Helsinki University of Technology

Systems Analysis Laboratory Research Reports A99, November 2007

MODELING AND ON-LINE SOLUTION OF AIR COMBAT OPTIMIZATION PROBLEMS AND GAMES

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Dissertation for the degree of Doctor of Technology to be presented with due permission for public examination and debate in Auditorium E at Helsinki University of Technology, Espoo, Finland, on the 18th of January, 2008 at 12 o'clock noon.

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ISBN 978-951-22-9107-6
ISBN 978-951-22-9108-3 (PDF)
ISSN 0782-2030

Multiprint Oy
Espoo 2007

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Date: November 2007

Abstract: The thesis deals with the guidance and control of a fighter aircraft in air combat. From the modeling perspective, the thesis formulates realistic air combat optimization problems and games in which inherent uncertainties of air combat are taken into account. From the solution perspective, the thesis develops on-line methods for the near-optimal feedback solution of the constructed models. The solution methods are based mostly on receding horizon control, where computational savings are achieved by using a truncated planning horizon. Considering single-sided optimization problems, the thesis presents a new approach for the guidance of an aircraft avoiding a homing air combat missile. The approach is applicable with various avoidance criteria that exploit different weaknesses of the missile system. In addition, a novel way based on Bayesian reasoning for taking into account the target's uncertainty about the guidance law of the missile is introduced. An approach and its software implementation for the user-oriented computation of realistic near-optimal aircraft trajectories are presented as well. The software can be used to assess the quality and realism of the solutions provided by the introduced guidance schemes. Considering games, the thesis presents an influence diagram game modeling a dogfight between two aircraft and develops a method for the on-line optimization of the aircraft's controls in such a setting. The game formulation enables the consideration of preferences, perception, and beliefs in air combat. In addition, a new game model and an on-line solution scheme providing game optimal support time of the missile in a missile duel is introduced. Considering practical implementation, the introduced models and solution methods could be further developed and consolidated in an onboard guidance system or in a pilot advisory system.

Keywords: air combat, missile avoidance, computational optimal control, game theory, receding horizon control, adaptive control, Bayesian reasoning, influence diagram, inverse simulation

Otsikko: Ilmataisteluun liittyvien optimointitehtävien ja pelien mallintaminen ja reaaliaikainen ratkaiseminen

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Tiivistelmä: Väitöskirja käsittelee hävittäjän ohjaamista ilmataistelussa. Väitöskirjassa konstruoidaan ilmataistelua kuvaavia optimointi- ja pelimalleja, joissa otetaan huomioon ilmataisteluun liittyviä epävarmuustekijöitä. Lisäksi kehitetään reaaliaikaisia laskentamenetelmiä mallien lähes-optimaalisten takaisinkytkettyjen ratkaisuiden saamiseksi. Ratkaisumenetelmät perustuvat etupäässä etenevän suunnitteluhorisontin säätöön. Metodiikkaa käytetään ohjuksenväistöön soveltuvassa hävittäjän ohjausmenetelmässä, jota voidaan soveltaa monilla ohjussysteemin eri heikkouksia hyödyntävillä väistökriteereillä. Menetelmä laajennetaan adaptiiviseksi, Bayes-päättelyä hyödyntäväksi ohjausmenetelmäksi, joka ottaa huomioon hävittäjän epävarmuuden ohjuksen ohjauslaista. Ohjuksen väistöongelmaa tarkastellaan myös pelinä, jonka ratkaisuna saadaan ilmataisteluohjuksen optimaalinen tukemisaika. Toisessa peliasetelmassa tarkastellaan kahden hävittäjän välistä kaartotaistelua, joka mallinnetaan hävittäjälentäjien preferenssit sekä epävarmuudet uhkatilasta huomioon ottavana vaikutuskaaviopelinä. Peli ratkaistaan laskentamenetelmällä, jonka avulla kaartotaistelua käyvän hävittäjän ohjaukset voidaan optimoida reaaliaikaisesti. Lisäksi väitöskirjassa esitellään menettelytapa sekä helppokäyttöinen ohjelmisto, jolla voidaan ratkoa realistisia lähes-optimaalisia lentoratoja. Ohjelmistolla voidaan arvioida muun muassa edellä mainittujen ohjausmenetelmien tuottamien ratkaisuiden laatua ja realistisuutta. Esiteltyjä menetelmiä on mahdollista jatkokehittää käytännön toteutusta varten ja ne voitaisiin yhdistää lentolaitteen ohjausjärjestelmään tai lentäjän päätöstukijärjestelmään.

Avainsanat: ilmataistelu, ohjuksenväistö, laskennallinen optimisäätö, peliteoria, etenevän suunnitteluhorisontin säätö, adaptiivinen säätö, Bayes-päättely, vaikutuskaavio, käänteissimulointi

Academic dissertation

Systems Analysis Laboratory
Helsinki University of Technology

Modeling and on-line solution of air combat optimization problems and games

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Official opponent:	Dr. Tal Shima, Technion – Israel Institute of Technology

Publications

The dissertation consists of the present summary article and the following papers:

[I] Karelahti, J., Virtanen, K., and Raivio, T., "Near-Optimal Missile Avoidance Trajectories via Receding Horizon Control," *Journal of Guidance, Control, and Dynamics*, Vol. 30, No. 5, 2007, pp. 1287-1298.

[II] Karelahti, J. and Virtanen, K., "Adaptive Controller for the Avoidance of an Unknownly Guided Air Combat Missile," *Proceedings of the 46th IEEE Conference on Decision and Control*, 2007.

[III] Karelahti, J., Virtanen, K., and Öström, J., "Automated Generation of Realistic Near-Optimal Aircraft Trajectories," *Journal of Guidance, Control, and Dynamics*, accepted for publication.

[IV] Virtanen, K., Karelahti, J., and Raivio, T., "Modeling Air Combat by a Moving Horizon Influence Diagram Game," *Journal of Guidance, Control, and Dynamics*, Vol. 29, No. 5, 2006, pp. 1080-1091.

[V] Karelahti, J., Virtanen, K., and Raivio, T., "Game Optimal Support Time of a Medium Range Air-to-Air Missile," *Journal of Guidance, Control, and Dynamics*, Vol. 29, No. 5, 2006, pp. 1061-1069.

Contributions of the author

The author had the principal contribution in developing the missile avoidance problems and the receding horizon control based solution methods in papers I and II. The development of the automated solution approach introduced in paper III has been carried out in collaboration with Dr. Kai Virtanen. The influence diagram game and its on-line solution method in paper IV have been developed in co-ordination with Dr. Kai Virtanen and Dr. Tuomas Raivio. The author was the main contributor in generation of the support time game model and the on-line solution scheme introduced in paper V. The implementation of the methods as well as the computation and the analysis of the results have been carried out by the author, except for paper III, where the programming of the inverse simulation method and the animation has been performed in collaboration with Mr. John Öström. The author was the principal writer in papers I-III and V and had an active role in the writing of paper IV.

Preface

This thesis has been carried out in the Systems Analysis Laboratory of Helsinki University of Technology. I am grateful to Professor Raimo P. Hämäläinen, head of the laboratory, for the possibility to work and accomplish the thesis in the laboratory. I also thankfully acknowledge him for the guidance and lessons about scientific working in general that have evidently impacted this thesis as well. I am overwhelmingly indebted to my principal supervisor, Dr. Kai Virtanen, for the incessantly pedant feedback and guidance during the thesis process. I equally appreciate my other supervisor, Dr. Tuomas Raivio, for constructive feedback and enlightening discussions. I dare to say that I have gained a lot of scientific discipline and patience because of you two. I also appreciate the preliminary examiners Professor Fumiaki Imado and Professor Geert Jan Olsder for constructive comments.

I greatly acknowledge the cooperation with the Finnish Air Force. I specifically wish to thank Mr. Ilpo Paukkeri for the open and flexible collaboration. I also gratefully acknowledge Mr. John Öström, a co-author of mine, for educating me about fighter aircraft and flight mechanics. In addition, I thank Professor Jaakko Hoffren and Mr. Timo Sailaranta for instructive lessons about aerodynamics and missile systems.

I would also like to thank Professor Harri Ehtamo for introducing me to the field of Operations Research in the first place. Similarly, I thank Professor Ahti Salo for inviting me on the board of the Finnish Operations Research Society, which has provided an excellent vantage point to the field of Operations Research in Finland. Moreover, I exalt the whole staff of the laboratory for providing a creative, intelligent, and dynamic environment to work in.

In addition, my friends deserve huge thanks for providing me also other activities besides writing this thesis. Especially, the co-members of the Strategy Game Association RISK are greatly acknowledged, and their value as top-class players of strategy games is hereby recognized. Finally, words fail me in thanking my wife Johanna for her constant support and patience during the thesis process.

Tampere, November 2007

Janne Karelahti

1 Introduction

Modern air combat is carried out by air-to-air missiles and guns, and an aircraft may be considered as a weapons platform whose purpose is to bring the armament into a suitable launch condition (Shaw, 1985). Two main elements affecting the success in air combat are the performance of air combat arsenal and tactics. From the mathematical point of view, the above elements can be analyzed by the means of constructive simulation (Stenvens and Lewis, 1992; Davis, 1995), optimal control theory (Bryson and Ho, 1975), and game theory (Fudenberg and Tirole, 1991; Başar and Olsder, 1995). For a comprehensive review of principles of mathematical modeling, simulation, and analysis in the field of air combat, see Feuchter (2000).

Considering air combat optimization problems and games, the emphasis has traditionally been on the computation of optimal open-loop solutions associated with a specific set of initial conditions (see, e.g., Betts, 1998). Such solutions provide an insight about characteristics of optimal flight paths, but cannot however be applied, e.g., in the guidance of an aerial vehicle because of unpredictable disturbances and inherent uncertainties that tend to deviate the vehicle from its nominal flight path. The trend has been changing due to the ongoing increase in computational resources, which enables the utilization of efficient numerical methods in the on-line solution of near-optimal feedback controls associated with the current state and time (see, e.g., Mayne et al., 2000). Contrary to open-loop solutions, feedback solutions can reckon with ambiguity, which makes them suitable for the guidance purposes.

The thesis at hand introduces new modeling approaches and computational methods for various aircraft trajectory optimization and one-on-one air combat scenarios. The main emphasis is on the on-line solution of the constructed models in feedback form. The analyses are carried out in three dimensions by mainly using point-mass models which generally can describe realistic vehicles that consider both translational and rotational motions. Consequently, the introduced approaches provide the basis for the onboard guidance system of an aerial vehicle or a pilot advisory system.

For missile avoidance problems between a guided missile and a fighter aircraft, the thesis develops an on-line aircraft guidance scheme based on receding horizon control (RHC) (García et al., 1989; Mayne et al., 2000). Compared to related approaches, the scheme can be utilized with various performance measures exploiting different weaknesses of the missile system, and the guidance law of the missile can be selected freely. Moreover, the target's uncertainty about the guidance law of the missile is taken into account by using a belief model based on Bayesian reasoning (see, e.g., Ross, 2006). As a result, an adaptive controller for missile avoidance is obtained.

The thesis also introduces a new approach and its software implementation for the user-oriented solution of realistic near-optimal aircraft trajectories. In the approach, the optimal open-loop trajectory is first solved off-line for a coarse aircraft model, after which the obtained trajectory is inverse simulated (see, e.g., Hess et al., 1991) on-line with a more delicate aircraft model. The resulting trajectory is realistic in a sense that it could be flown by a real aircraft, and near-optimal in a sense that it follows closely the optimal open-loop trajectory computed with the coarse model. With the current implementation, both aircraft minimum time and missile avoidance problems can be solved. Contrary to existing trajectory optimization softwares, the implementation of the approach provides a way to assess the realism of the obtained trajectories.

For one-on-one air combat, the thesis introduces an influence diagram (Howard and Matheson, 1984, 2005, see the latter reference for a reprint) game that models pilots' maneuvering decisions in a dogfight. A computational method based on RHC for the near-optimal solution of the game model is introduced as well. Overall, the work extends the seminal work carried out by Virtanen et al. (1999b, 2003, 2004) by elaborating and implementing the earlier ideas in practice. In addition, a game model providing game optimal support times of air-to-air missiles in a missile duel is constructed. The support time refers to a period for which the aircraft relays target information to the missile after the launch. A computational method for the on-line calculation of approximate game optimal support times associated with a given set of launch conditions is introduced as well.

In single-sided air combat settings containing only one actor, the best possible course of action with respect to a given performance measure can be determined using optimal control theory (Bryson and Ho, 1975). Typical missions include aircraft minimum time problems where the objective of the aircraft is to reach a given target set in minimum time (Bryson and Denham, 1962; Bryson et al., 1969), which is considered in paper III of the thesis as well. The feasible flight region may also be restricted by no-fly zones (Bellingham et al., 2002) or the topography (see, e.g., Anisi et al., 2006). In addition, the capability to turn in minimum time (Well and Berger, 1982) or with minimum consumed fuel (Ringertz, 2000b) can be crucial in air combat. More complicated missions include, e.g., hostile radar stations that must be approached such that the inevitable detection occurs as near the target station as possible (see Norsell, 2003, 2005). Missile avoidance problems in which maneuvering commands of the missile are given by a control law like in papers I–III and V can be formulated as optimal control problems as well (see, e.g., Imado and Miwa, 1986).

If the engagement consists of two actors with conflicting goals, the duel can be modeled using concepts of noncooperative game theory (Fudenberg and Tirole, 1991). In a static game, the order in which the actors make their decisions is not important, whereas in a dynamic game it is. Moreover, if the evolution of the engagament is governed by a differential equation, a dynamic game is called a differential game and the underlying mathematical framework is referred to as the theory of differential games (Başar and Olsder, 1995). Air combat duels are typically modeled as differential games. For example, missile avoidance problems have been traditionally described as pursuit-evasion games (Isaacs, 1975) where the roles of the players are fixed. In the game model, the pursuing player, the missile, tries to capture the aircraft designated as the evader. The analysis of more complicated encounters such as dogfights and missile duels where the roles of the players cannot be fixed is also possible. Such duels can be modeled as two-target games in which each player tries to catch the other one while preventing to be capured (Blaquière et al., 1969; Grimm and Well, 1991). In papers IV and V, approaches for the approximate on-line solution of two-target games are introduced.

In practice, at most optimal open-loop solutions or, in case of pursuit-evasion games, open-loop representations of feedback solutions (Başar and Olsder, 1995) for the above formulations can be obtained when simplistic vehicle models are applied. The computations must be performed off-line, whereby the solutions are not applicable for the guidance purposes as such. Nevertheless, optimal open-loop solutions give information about efficient flight paths and performance limits of the aircraft (see, e.g., Ringertz, 2000a). These aspects are considered in paper III of the thesis, where optimal open-loop trajectories are solved and analyzed for aircraft minimum time and missile avoidance problems.

A drawback of open-loop solutions is that sources of uncertainty arising in practical situations are not taken into account, which justifies the development of approximate feedback methods introduced in the thesis. For practical purposes, several methods for the on-line computation of near-optimal feedback solutions are available. In case of tracking problems where the objective is to follow a given flight path (see, e.g., Kirk, 1970), approximate feedback controls can be computed using inverse simulation (see, e.g., Hess et al., 1991; Gao and Hess, 1993). This approach is utilized in paper III of the thesis. Neural networks (Goh et al., 1993; Järmark and Bengtsson, 1994; Pesch et al., 1995), neighboring extremals (Pesch, 1989a), and RHC (Shinar and Glizer, 1995; Cruz et al., 2001, 2002; Virtanen et al., 2003, 2004) have been applied in the approximate solution of various air combat problems as well. In the first approach, the controls are approximated on the basis of a set of off-line computed optimal open-loop solutions. This idea is exploited in the approximate solution of the support time game constructed in paper V. In the second approach, feedback controls are approximated by repeatedly solving the optimal open-loop controls associated with the current state and time as the situation evolves. In RHC, which is applied in papers I–IV, near-optimal controls are optimized on-line by using a truncated planning horizon and approximating the optimal cost over the remaining interval by a suitable cost-to-go function (see, e.g., Bertsekas, 2001).

Due to the on-line computation, auxiliary methods for handling ambiguities can be integrated into the RHC scheme. For example, the uncertainty about unknown system parameters can be modeled using Bayesian reasoning. This refers to the updating of the subjective probabilities assigned to given events on the basis of the observed evidence using Bayes' theorem (see, e.g. Ross, 2006). The degrees of belief in the values of a particular parameter can be represented as a discrete probability distribution over the range of the parameter. As the situation evolves, the probability distribution is updated on the basis of particular state measurements (Bertsekas, 2000). In paper II, the above approach is utilized in the updating of the target's belief in the guidance law of the missile.

Decision theoretical frameworks such as the influence diagram introduced by Howard and Matheson (1984) enable the structuring and solving decision problems under conditions of uncertainty. The influence diagram provides a graphical method for a compact presentation of relationships between uncertain variables and decisions in the form of an acyclic graph. It also allows the solution of optimal decisions of the actor. Overall, the framework offers an intuitive way to incorporate expert knowledge such as pilots' combat insight into the modeling process (Shachter, 1986). This allows the consideration of preferences, perception, and beliefs in air combat. It is noteworthy that influence diagrams incorporate Bayesian reasoning in the evaluation of uncertain variables within the diagram. Virtanen et al. (1999b, 2004) model the pilot's maneuvering decisions in one-on-one air combat using influence diagrams, where the pilot's belief in the threat situation of the combat is updated using Baysian reasoning. Virtanen et al. (2003) extend these ideas to a two-sided setting by constructing an influence diagram game, which paves the way for the approximate solution of a two-target game. Although the complete solution of the influence diagram game is computationally infeasible, it can be solved approximately using RHC. These ideas are further elaborated and implemented in paper IV of the thesis.

The summary article is structured as follows. In the following section, the vehicle models utilized in the thesis are briefly explained. In Section 3, solution methods for optimal control problems, approaches for handling uncertainty, representative military aviation applications, and papers I–III are reviewed. In Section 4, air combat game models, their solution methods, and uncertainty models in games are reviewed along with the presentation of papers IV and V. Section 5 concludes the summary and proposes topics for future research.

2 Aircraft and missile models

For the purposes of trajectory optimization and trajectory simulation, the dynamic models describing the motions of the vehicles must be available. The aircraft and missile models utilized in the thesis are described next. Although a three-degree-of-freedom (3-DOF) point-mass model (Miele, 1962) employs only translational equations of motion, it is usually considered sufficiently accurate for trajectory optimization and performance analysis, at least in planar studies (Bryson et al., 1969). Importantly, the model is also computationally tractable for trajectory optimization due to the neglected moment equations. The complete 3-DOF aircraft and missile models utilized in the thesis are described in paper I. The position of the point-mass in the three dimensional space is determined by two horizontal coordinates and the altitude, whereas the direction of the velocity vector is given by the flight path and heading angles. The model contains three kinematic equations for the position variables and three dynamic equations for the flight path angle, heading angle, and velocity.

The 3-DOF aircraft model is controlled with the angle of attack, bank angle, and throttle setting. The first two variables control the lift force and its direction, whereas the last variable controls the tangential acceleration. The aircraft is subject to control and path constraints that prevent the violation of the minimum altitude, stalling, and exceeding of the structural and dynamic pressure limits. For more realistic modeling of rotational motion, the angle of attack and the bank angle are constrained by imposing limits on the angular rates (Raivio and Ranta, 2002) and angular accelerations of the aircraft. This enhanced 3-DOF aircraft model is applied in papers I–III of the thesis.

The 3-DOF missile model has skid-to-turn configuration (Blakelock, 1991) which means that there is no roll motion about the centerline of the missile. The missile is controlled by the pitch and yaw acceleration commands where the pitch and yaw are the angular motions about the lateral and vertical axes of the missile, respectively. It is assumed that the missile has a single-lag guidance system, and the guidance channels are decoupled from each other (Zarchan, 1997). In other words, a command for yaw results in yaw motion only and vice versa. The acceleration commands that are constrained by the stall and structural limits are given by the utilized guidance law. Common guidance laws utilized by air combat missiles include, e.g., proportional navigation (PN) presumably invented by Yuan (1948), pure pursuit (PP), and command to line-of-sight (CLOS). These guidance laws are applied in the thesis as well and summarized in paper III. For detailed descriptions of the operational principles of the above guidance laws, see, e.g., Zarchan (1997) and Shneydor (1998).

In trajectory simulation, more accurate vehicle models can be used due to the significantly smaller computational load compared to trajectory optimization. 6-DOF models (Stenvens and Lewis, 1992) describing both translational and rotational motions of the vehicle provide the means for the performance analysis of the highest fidelity. A downside of 6-DOF models is the necessity for a large amount of data required in the computation of the attitude changes via moment equations. Moreover, the data cannot be reliably estimated for unfamiliar vehicle types (Hoffren and Sailaranta, 2001).

A more straightforward course of action is to apply performance models where the attitude dynamics are described by response functions characterized by the maximum attainable agility of the vehicle. Although the moment equations of the 6-DOF model are omitted, performance models are comparable in realism to 6-DOF models when the fundamental dynamics are modeled correctly. In addition, the amount of data required in the computation is much smaller than with 6-DOF models, and the data can be estimated straightforwardly for various vehicle types (Hoffren and Sailaranta, 2001). In the inverse simulation method presented in paper III, a 5-DOF performance model (Hoffren and Sailaranta, 2001) is utilized for describing the motion of the aircraft. In the particular aircraft model, the pitch, roll, and tangential acceleration commands are given by the load factor, roll rate, and throttle setting control parameters, respectively. Although similar performance model could be applied for the missile as well, the 3-DOF missile model is considered sufficiently accurate due to the fast rotational dynamics of the missile.

In all the applied models, the lift and drag coefficients of the vehicles as well as the maximum thrust level and roll rate of the aircraft over the respective flight envelopes are provided by the vehicle type specific parameters. The parameters are given as tabular data that are interpolated as continuous and smooth functions necessary for the numerical integration and optimization algorithms. The parameters describe also reference wing areas and initial masses of the vehicles, requisite time constants, and thrust profile of the missile.

3 Air combat optimization problems

As stated in the introduction, optimal trajectories with respect to the objective of a single actor can be obtained using optimal control theory (Bryson and Ho, 1975). If the optimal controls are given as a function of the initial state and the current time, the solution is said to be open-loop form. Optimal open-loop solutions provide information about characteristics of optimal flight paths with respect to a given performance measure. In principle, they may be implementable if the sources of uncertainty are negligible, which is, however, not the case in military aviation. On the other hand, feedback optimal controls are given as a function of the current state and the current time, whereupon they are usually preferred to open-loop solutions. Nevertheless, the traditions of aircraft trajectory optimization rest mainly on the open-loop solution of the problems of interest, following the fact that optimal feedback solutions are obtainable for simplistic formulations only.

The section proceeds by first reviewing numerical methods for the open-loop solution of an optimal control problem. Thereafter, approaches for obtaining optimal and near-optimal feedback solutions with special attention to RHC are dissected, followed by an introduction of approaches enabling the consideration of uncertainty in air combat. In the end of the section, papers I–III are shortly reviewed. Representative military aviation applications are covered along the presentation.

3.1 Open-loop solution of optimal control problems

Optimal open-loop solutions are obtained by Pontryagin's Maximum Principle providing the necessary optimality conditions that must be satisfied by an optimal solution (Pontryagin et al., 1962). These conditions consitute a two- or multipoint boundary value problem (BVP) which is generally analytically intractable for high-dimensional nonlinear systems. Temporal discretization of the necessary optimality conditions transforms the BVP to a set of nonlinear equations that can be solved using a root-finding algorithm. These kinds of solution approaches are known as indirect methods (Betts, 1998). In general, the solutions provided by indirect methods are considered accurate. On the downside, convergence domains of root-finding algorithms are known to be small, which means that a high-quality initial guess for an optimal solution is mandatory (Betts, 2001).

In direct methods, the dynamics of the problem are discretized in time, and the control and path constraints are evaluated at discrete instants called nodes. The performance measure is then directly optimized subject to the constraints with a nonlinear programming (NLP) solver (Hull, 1997). Usually, sequential quadratic programming (Bertsekas, 1995) (SQP) methods are applied due to their superiority in the solution of discretized optimal control problems (Gill et al., 1994). Direct methods have proven to be substantially more robust and easier to apply than indirect ones, whereupon they have become the methods of choice in contemporary trajectory optimization (Betts, 2001; Paris et al., 2006), and utilized in the thesis as well.

A number of discretization schemes are available for carrying out the discretization (see, e.g., Hull, 1997). Basically, the schemes are applicable for both indirect and direct formulations (Betts, 1998), although only direct ones are considered here. Depending on the scheme, either the control variables, the state variables, or both of them are treated as unknown parameters of the transcribed NLP problem to be solved. In the direct shooting method, only the controls are parameterized and the state equations are integrated explicitely using the parameterized controls. The size of the resulting NLP problem is typically small, so the method is robust and fast with a modest amount of nodes. For large-scale problems, the robustness is maintained by breaking the trajectory into several segements and applying the shooting method within each segement, resulting in the multiple shooting method (Keller, 1968). In collocation methods (Russell and Shampine, 1972), the state and control trajectories are approximated by piecewise polynomials of a given degree. With polynomials of the third degree, the widely used direct Hermite-Simpson collocation follows (Hargraves and Paris, 1987; von Stryk, 1993). In differential inclusion (Seywald, 1994), the control variables are eliminated from the state and constraint equations, whereupon the NLP variables correspond to the state variables only.

Basically, the feasibility of a discretization scheme depends on the problem under consideration. For example, the attractiveness of differential inclusion is reduced if the controls cannot be eliminated analytically (Hull, 1997). For high-dimensional systems, collocation methods that require the parametrization of the state variables at each node are most likely infeasible due to the massiveness of the resulting NLP problem. If the number of control variables is however small, shooting methods become computationally appealing. This is the case with the missile avoidance problems considered in the thesis, whereupon small- and large-scale optimal control problems are solved using direct shooting and direct multiple shooting, respectively. It should be however noted that not only the size, but also the structure of the problem affects the computational complexity of an NLP problem (Betts, 2001). Thus, no a priori guarantee about the suitability of a particular discretization scheme for a given problem type cannot be given, but each case should be analyzed individually.

Conventionally, the discretization is carried out by using equally spaced nodes. Depending on the dynamics of the problem, the accuracy of the solution may however be improved with a mesh refinement scheme. In the refinement schemes, the basic idea is to estimate the discretization errors at each interval after the SQP iteration, and add nodes within the intervals with the largest errors. The problem is then solved again by using the refined mesh of nodes (Betts and Huffman, 1998; Betts, 2001). In principle, an indirect method can be used to improve the solution given by a direct method as well (Pesch, 1994b).

The above mentioned discretization methods have been widely utilized in aircraft trajectory optimization. A typical mission is to transfer the aircraft in minimum time to a given destination that corresponds to a fixed or moving point in the space. In a minimum time climb problem (Bryson et al., 1969), the destination corresponds to a particular final altitude and velocity. Nowadays, direct Hermite-Simpson collocation first introduced by Hargraves and Paris (1987) is probably the most popular solution approach for these problems (see, e.g. Raivio et al., 1996; Virtanen et al., 1999a; Ringertz, 2000a; Paris et al., 2006). In paper III, similar problems are solved using direct multiple shooting. Järmark (1986b) solves a minimum time turn problem on a plane using an indirect method, whereas Walden (1994) and Grimm and Hans (1998) derive analytic feedback solutions for the same problem.

Other typical objectives are the maximization of the aircraft's flight range or the minimization of the consumed fuel (Bryson et al., 1969). Seywald et al. (1994) solve range-optimal trajectories using an indirect method. Ringertz (2000b) solves optimal aircraft trajectories for minimum fuel turn using direct Hermite-Simpson collocation. Likewise, Norsell (2003) applies the same approach in the solution of a multiobjective mission, where the aircraft must reach a hostile radar station such that a weighted sum of the detection time and the consumed fuel is minimized. Norsell (2005) extends the mission by including waypoints and an intermediate radar station.

Also pursuit-evasion problems between a missile and an aircraft like those considered in papers I–III and V can be formulated as optimal control problems if the missile is guided by a control law such as PN. A typical performance measure maximized by the aircraft is the miss distance which corresponds to the distance between the missile and the target at the moment of the closest approach. Imado and Miwa (1986) reduce a planar miss distance maximization problem to a two-point BVP and solve it using a steepest ascent method introduced by Bryson and Denham (1962). The maximization of the gimbal angle is considered as well. Imado and Miwa (1994) and Imado and Uehara (1998) extend the analysis to three dimensions and compare the performance of a high-g barrel roll to the optimal evasive maneuver. If the kinematics are linearized (Shinar and Steinberg, 1977; Shinar et al., 1979) or the engagement is constrained on a plane and velocities are assumed constant (Shinar and Tabak, 1994), miss distance maximizing maneuvers can be calculated also in a more straightforward manner than by solving the related BVP. With more realistic vehicle models, optimal open-loop solutions can be obtained with direct methods. Ong and Pierson (1996) apply direct shooting in the solution of a planar miss distance maximization problem, whereas Raivio and Ranta (2002) solve the miss distance maximizing endgame maneuvers in three dimensions using a direct multiple shooting method in which separate meshes are applied for the aircraft and the missile.

3.2 Feedback solution of optimal control problems

Dynamic programming (DP) developed by Bellman (1957) as well as its continuous time counterpart, the Hamilton-Jacobi-Bellman (HJB) partial differential equation, provide the means for the feedback solution of an optimal control problem. Unfortunately, the curse of dimensionality severely impedes the applicability of these methods, whereupon they are generally feasible for simple and low-dimensional problems only. With linear dynamics and quadratic performance measure, a closed form solution being a linear state feedback is available (Bryson and Ho, 1975). This solution provides the basis for numerous guidance schemes of the missile (see, e.g., Hull et al., 1990; Shneydor, 1998; Vergez, 1998; Ben-Asher and Levinson, 2003). In this case, a natural performance measure is a combination of the miss distance and the control effort of the missile. When the target maneuvers and missile dynamics are ignored, the minimization of the above performance measure results in the well-known PN guidance law (Bryson and Ho, 1975) utilized also in the thesis.

Although optimal feedback solutions are much more difficult to obtain than open-loop ones, they are normally preferred in engineering applications. From the practical point of view, global optimality is however not of the utmost necessity, but feasible near-optimal solutions are usually considered sufficient for practical purposes. This justifies the development of suboptimal feedback controllers whose principles are reviewed next.

3.2.1 Inverse simulation

In case of tracking problems where the objective is to follow a reference flight path, near-optimal feedback controls can be obtained, e.g., by using inverse simulation (see, e.g., Hess et al., 1991). In the inverse simulation, controls that reproduce the reference trajectory as well as possible are determined. Being computationally less demanding than optimization based approaches, inverse simulation allows the utilization of higher-fidelity aircraft models in the approximate feedback solution of tracking problems. In paper III of the thesis, optimal open-loop trajectories computed with the enhanced 3-DOF aircraft model are tracked on-line with the 5-DOF performance model by using inverse simulation.

Inverse simulation approaches include, e.g., the manual construction of near-optimal controls on the basis of the optimal solution (Hoffren and Raivio, 2000), differentiation inverse method (Kato and Sugiura, 1986, 1990), and integration inverse method (Hess et al., 1991; Gao and Hess, 1993). The first approach calls for intensive user interference and expertise casting it infeasible considering real-time implementation, whereas the second one requires an accurate initial guess for the solution. On the other hand, the integration inverse method, which is applied also in paper III of the thesis, has appeared to be robust, self-contained, and computationally efficient (see Öström, 2005; Öström and Hoffren, 2006). The basic idea of the method is to find controls that nullify the error between the desired and achieved outputs at the next instant. Such controls can be solved, e.g., using Newton's method (see, e.g., Kelley, 2003).

3.2.2 Open-loop optimization based methods

One way to produce near-optimal feedback controls is to exploit an obvious fact that for a deterministic optimal control problem, optimal open-loop and feedback solutions associated with particular state and time are the same (see, e.g., Başar and Olsder, 1995). The idea of utilizing this fact in the on-line solution of feedback controls was already stated by Lee and Markus (1967). Practical implementations exploiting the above connection are, however, more recent.

One approach is to approximate the optimal controls associated with the current state and time on the basis of a set of off-line computed optimal open-loop solutions over the state space. This principle is exploited in the real-time scheme for obtaining a game optimal support time of the missile introduced in paper V of the thesis. A common way to carry out the approximation is to use a neural network. First, optimal open-loop solutions associated with a representative set of initial conditions are computed off-line, after which the network is trained by using optimal open-loop trajectories as inputs and the respective optimal control histories as outputs (see, e.g., Goh et al., 1993). A downside of neural networks is that for high-dimensional systems, a plethora of optimal open-loop solutions over the state space must be solved off-line and stored for the adjustment of the network's parameters. Hence, efficient methods for the open-loop optimization are mandatory considering the practicality of the approach. Nevertheless, neural networks have been applied in the approximate solution of problems considering, e.g., the maximum energy climb and minimum time flight (McKelvey, 1992), rendevouz with an intruder as far as possible (Järmark and Bengtsson, 1994), and re-entry of a space shuttle to Earth (Breitner, 2000).

In the repeated correction method based on the theory of neighboring extremals (see Pesch, 1989a), the off-line computed optimal open-loop solution is adjusted on-line to correspond to the current state and time as the state of the system evolves. Real-time computational performance is gained via the linearization of the necessary conditions which results in an efficiently solvable linear multipoint BVP (Pesch, 1989b, 1994a). A drawback of the approach is the requirement for the optimal open-loop solution associated with the initial conditions, which must be solved off-line.

3.2.3 Receding horizon control

Yet another course of action for obtaining near-optimal feedback controls is to use RHC, also known as moving horizon control and model predictive control (Camacho and Bordons, 1999), in which the controls are optimized on-line by using a limited planning horizon and approximating the optimal cost to go. Compared to the repeated correction method by Pesch (1989b), computational savings are achieved at the expense of accuracy by truncating the optimization interval. RHC is the principal methodology applied in the thesis, which is due to the flexibility of the approach: It provides a suitable basis for auxiliary methods capable of coping with ambiguities and allows the change of the utilized performance measure during the mission, if necessary. Moreover, RHC is applicable for the approximate solution of complicated dynamic games as well. The methodology is applied in the on-line solution of the air combat optimization problems and games considered in papers I–IV.

The basic idea of RHC is presented in Figure 1. Based on the measurements received at the instant t_k, the future behaviour of the system is predicted and the open-loop controls over the planning horizon T are determined on-line such that a given performance measure is optimized. Since the utilized model differs most likely from the true system, the open-loop controls are implemented only till the next measurement at t_{k+1}. This provides a feedback mechanism that takes into account disturbances and uncertainty regarding the true system. Although a continuous and smooth control trajectory over the planning horizon could be required, usually piecewise constant controls are applied for numerical reasons. RHC allows the optimization of an arbitrary performance measure and the consideration of control and path constraints which makes the approach attractive for controlling complex dynamic systems (Mayne et al., 2000).

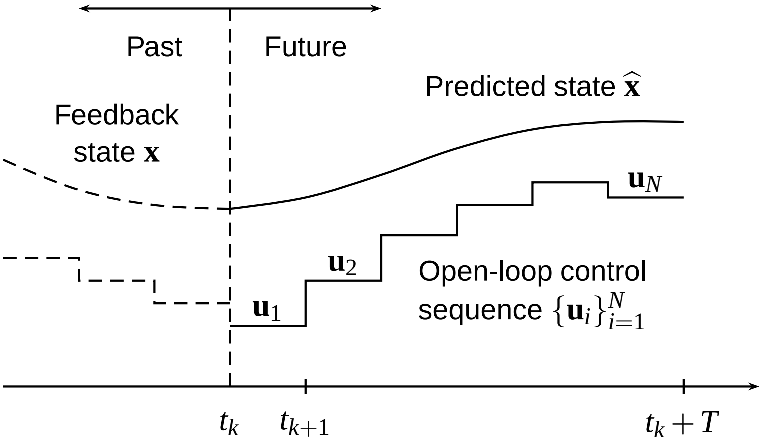


Figure 1: Principle idea of receding horizon control.

In RHC, the performance measure consists of the original performance over the planning horizon appended with a suitable cost-to-go function approximating the optimal cost over the remaining portion of the trajectory. Instead of approximating well the optimal cost to go, it is more important that the differences of the true cost-to-go function over the state space are properly approximated. Obviously, cost-to-go approximations are problem specific, and there are numerous ways for obtaining them (see, e.g., Bertsekas, 2000). It should be noted that even though the model would correspond the true system perfectly, the obtained solution is not necessarily globally optimal due to the limited planning horizon. The issue is naturally related to the quality of the cost-to-go approximation. For example, with the rolling horizon approach where the cost-to-go approximation is just zero, a longer planning horizon does not necessarily result in a better outcome than a shorter one (see Bertsekas, 2000).

Although first adopted in control applications of the process industry (Richalet et al., 1976, 1978), RHC has been recently applied in the approximate solution of various air combat optimization problems as well. Virtanen et al. (1999b, 2004) construct a realistic one-on-one air combat model and solve the controls of the aircraft against a predetermined trajectory of the adversary. In the latter paper, comparison to the optimal open-loop solution is carried out, too. Bellingham et al. (2002) consider a planar minimum time problem where the flight region contains no-fly zones. Singh (2004) studies a missile avoidance problem where the aircraft tries to maximize the tracking rate of a PN guided missile. Anisi et al. (2006) and Williams (2006) consider a terrain following problem where in addition to the flight time minimization, another goal is to keep the aircraft near the surface of the earth. Kang and Hedrick (2006) study a tracking problem where an unmanned aerial vehicle tries to follow a predetermined flight path, whereas Singh and Fuller (2001) and Shim and Sastry (2006) develop an RHC scheme for the guidance of an unmanned aerial vehicle via given waypoints such that no-fly zones are avoided. Geiger et al. (2006) consider the following of a stationary or moving target with an additional goal to keep the target within the field of view.

3.3 Uncertainty models

Air combat is an inherently stochastic event with numerous uncertainties, of which the most essential ones include random disturbances in the atmosphere such as wind, imperfect state observations, and unknown performance parameters and behavior of the adversary. In addition to deterministic problems, DP and the HJB equation allow the solution of simplistic stochastic optimal control problems as well (Bertsekas, 2000; Kamien and Schwartz, 1991). This gives rise to linear quadratic Gaussian guidance laws (see, e.g., Cloutier et al., 1989; Shneydor, 1998) and optimal state estimation such as Kalman filtering (Kalman, 1960; Kalman and Bucy, 1961) capable of coping with Gaussian process and measurement noises. However, uncertainties have been typically ignored in more complicated air combat optimization applications.

If anything, constructive simulations such as stochastic simulation in which uncertainties are explicitely incorporated via probability distributions allow comprehensive consideration of ambiguities in air combat (see, e.g., Davis, 1995). Batch air combat simulators (Goodrich et al., 1995; Goodrich and McManus, 1989; Glærum, 1999) are suitable for analyzing tactics and aircraft performance with multiple actors, whereas real-time man-in-the-loop simulators (Goodrich and McManus, 1989; McManus and Goodrich, 1989) provide a realistic training environment for human pilots. They also produce knowledge for improving the guidance logics of computer guided aircraft in the simulators. A downside of the existing guidance logics is simplistic prediction of the adversary's maneuvers. This issue is tackled in paper IV of the thesis, where the adversary is assumed to maneuver optimally.

As remarked above, the feedback mechanism of RHC allows the consideration of random disturbances and uncertainty regarding the mismatch between the model and the true system. Importantly, it enables the inclusion of auxiliary methods capable of coping with essential sources of uncertainty such as unknown performance parameters of the adversary and threat situation of the combat. The uncertainty regarding these issues can be handled, e.g., by using Bayesian reasoning and decision theoretical frameworks such as influence diagrams that are explained next. For a comprehensive review of applying decision theoretical frameworks in air combat modeling, see the summary of Virtanen (2004).

3.3.1 Bayesian reasoning

In Bayesian reasoning, the degrees of belief in given hypotheses, or subjective probabilities, are updated on the basis of the observed evidence using Bayes' theorem (see, e.g., Bertsekas, 2000; Ross, 2006). The evolution of the degrees of belief is affected by the likelihood function that gives the likelihoods of the hypotheses under the observed evidence. Essentially, the likelihood function determines how much the observed evidence alters beliefs in the hypotheses. In any case, the likelihood function is such that the degree of belief in the true hypothesis is fed, whereas those correponding to the false ones are decreased. Bayesian reasoning is a popular approach for carrying out the estimation of unknown parameters involved in a dynamic system. The approach is adopted also in paper II of the thesis which considers the identification of the missile's guidance law. In that work, the hypotheses correspond to guidance laws, and the observed evidence equals particular state measurements.

3.3.2 Influence diagrams

As stated in the introduction, the influence diagram allows the description of decision problems under conditions of uncertainty, which is understandable for both computers and people without expertise in decision analysis. The framework is utilized in paper IV of the thesis which considers a dogfight between two aircraft. In the influence diagram, the relationships between the elements of the problem are shown at three levels of specification: relation, function, and number (Howard and Matheson, 1984). At the level of relation, influence diagrams depict the interrelationships of the decisions and variables in the form of acyclic graphs. At the level of function, the relationships of the nodes of the graph are specified by using functional forms. At the level of number, the numbers associated with the functions are specified (Diehl and Haimes, 2004).

The nodes of the acyclic graph correspond to decision, random, and deterministic variables. The utility to be optimized is depicted by a specific node as well. The relationships between the nodes are shown by arcs connecting them. Arcs pointing to a decision node mean that the values of the source nodes are known when the decision is made, whereas arcs directed at a random node means that the node depends conditionally on the input nodes. It is noteworthy that influence diagrams incorporate Bayesian reasoning in the evaluation of the conditional probabilistic dependencies. Finally, arcs leading to a deterministic or value node imply dependence from the input nodes. At the level of relation, influence diagrams provide a compact and easily understandable presentation of a decision problem. At the level of number, they allow the solution of optimal decisions of the actor.

Originally, influence diagrams deal with static decision problems. Virtanen et al. (1999b) extend the application of influence diagrams to a dynamic setting by modeling the pilot's maneuvering decisions in one-on-one air combat using influence diagrams, and develops an RHC based method for the approximate solution of the model in feedback form. In the model, the adversary is assumed to fly along a predefined trajectory. Virtanen et al. (2004) widen the analysis by solving the model in open-loop form, and compares the optimal open-loop solution to the near-optimal one obtained by the RHC based method. Overall, the modeling approach allows the consideration of preferences, perception, and beliefs in air combat.

3.4 RHC based controller for missile avoidance

In paper I, RHC is utilized in the solution of various missile avoidance problems where the missile is guided by PN. Compared to related approaches (see, e.g., Singh, 2004), the controls of the aircraft are now optimized with respect to various performance measures including capture time, closing velocity, miss distance, gimbal angle, tracking rate, and control effort of the missile. Suitable closed form cost-to-go approximations for each performance measure are constructed on the basis of engineering intuition and computational experience.

In the introduced RHC based controller, optimal open-loop controls over the planning horizon are solved using the direct shooting method, and the resulting NLP problem is solved with SNOPT (Gill et al., 2005) solver which uses an SQP algorithm. Here, the conversion results in small-scale NLP problems although the number of state variables is large. To achieve longer planning horizons without overly expanding the NLP problem, the time intervals between the nodes are linearly increased towards the end of the planning horizon.

The overall performance of the introduced controller is validated by comparing the obtained near-optimal solutions to the respective optimal open-loop solutions computed by direct multiple shooting. This is performed for numerous initial states for each of the above mentioned performance measures. Consequently, optimal values of the performance measures for a set of initial states over the state space are obtained. This information can be utilized in the selection of the most suitable performance measure for a given combat state.

Figure 2 presents capture time maximizing open-loop trajectories of the aircraft and the missile for an example case where the missile is launched towards the aircraft at the range of 10 km. The initial altitudes and velocities of the vehicles are 6 km and 250 m/s, respectively, and the aircraft flies towards the missile at the aspect angle of 45 deg. The aircraft first turns away from the missile and tries to outrun it by performing a dive manuever. This is a typical evasive maneuver with many performance measures and is in accordance with evasion tactics applied by fighter pilots, since the ability of the aircraft to avoid the missile depends strongly on the closing velocity of the missile. By turning away and diving, the aircraft induces the missile to a lower altitude which decelerates the missile due to a higher drag. On the other hand, the aircraft avails larger thrust at lower altitude (Shaw, 1985). Figure 3 presents the convergence of near-optimal controls computed by the controller towards the optimal open-loop ones that are shown on the right. In this case, near-optimal solutions are obtained even with short planning horizons.

The numerical examples indicate that in general, the solutions obtained by the RHC based controller are close to the optimal open-loop ones. Moreover, the solution times are feasible for the real-time calculation of the avoidance maneuvers.

3.5 Modeling target's uncertainty about the guidance law

In paper II, the RHC based controller described above is extended to an adaptive one by treating the guidance law of the missile as an unknown parameter to the target aircraft. The target's belief in the guidance law is represented as a discrete probability distribution over a predefined set of guidance laws. In the paper, ideal PN (Yuan and Chern, 1992), PP, and CLOS are considered. Open-loop controls over the planning horizon are optimized with respect to the expected value of the performance measure. As the missile closes on the aircraft, the target's belief in the guidance law is updated using Bayesian reasoning. Opposed to existing identification approaches (see, e.g., Lin et al., 2005), the target actively avoids the closing missile instead of passive identification of the missile's performance parameters only.

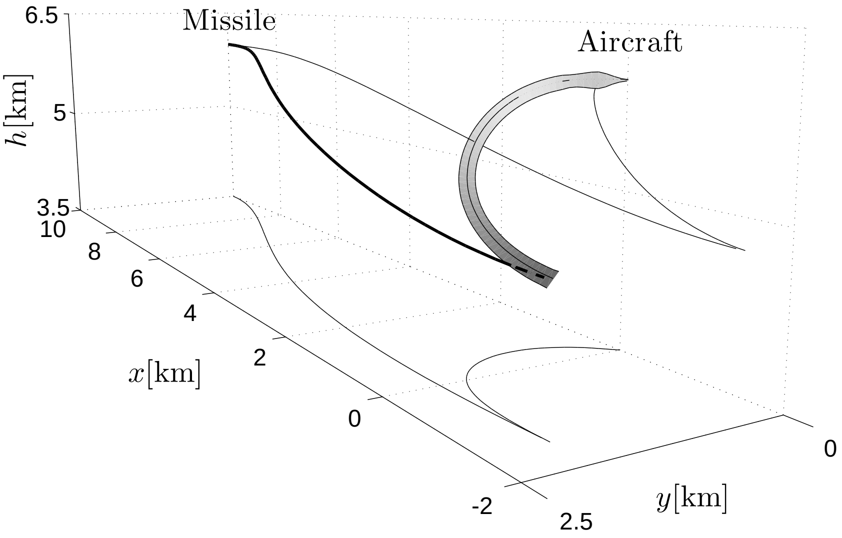
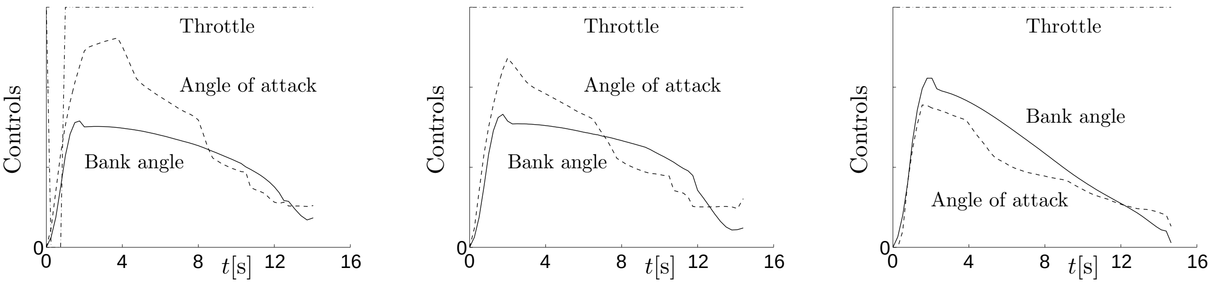


Figure 2: Optimal flight path in capture time maximization. Figure from paper I.



(a) $T = 0.56$ sec, $t_f = 14.05$ sec (b) $T = 1.38$ sec, $t_f = 14.40$ sec (c) $T = t_f$, $t_f = 14.64$ sec

Figure 3: Control histories of the aircraft with various planning horizons T. Figure from paper I.

Specifically, the target's belief serving as the prior probability distribution is updated on the basis of particular state measurements using Bayes' theorem. The measured states correspond to the angle between the missile's velocity and line-of-sight to the target vectors, and the distance of the missile from the guideline between the launcher and the target. Since the above variables obtain typically distinct values for different guidance laws, they provide the basis for the identification. The likelihood function of the guidance law applied in Bayes' formula is constructed on the basis of the operational principles of the considered guidance laws.

Altogether, the utilized modeling approach provides a flexible way to expand the repertoire of guidance laws of the missile in the adaptive controller. The numerical examples presented in the paper indicate that the guidance law can be identified by the endgame, allowing the utilization of efficient last-ditch evasive maneuvers.

3.6 Automated approach for aircraft trajectory optimization

Paper III introduces a new approach and its software implementation, Ace, for the user-oriented computation of realistic near-optimal aircraft trajectories. In the approach, the optimal trajectory is first solved using the enhanced 3-DOF aircraft model. Then, the obtained trajectory is inverse simulated with a more delicate 5-DOF performance model. If the optimal and inverse simulated trajectories are sufficiently similar, the inverse simulated trajectory can be considered a realistic near-optimal trajectory that could be flown by a real aircraft. Otherwise, the parameters affecting the computations are adjusted such that more realistic trajectories will result, and the optimization and inverse simulation are repeated. Contrary to existing optimal control software packages (see, e.g., Virtanen et al., 1999a; ASTOS, 2006; SOCS, 2006; Whiffen, 2006), the introduced approach enables the assessment of the realism of the obtained aircraft trajectories.

The optimal open-loop solutions are computed using direct multiple shooting. The approach is particularly suitable for solving missile avoidance problems where the number of state variables is large in relation to the number of control variables. The initial guess of the solution required by the SQP method is computed using the controller presented in paper I of the thesis. The inverse simulation is performed on-line using a modified version of the integration inverse method introduced by Hess et al. (1991). Opposed to differentiation inverse methods (Kato and Sugiura, 1986, 1990), the integration based approach has proven to be robust and fast (see Öström, 2005; Öström and Hoffren, 2006). Unlike the approaches based on the manual construction of controls on the basis of optimal trajectories (see, e.g., Hoffren and Raivio, 2000), the utilized method requires minimal user intervention which makes it suitable for the automated approach.

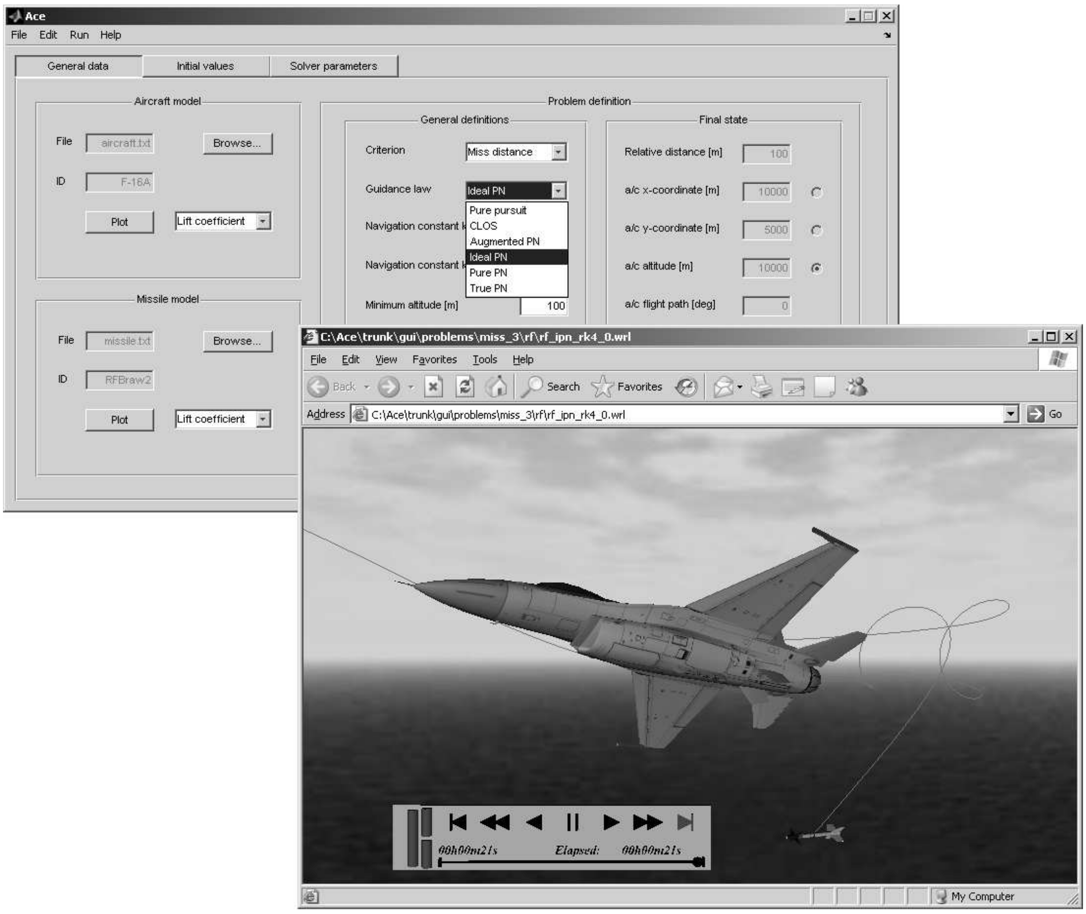
With the Ace software, both aircraft minimum time and missile avoidance problems can be solved. In addition to the guidance laws used in papers I and II, three variants of PN including augmented, true, and pure PN (see, e.g., Shneydor, 1998) can be selected. The mission is defined via a graphical user interface by using which vehicle types, performance measure, guidance law, specific limits, and necessary boundary conditions can be defined in a user-oriented way. The lift coefficients, drag coefficients, and maximum thrust profiles of the vehicles can then be studied via surface plots. After the optimization and inverse simulation, the obtained results can be analyzed via various graphs. It is possible to visualize the encounter as a real-time 3-D animation as well. Figure 4 presents the problem definition panel of Ace and a screenshot from the 3-D animation for a missile avoidance problem.

Overall, the obtained computational experiences underpin the feasibility of the approach and the software implementation for solving realistic near-optimal aircraft trajectories. The obtained numerical results also indicate that optimal trajectories computed using the enhanced 3-DOF aircraft model are possible to attain by a higher-fidelity model using inverse simulation.

4 Air combat games

When the engagement comprises two independent actors with conflicting interests, optimal maneuvering can be analyzed using noncooperative game theory (Fudenberg and Tirole, 1991). In contrast to one-sided settings where the concept of optimality is clear, the solution now depends on the information structure of a game such as an open-loop or a feedback pattern (Başar and Olsder, 1995). Hence, depending on the information available to each player and the payoff functions of the players, a game admits different equilibrium solutions. The essence of an equilibrium is such that each player suffers by deviating from the equilibrium alone. If the sum of the players' payoffs is zero meaning that a gain of one player is a loss of the other player, a game is zero-sum, and its solution is known as the saddle-point equilibrium solution. On the other hand, if the above sum is not a constant, a game is nonzero-sum, and it admits a Nash equilibrium solution.

In case of games, it is sensible to talk about strategies that map the available information



Aircraft model: File aircraft.txt; ID F-16A; Lift coefficient

Missile model: File missile.txt; ID RFBraw2; Lift coefficient

Criterion: Miss distance

Guidance law: Ideal PN

Minimum altitude [m]: 100

Relative distance [m]: 100

a/c x-coordinate [m]: 10000

a/c y-coordinate [m]: 5000

a/c altitude [m]: 10000

a/c flight path [deg]: 0

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Figure 4: Screenshots from Ace. Figure from paper III.

to controls. In a static game, the players act independently of each other. A classical way to describe such a game is to use a normal or strategic form representation, which results in a matrix structure in case of two players (Fudenberg and Tirole, 1991). In a dynamic game, a player is allowed to use a strategy that depends on previous controls (Başar and Olsder, 1995). Such a game can be represented intelligibly in extensive form pioneered by von Neumann and Morgenstern (1947), where the evolution of the game is described with a tree structure. Moreover, if the evolution of the engagement is governed by a differential equation, a dynamic game is called a differential game. The underlying theory, the theory of differential games (Krasovskii and Subbotin, 1988; Başar and Olsder, 1995), can be viewed as a crossing of game theory and optimal control theory.

Two central game models applied in air combat modeling are pursuit-evasion games and two-target games, which are basically differential games. The first ones are classical models for describing pursuit-evasion settings between, e.g., a missile and an aircraft. Contrary to optimal control problem formulations where the pursuer is guided by a guidance law such as those considered in papers I–III of the thesis, the pursuer is now assumed to maneuver optimally. On the other hand, two-target games are applicable for modeling dogfights and missile duels between two aircraft like those studied in papers IV and V. In the following, pursuit-evasion games, their suitable solution techniques, and representative applications are dissected at first. This is followed by a similar discussion on two-target games and a review of papers IV and V.

4.1 Pursuit-evasion games

In pursuit-evasion games, the players have fixed roles, the pursuer and the evader, of which the first player tries to capture the other one. A capture means that the state of the game enters a target set of the game which typically corresponds to an evader centered sphere with a given radius. The players have a common performance measure that is minimized by one player and maximized by the other one, whereupon the game is zero-sum. A common choice is the final time which is minimized by the missile and maximized by the aircraft (see, e.g. Breitner et al., 1993a; Pesch, 1994c; Lachner et al., 1995; Ehtamo and Raivio, 2001). The game admits a saddlepoint equilibrium solution that can be obtained either as the solution of the related Hamilton-Jacobi-Isaacs partial differential equation, or by solving the necessary conditions of an open-loop representation of a feedback saddlepoint solution (Isaacs, 1975).

The qualitative solution of a pursuit-evasion game divides the state space into domains of initial conditions leading to a capture or evasion supposed that the players maneuver optimally. For an example, see Raivio (2001) where capture zones of an optimally guided missile pursuing a fighter aircraft are computed. The quantitative solution determines game optimal strategies of the players within the capture zone (see Ehtamo and Raivio, 2001).

Breitner et al. (1993a,b) study a realistic pursuit-evasion game for a missile versus an aircraft in a vertical plane, and solve the multipoint BVP given by the necessary conditions using multiple shooting. Lachner et al. (1995) extend the analysis to three dimensions. Imado and Kuroda (2004) proceed similarly but consider a planar miss distance maximization instead of the capture time. Recently, also direct methods have been applied in the numerical solution of pursuit-evasion games, where the basic idea is to alternately optimize the player's controls against the fixed trajectory of the adversary. For realistically modeled missile-aircraft engagements solved using direct methods, see Raivio and Ehtamo (2000); Ehtamo and Raivio (2001).

As with optimal control problems, feedback solutions are available for linear-quadratic game formulations. These solutions have been utilized in the guidance law design, too (Bryson and Ho, 1975; Shneydor, 1998; Ben-Asher et al., 2004; Oshman and Arad, 2006). The approaches for obtaining approximate feedback solutions are similar to those for optimal control problems. Based on a fact that in case of pursuit-evasion games, the optimal open-loop and feedback solutions generate the same trajectories (see, e.g., Başar and Olsder, 1995), a course of action is to approximate optimal strategies associated with the current state and time using a neural network (see, e.g., Pesch, 1994c; Pesch et al., 1995). Compared to optimal control applications, yet more massive amount of optimal open-loop solutions covering the appropriate regions of the state space must be solved off-line. Another approach is to solve the optimal open-loop strategies associated with the current state and the current time on-line. This can be done by updating the reference BVP solution on the basis of the error between the current state and the state of the reference solution as the encounter evolves (see, e.g., Anderson, 1977; Järmark, 1986a; Shinar et al., 1986, and the references cited therein). Obviously, a fast method for solving the BVP associated with the current state is imperative. RHC, also, provides an approximate solution method for pursuit-evasion problems (see, e.g., Shinar and Glizer, 1995).

4.2 Two-target games

Not all one-on-one air combat scenarios can be modeled as pursuit-evasion games with the fixed roles of the players. For example, in a dogfight between two aircraft considered in paper IV, the roles of the players may alternate several times from the pursuer to the evader and vice versa. The analysis of such an engagement can be carried out using two-target game models (Blaquière et al., 1969; Olsder and Breakwell, 1974; Grimm and Well, 1991). In these games, the players aim to drive the state of the game to their own target sets while avoiding that of the adversary. The four possible outcomes of the game are the capture by either player, joint capture, or draw.

As with pursuit-evasion games, the qualitative solution divides the state space into regions of different outcomes under optimal strategies of the players. For an example, see Davidovitz and Shinar (1989) where a two-target game between two aircraft maneuvering with constant velocities on a plane is analyzed. Quantitative solutions for two-target games would give optimal strategies of the players in different regions of the state space.

Although some theoretical developments have been made, several unresolved modeling issues are remaining. In the combat game approach (Ardema et al., 1985), the player is constrained from entering the adversary's target set by an ill-posed state inequality constraint that involves both players but does not restrict the actions of the adversary. Ghose and Prasad (1989) consider a two objective game where the criteria correspond, e.g., to the distances between the state of the players and the target sets. Several solution concepts are defined on a theoretical level (see Grimm and Well, 1991), but practical applications are missing. Another issue is the nonuniqueness of the optimal solution in the regions leading to a joint capture and to a draw.

Yet more complicated game models involve both the aircraft and the missiles. Järmark (1985) constructs a zero-sum differential game model of a three dimensional duel with fire-and-forget missiles and solves the game in open-loop form using differential dynamic programming (Jacobson and Mayne, 1970). In some applications like in a missile duel analyzed in paper V, the aircraft can improve the performance of the missile by relaying it target information for a given support time. Moritz et al. (1987) introduce zero- and nonzero-sum differential game formulations of a 3-D duel with the compulsory support of the missile till the lock-on range, and solve the games in open-loop form using a direct method.

Due to their complexity, complete feedback solutions of two-target games are beyond reach. For the sake of computational tractability, mostly planar engagements with discrete control alternatives have been studied. Kelley and Cliff (1980) propose reprisal strategies where the nonoptimal behavior of the adversary is utilized in order to reach one's winning zone before the adversary. Neuman (1990) uses suboptimal feedback laws as decision alternatives of the aircraft, and solves the best guidance laws associated with a given initial state from a matrix game. The missiles are of the fire-and-forget type. Shinar et al. (1988) and Le Ménec and Bernhard (1995) combine artificial intelligence techniques with pursuit-evasion game solutions in the pilot decision support systems that recommend favorable launch and evasion moments in a missile duel. Like with single-sided optimization and pursuit-evasion problems, RHC has been applied in the approximate feedback solution of two-target games as well. Austin et al. (1990) study one-stage matrix games with a heuristic payoff. Katz and Ross (1991) and Katz (1994) model a helicopter duel as a myopic game in extensive form in which each node is associated with a score computed on the basis of the players' kill and survival probabilities.

4.3 Uncertainty models

In a dynamic game, uncertainty can be incorporated in the extensive form representation of the game by specific chance moves controlled by the chance player or nature. For example, in the myopic game of Katz and Ross (1991) and Katz (1994), each player's probability of survival decays exponentially as a function of the time spent in the adversary's target set. These probabilities are then propagated in the payoff functions of the players. In case of differential games, the evolution of the conflict can be described by a stochastic differential equation, where the actions of the chance player follow a stochastic process such as the Wiener process (see Başar and Olsder, 1995). Although theoretical foundations of stochastic differential games are sound, such game formulations have been of little practical use from the viewpoint of air combat modeling.

Although influence diagrams originate from decision problems comprising only one decision maker, their foundations have been recently expanded to cover game situations as well. This research direction, although first proposed by Shachter (1986), was explored by Koller and Milch (2001, 2003) who introduced the multi-agent influence diagram framwork for describing and solving multiple decision maker problems by influence diagrams. Contrary to strategic or extensive form representations of games, the framework allows compact representation and efficient solution of noncooperative static games by the means of influence diagrams. However, the framework is not suitable for describing dynamic settings governed by differential equations such as a dogfight between two aircraft studied in paper IV of the thesis.

Virtanen et al. (2003) extend the analysis of Virtanen et al. (1999b, 2004) to a two-sided setting by constructing an influence diagram game modeling pilots' maneuvering decisions in one-on-one air combat. The game model essentially combines two separate single actor influence diagrams that incorporate also the dynamics of the aircraft. The game is solved approximately using RHC with a planning horizon of one decision stage.

4.4 Influence diagram game modeling one-on-one air combat

In paper IV, the seminal ideas introduced by Virtanen et al. (1999b, 2003, 2004) are elaborated and implemented. The paper unifies the ideas of applying influence diagrams for a dynamic setting (Virtanen et al., 1999b, 2004) and for multiple decision maker problems (Koller and Milch, 2003). In the introduced influence diagram game that models a dogfight between two aircraft, the players optimize their controls with respect to the expected utilities that capture the preferences of the players in different combat situations. The players' beliefs in the threat situation of the combat are updated using Bayesian reasoning. The utility and likelihood functions of the players are constructed such that the game model corresponds to a two-target game. For example, in an advantageous threat situation the players prefer to pursue the adversary, whereas in a disadvantageous situation, to evade. It should be noted that contrary to separated determination of the qualitative and quantitative solutions of the game, the two stages have been unified in the influence diagram game.

As such, the game is nonzero-sum, and its solution corresponds to a Nash equilibrium solution. Since the computation of a complete feedback Nash equilibrium solution of the game is infeasible, the controls of the players are solved using RHC with the planning horizon of several stages. For the sake of computational tractability, discrete control alternatives are used, whereupon an equilibrium solution of the truncated horizon game can be computed using dynamic programming. Cruz et al. (2001, 2002) apply similar technique in the solution of a complex military air operation game. It is noteworthy that the controls of the players over the planning horizon are computed in feedback form instead of the more commonly used open-loop optimization (see Mayne et al., 2000).

The game model and the RHC based solution method are demonstrated by numerical examples where representative inital conditions of a dogfight are applied. Figure 5 presentes 3-D trajectories of two aircraft engaged in a dogfight for an example case. In the particular case, a planning horizon of three stages has been used. The combat situation is initially advantageous for red, which is modeled less agile than blue. In the end, however, the more agile blue captures red in 21 seconds. In general, a longer planning horizon appears to benefit usually the more agile player.

The numerical examples presented in the paper indicate that the game model and the solution approach are feasible for the near-optimal solution of a two-target game. Overall, the solution approach provides a transparent way to analyze the effect of the players' preference and belief models on the solution of the game. According to the numerical examples, the computation times are feasible considering the real-time implementation. Consequently, the approach provides a means for the on-line optimization of air combat maneuvers in a dogfight between two aircraft.

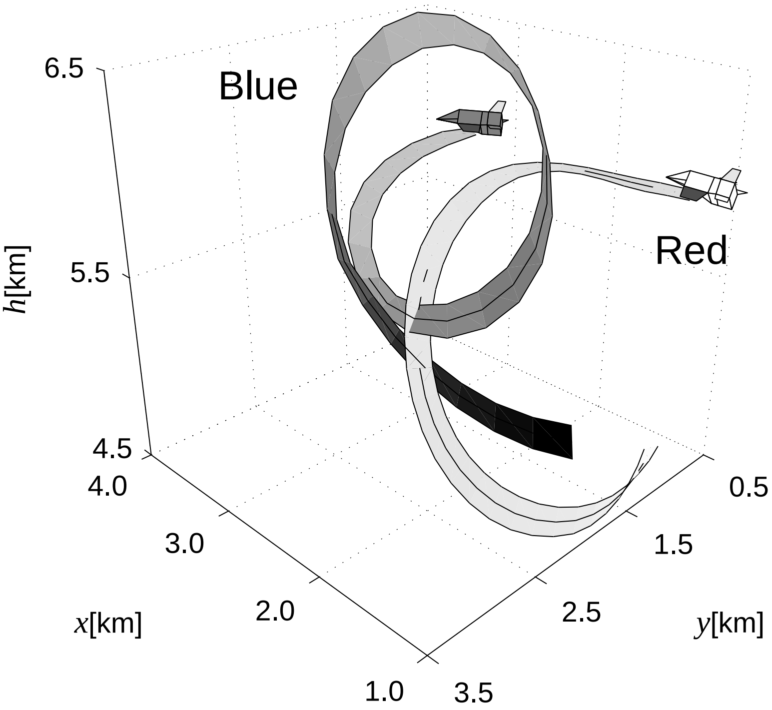

Figure 5: Dogfight between two aircraft. Figure from paper IV.

4.5 Game optimal support time of an air-to-air missile

In paper V, a missile duel between two aircraft equipped with one missile each is considered. In the game setting, each aircraft can relay target information to the respective missile for a selectable support time, after which the missile has to extrapolate the target position until the lock-on. The longer the aircraft supports the missile, the more accurate target information the missile receives which improves the missile's probability of hit to the adversary. On the other hand, the aircraft must fly towards the adversary during the support phase which decreases the aircraft's probability of survival against the enemy missile. Overall, the support time game can be considered as a special case of a two-target game. For closely related works in the open literature, see Järmark (1985); Moritz et al. (1987); Shinar et al. (1988); Le Ménec and Bernhard (1995). However, none of the listed works consider the support time as a variable to be optimized.

By applying suitable simplifications, the complicated differential game described above is converted to a computationally tractable nonzero-sum static game providing optimal support times of the missiles. In this game, each player maximizes a weighted sum of the probabilities of hit to the adversary and own survival, where the weights reflect the risk attitudes of the players. For each player, these probabilities are obtained on the basis of solutions of a set of pursuit-evasion optimal control problems between the missile and the aircraft. Game optimal support times of the missiles are given as a Nash equilibrium solution of the static game computed by fixed point iteration (Başar and Olsder, 1995).

Figure 6 illustrates the optimal trajectories of the aircraft and the missiles for a given initial state. The initial altitudes and velocities of the blue and red players are 9750 m, 9500 m, 275 m/s, and 250 m/s, respectively, whereas the initial range between the players is 18 km. In the particular example, blue is utmost risk aversive and considers only his own survival, whereas red weights evenly the probabilities of hit to the adversary and own survival. The obtained solution is plausible, for risk aversive blue evades immediately, whereas risk prone red supports the respective missile for a considerable time.

Furthermore, a real-time computation scheme for obtaining approximate game optimal support times related to a given set of launch conditions is introduced. The scheme is based on the linear interpolation of the off-line computed solutions of the optimal controls problems over the state space. The obtained computational experiences indicate that the approach is feasible for the determination of game optimal support times of the missile. It is noteworthy that the game model provides optimal trajectories of the aircraft as well. In addition, solutions computed by the real-time scheme are close to the game optimal ones.

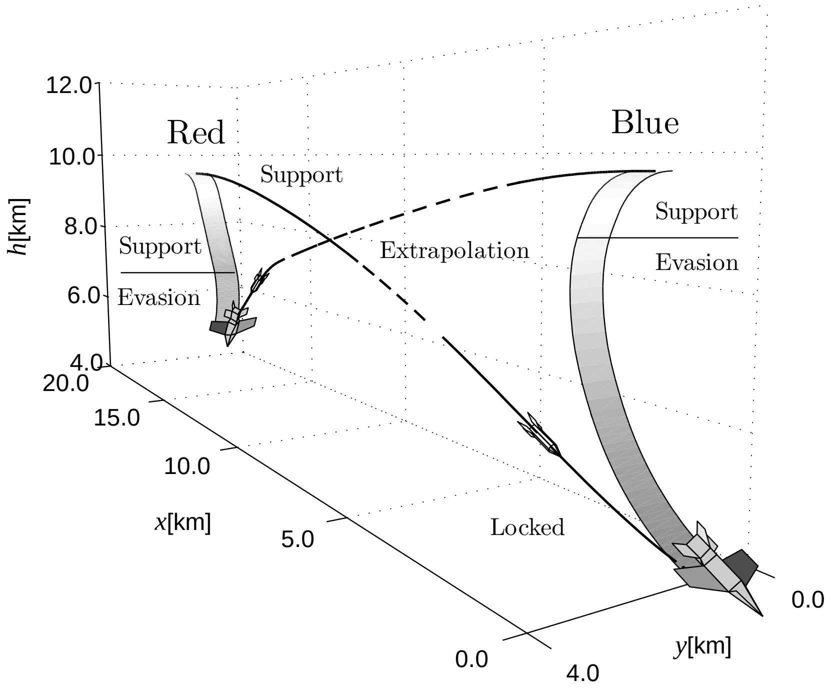

Figure 6: Illustration of optimal flight paths in the support time game. Figure from paper V.

5 Conclusions and future work

The thesis comprises five individual papers. The first two papers consider missile avoidance and introduce novel on-line aircraft guidance schemes for pursuit-evasion problems between the missile and the aircraft. The third paper concerns aircraft trajectory optimization and proposes a new approach and its software implementation for the user-oriented solution of realistic near-optimal aircraft trajectories. The last two papers consider one-on-one air combat and introduce game models along with computational methods for the on-line solution of the pilot's maneuvering decisions in a dogfight and the optimal support time of an air-to-air missile.

The computational results presented in the thesis indicate that RHC and direct methods provide suitable tools for the on-line solution of complicated air combat optimization problems and games. Apparently, RHC serves as a suitable platform for auxiliary methods such as Bayesian reasoning that enable the consideration of uncertainties in air combat. Consequently, a practical framework for adaptive control capable of coping with uncertain dynamics, human factors, and unpredictable disturbances is obtained. The results obtained from the inverse simulation also indicate that the enhanced point-mass aircraft model that takes into account also rotational kinematics is sufficiently realistic for the computation of optimal trajectories, on the basis of which actual control commands of an aerial vehicle could be solved on-line. Hence, the modeling and solution approaches introduced in the thesis provide the basis for the onboard guidance system of an aerial vehicle committing air combat or a pilot advisory system.

Considering the real-life implementation of the presented approaches, several issues are still open for future research. For example, the performance of the controllers developed in papers I and II with imperfect state information is essential in real-life situations, which raises a need for the development of a suitable state estimator to be incorporated into the controllers. Another key issue is the computational efficiency, which calls for the determination of optimal planning horizons for each performance measure with respect to computation times and the quality of solutions. The unification of the presented on-line solution approaches into a single guidance system invokes further research as well. The integration of the controllers and the support time game of paper V is one theme. Another topic is the extension of the influence diagram game presented in paper IV to cover longer range missile duels, which calls for the development of preprogrammed control sequences enabling longer planning horizons and combat ranges.

To summarize, the thesis introduces new modeling approaches and on-line solution methods for complicated air combat settings. The applied techniques enable the consideration of uncertainties in air combat and the utilization of realistic vehicle models, whereby the presented approaches provide the basis for the onboard guidance system of an aerial vehicle. It is, however, evident that due to the technological development of air combat arsenal, the complexity of air combat just increases. Thus, it appears unlikely that automated guidance systems would totally override human pilots at some point in future. Nevertheless, such systems provide invaluable decision support for fighter pilots whose capabilities to observe and process the abundant information on the state of the combat are necessarily limited. Altogether, the thesis closes the gap between the analysis of air combat tactics and reality by presenting truly practical approaches for the modeling and on-line solution of fundamental air combat scenarios.

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