

## **ELECTRICITY DERIVATIVE MARKETS: INVESTMENT VALUATION, PRODUCTION PLANNING AND HEDGING**

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**Title:** Electricity Derivative Markets: Investment Valuation, Production Planning and Hedging

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**Abstract:** This thesis studies electricity derivative markets from a view point of an electricity producer. The traditionally used asset pricing methods, based on the no arbitrage principle, are extended to take into account electricity specific features: the non storability of electricity and the variability in the load process. The sources of uncertainty include electricity forward curve, prices of resources used to generate electricity, and the size of the future production. Also the effects of competitors' actions are considered. The thesis illustrates how the information in the derivative prices can be used in investment and production planning. In addition, the use of derivatives as a tool to stabilize electricity dependent cash flows is considered. The results indicate that the information about future electricity prices and their uncertainty, obtained from derivative markets, is important in investment analysis and production planning.

**Keywords:** Asset Pricing, Real Options, Portfolio Selection, Electricity Markets, Forward Curve



## Academic dissertation

Systems Analysis Laboratory  
Helsinki University of Technology

### Electricity Derivative Markets: Investment Valuation, Production Planning and Hedging

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## Publications

The dissertation consists of the present summary article and the following publications:

- [I] Murto P., Näsäkkälä E., and Keppo J., 2004, Timing of Investments in Oligopoly under Uncertainty: a Framework for Numerical Analysis, *European Journal of Operational Research*, Vol. 157 (2), p. 486-500
- [II] Näsäkkälä E., Keppo J., 2005, Electricity Load Pattern Hedging with Static Forward Strategies, *Managerial Finance* Vol. 31 Number 6, special issue on Energy Pricing and Risk Management
- [III] Näsäkkälä E., Fleten S. E., 2004, Gas Fired Power Plants: Investment Timing, Operating Flexibility and Abandonment, Submitted Manuscript, Working Paper 04-03, Department of Industrial Economics and Technology Management, Norwegian University of Science and Technology
- [IV] Näsäkkälä E., Fleten S. E., 2004, Flexibility and Technology Choice in Gas Fired Power Plant Investments, *Review of Financial Economics*, to appear
- [V] Näsäkkälä E., Keppo J., 2004, Hydropower Production Planning and Hedging under Inflow and Forward Uncertainty, *Systems Analysis Research Report E15*, Helsinki University of Technology

## Contributions of the author

I am the principal author in the papers [II], [III], [IV], and [V]. In these papers I have been responsible of the writing, development and implementation of the models, and analysis of the results. The initial ideas of the papers [II] and [IV] are mine, whereas the initial ideas of the papers [III] and [V] are of the co-authors. In paper [I] I carried out the model implementation, helped to analyze the results and write the paper.



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# 1 Introduction

Since the beginning of the 1990's the energy, particularly electricity, markets around the World have started to move from centrally planned monopolies towards deregulation. In the deregulation process the pioneering countries have been the United Kingdom, New Zealand, the Nordic countries, and USA. Also the European Union has adopted directives to deregulate markets for electricity and natural gas (directives 96/92/EC and 98/30/EC). In a deregulated electricity market there is a publicly quoted market price set by the supply and demand of electricity. The emergence of the electricity markets has given birth to electricity derivative markets. An electricity derivative is a financial contract whose value depends on electricity price. For example, a forward contract is an obligation to buy or sell electricity for a predetermined price at a predetermined future time. Trading of electricity together with forward and option trading is done on electricity exchanges, e.g., Nord Pool (Scandinavia), APX (Netherlands), NYMEX (New York), and EEX (Germany). This thesis concentrates on the electricity derivative markets by taking a view point of a company whose earnings depend on the electricity prices. The main contribution of this thesis is the derivation of investment valuation and production planning methods that capture the typical characteristics of electricity price dynamics. The thesis illustrates how the price dynamics can be obtained from electricity derivative markets and how electricity derivatives can be used to stabilize electricity dependent cash flows. Before explaining the contribution of the thesis in detail, let me briefly summarize the basic theory in the pricing of derivative instruments.

The seminal work in the analysis of derivative instruments was done by Black and Scholes (1973) and Merton (1973) when they derived a method to calculate the value of a European call option, i.e. a right to buy one stock at a future date with a price specified in advance. According to their idea a stock option can be valued by constructing a replicating portfolio. A replicating portfolio consists of positions in the stock and cash markets giving exactly the same payoff as the option. As the payoffs are identical the option must be worth as much as the replicating portfolio. Otherwise, by trading the option, the stock, and cash traders could make earnings without initial investments. Note that such a trading brings the prices back into the line. This simple idea, called absence of arbitrage, has led to the theory of risk neutral valuation, which is formalized in a more general case, e.g., in Cox and Ross (1976), Harrison and Kreps (1979), Harrison and Pliska (1981), and Kreps (1981). The main result of risk neutral valuation is that in a complete market the relationship between risk and return of a financial asset can be characterized by a unique linear pricing function. The existence of a unique linear pricing function is equivalent to the existence of a martingale measure  $Q$  under which the expected returns of traded non-dividend paying financial assets are equal to the risk-free interest rate. In complete markets there are as many tradable assets as there are sources of uncertainty. When there are more sources of uncertainty than traded assets the markets are incomplete. In incomplete markets the linear pricing function, i.e. the pricing measure  $Q$ , is not unique.

Electricity is a non storable commodity and thus electricity derivatives cannot be replicated by trading electricity. In other words, electricity is not a financial asset and its expected returns under  $Q$  are not usually equal to the risk-free interest rate. On the other hand, the derivatives written on electricity spot price are storable, i.e. they are financial assets, and thus their returns should be equal to the risk-free interest rate. When electricity derivative markets are complete, i.e. each derivative written on electricity price can be replicated with some other derivative, the pricing measure  $Q$  for electricity dependent assets is unique and it can be estimated from derivative prices. The simplest derivative is a forward contract paying the difference of current forward price and the realized spot price at maturity.

The electricity derivatives can be used to stabilize future cash flows, for example the cash flows from future electricity production can be stabilized by selling a portfolio of electricity forwards that replicates the future cash flows. Typically, the electricity consumption and production processes are given by an exogenous stochastic process. For example, in the case of wind power production the total electricity production varies as a function of weather. This means that usually the electricity producers do not exactly know the size of their future production. Due to this additional source of uncertainty there are more sources of uncertainty in the electricity portfolio than there are derivative instruments available,

and thus the electricity hedging needs to be done in an incomplete market. The literature studying portfolio selection in incomplete markets originates from Karatzas et al. (1991), Cuoco (1997), and Cvitanic et al. (1997).

Derivative instruments have an analogy with investment opportunities. Most of the investment opportunities share the following three characteristics:

- The investment is partially or completely irreversible.
- The future cash flows from the investment contain uncertainty.
- The timing of the investment contains some leeway.

Investment opportunities satisfying the above mentioned three characteristics can be seen as American call options where the underlying product is the investment project. McDonald and Siegel (1985), Brennan and Schwartz (1985), McDonald and Siegel (1986), Pindyck (1988), and Dixit (1989) were the first to analyze physical investments using the option pricing theory. Their ideas led to the theory of investment under uncertainty. This theory is often called real options theory, due to the analogy between investments and derivative instruments.

This thesis contributes both to real options and portfolio selection literature. The papers [III] and [IV] use the real options theory for natural gas fired power investments when also gas prices are uncertain. The introduced simple but realistic price process enables the calculation of investment thresholds above which investing is optimal. The paper [I] belongs to the literature stream combining real options analysis and game theory. The proposed numerical method can be used to calculate optimal investment strategy when there are several investment opportunities and the competitors' actions affect the profitability of each investment. Generally, these three papers show that neither the competitors' actions nor the uncertainties in the fuel prices should be disregarded when investment to electricity production units are considered. Papers [II] and [V] belong to the portfolio selection literature as they study electricity production planning and hedging by using electricity forwards. In paper [II] the distribution for the value of stochastic electricity load pattern is calculated analytically and then the distribution is used to calculate optimal hedging strategy. The proposed method simplifies the calculation of optimal hedging strategies considerably and thus it can be used to analyze the behavior of different electricity market participants. Paper [V] illustrates how electricity forwards can be used in hydropower production planning. Instead of the traditionally used spot price models, in [V] the electricity forwards and their dynamics are used as a price process. The bench marking of the method reveals that the profits of an actual hydropower producer would have increased if the information in electricity forward markets had been fully utilized. The papers [I], [III], [IV] can be used when investments to electricity production units are considered, whereas the papers [II] and [V] are intended to help every day operations of an electricity producer. Even though the papers contribute to slightly different fields of application they all use the same idea that the electricity forward markets can be used to obtain information about future price and risk. Thus, generally this thesis consists of studies that illustrate how the electricity production industry can utilize electricity derivative markets. The contributions of each article are described more thoroughly in Section 5, however before that, in sections 3 and 4 the basic theory of investment valuation and production planning is introduced. In Section 2 the source of price information, i.e. the forward curve and its dynamics are introduced in more detail.

I want to motivate this thesis by answering to some of the criticism that I have faced during this project. First, the research on derivative markets is sometimes questioned that it is about trying to be cleverer than the other market participants. In this thesis I am not trying to predict the future changes in the market prices. The developed models just take the existing information about the market prices and their risks and derive the optimal strategies based on this information and the no arbitrage condition. Thus, I just try to characterize the rational behavior of a value maximizing firm operating on the electricity spot and derivative markets. Second, the production hedging has been criticized that since trading derivative instruments does not produce or consume any electricity it is nothing but a zero-sum game. The role of the derivative markets is to facilitate the flow of capital to electricity industry with the least possible costs. For example, consider an electricity producer borrowing capital to expand its

production capacity to meet the increased demand. By hedging the future production the company can reduce the uncertainty in the future cash flows and thus decrease the possibility of not being able to pay back the borrowed capital. The reduction in the risks of the future cash flows gives better terms for the loan.

This summary article is structured as follows. Section 2 introduces the electricity forward curve dynamics. In Section 3 the characteristics of electricity production investments are summarized. In Section 4 production planning and hedging in the electricity markets are discussed. Section 5 reviews the thesis shortly and summarizes the contributions of each article. Finally, Section 6 concludes this summary article.

## 2 Forward curve dynamics

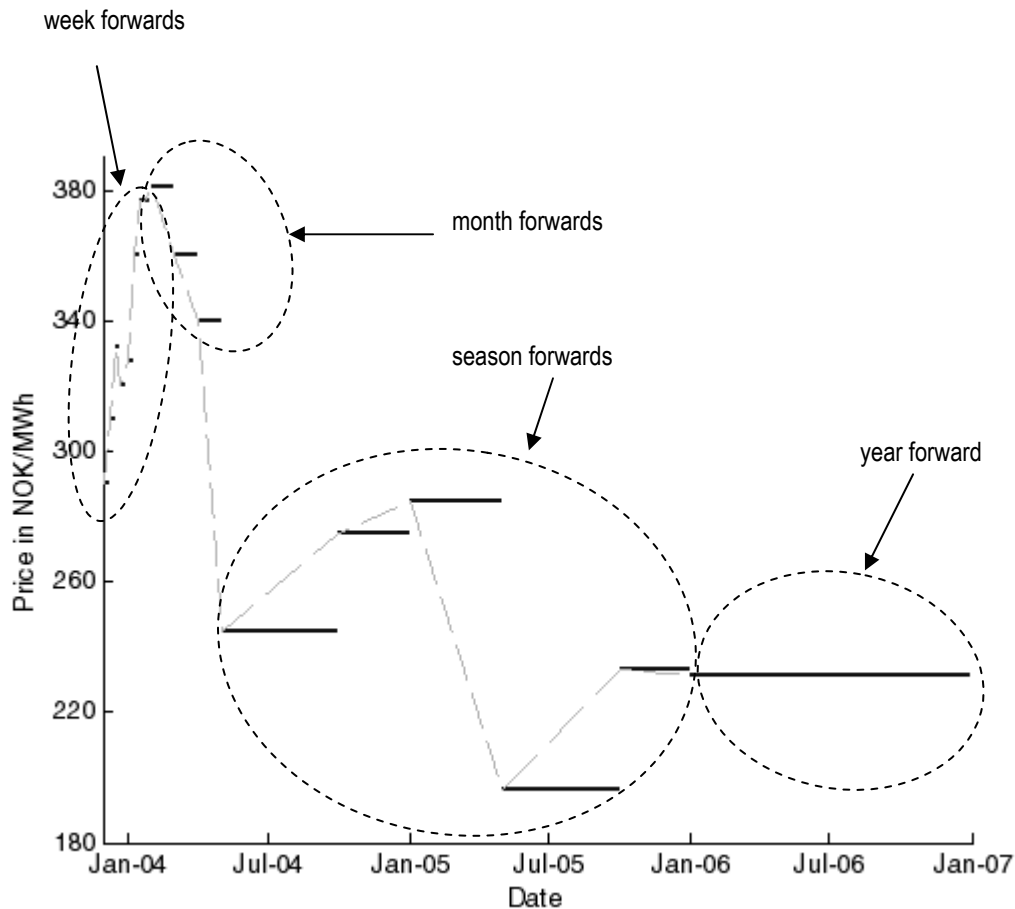
The main contribution of this thesis is to illustrate models for electricity investment valuation and production planning based on the information in electricity derivative markets. Electricity forwards are the simplest and most common derivatives and thus the modeling of electricity forward curve dynamics forms the base for the whole process. When a forward contract is traded, there is a market participant willing to buy electricity at a specified future date with the price dictated by the forward contract. On the other hand, there is also a market participant that is willing to sell electricity with the same price. This means that the forward prices reflect the market's expectations about the future supply, demand and risk of electricity, i.e. the forward prices can be seen as the risk-adjusted expected future spot prices. The time to the delivery of the electricity is called maturity. The different maturity electricity forwards form an electricity forward curve. There are cycles and peaks in the forward curve due to the seasonality in the supply and demand of electricity. The starting point of the forward curve approaches electricity spot price as the forward maturity goes to zero. Figure 1 illustrates the electricity forward curve in Nord Pool on the 30th of November 2003. The currency used in Nord Pool is a Norwegian krone (*NOK*), whose exchange rate is around  $7 \text{ NOK/USD}$ . There are week, month, season, and year forwards in Nord Pool. The delivery period, i.e. the duration of the forward contracts increases as a function of the maturity. Week contracts are traded for nearest 8 weeks, month contracts for nearest 6 months, season contracts for nearest 6 seasons, and year contracts for nearest 3 years. Each year is divided into three seasons: late winter (January- April), summer (May- September), and early winter (October- December). The forward curve in Figure 1 illustrates that the market expects electricity prices to increase in December and January and then decrease towards summer. Due to the cold winter in Scandinavia the prices in January- April are traditionally higher than in summer and autumn. The Nord Pool's forward curve gives information about the expected weekly variations only for the nearest 8 weeks. In many cases a forward curve with weekly granularity is needed over a longer period than these 8 weeks. For example, Fleten and Lemming (2003) present a method to estimate a weekly forward curve based on longer-term forwards and forecasts generated by bottom-up models.

Björk and Landén (2000), Miltersen and Schwartz (1998), and Clewlow and Strickland (1999), among others, consider the modeling of electricity forward curve dynamics. Koekebakker and Ollmar (2001) and Audet et al. (2003) report that the key risk factors in the variations of the electricity forward curve are:

- The spot volatility curve.
- The volatility curve's maturity effect.
- The forward curve's correlation structure.

The spot volatility curve captures the seasonal variations in the spot price uncertainty. Forwards whose delivery date is further ahead tend to vary less than forwards with shorter maturity. This decrease in forward volatility as a function of forward maturity is described by maturity effect. Forwards whose maturity dates are closer to each other correlate more than forwards whose delivery dates are farther apart. The forward curve's correlation structure captures this phenomenon. In this thesis paper [II]

studies static forward hedging strategies for uncertain electricity production/consumption processes when the parameterization of the forward curve dynamics is similar to Koekebakker and Ollmar (2001) and Audet et al. (2003). In paper [V] optimal hydropower production and hedging strategies under similar forward curve parameterization are studied.



**Figure 1:** Nord Pool's forward curve on the 30th of November 2003

When the valuation of electricity dependent assets is based on the electricity forward curve there is no possibility for arbitrage. The idea is similar to the Heath et al. (1992) framework, which was originally used for interest rate markets. The following example motivates the importance of the forward curve as a source of future price estimates when electricity dependent assets are valued. A company owns 16 MWh of electricity production capacity. The next week's production will be sold on an electricity market, where the next week's spot price is 250 NOK/MWh with probability 0.5 and 160 NOK/MWh with probability 0.5. On the forward markets the next week's forward price for electricity is 200 NOK/MWh. Note that there is nothing inconsistent in  $200 \neq 0.5 \cdot 250 + 0.5 \cdot 160$ . The next week's spot price estimates are under the objective measure  $P$ , whereas the forward price is under the pricing measure  $Q$ . The forward markets have priced the risk associated with the next week's spot price. For simplicity, I disregard discounting in this example and thus the value of the next week's production, under measure  $Q$ , is 3200 NOK. Under the objective measure  $P$  the value is 3280 NOK. The market value of the next week's production is the one given by the forward prices not the one given by the spot price estimates. The company can sell the next week's production on the forward markets with 3200 NOK not with 3280 NOK. Correspondingly, if the value under  $P$  is higher, of course the company is not going to sell the production below what can be obtained on forward markets.

### 3 Investment analysis

When long-term commodity projects are valued, the forward curve dynamics can be approximated with spot price models using stochastic convenience yield (see, e.g., Schwartz, 1997 and 1998). Traditionally, in the real options literature geometric Brownian motion is used to describe the price

variations. Laughton and Jacoby (1993, 1995), Cortazar and Schwartz (1994), and Smith and McCardle (1999), among others, argue that mean-reverting price models describe better variations in the commodity prices than the geometric Brownian motion models. In an equilibrium setting increase in electricity prices attracts high cost producers to the market putting downward pressure on prices. Conversely, when prices decrease some high cost producers will withdraw capacity temporarily, putting upward pressure on prices. As these entries and exits are not instantaneous, prices may be temporarily high or low, but will tend to revert toward equilibrium price. Based on this intuition Schwartz and Smith (2000) develop a two-factor model for commodity spot prices where the short-term deviations are modeled with a mean-reverting process and the equilibrium price evolves according to a Brownian motion. Other two-factor models with long- and short-term factors are, for example, Ross (1997) and Pilipović (1998).

The value of an electricity production unit is usually not just a function of electricity price, as also the fuels used for electricity generation have a market price. The spread between the electricity price and the cost of fuel used to generate electricity is called spark spread. For example, Deng et al. (2001) and Deng (2003) study spark spread based valuation of electricity generation and transmission assets. In this thesis paper [III] studies investments in natural gas fired power plants using a mean-reverting two-factor model for spark spread between electricity and natural gas. The example in paper [III] illustrates that when the decision to build a gas fired power plant is considered, the abandonment option does not have significant value, whereas the operating flexibility and the time-to-build option have significant effect on the building threshold. The value of the operational flexibility is caused by the plant's ability to react to short-term variations in the spark spread. This option to alter the plant's operating scale partly compensates the option to temporarily shut down the plant and thus the value of the abandonment is insignificant.

During the last decade concerns about the environmental impacts of the fossil fuel based energy production has increased. The most alarming issue is the global warming caused by the emission of green house gases, in particular CO<sub>2</sub>. To reduce the green house gas emissions, without considerably reducing consumption, the existing fossil fuel based production capacity should be upgraded to production capacity with fewer emissions. To foster these changes in European Union the trading of emission allowances started in October 2003. The value of an emission allowance affects the cash flows of a fossil fuel fired power plant over the entire lifetime of the plant. In this thesis paper [III] studies how the emission costs and their uncertainty affect the natural gas fired power plant investments. Intuitively, change in emission costs can be seen as a change in the spark spread and hence increase in emission costs decreases spark spread whereas increase in the uncertainty of the emission costs increases the variance of the spark spread.

The electricity production units have often flexibility in the selection of the production strategy. The real options theory can be used to value this operational flexibility. For example, Kulatilaka (1993) studies the flexibility to switch between two fuels in an industrial steam boiler, whereas Brekke and Schieldrop (2000) study the input flexibility in a power plant when the plant can be either gas or oil fired, and He and Pindyck (1992) study the output flexibility for a firm having two possible products to produce. Paper [IV] in this thesis studies the flexibility in the choice of optimal technology for a gas fired power plant when the spark spread between electricity and gas follows a mean-reverting two-factor process. The plant can be built either as a peak or as a base load plant. A peak load plant generates electricity when spark spread exceeds emission costs, whereas a base load plant generates electricity at all levels of spark spread. A base load plant can be upgraded to a peak load plant. The numerical example in paper [IV] illustrates that increase in the spark spread's short-term variations makes a peak load plant more attractive choice by increasing the value of a peak load plant. On the other hand, an increase in the long-term uncertainty makes base load plant more favorable by postponing the upgrading decision.

Due to the strict restrictions on the investments in electricity production units markets for such investments consists of a relatively small number of competing firms. In such oligopolistic settings strategic interactions between the firms must be explicitly accounted for. For example, Smit and Ankum (1993), Kulatilaka and Perotti (1998), Grenadier (1996), Lambrecht (1999), and Murto (2004) study

single investments under competition by combining real options theory and game theory. Baldursson (1998) and Williams (1993) study the dynamics of incremental investments in similar setting. In this thesis paper [I] extends the literature stream combining real options and game theory to the timing of lumpy investment projects under uncertainty and oligopolistic competition by introducing a numerical method to study investment dynamics in an oligopoly when there are multiple lumpy investment opportunities. The numerical example in paper [I], illustrating the trade-off between the value of flexibility and economies of scale under competitive interaction, is given in building of telecommunications capacity. However, the model can also be applied for electricity, e.g. wind power or geothermal power investments.

## 4 Production planning and hedging

Production hedging is the process of reducing the risks in the cash flows of future production. One way to do this is to find a portfolio of standard derivative instruments that replicates the future cash flows. Selling this portfolio provides the hedge. The optimal portfolio choice in complete markets is studied, for example, in Cox and Huang (1989), Karatzas et al. (1987), Cvitanic and Karatzas (1996), and Shreve and Soner (1994). These models consider continuous hedging involving constant trading. In reality transaction costs and illiquidity concerns can make the use of dynamic hedging strategies expensive and difficult. Carr et al. (1998), and Carr and Wu (2002), among others, study static portfolios of standard derivatives replicating the payoffs of a given derivative. These hedging strategies are called static hedging strategies as the optimal weights of the derivatives in the portfolio do not change when the underlying asset changes. Hence, the replicating portfolio does not need to be adjusted dynamically. When static hedging strategies are used the transaction costs are lower and the implementation of the hedging strategies is easier. For example, Cvitanic et al. (1999) have shown that under proportional transaction costs the minimal super-replicating strategy for European type contingent claims is the least expensive buy and hold strategy.

From a view point of a market participant one major difference between electricity and stock markets is that electricity consumption/production is given by an exogenous process while a stock investor can decide asset holdings himself. For example, in the case of hydropower production the total electricity production depends on the inflow to the hydro reservoirs, which depends on the amount of rainfall. Often the uncertainty in the production/consumption process does not perfectly correlate with the electricity derivative prices. This means that the load uncertainty cannot be totally hedged with those derivatives. There is always some load risk that the producer can not hedge.

The portfolio management in electricity markets has been studied, for example, in Pilipović (1998), Fleten et al. (2002), and Vehviläinen and Keppo (2003). Following simple example illustrates the electricity production hedging. A company owns a production capacity of 16 *MWh*. The next week's forward price is 200 *NOK/MWh*. In addition, the company knows that the next week's spot price is 150 *NOK/MWh* with probability 0.5 or 250 *NOK/MWh* with probability 0.5. For simplicity, I assume in this example that the objective measure  $P$  is equal to the pricing measure  $Q$ . For same reason I also disregard discounting in this example. By selling 16 *MWh* of next week's forwards the company receives certainly 3200 *NOK* of its production. On the other hand, if the company does not sell forwards it receives 4000 *NOK* if the price increases but only 2400 *NOK* if the price decreases. Thus, in both cases the expected value of the future production is 3200 *NOK*. However, by selling the forwards the portfolio variance decreases to zero.

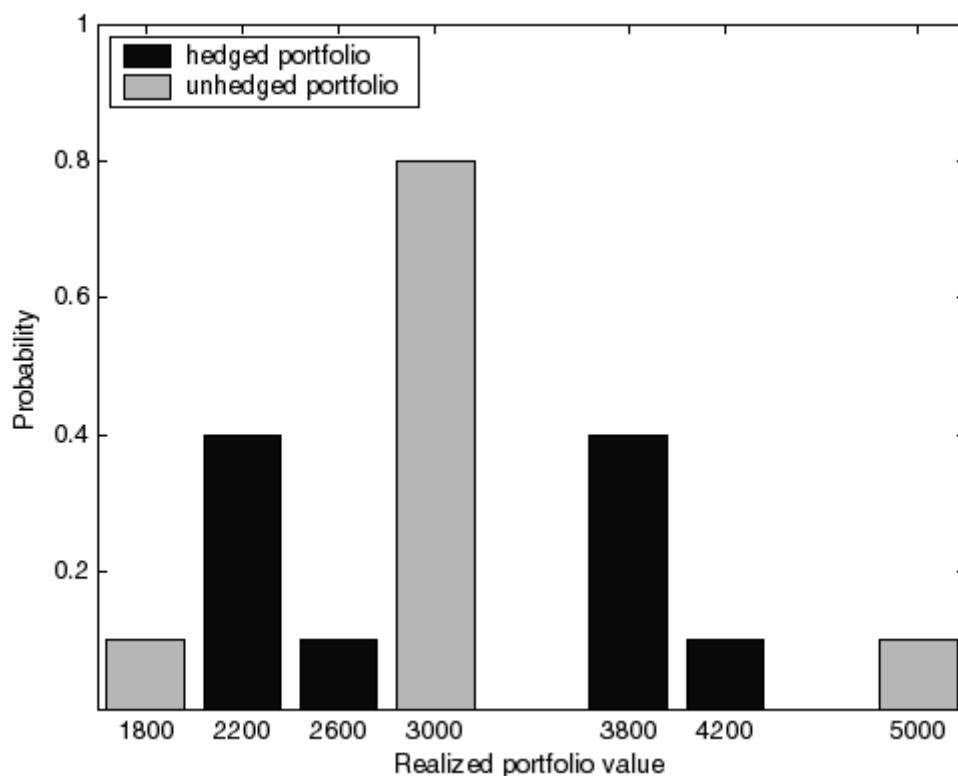
In the previous case the size of the production did not have any uncertainty, i.e. the hedging decision was done in a complete market. In the next example also the production has uncertainty which partly correlates with price estimates. More precisely, there are two possible sizes for the production and two different values for the spot price, i.e. there are four possible production-price scenarios for the next week. The probabilities of different scenarios are given in Table 1. The production and price have a negative correlation, i.e. when the price is low the production is 12 *MWh* with probability 0.2 and 20

MWh with probability 0.8. Note that in this case the expected value of the production is 16 MWh, which is equal to the deterministic production in the previous case.

**Table 1:** The probabilities of different production price scenarios

Price/Production	150 NOK/MWh	250 NOK/MWh
12 MWh	0.1	0.4
20 MWh	0.4	0.1

The next week's forward price is again 200 NOK/MWh. In Figure 2 the probabilities of different portfolio values, when the production is hedged by selling 16 MWh of forwards, are illustrated with black bars. A portfolio which is not hedged is illustrated with grey bars. As there is also uncertainty in the production process the hedging does not totally remove the uncertainty in the future cash flows. The future value of the unhedged portfolio varies between 1800-5000 NOK, whereas the future cash flows of the hedged portfolio vary between 2200-4200 NOK. The standard deviation of the unhedged portfolio is 733 NOK and the standard deviation of the hedged portfolio is 816 NOK. Hence, in this example hedging using the expected production value increases the standard deviation of the portfolio. This illustrates how the uncertainty in the production process complicates the production hedging decisions considerably. Paper [II] in this thesis studies static hedging strategies for uncertain electricity production/consumption processes. In paper [II] the distribution of a log-normally distributed production/consumption process is calculated analytically when the electricity forward curve has dynamics similar to Koekebakker and Ollmar (2001) and Audet et al. (2003). The solved distribution is then used to derive conditions for the portfolio's optimal hedge ratio and hedging time. Paper [V] in this thesis studies forward hedging of hydropower plant under inflow and price uncertainty. Paper [V] shows that an optimal production strategy for a value maximizing hydropower producer is a bang-bang strategy, which can be characterized by a threshold function for electricity spot price. In paper [V] this threshold is used to calculate the optimal forward hedging strategy for the hydropower plant.



**Figure 2:** Probabilities of different portfolio values

## 5 The thesis

This thesis can be divided into two different parts. The first part, consisting of papers [I], [III], and [IV], studies investments in electricity production units. These models can be used when electricity production investments are considered. These articles can be seen as applications of the real options literature. The second part consists of models for production planning and hedging in the electricity markets, i.e. papers [II], and [V]. These models can be used in the every day operations of an electricity producer/consumer. The articles [II] and [V] can be seen as applications of portfolio selection literature. In all of the papers the electricity forwards are used to estimate the future price dynamics. Next, the contributions of each article are summarized.

### **[I] *Timing of Investments in Oligopoly under Uncertainty: a Framework for Numerical Analysis***

This paper presents a modeling framework for the analysis of investments in an oligopoly market for a homogenous commodity. Smit and Ankum (1993), Kulatilaka and Perotti (1998), Grenadier (1996), Lambrecht (1999), and Murto (2004), among others, study single investments under competition by combining the real options theory and game theory. Baldursson (1998) and Williams (1993) study the dynamics of incremental investments in similar setting. This paper extends the above mentioned literature stream to the timing of lumpy investment projects under uncertainty and oligopolistic competition. This paper presents a systematic modeling framework to analyze a market with the following properties. There are several large firms producing a homogeneous and non-storable commodity, the demand for which evolves stochastically. The commodity price is influenced by two factors: demand uncertainty and new capacity investments. The firms have a number of investment opportunities available, which they can use to adjust their production cost functions or maximum capacities. In choosing the timings for such investments, the firms take into account the demand uncertainty and the actions of the other firms. Thus, the market can be seen as a game with two opposite factors affecting the optimal investment timing. On one hand, uncertainty and irreversibility give an incentive to postpone the investment. On the other hand, the firms recognize the preemptive effect of the investment against the other investors, which increases the incentive to act quickly. The model is illustrated with an example measuring the trade-off between the value of flexibility and economies of scale under competitive interaction.

### **[II] *Electricity Load Pattern Hedging with Static Forward Strategies***

This paper considers the partial hedging of stochastic electricity load pattern with static forward strategies. Fleten et al. (2002) study a scenario-based approach for solving the optimal portfolio management problem in electricity markets, and Vehviläinen and Keppo (2003) use Monte Carlo simulation in the managing of electricity market price risk. In this paper, instead of utilizing numerical techniques, the distribution of the portfolio value is solved analytically. Methods to calculate optimal hedge ratio and optimal hedging time for a stochastic electricity load pattern are given when the forward curve dynamics have a parameterization similar to Koekebakker and Ollmar (2001) and Audet et al. (2003). In contrast to Caldentey and Haugh (2003) that consider combined production and dynamic hedging decisions, in this paper the use of static hedging strategies are studied. In the example section the hedging behavior of different electricity market participants are analyzed. The numerical results illustrate that agents with high load volatility should hedge later than agents that have low load volatility and that negative correlation between forwards and electricity load pattern should postpone the hedging timing.

### **[III] *Gas Fired Power Plants: Investment Timing, Operating Flexibility and Abandonment***



This paper analyzes investments in gas fired power plants under stochastic electricity and gas prices. The gas fired power plant's operating cash flows depend on the spark spread, defined as the difference between the price of electricity and the cost of gas used for the generation of electricity. This paper makes some extensions to the existing literature studying spark spread based valuation of power plants (see, e.g., Deng et al., 2001 and Deng, 2003). First, the use of a two-factor price process, similar to that of Schwartz and Smith (2000), enables the incorporation of the typical characteristics of the non-storable commodity prices, i.e. short-term mean-reversion and long-term uncertainty. The short-term dynamics are needed for the valuation of the plant's operational flexibility, i.e. the plants ability to change production capacity according to variations in the spark spread. Second, this model takes into account the option to postpone investment decisions. The numerical example illustrates that when the decision to build a gas fired power plant is considered, the abandonment option does not have significant value, whereas the operating flexibility and the time-to-build option have significant effect on the building threshold. The value of the operational flexibility is caused by the plant's ability to react to short-term variations in the spark spread. This option to alter the plant's operating scale partly compensates the option to temporarily shut down the plant and thus the value of the abandonment is insignificant.

#### ***[IV] Flexibility and Technology Choice in Gas Fired Power Plant Investments***

This paper analyzes the flexibility in gas fired power plants under stochastic electricity and gas prices. He and Pindyck (1992) study the output flexibility for a firm having two possible products to produce. Brekke and Schieldrop (2000) study the input flexibility in a power plant when the plant can be either gas or oil fired. In this paper, the flexibility is in the choice of production strategy, not in the choice of input or output. A peak load plant generates electricity only when spark spread exceeds emission costs, whereas a base load plant generates electricity at all levels of spark spread. A plant operating as a base load plant can be upgraded to a peak load plant by changing into a more flexible gas supply system. The optimal technology for a gas fired power plant together with upgrading threshold and building threshold for the investment costs are solved. The numerical example illustrates that increase in the spark spread's short-term variations makes a peak load plant more attractive choice by increasing the value of a peak load plant. On the other hand, an increase in the long-term uncertainty makes base load plant more favorable by postponing the upgrading decision.

#### ***[V] Hydropower Production Planning and Hedging under Inflow and Forward Uncertainty***

This paper studies production planning and hedging in a hydropower plant under inflow and price uncertainty. For example, Gjelsvik and Wallace (1996), Fosso et al. (1999), Mo et al. (2001), and Fleten et al. (2002) use stochastic programming to hydropower production planning in deregulated electricity markets. This paper takes a slightly different approach. Instead of traditionally used stochastic programming methods the optimal production strategy is derived by applying the martingale method of optimal consumption and portfolio selection (see, e.g., Cox and Huang 1989, and Karatzas et al., 1987). The paper shows that an optimal production strategy for a firm that maximizes its value is a bang-bang strategy, which can be characterized with a threshold function for electricity spot price. A simple and intuitive parameterization for the production threshold is suggested when the electricity price dynamics are characterized by the parameterization of electricity forward dynamics given in Koekebakker and Ollmar (2001) and Audet et al. (2003). The bench marking reveals that during winters 1997-2003 an actual hydropower producer could have increased its earnings by more than 4.2% if the information in the electricity derivative markets had been fully utilized.

## **6 Conclusions**

This thesis consists of five articles that consider investment and production decisions under stochastic electricity prices. The models presented try to help electricity producers to understand the risks and opportunities caused by the variations in the future electricity prices. Also methods to hedge against

adverse price changes are introduced. In addition to price uncertainty, electricity market participants typically face other sources of risks that partly correlate with the price risk. Paper [I] considers the effects of competitors' actions on the optimal investment strategy. Papers [III] and [IV] study investments to natural gas fired power plants when also gas prices are uncertain. In papers [II] and [IV] the other source of uncertainty is the size of the future production.

In this thesis I have assumed that the future price estimates can be obtained from forward markets. This is not always the case, for instance plant's location in a rural area can cause price disparity between the closest forward market and the realized electricity price. However, with small modifications the models presented are also applicable when forward information is not available. In this case the future electricity prices need to be estimated and the risk adjustment inherently present in the forward prices needs to be accounted for by using a risk-adjusted discount rate. An estimate for future electricity prices can be obtained, for example, as an equilibrium between future supply and demand estimate.

Concerns about the environmental impacts of the fossil fuel based energy production are becoming more and more important. In a deregulated electricity market each market participant makes investment and production decisions based on its own preferences. In other words the electricity markets do not give incentives for value maximizing companies to build production capacity with fewer emissions unless production capacity with fewer emissions is made price competitive. This means that the policy makers and market regulators, whose objective is to maximize the social surplus from electricity production industry need to design market rules that give incentives for the market participants to end up in decisions that help to reach the sustainable emission levels. Based on this kind of thinking several countries have started, or are planning to start, projects making new production technologies price competitive. For example, in European Union the trading of emission allowances started in October 2003. In addition to emission allowances, also investment subsidies and tax incentives are often used to subsidize emerging technologies. Thus, one possible field of application for this thesis is to study how investment subsidies, tax incentives, emission allowances, and their uncertainty together with the uncertainty in the technological development affect the behavior of a value maximizing investor.

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