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Independent postulates for subjective expected utility

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Abstract

Although the subjective expected utility (SEU) theory is more than 60 years old, it was recently discovered by Hartmann (Econometrica 88(1):203–205, 2020, https://doi.org/10.3982/ECTA17428) that one of the original seven postulates is redundant, i.e., it is implied by the other six postulates. In this brief communication, we show that this redundant axiom is the only one that is implied by the other axioms, thereby establishing that the remaining six postulates form an independent axiomatic system. This result further streamlines the preference assumptions underlying the SEU theory.

Keywords Decision analysis · Decision theory · Subjective expected utility · Subjective probability

1 Introduction

Subjective expected utility (SEU; Fishburn 1970; Savage 1954) is perhaps the most widely used theory for rational decision making under uncertainty. The formalization of SEU in Savage (1954) is based on seven assumptions, referred to as postulates, regarding the preferences of a rational decision maker (DM). The DM's preferences satisfy these postulates if and only if there is an expected utility representation of these preferences. This representation is defined by (i) a subjective probability measure across the states of the world and (ii) a utility function capturing the desirability of different consequences. A rational DM should thus choose the decision alternative, or act in Savage's terminology, that maximizes the expected utility under the subjective probability measure.

Although the theory of SEU is more than 60 years old, it has remained relevant to this day and continues to provide the core for research on decision making under uncertainty (see, e.g., Abdellaoui and Wakker 2020; Karni 2014; Shafer 1986; Wakker 1993). The aforementioned articles also include literature reviews on SEU, something that is omitted from this brief communication. Regarding the axiomatic

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foundations of SEU, it has recently been discovered that one of the original seven postulates presented by Savage (1954) is redundant. Specifically, Hartmann (2020), amended by Frahm and Hartmann (2023), shows that preferences always satisfy the third postulate if they satisfy the other six postulates. This surprising discovery raises the question whether there are further redundancies in the axiomatization of SEU.

In this brief communication, we show that the third postulate is the only one in Savage's axiomatizaton that is implied by the others. In other words, removing any of the other postulates would not result in a set of sufficient preference conditions for SEU. This result implies that the six postulates other than the third one must all be independent. To our knowledge, this is the first time that the independence of the postulates is formally verified. Postulates 1, 2 and 4–7, therefore, provide a set of sufficient, necessary and independent postulates on top of which SEU theory and consequent research efforts in decision making under uncertainty can be based upon.

2 Subjective expected utility theory

An axiomatization for SEU is presented by both Savage (1954) and Fishburn (1970). The two differ somewhat with respect to notation. There is also a more fundamental difference in that Savage bases his system on weak preference while Fishburn bases his system on strict preference. The only significant difference from a mathematical point of view, however, is in the seventh postulate. Fishburn's formulation of the postulate assumes less from the DM's preferences. It is known, however, that the two formulations are equivalent when the other postulates hold. Here, we use Fishburn's version of the postulate, but all of our results apply equally with Savage's version as well.

In the SEU model, the set of all possible states of the world is denoted by S. The states $s \in S$ capture possible realizations of all relevant uncertainties. The true state of the world is one of the elements $s \in S$, but the DM does not know which one. Subsets of S are referred to as events. Each decision alternative, or act, corresponds to a mapping $f : S \rightarrow X$ that assigns a consequence x from the set of all consequences X to each state $s \in S$. Specifically, $f(s) \in X$ is the consequence when the DM chooses act f and s is the true state. The set of all acts, i.e., all mappings from S to X, is denoted by $F = \{f : S \rightarrow X\}$. Note that the set F includes not only the actual decision alternatives among which the DM has to make a choice (i.e., concrete acts; Shafer 1986), but also all hypothetical acts. Moreover, this model does not set explicit requirements on the cardinality of the set of states S or the set of consequences X, but the postulates imply that S is infinite.

The DM's preference between any two acts in the set F is represented by the binary relation $\prec$. In particular, $f_1 \prec f_2$ denotes that the DM (strictly) prefers act f_2 to act f_1. If neither $f_1 \prec f_2$ nor $f_2 \prec f_1$ holds, the DM is said to be indifferent between the two acts, denoted by $f_1 \sim f_2$. The DM weakly prefers f_2 to f_1, denoted by $f_1 \precsim f_2$, if $f_1 \prec f_2$ or $f_1 \sim f_2$.

The SEU postulates utilize two special types of acts: constant acts and binary acts. A constant act assigns the same consequence to all states of the world. Such an act is denoted by the corresponding consequence in bold. A binary act is identical to one act in some event and to another act in the complement of this event.

Definition 1 (*Constant act*) For any consequence $x \in X$, the constant act $\mathbf{x} \in F$ is an act such that $\mathbf{x}(s) = x$ for every state $s \in S$.

Definition 2 (*Binary act*) For any acts $f_1, f_2 \in F$ and event $A \subseteq S$, the binary act $f_{1A}f_2 \in F$ is an act such that

$$f_{1A}f_2(s) = \begin{cases} f_1(s), & \text{if } s \in A, \\ f_2(s), & \text{otherwise.} \end{cases}$$

We introduce conditional preferences to enable a more compact presentation of the postulates. The DM conditionally prefers act f_1 to f_2 given event A if they prefer f_1 whenever the two acts' consequences are modified to be identical in the complement of A.

Definition 3 (*Conditional preference*) For any acts $f_1, f_2 \in F$ and event A,

$$\begin{aligned} &f_1 \prec_A f_2 \text{ if } f_{1A}f' \prec f_{2A}f' \text{ for all } f' \in F, \\ &f_1 \sim_A f_2 \text{ if neither } f_1 \prec_A f_2 \text{ nor } f_2 \prec_A f_1, \\ &f_1 \precsim_A f_2 \text{ if } f_1 \prec_A f_2 \text{ or } f_1 \sim f_2. \end{aligned}$$

Finally, one of the postulates makes use of the concept of null events. An event is said to be null if the DM is indifferent between any acts whose consequences differ only in this event.

Definition 4 (*Null event*) The event $A \subseteq S$ is null if $f_{1A}g \sim f_{2A}g$ for all acts $f_1, f_2, g \in F$.

The axiomatization of SEU theory by Fishburn (1970) consists of the following seven postulates. The descriptive titles of these postulates were introduced by Shafer (1986).

Postulate 1 (The existence of a complete ranking) The relation $\prec$ is asymmetric (for any acts $f_1, f_2 \in F$, $f_1 \prec f_2$ and $f_2 \prec f_1$ cannot both hold) and negatively transitive (for any acts $g_1, g_2, g_3 \in F$, if neither $g_1 \prec g_2$ nor $g_2 \prec g_3$, then $g_1 \prec g_3$ cannot hold).

Postulate 2 (The independence postulate) For any acts $f_1, f_2, g_1, g_2 \in F$ and any event $A \subseteq S$, $f_{1A}g_1 \prec f_{2A}g_1 \Leftrightarrow f_{1A}g_2 \prec f_{2A}g_2$.

Postulate 3 (Value can be purged of belief) For any consequences $x, y \in X$ and any event $A \subseteq S$ that is not null, $\mathbf{x} \prec_A \mathbf{y} \Leftrightarrow \mathbf{x} \prec \mathbf{y}$.

Postulate 4 (Belief can be discovered from preference) For any consequences $x_1, x_2, y_1, y_2 \in X$ such that $\mathbf{x_1} \prec \mathbf{x_2}$ and $\mathbf{y_1} \prec \mathbf{y_2}$ and any events $A, B \subseteq S$, $\mathbf{x_1}_A\mathbf{x_2} \prec \mathbf{x_1}_B\mathbf{x_2} \Leftrightarrow \mathbf{y_1}_A\mathbf{y_2} \prec \mathbf{y_1}_B\mathbf{y_2}$.

Postulate 5 (The nontriviality condition) There exist consequences $x, y \in X$ such that $\mathbf{x} \prec \mathbf{y}$.

Postulate 6 (The continuity condition) For any acts $f, g \in F$ such that $f \prec g$ and any consequence $x \in X$, there is a finite partition $\{A_1, \ldots, A_n\}$ of S such that $\mathbf{x}_{A_k} f \prec g$ and $f \prec \mathbf{x}_{A_k} g$ for every $k \in \{1, \ldots, n\}$.

Postulate 7 (The dominance condition) For any acts $f, g \in F$ and any event $A \subseteq S$, if $f \prec_A \mathbf{x}$ for every consequence $x \in \{g(s) \mid s \in A\}$, then $f \precsim_A g$, and if $\mathbf{x} \prec_A g$ for every $x \in \{f(s) \mid s \in A\}$, then $f \precsim_A g$.

Fishburn (1970) shows that the DM's preferences satisfy Postulates 1–7 if and only if there exist a non-atomic finitely additive probability measure P defined on all the subsets of S and a non-constant bounded function $u : X \to \mathbb{R}$ such that for any acts $f_1, f_2 \in F$

$$f_1 \prec f_2 \Leftrightarrow \int u(f_1(s))dP(s) < \int u(f_2(s))dP(s). \tag{1}$$

Furthermore, the probability measure P is unique and the function u is unique up to a positive affine transformation. This result implies that a rational DM should choose the act f that maximizes the expected utility

$$U(f) = \int u(f(s))dP(s).$$

3 Independence of Postulates 1, 2 and 4–7

This section shows that Postulates 1, 2 and 4–7 are each independent of the other six postulates including Postulate 3. For each Postulate 1, 2 and 4–7, we provide an example preference relation ($\prec^i$) that violates the postulate in question (i), but satisfies the others, including Postulate 3. Each example shows that the corresponding postulate cannot be derived from the other six postulates and is, therefore, not redundant. Together, these examples show that removing any of the other postulates instead of Postulate 3 would not result in a set of sufficient preference conditions for SEU.

For brevity, we only provide an overview of the example preferences, and provide some insight into why these preferences violate one postulate but satisfy the others. A formal examination of the example preferences is found in the electronic supplement. Note that these examples have been constructed to be mathematically straightforward in order to clearly demonstrate the independence of the postulates rather than to represent some empirically motivated decision behavior.

In each of the six examples, both the states of the world and the consequences correspond to the positive integers, i.e., $S = X = \{1, 2, \ldots\}$. Moreover, the example preference relations utilize a probability measure π that satisfies

$$\pi(\{k, k+n, k+2n, \dots\}) = \frac{1}{n} \tag{2}$$

for all positive integers k and n. For instance, this measure assigns a 50% probability for the event consisting of all positive odd integers, i.e., $\pi(\{1, 3, 5, \dots\}) = 1/2$. The existence of measures that satisfy (2) has been established by, e.g., Kadane and O'Hagan (1995). Notably, the measure π is non-atomic.

To provide intuition on why the example preferences satisfy all but one of the postulates, we compare each of them to a reference relation $\prec^0$ that satisfies all seven postulates. Formally, this reference relation is defined as

$$f \prec^0 g \Leftrightarrow V^0(f) < V^0(g),$$

where $V^0(f) = \int v^0(f(s))d\pi(s)$ and $v^0 : X \to \mathbb{R}$ is an arbitrary non-constant bounded function. The definition of $\prec^0$ matches (1), so the relation is known to satisfy all seven postulates.

To construct preferences that violate Postulate 1 ('The existence of a complete ranking'), suppose the DM seeks to minimize the probability of obtaining the consequence $x = 1$. Furthermore, the DM prefers act g to act f only if this probability is at least 75% higher for f than for g. Formally, preferences $\prec^1$ are defined by

$$f \prec^1 g \Leftrightarrow \pi(\{s \in S \mid g(s) = 1\}) < \pi(\{s \in S \mid f(s) = 1\}) - \frac{3}{4}.$$

This is an exaggerated case of the DM being indifferent about small differences in the values of alternatives. It is straightforward to construct acts $f_1, f_2, f_3 \in F$ such that the probabilities of obtaining the consequence $x = 1$ from each act are 1, 1/2 and 0, respectively. Then, neither $f_1 \prec^1 f_2$ nor $f_2 \prec^1 f_3$, but $f_1 \prec^1 f_3$ does hold, which clearly violates the negative transitivity requirement of Postulate 1.

The other postulates are satisfied by $\prec^1$, as its definition is similar to that of $\prec^0$, but requires the value difference between the alternatives to exceed 3/4 rather than 0. This change in the threshold has no affect on Postulates 2 and 7. Postulates 3 and 4 are also unaffected, as there are effectively only two consequences, namely $x = 1$ and $x > 1$. Postulate 5 is satisfied as the new threshold is smaller than $\pi(S) = 1$. Finally, Postulate 6 is satisfied since the definition of $\prec^1$ requires strict inequality to hold between the alternatives' values.

Postulate 2 ('The independence postulate') is violated if the consequences from different states of the world are not aggregated linearly. Suppose, for instance, that the DM assigns values 0, 2 and 5 for the consequences $x = 1$, $x = 2$ and $x \geq 3$, respectively, but also prefers having a high probability for one of these consequences. Specifically, preferences $\prec^2$ are represented by

$$\begin{aligned} V^2(f) &= 0\pi(\{s \in S \mid f(s) = 1\}) + 2\pi(\{s \in S \mid f(s) = 2\}) + 5\pi(\{s \in S \mid f(s) \geq 3\}) \\ &\quad + \max\{\pi(\{s \in S \mid f(s) = 1\}), \pi(\{s \in S \mid f(s) = 2\}), \pi(\{s \in S \mid f(s) \geq 3\})\}, \end{aligned}$$

i.e., $f \prec^2 g$ holds if and only if $V^2(f) < V^2(g)$ holds. Due to the non-linear term in V^2, improving consequences of act f from $x = 1$ to $x = 2$ in some states can result in different increases in the value $V^2(f)$ depending on the consequences of f in other states. This is a violation of Postulate 2, which requires that the resulting increase should be independent of the consequences in other states.

The non-linear maximization term in V^2 is the only difference between preferences $\prec^2$ and preferences $\prec^0$, which satisfy all of the postulates. Thus, $\prec^2$ satisfies Postulates 1, 5 and 6 as these postulates are unaffected by such a term. Furthermore, Postulates 3, 4 and 7 are also satisfied, because the term is small enough relative to the values of the consequences.

Postulate 4 ('Belief can be discovered from preference') is violated if the DM assigns different values for consequences depending on the state of the world. Suppose, for instance, that the DM assigns values 0 and 3 for the consequences $x = 1$ and $x \geq 3$, respectively, while the value assigned for the consequence $x = 2$ depends on the state: it is 1 in odd states and 2 in even states. Formally, preferences $\prec^4$ are represented by

$$\begin{aligned} V^4(f) = {} & 0\pi(\{s \in S \mid f(s) = 1\}) + 1\pi(\{s \in \{1, 3, \ldots\} \mid f(s) = 2\}) \\ & + 2\pi(\{s \in \{2, 4, \ldots\} \mid f(s) = 2\}) + 3\pi(\{s \in S \mid f(s) \geq 3\}). \end{aligned}$$

These preferences make it impossible to determine whether preference for a particular act f is the result of the high utility of its consequences ($u(x)$), or the high probability of obtaining these consequences ($\pi(\{s \in S \mid f(s) = x\})$).

To confirm that $\prec^4$ satisfies the other postulates, note that the only difference between $\prec^4$ and $\prec^0$ is that in the former, the value of the consequence $x = 2$ depends on the state of the world s. Since the preference order of consequences $x = 1$, $x = 2$, and $x \geq 3$ remains the same, this modification does not affect any of the other postulates.

Postulate 5 ('The nontriviality condition') is violated if the DM is indifferent between all acts. In particular, assuming that $f \prec^5 g$ does not hold for any $f, g \in F$ implies that there are no consequences $x, y \in X$ such that $\mathbf{x} \prec^5 \mathbf{y}$. Moreover, since the DM is indifferent between any pair of acts, it is straightforward to establish that the other postulates hold.

Preferences $\prec^5$ can be derived from $\prec^0$ by allowing the consequence value function v^0 to be constant. As none of the other postulates contradict a constant consequence value function, they are satisfied by preferences $\prec^5$.

Postulate 6 ('The continuity condition') is violated if the DM evaluates acts solely based on the consequences they yield in a specific state. For instance, if the DM assumes that the true state of the world is $s = 1$ and prefers a higher consequence in this state to a lower one, then such preferences $\prec^6$ are defined by

$$f \prec^6 g \Leftrightarrow f(1) < g(1).$$

Clearly, the DM prefers the constant act **2** over **1**. However, changing the consequences of act **1** in any event that includes the state $s = 1$ results in an act that is weakly preferred to the constant act **2**. This violates Postulate 6.

The difference between $\prec^6$ and $\prec^0$ is that latter uses the non-atomic measure π given by (2), while the former implicitly uses the atomic measure $\pi^*(S')$ defined such that $\pi^*(S') = 0$ when $1 \notin S'$ and $\pi^*(S') = 1$ when $1 \in S'$. As none of the postulates except for Postulate 6 contradict an atomic probability measure, preferences $\prec^6$ satisfy the other postulates.

To construct a preference relation that violates Postulate 7 ('The dominance condition') but satisfies all others, we adapt an example in Section 5.4 of Savage (1954). Suppose the DM evaluates the probability that the consequence of an act f is higher than some threshold limit m, and values f based on the limit of this probability when m approaches infinity (i.e., $\lim_{m\to\infty} \pi(\{s \in S \mid f(s) \geq m\})$). This limit is zero for all constant acts and hence, in order to satisfy Postulate 5 ('The nontriviality condition'), we augment the limit by subtracting the probability of obtaining the least preferred consequence $x = 1$. Formally, preferences $\prec^7$ are represented by

$$V^7(f) = \lim_{m\to\infty} \pi(\{s \in S \mid f(s) \geq m\}) - \pi(\{s \in S \mid f(s) = 1\}).$$

Since the limit is zero for all constant acts, it is easy to construct acts $f, g \in F$ such that g is preferred to f, but both are preferred to any constant act. This implies that f is preferred to the constant act $\mathbf{g}(\mathbf{s})$ for every state s, which is a violation of Postulate 7.

Compared to the reference preferences $\prec^0$ satisfying all of the postulates, $\prec^7$ introduces a limit-term into the value function. This term does not, however, cause a violation of any of the other postulates. Postulate 1 is satisfied by any preferences represented by a value function. Postulates 4 and 5 are also satisfied, as the term in question is always 0 for acts consisting of a finite number of consequences. Finally, because the term is additive over the states of the world, i.e., $\lim_{m\to\infty} \pi(\{s \in S \mid f(s) \geq m\}) = \lim_{m\to\infty} \pi(\{s \in S' \mid f(s) \geq m\}) + \lim_{m\to\infty} \pi(\{s \in S \backslash S' \mid f(s) \geq m\})$ for any $S' \subseteq S$, Postulates 2, 3 and 6 are also satisfied.

4 Conclusion

The redundancy of Postulate 3 shown by Hartmann (2020) together with the preference relations developed in Sect. 3 make it possible to establish a simplified representation theorem for SEU as follows.

Theorem 1 *The DM's preferences satisfy Postulates* 1, 2 *and* 4–7 *if and only if there exist a non-atomic finitely additive probability measure P defined on all the subsets of S and a non-constant bounded function $u : X \to \mathbb{R}$ such that for any acts $f_1, f_2 \in F$*

$$f_1 \prec f_2 \Leftrightarrow \int u(f_1(s))dP(s) < \int u(f_2(s))dP(s), \tag{3}$$

where the probability measure P is unique and the function u is unique up to a positive affine transformation. Furthermore, any proper subset of Postulates 1–7 *other than* 1, 2 *and* 4–7 *is not sufficient to imply the existence of representation* (3).

Theorem 1 has several appealing properties. First, it includes only six postulates as opposed to the previously used seven. Second, the examples in Sect. 3 imply that the representation theorem cannot be simplified further by removing any of the postulates. Third, the postulates used in this representation theorem do not utilize null events, thus reducing the number of concepts required in developing and deploying SEU models.

Although the third postulate in the axiomatization of SEU is indeed redundant, as shown by Hartmann (2020), there are intuitive explanations why it was originally included. In Savage (1954), it is shown that the first six postulates are sufficient to establish SEU in the special case where there is only a finite number of possible consequences. These six postulates are also sufficient to provide a definition for subjective probability which arguably was one of Savage's research objectives (Hartmann 2020; Abdellaoui and Wakker 2020). The seventh postulate is then introduced to extend SEU to the general case that does not impose any limitations on the set of consequences. However, the introduction of the seventh postulate also renders the third postulate redundant.

Why has this observation gone unreported for so long until the recent work of Hartmann (2020)? It seems that researchers have simply assumed that the independence of the postulates has already been established, even though this is not the case. For instance, Baccelli (2017) states that *"Presumably, Savage and others have checked the logical independence of the [third and fourth postulate], but they may have thought that the mathematical observation underlying this subsection did not deserve to be reported"*. Another reason might be that SEU is usually taught using a setting in which acts can only yield a finite number of different consequences. As mentioned, in this setting only the first six postulates are needed and the third postulate is then independent of the other five.

Our results show that the six postulates are enough to establish a solid and irreducible axiomatic foundation for SEU. Based on the redundancy of the third postulate and the independence of the remaining postulates, we established a simplified representation theorem for SEU. This theorem is compact in the sense that removing any additional postulates would result in a set of assumptions that does not guarantee SEU representation of preferences. Moreover, the third postulate is the only postulate that utilizes the somewhat unintuitive concept of null events. Thus, the simplified axiomatization does not require the introduction and use of null events.

In this brief communication, we considered the postulates of Savage (1954) and showed that removing Postulate 3 is the only simplification of the preference assumptions that can be made. It may still be possible to simplify Savage's axiomatization of SEU further, however, by modifying the postulates rather than removing any of them. This is an intriguing avenue for future research. Indeed, Baccelli and Hartmann (2023) consider this question in the case where the set of consequences is finite and Postulate 7 can therefore be dropped instead of Postulate 3.

The results of Hartmann (2020) and this brief communication also affect other preference models that make use of SEU. For instance, the consequence consistency axiom in the spatial decision analysis model in Harju et al. (2019) is redundant on precisely the same grounds as the third postulate in SEU. Indeed, an obvious step in future research would be to examine the implications the redundancy of the third postulate has on the plethora of preference models that build on subjective expected utility.

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Declarations

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