives an interval for the overall importance index of each hostile aircraft by aggregating the individual importance intervals. This allows the ranking of the state information of the aircraft in an order of importance.

The intervals can also be utilized in sensitivity analysis in which the impacts of different factors affecting the prioritization can be found out. With the help of the sensitivity analysis, one can obtain important validation information for supporting an elicitation process in which air combat experts are assessing the parameters of the value function. An example sensitivity analysis is described in Paper III.

In Paper III, the prioritization approach is implemented in a Matlab-based (Anon., 1998) software called Information Prioritization in Air Combat (IPAC) that can be used in the assessment and validation of value functions and their weights. The software simulates and visualizes the signaling and flight paths of the involved aircraft in a multiplayer air combat game. IPAC also offers features for eliciting preferences from pilots into the value function. The graphical user interface of IPAC is shown in Fig. 4.

The prioritization approach can be utilized in the implementation of the data link system as well as in a data link model included in decision-making systems of air combat simulators. In the preliminary validation experiments, signaling strategies are evaluated in different air combat scenarios with the IPAC software. The validation results imply that the new approach produces significantly better strategies than existing prioritization techniques implemented in the data link system. Hence, the approach seems to provide a possibility to improve the situation assessment capabilities of the pilots in air combat. This supports more collaborative decision-making between the friendly pilots.

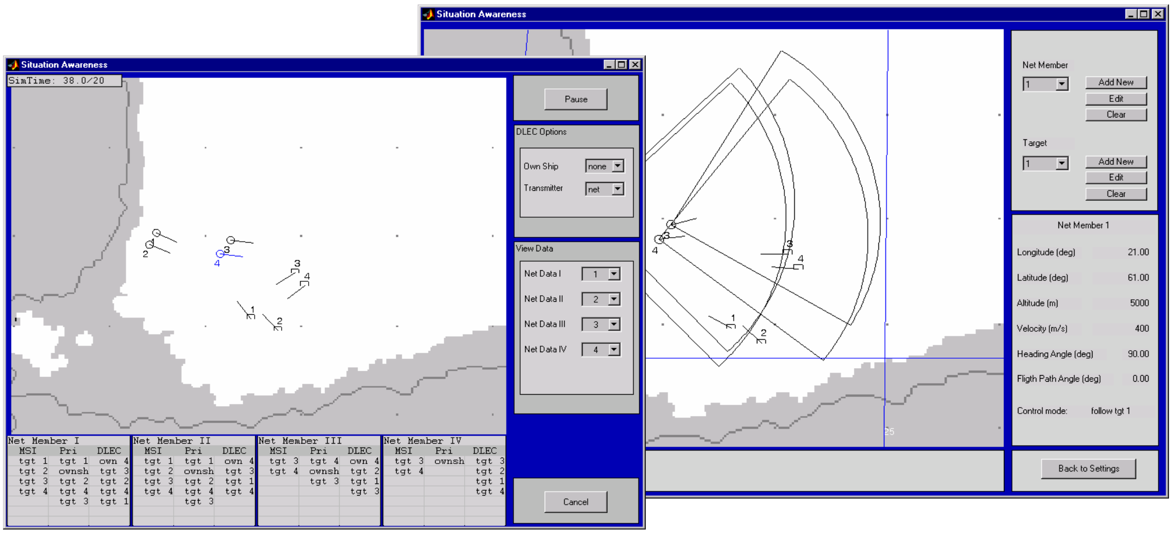

Figure 4. The user interface of IPAC.

3.2 Optimal flight trajectory planning

3.2.1 Numerical solution of optimal control problems

When describing methods for solving deterministic continuous time optimal control problems that are considered in Paper IV, a solution technique is classified as either a direct method or an indirect method. Both of them give optimal controls in an open-loop form. The indirect methods are based on the solution of the necessary conditions for optimality given by the Pontryagin's Maximum Principle (Pontryagin et al., 1962). The necessary conditions constitute a multipoint boundary value problem that can be solved using, e.g., multiple shooting methods (e.g., Pesch, 1994). In the direct methods, the original optimal control problem is replaced with a finite dimensional approximation by discretizing time. The resulting parameterized optimization problem can be solved with nonlinear programming (e.g., Bertsekas, 1995). Several methods exist for the discretization, see the survey papers by Hull (1997) and Betts (1998) as well as a book by Betts (2001). A suitable discretization scheme is a direct transcription formulation (e.g., Bulirsch and Kraft, 1994; Betts, 2001) in which the values of both control and state variables at discrete grid points are taken as the decision variables of the nonlinear optimization problem.

The indirect techniques produce more accurate results than the direct ones. However, nowadays computational experience supports a view that the direct methods have several benefits compared to the indirect methods (e.g., Betts, 2001). The convergence domain of nonlinear programming methods is large. Thus, the direct methods provide reliable convergence and the initial estimate of iteration does not have to lie close to the optimal solution. The direct methods do not require the construction of the necessary conditions for optimality that would need a priori estimate for the sequence of unconstrained and constrained arcs of the solution trajectory as well as an initial estimate for adjoint variables.

Furthermore, there are available several versatile and effective implementations of nonlinear programming methods that can be utilized when solving discretized optimal control problems. Due to the advantages of the direct methods, they are convenient for automating the solution process of optimal control problems when moderate precision of optimal solutions suffices. This fact is also recognized in the automated solution approach introduced in Paper IV.

Feedback or closed-loop solutions for deterministic optimal control problems containing a simple aircraft model can be computed by using the method of dynamic programming (Bellman, 1957). The role of dynamic programming is more important in stochastic optimal control problems (e.g., Bertsekas, 2001) in which, for instance, the time development of a state is affected by a random event or the state is partially observable. The stochastic flight trajectory optimization problem of this type is considered in Paper V. Dynamic programming allows the determination of optimal controls for problems where a performance index is quadratic and a linear stochastic difference or differential equation represents the evolution of the decision environment in time. More complex problems can only be solved approximately by using, e.g., neuro-dynamic programming (see, e.g., Bertsekas and Tsitsiklis, 1996). Paper V elaborates an alternative approach based on influence diagrams for representing stochastic factors in the context of optimal air combat maneuvering. In this approach, computational difficulties are avoided by restricting the analysis to open-loop preference optimal controls only.

3.2.2 Approach towards the automated solution of optimal flight trajectories

The numerical solution of optimal flight trajectories with the theory of optimal control is not a straightforward task. One has to construct the equations of motion of an aircraft as well as to select appropriate control and state constraints. The state equations contain aircraft specific parameters whose values are known only as tabular data. The parameters must be approximated by using continuous and smooth functions that are required in the numerical solution. In addition, the necessary conditions for optimality or a parameterized optimization problem must be formulated. These formulations must be implemented and linked to a suitable boundary value problem solver or a nonlinear optimization package. Finally, resulting optimal trajectories should be visualized to facilitate their interpretation.

In order to be able to produce optimal flight trajectories rapidly and easily, the solution of optimal control problems should be automated. The automated approach is also needed when nonliterate persons without any specific background in optimization and mathematical modeling would like to generate and utilize optimal trajectories in analyses of air combat. Several software packages for automating the numerical solution process of optimal control problems are available. The implementations are constructed so that their user has to define an optimal control problem to be solved by modifying source code (e.g., Hargraves and Paris, 1987; von Stryk, 1993; Kraft, 1994; Schwartz, 1996) or a text file (e.g., Bless et al., 1993) and then the whole code must be recompiled. Some packages also offer a graphical user interface (Dolezal and Fidler, 1996; Gath and Well, 2001) but the modification of source code is still needed. Hence, the user of these implementations must be familiar with optimal control theory but, on the other hand, they allow the solution of different types of optimal control problems that are not restricted to a specific application area. An alternative design principle is to fix available optimal control problems by saving alternative state equations, constraints, and performance indices in a model library (Gath and Well, 2001).

Then, the user can define an optimal control problem under consideration simply by selecting appropriate components from the library.

In Paper IV, an interactive automated solution concept is developed. It outlines aspects that must be concerned when designing an aircraft trajectory optimization software that can be operated by a user without knowledge of the methods of optimal control theory, mathematical modeling, and programming. Following these aspects, an example implementation called Visual Interactive Aircraft Trajectory Optimization (VIATO) is constructed in Paper IV. This MS Windows-compatible software solves minimum time climb problems, minimum time trajectories to a fixed or moving target on the vertical plane, and interception problems in three dimensions. The software is easy-to-use because its graphical user interface hides the underlying mathematical theory.

The main design principle behind VIATO is to specify a class of trajectory optimization problems to be solved in advance by allowing only the use of a 3-DOF aircraft model and certain state and control constraints. Since the performance index, i.e., the total flight time, is also specified, the user avoids the modeling and explicit formulation of optimal control problems. However, it is possible to use different aircraft types that are distinguished with a set of aircraft specific parameter data saved in a model library. From this data, VIATO creates automatically approximations for drag coefficients and maximum thrust force that are needed in the optimization.

In an automated optimization generator, the exact accuracy of optimal solutions is not always as important as are reliable convergence features of a solution procedure and reasonable execution times. Hence, a direct solution technique is a suitable choice. In VIATO, the discretization of the trajectory optimization problems is carried out with direct collocation (Hargraves and Paris, 1987) or a scheme based on differential inclusions (Seywald, 1986) that are described in more detail in Paper IV. The methods are compared, e.g., in Raivio et al. (1996). The resulting parameterized optimization problems are solved by utilizing a nonlinear programming package NPSOL (Gill et al., 1986) that is an implementation of sequential quadratic programming (e.g., Bertsekas, 1995). Due to the large convergence domain of the optimization algorithm, the initial estimate of the decision variables can be generated automatically by choosing it from a straight line connecting given initial and terminal conditions. In addition, the scaling of the variables affecting the convergence of iteration can be made such that each state and control variable is normalized by the absolute maximum value of the initial guess.

The user of VIATO can solve an optimal flight path simply by choosing the parameters of an appropriate aircraft from the model library and selecting the problem type from among four optimization tasks mentioned above. Then, the user has to specify the initial and terminal states of the aircraft. Once the required information has been defined, the user interface calls the optimization routine. After the solution, the resulting optimal control and state histories are visualized in the user interface that is shown in Fig. 5. With the help of it, the results can be analyzed in different plots. Furthermore, the sensitivity of optimal trajectories with respect to the maximum thrust force, the drag coefficients as well as to the initial and terminal states can be studied.

A drawback in VIATO is that the resulting optimal flight trajectories are approximate due to discretization. The accuracy of solutions can be increased by adding the number of discretization points or by using a nonequidistant mesh (see, e.g., Betts and Huffman, 1998). Solutions obtained with VIATO can also be used as an initial estimate when resolving a problem by an indirect technique. On the other hand, the use of the 3-DOF point-mass model might lead to unrealistic flight paths. In spite of this, the point-mass assumption is widely used and accepted in flight trajectory optimization. The accuracy and validity of an optimal trajectory could be analyzed by reproducing the solution trajectory with a simulation procedure in which more detailed than the 3-DOF model is applied. The analysis of this type is carried out in Hoffren and Raivio (2000).

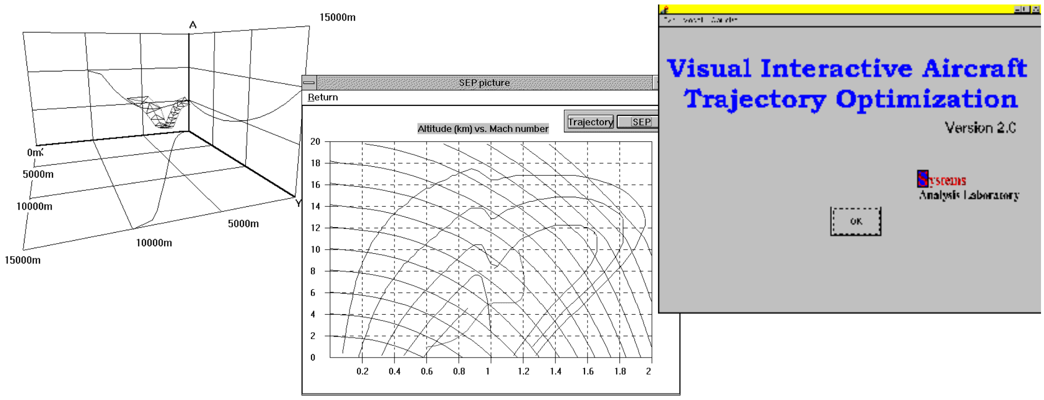

Figure 5. The user interface of VIATO.

3.2.3 Flight trajectory planning under uncertainty

The literature on flight trajectory optimization as well as Paper IV of the thesis have focused on deterministic problems. A main reason for this is that stochastic formulations are intractable due to the computational restrictions posed by the method of dynamic programming. However, for an optimization problem to produce realistic air combat trajectories, it should take into account the conflicting objectives and preferences of a pilot as well as uncertainties in the decision environment. These aspects are handled in the myopic influence diagrams presented in Papers I and II in which a single maneuvering decision is not affected by the upcoming decisions since the future states of the aircraft are anticipated for a short planning horizon ahead. In order to compute control sequences that are better with respect to the overall goals over the total flight time, the influence diagram must be able to predict the upcoming states of the combat further than one decision stage ahead.

In Paper V, the interaction of several successive maneuvering decisions is addressed in the context of influence diagrams. The paper formulates a multistage influence diagram that contains components describing the preferences of the pilot under uncertainty as well as the decision and aircraft dynamics. The multistage influence diagram is constructed by connecting several myopic decision models that are a simplified version of the influence diagram proposed in Paper I. The multistage model offers a novel way to incorporate preference and uncertainty models into flight path optimization problems. On the other hand, Paper V extends the formalism of influence diagrams into trajectory planning by combining a multistage influence diagram with an explicit model of the dynamic decision environment, which is represented by a set of difference equations. In addition, a new off-line solution procedure based on nonlinear programming for such a multistage model is introduced in Paper V. By using this procedure, one obtains optimal long-sighted flight paths with respect to an elicited preference representation.

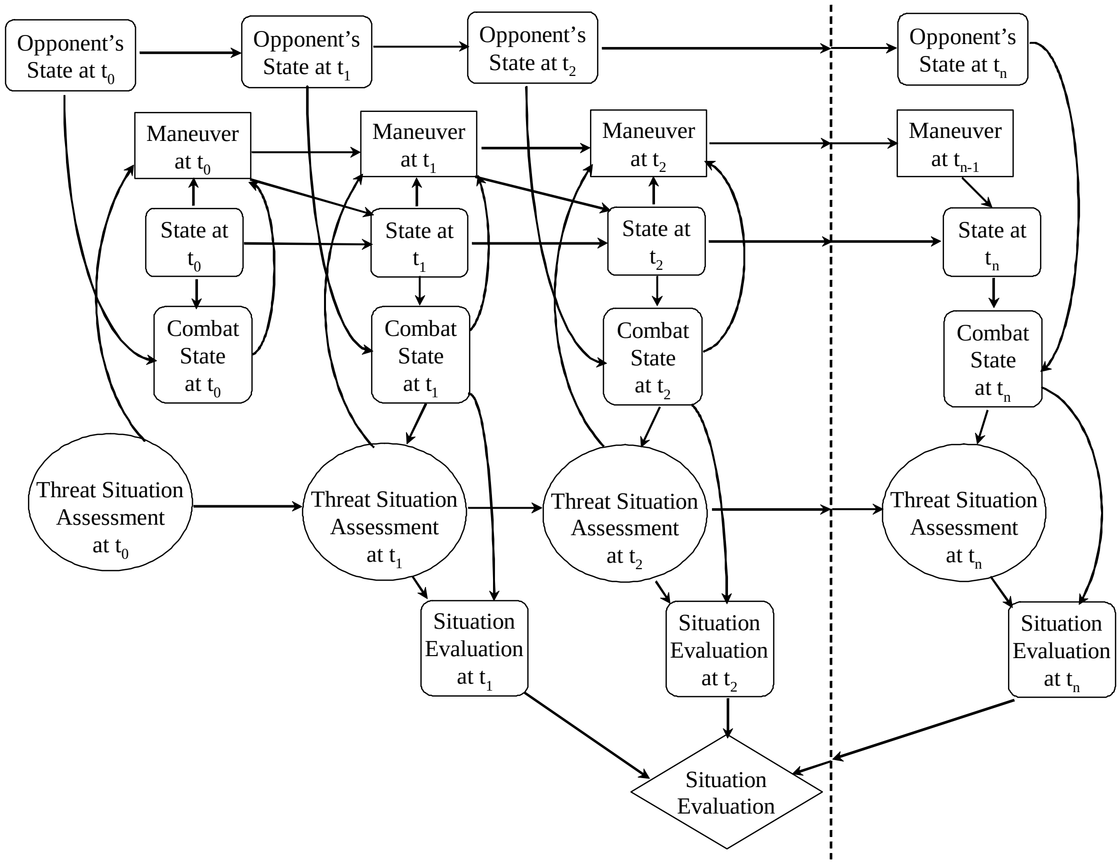

Figure 6. A multistage influence diagram for the sequence of the maneuvering decisions.

The constructed multistage model is shown in Fig. 6. It describes the sequential maneuvering decisions of the pilot in a one-on-one air combat setting in which an adversary aircraft is obeying a given air combat maneuver. It is assumed that the pilot is selecting controls such that he or she tries to avoid the adversary's weapons and, on the other hand, tries to achieve a firing position. Each decision node has a continuous control vector containing the control variables of a 3-DOF point-mass model. The current state of the pilot depends on the control and the state at the previous decision instant. This relationship is represented with the equations of motion that are here discretized by using Euler's method. The trajectory of the adversary is defined in deterministic nodes that contain the state vector of the adversary at each time instant. The momentary state of the combat is determined by the states of the players.

At each decision instant, the pilot is aware of the state of his or her own aircraft and the state of the combat. In addition, the pilot infers the threat situation. The threat situation assessment is modeled with a discrete random variable whose outcomes are associated with the relative geometry between the aircraft of the players. The probabilities of the outcomes are calculated on the basis of the current combat state and the probabilities of the previous threat node by using Bayes' theorem.

In Paper V, the combat situation is evaluated separately at each decision stage by a utility function. It reflects the pilot's preferences and, in practice, evaluates the possible consequences of the controls. Because the chain of controls maximizing the expected value of the cumulative utility over all the decision stages is computed, the overall utility node contains the sum of the single stage utilities.

Traditional solution methods of influence diagrams produce the best value for the decision variable as a function of the information available at the decision instant, i.e., the solution is in a closed-loop form. However, these solution methods require an enormous computational effort when a multistage influence diagram with several decision stages and alternatives or continuous control variables is to be solved. In Paper V, the computational difficulties are overcome by converting the multistage influence diagram into a discrete time optimal control problem that can be solved off-line using nonlinear programming methods. The sequence of myopic closed-loop solutions is used as an initial estimate for the decision variables of the optimization problem whose solution provides preference optimal maneuvering decisions maximizing the overall expected utility. The best controls are chosen without knowing the exact outcomes of the different uncertainties at the decision stages, i.e., the preference optimal solution is in an open-loop form.

Although the trajectory of the adversary must be predetermined in the multistage influence diagram elaborated in Paper V, this new modeling approach supports an air combat analysis in which the adversary is assumed to conduct a given tactics or basic fighter maneuver and one aims at design an optimal counter flight strategy with respect to the goals of the combat. Such optimal response flight paths obtained in different combat situations could provide a basis for the development of new combat maneuvers and give information for the planning of tactics. It should also be noted that because the solution procedure developed in Paper V generates an initial estimate with a myopic influence diagram utilizing feedback state information, the adversary's trajectory must not necessarily be fixed in advance. The adversary's trajectory can be determined during the generation of the myopic initial estimate by using, e.g., a batch air combat simulator or even a man-in-the-loop simulator. Then, the preference optimal long-sighted flight path against this adversary's trajectory is obtained by solving the discrete time optimal control problem corresponding to the multistage influence diagram, with the myopic solution as the starting point of iteration.

4. Concluding remarks and future research

The thesis consists of studies that concern optimal pilot decision-making and trajectory planning in air combat. The first three papers deal with the synthesis and analysis of combat decisions. Paper I focuses on the modeling of the maneuvering and missile launching decisions in on-one-on air combat. It develops an influence diagram that takes into account the inherent features – dynamics, uncertainty, and multiple conflicting objectives – appearing in air combat. Paper II takes another perspective on the same modeling problem and formulates the maneuvering decisions of the pilots involved in one-on-one combat as an influence diagram game. The elaborations presented in Papers I and II are the first applications of the methodology of influence diagrams that address a dynamic decision environment. Paper III focuses on team decision-making in a multiple aircraft scenario by constructing a value-focused prioritization model where issues on uncertainty are tackled with interval analysis. This modeling work provides a novel way to prioritize and select tactical data included in messages that are transmitted within a team of pilots by a data link system.

The two remaining papers deal with trajectory planning. Paper IV concerns the automated solution concept of deterministic optimal control problems. The paper gives design principles for a flight trajectory optimization software that can be run automatically by nonexpert users. According to these principles, an example implementation – Visual Interactive Aircraft Trajectory Optimization software – is programmed and reported. Paper V considers the representation of preferences and uncertainties in the design of optimal air combat maneuvering with the help of influence diagrams. A multistage influence diagram representing the series of maneuvering decisions is constructed and the new solution procedure for such a model is presented. This influence diagram framework enables a single pilot to find an optimal flight path with respect to a given preference model against a presupposed maneuver of the adversary. The work reported in Paper V extends the use of the formalism of influence diagrams into dynamic multistage decision situations such as trajectory planning problems.

In the thesis, the influence diagrams are applied for representing the actions of pilots in both short-sighted and long-sighted decision settings as well as in a game setting. Contrary to the current air combat modeling approaches, the influence diagrams unify analysis aspects with the simulation synthesis to provide models that are capable of answering analysis questions and synthesis needs in a concise, traceable, and transparent manner. They separate the structural and uncertainty aspects of a specific decision problem from aspects related to opinions and preferences of a human decision maker. The structured and well-laid foundations of the methodology of influence diagrams make the modeling process understandable for the experts of the substance area, too. Considering the contributions of the thesis, the work on influence diagrams can also be seen from another perspective. The thesis presents the extension of the methodology to dynamically evolving decision situations involving possibly multiple actors with conflicting objectives.

From the practical point of view, all the synthesis models proposed in the thesis can be utilized in decision logics of air combat simulators that enable versatile possibilities for analyzing air combat tactics and technologies. Such analyses can also be supported with optimal flight trajectories that are provided by the trajectory planning frameworks described in the thesis. On the other hand, the information prioritization approach can be implemented in an onboard data link system for improving the situation assessment capabilities of pilots in air combat. The thesis may also give insights into issues on the development of technologies required for commanding and controlling unmanned aerial vehicles that are an alternative to manned aircraft for conducting critical air missions in the future.

One of the main emphasis in the thesis is the application of methods originating from decision analysis in the quantitative formulation and analysis of air combat. The resulting decision models are prescriptive in their nature. That is, they support decision makers in making better decisions. However, if one accepts the underlying axioms of decision theory as principles for the rational choice, the models can be considered normative. The nature of optimization models is also normative, i.e., they provide the best solutions to specific problems with respect to given goals. The character of decision models should be descriptive when they aim at mimicing human decision-making, e.g., in a man-in-the-loop air combat simulation. Despite the prescriptive nature of the decision theoretical models, they also have the flexibility to represent some issues in a descriptive way, too. The construction of the models is, of course, completely different. Rather than decision theoretic techniques, behavioral studies are needed, which could be a research theme in the future.

The thesis also raises other research topics that should be considered in the future. Papers II and V provide a good basis for constructing a multistage influence diagram game that would represent the sequential decision-making processes of players in a one-on-one air combat (see Virtanen et al., 2004). Furthermore, there are several ways to refine the available influence diagrams. All the preference representations discussed in the thesis are assigned rather informally. Although the models would not be used for descriptive purposes, the preferences and behavior of pilots should be elicited by assessing required utility and value functions as well as probability distributions in cooperation with pilots. The structures of the influence diagrams can also be improved. One should consider questions related to incorporating features describing team decision-making in the influence diagrams as well as to representing elements involved in a combat between aircraft carrying missiles. It should also be noted that the myopic influence diagrams would allow the use of more detailed flight mechanical models than three-degrees of freedom. On the other hand, the dynamics in air combat should be taken into account in future work on the prioritization of tactical data.

The interactive automated solution concept of optimal control problems proposed in the thesis can be extended into other air combat related tasks and other application areas as well. In fact, such a refined implementation is already available (Järvenpää, 2003). In addition to minimum time trajectories, this software, whose user interface is shown in Fig. 7, solves an endgame setting in which the optimal avoidance maneuver for an aircraft pursued by a missile employing a given guidance law is determined. On the other hand, the effects of preferences on optimal trajectories could be analyzed effectively, if elicitation tools extracting preferences from pilots into the multistage influence diagram are combined with a trajectory optimization software. Besides optimal control, the automated solution approach could also be applied for pursuit-evasion games because they can be resolved robustly with a direct solution method developed recently (Raivio and Ehtamo, 2000a; Ehtamo and Raivio, 2001).

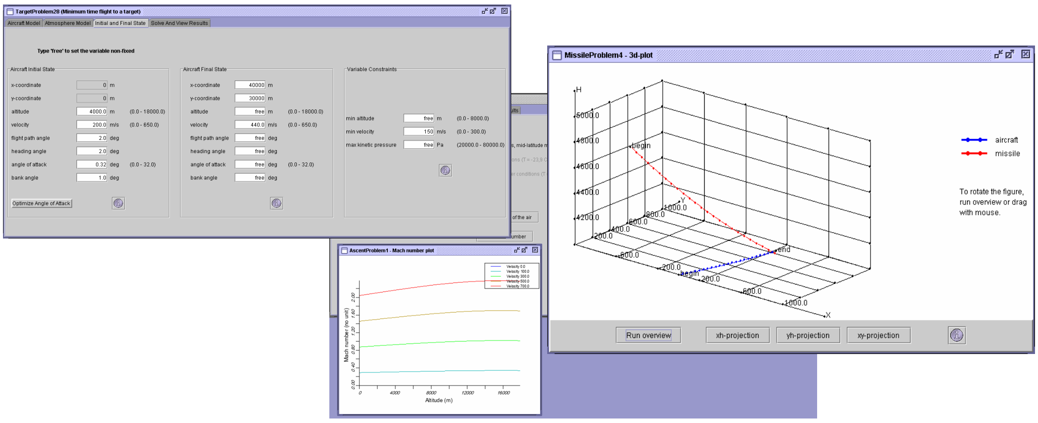

Figure 7. The user interface of the Missile Avoidance Trajectory Optimization software.

The problem area of the thesis – optimal air combat decision-making and maneuvering – is challenging because of difficulties in the formulation of air combat models that are both interesting and feasibly solvable. Optimal air combat tactics leading to the ultimate goal “win the war” are, and will be, unobtainable. Hence, one could ask about the pragmatic value of the thesis: Can we really improve our understanding of air combat by utilizing the ideas and the models innovated in the thesis? Although the new approaches offer several benefits compared to the current air combat modeling frameworks, it is impossible to give an unambiguous answer within the scope of this thesis. Therefore, more verification and validation studies should be conducted in close cooperation with air combat experts. Nevertheless, in the author's opinion, the thesis has taken the state of the art one step further by introducing a set of new ways of treating uncertainties in air combat modeling as well as in other dynamic decision settings. Due to the improvements on, e.g., stealth and unmanned aerial vehicle technologies, such uncertainty issues are of particular interest in the modeling of tomorrow's air-to-air combat.

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