# Chapter 1

# Preference Programming – Multicriteria Weighting Models under Incomplete Information

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**Abstract** Useful decision recommendations can often be provided even if the model parameters are not exactly specified. The recognition of this fact has spurred the development of multicriteria methods which are capable of admitting and synthesizing incomplete preference information in hierarchical weighting models. These methods share similarities in that they (i) accommodate incomplete preference information through set inclusion, (ii) offer decision recommendations based on dominance concepts and decision rules, and (iii) support the iterative exploration of the decision maker's preferences. In this Chapter, we review these methods which are jointly referred to by the term 'preference programming'. Specifically, we discuss the potential benefits of using them, and provide tentative guidelines for their deployment.

## 1.1 Introduction

Hierarchical weighting methods–such as value trees [31] and the Analytic Hierarchy Process [54]–are widely used in the analysis of decision problems that are characterized by incommensurate objectives, competing alternatives and conflicting stakeholder interests (see, e.g., [23, 29]). In these methods, the decision maker (DM) is engaged in a process where the decision objectives are structured as a hierarchy of attributes. This phase is often one of the most instructive phases of problem solving due to the insights that it may give [8, 30]. In effect, the resulting hierarchical

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problem representation provides a framework for synthesizing information about (i) how the alternatives perform on the attributes (=scores) and (ii) how important the attributes are (=weights). Based on these two forms of statements, an overall performance measure can be attached to each alternative.

Yet, most multicriteria methods are based on the assumption that complete information about the model parameters (scores, attribute weights) need to be elicited as 'exact' point estimates. This assumption can be questioned on many grounds. For example, it may be impossible to obtain exact *ex ante* estimates what impacts the alternatives will have on the attributes; and even if such information can be obtained, it may come at a prohibitively high cost, which makes it of interest to examine what tentative conclusions are supported by the information that can be acquired at an affordable cost [34, 73]. This is the motivation for many of the rank-based methods such as SMARTER [17] which converts the DMs ordinal statements into centroid-weights; however, in doing so, an initial incomplete (ordinal) preference specification is essentially transformed into 'exact' cardinal weights which no longer reflect this incompleteness (see [6, 24]) . Furthermore, if the DMs are not confident with providing a complete specification of their preferences, they may regard the results with untrustworthy if they are not confident with the inputs. From the viewpoint of sensitivity analysis, too, it is advisable to examine how recommendations will change when the model parameters vary within plausible limits ([46, 53]).

The above concerns have motivated the development of methods which accommodate incomplete preference information in hierarchical weighting models (see, e.g., [2, 18, 20, 34, 35, 39, 42, 43, 50, 51, 52, 60, 61, 62, 66, 73, 75]). Even though these methods differ in their details, they share many similarities: in particular, they (i) model incomplete preference information through set inclusion, (ii) apply dominance structures and decision rules to derive decision recommendations and, in many cases, (iii) guide the DM during the iterative phases of preference elicitation. In view of these similarities, we therefore employ 'preference programming'–a term that was coined by Arbel [5]–as a general term for all methods which fulfil at least the two first of the above conditions. This term seems pertinent also because these methods engage the DM in an interactive exploration of preferences and offer intermediate results by solving mathematical programming problems.

Apart from numerous incremental methodological contributions, there is a growing number of papers that describe promising real-life applications of preference programming methods (see, e.g., [19, 26, 28, 48]. Yet, not much work has been done to synthesize 'lessons learned' from this applied work. Nor has it been examined in what decision contexts preference programming methods work best, or how they should be best employed in such contexts. We therefore give a structured review of these methods and identify conditions which suggest that the modeling of incomplete information can be particularly helpful. We also argue that, in some conditions, preference programming methods may outperform 'conventional' approaches, particularly if the costs of preference elicitation are high, or if there is a need to focus the analysis on the few most preferred alternatives.

This review Chapter is structured as follows. Section 2 describes the essential features of preference programming. Section 3 reviews selected applications and presents relevant software tools. Section 4 considers what problem characteristics may call for the deployment of preference programming methods. Section 5 provides some tentative guidelines for preference elicitation.

## 1.2 Key Characteristics of Preference Programming

Among the different hierarchical weighting methods, the multiattribute value theory (MAVT) [31] has a strong axiomatic foundation in measurement theory. Specifically, if the DM's preference relation satisfies axioms that characterize rational decision making, this relation has a *value function* representation with the help of which the overall value of an alternative can be expressed as the attribute-weighted sum of its attribute-specific values (i.e., scores).

In terms of the aggregation of the overall performance measure, MAVT shares similarities with the Analytic Hierarchy Process [54] where the overall priority weight of an alternative is expressed as the weighted sum of its local priorities with regard to the attributes at the lowest level of the hierarchy of objectives. In view of these similarities, we therefore provide the following generic formulation of additive preference models, in the understanding that this formulation can be interpreted in the context of MAVT and AHP models. A more detailed comparative analysis of the two methodologies can be found in [63].

### *1.2.1 Additive Preference Representation*

We assume there are $n$ attributes at the lowest level of the hierarchical representation of decision objectives. The importance of the $i$-th attribute is indicated by a non-negative *weight* $w_i \in [0,1]$. These attribute weights are normalized so that they add up to one, i.e., $\sum_{i=1}^n w_i = 1$.

There are $m$ alternatives $x^1,\ldots,x^m$. The achievement level of the $j$-th alternative on the $i$-th attribute is denoted by $x_i^j$ (for instance, this could be the fuel consumption of a car). The single-attribute value associated with this achievement level is called the *score* $v_i(x_i^j) = v_i^j \in [0,1]$. These scores map the actual achievement levels onto a possibly non-linear scale of subjective value. The overall value of alternative $x^j$ is expressed by the sum $V(x^j) = \sum_{i=1}^n w_i v_i^j$ which is based on the model parameters (i.e., weights $w = (w_1,\ldots,w_n) \in W = \{w \mid w_i \geq 0, \sum_{i=1}^n w_i = 1\}$ and scores

$v^j = (v_1^j, \ldots, v_n^j), j = 1, \ldots, m$). In preference programming, incomplete preference information is typically modeled through set inclusion. Specifically, the DM's preference statements are transformed into constraints on the model parameters (i.e., attribute weights and score vectors). For instance, as shown in Figure 1, the DM could state that the score of an alternative like Job B is between 50 % and 70 % of the maximum score of 1; or that the weight of attribute B is at least half of the weight of attribute A, but at most twice as high as the weight of attribute A.

These kinds of constraints define sets of feasible weights and score vectors $S_w, S_{v^j}$ (where $w \in S_w \subset W$ and $v^j = (v_1^j, \ldots, v_n^j) \in S_j \subset [0,1]^n, j = 1, \ldots, m$). For any consistent set of DM's preference statements, the resulting *feasible sets* will be non-empty. If the DM's preference statements are not very informative, these feasible sets will be large because the corresponding constraints will be satisfied by many parameters.

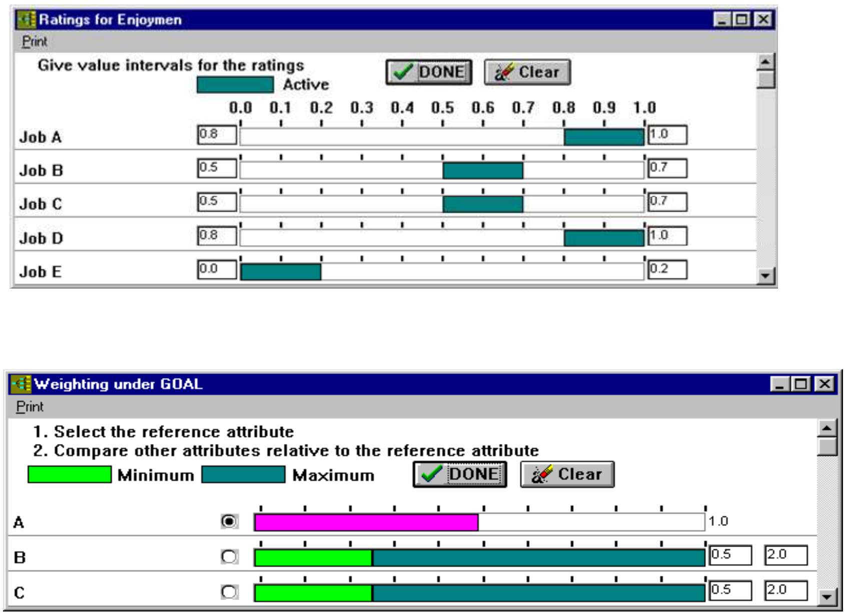


Figure 1. Elicitation screens for the specification of incomplete information about scores and attribute weights in WinPRE.

Table 1 gives an overview of selected preference programming methods, with particular attention to preference elicitation and the specific characteristcs of the methods. The evolution of methods has progressed from the mere incorporation of incomplete information towards the delivery of supplementary information–such as consistency bounds, measures of incompleteness, and decision rules–that help the DM to decide whether or not the elicitation phase should be continued and, if so, how the continuation of the process should be organized. Another trend is the increasing availability of decision support tools for interactive decision support processes.

It is worth noting that even other terms–such as 'imprecise information' (e.g., [18, 61] or 'partial information' (e.g., [9])–have been employed when referring to set inclusion. Yet we feel that 'incompleteness' is more adequate as a term, because it stresses that while the available preference information–as captured by the sets $S_w$ and $S_j$–may not imply a full ranking of alternatives, such information could, in principle, be completed by eliciting further preference statements. 'Incompleteness' also appears better on the grounds that the constraints on feasible weights and scores are not 'imprecise': for instance, the lower and upper bounds of interval-valued ratio statements in weight elicitation are crisp numbers with no associated uncertainties.

Although we do not consider approaches based on probabilistic modelling, fuzzy sets (e.g., [9, 11, 58]) or outranking relationships (e.g., [68]), there are computational and other parallels to these other approaches. The consideration of incompletely specified probabilities in Bayesian updating, for instance, leads to related problem formulations [59]. Thus, many of our observations apply in other settings as well.

### *1.2.2 Preference Elicitation*

The elicitation techniques of preference programming methods often extend those employed by the more conventional decision analytic methods. For example, many preference programming methods allow the DM to provide interval-valued estimates instead of exact crisp numerical estimates. The popular ratio-based techniques–such as the AHP [54] and SMART [16]–have also been extended to methods where the DM may provide interval-valued statements about the relative importance of attributes [5, 44, 61, 62].

Park et al. [49] present the following taxonomy which illustrates different approaches to the elicitation of incomplete information. Specifically, in the context of attribute weights, they consider both interval-valued and ordinal preference statements defined by: (1) a weak ranking (i.e., $w_i \geq w_j$), (2) a strict ranking ($w_i - w_j \geq \alpha_{ij}$) (3) a ranking with multiples ($w_i \geq \alpha_{ij} w_j$), (4) an interval form ($\alpha_i \leq w_i \leq \alpha_i + \varepsilon_i$), (5) a ranking of differences ($w_i - w_j \geq w_k - w_l$ for $j \neq k \neq l$,); here $\alpha_{ij}, \varepsilon_i \; \forall \; i,k$. All of these statements correspond to linear constraints on attribute weights.

In an extension of these elicitation techniques, Salo and Punkka [66] develop the *Rank Inclusion in Criteria Hierarchies* (RICH) approach which allows the DM to

**Table 1.1** Selected examples of preference programming methods

| Method | Score elicitation | Weight elicitation | Remarks |
|---|---|---|---|
| ARIADNE, White et al. [75] | Upper and lower bounds on scores | Linear constraints on attribute weights | Eliminates inconsistencies through linear programming |
| Hazen [21] | Completely specified score information | Linear constraints on attribute weights | Gives an extensive mathematical treatment of optimality conditions |
| HOPIE, Weber [72] | Derived indirectly from holistic comparisons among alternatives | Derived indirectly from holistic comparisons among alternatives | Offers recommendations from the consideration of probability distributions over the alternatives' values |
| PAIRS [28] | Lower and upper bounds on score information | Interval-valued statements about ratios of attribute weights | Computes dominance structures through hierarhically structured linear optimization problems |
| Preference Programming [62] | Interval-valued ratio statements using AHP-style pairwise comparisons | Interval-valued ratio statements using AHP-style pairwise comparisons | Offers an ambiguity index for measuring the amoung of incompleteness in the preference specification |
| Ahn et al. [1] | Linear constraints on alternatives' scores | Linear constraints on attribute weights | Suggests the use of aggregated net preference as a decision rule |
| PRIME [60] | Upper or lower bounds on scores | Interval-valued statements about ratios of value differences | Introduces several decision rules and examines their computational properties |
| Eum et al. [18] | Both complete and incomplete score information considered | Several kinds of preference statements that correspond to linear constraints on weights | Offers a taxonomy of several forms of incomplete information |
| RICH, Salo and Punkka [66] | Lower and upper bounds on attribute-specific scores | Incomplete ordinal preference information about the relative importance of attributes | Introduces an incompleteness measure for ordinal preference information |
| Interval SMART/SWING [44] | Score intervals about the alternatives | Interval-valued ratio statements in SMART/SWING | Extends the SMART/SWING method by allowing the choice of reference attributes |
| Smart Swaps [45] | Complete score information | Dominance statements among alternatives | Supports the Even Swaps process by using preference programming to identify practically dominated alternatives and candidate attributes for the next swap |
| RPM, Liesiö et al. [40] | Lower and upper bounds on attribute-specific scores | Incomplete ordinal preference information and also other forms based on set inclusion | Extends dominance concepts to multicriteria portfolio selection problems |

provide incomplete ordinal information about the relative importance of attributes (e.g., ‘cost is among the three most important attributes’ or ‘the most important attribute is either cost or quality’). Such statements correspond to constraints that define possibly non-convex sets of feasible attribute weights. The resulting sets can be readily examined to obtain decision recommendations based on the application of dominance concepts and decision rules.

### *1.2.3 Dominance Structures*

Once the incomplete preference specification has been elicited (as characterized by feasible weights $S_w$ and scores $S_j$), it is of interest to examine what, if any, inferences can be made about what alternatives are ‘better’ than the others.

These inferences can be based on the concept of (pairwise) *dominance*. In particular, alternative $x^k$ dominates $x^l$ if the overall value of $x^k$ is higher than that of $x^k$ for all feasible model parameters and strictly higher for some parameters. The case of dominance can be checked by considering whether or not the inequality

$$V(x^k) = \sum_{i=1}^{n} w_i v_i^k \geq \sum_{i=1}^{n} w_i v_i^l = V(x^l), \tag{1.1}$$

holds for all combinations of feasible weights $w \in S_w$ and scores $v^j \in S_j$. This definition establishes a transitive and asymmetric binary relation among the alternatives. Moreover, if (1.1) holds, the value of alternative $x^k$ will be at least as high as that of $x^l$, even if additional preference statements were to be acquired until the sets of feasible weights and scores become singletons.

### *1.2.4 Decision Rules*

If there are several non-dominated alternatives, it is not possible to derive conclusive statements about which alternative is the ‘best’ one. This is because for any alternative $x^k$, there exists a combination of feasible weights and scores such that the overall value of some other alternative $x^l$ will be higher than that of $x^k$. In consequence, other principles–called *decision rules*–can be applied to derive a decision recommendation. Several such decision rules have been proposed:

1. **Choice of representative parameters:** Based on ‘representative’ parameters from feasible regions, the recommendation can be based on the comparison of

alternatives' overall values for some representative parameters. For instance, the PRIME method uses, as one of several possibilities, *central weights* that are near the center of the feasible weight set [64]. Even the approaches to the computation of rank based weights (e.g., rank sum, rank reciprocal, rank exponent, rank order centroid; see, [6, 17, 69]) can be viewed as ways of converting ordinal preference information into representative weight vectors.

2. **Alternatives' value ranges:** Recommendations can be based on an analysis of the ranges of values that alternatives may take. Examples of such rules include the *maximax rule* (i.e., choose the alternative which has the highest possible overall value), *maximin* (i.e., choose the alternative for which the smallest possible overall value is the highest among alternatives) and *central values* (i.e., choose the alternative for which the mid-point of the value interval is highest) (see [64]). The advantage of these rules is that they can be readily computed and communicated.

3. **Pairwise value differences between alternatives:** Decision rules can be based on measures on how well alternatives perform relative to each other. One such measure is the *maximum loss of value* which indicates how much *more* value the DM could at most acquire in comparison with $x^i$, if she would choose some other alternative [64]). The corresponding *minimax regret* decision rule recommends the alternative which has the smallest maximum loss value. This rule is appealing because it allows the DM to take an informed decision on whether or not the possible loss of value is small enough so that elicitation efforts can be stopped. Even measures of preference strength (see, e.g., [1])–which are computed by aggregating value differences across several alternatives–belong to this class of decision rules.

4. **Maximization of expected value:** If there are grounds for making plausible assumptions about how probable the feasible parameters are, it is possible to recommend the alternative with the highest expected overall value (see, e.g., [72]). Although this decision rule is conceptually appealing, it is not necessarily easy to apply because the elicitation of required probability distributions is likely to be a major effort. Also computational difficulties may be encountered.

5. **Likelihood maximization of potentially optimal alternatives:** If probability distributions on the feasible regions can be elicited, the alternative which has the highest probability of receiving the largest overall value can be offered as the decision recommendation. This approach is, in effect, the SMAA method [36] which in its basic formulation recommends potentially optimal alternatives. Subsequently, this method has been extended so that it considers not only non-dominated alternatives, but considers the alternatives' relative rankings and, based, on an analysis of these, may recommend alternatives that are not necessarily potentially optimal for any combination of feasible parameters (see, e.g., [37, 38].

Although several decision rules have been proposed, the literature does not offer conclusive guidance as to what decision rules should be applied in specific decision contexts. Simulation studies suggest that decision rules based on the use of central values tend to outperform others in terms of minimizing the expected loss of value [60]. But even this tentative conclusion depends on context-specific assumptions (e.g., absence of correlations among alternatives). It therefore appears that further computational and empirical studies are needed.

It is also possible, particularly in group decision making, that an examination of different recommendations based on different decision rules may provoke discussions about which decision rules are 'better'. Such discussions may be driven by strategically motivated arguments if the DMs defend certain decision rules on the grounds that these favor their own favorite alternatives. But rather than focusing on the comparative merits of decision rules, it may be instructive to examine several decision rules in parallel, or to agree what decision rules will be applied before the phases of preference elicitation and synthesis are started.

### *1.2.5 Management of Inconsistencies*

The derivation of decision recommendations from an incomplete preference specification presumes that the DM's preference statements are consistent and thus define non-empty sets of feasible weights and scores. Yet, without adequate decision support, the DM may be inclined to provide preference statements that are not consistent with the previous statement, in which case the set of feasible parameters would become empty. Two main approaches (which are also supported by software tools, see Section 1.3) have been proposed to avoid this possibility:

- **Consistency restoration:** Taking the set of conflicting constraints as a point of departure, the DM can be requested to modify or withdraw earlier statements until the remaining, possibly revised constraints are not in conflict with each other any more (see, e.g., [34, 75]).
- **Consistency preservation:** Before the elicitation of each new preference statement, full information about the implications of earlier preference statements can be computed and presented to the DM, to ensure that the new statement is not in conflict with the earlier ones (see, e.g., by [57, 61, 64]).

Consistency restoration may be problematic if the DM is not able or willing to revisit earlier statements. Furthermore, the withdrawal of earlier statements may undermine the credibility of the analysis, because it insinuates that there are 'errors' in some inputs without guaranteeing that the other inputs are less 'erroneous'.

Also, although automated procedures can be used to identify the least number of constraints that should be removed to re-establish consistency, such procedures are computational interventions with little interaction on the part of the DM (see, e.g., [56, 75]). At worst, this approach may thus lead to the removal of statements that the DM feels most confident about. Computationally, however, consistency restoration can be applied in a batch mode so that possible problems with inconsistencies–if they do arise–can be addressed after the preference elicitation phase.

Consistency preservation requires that the implications of earlier preference statements are presented to the DM whenever new preference statements are elicited. These implications can be presented through the *consistency bounds* which convey the smallest and upper bounds that previously entered ratio-statements imply for the ratio statement that is to be elicited next [61]. For instance, in Figure 2, the two statements 'neither attribute A nor attribute B is more than two times more important than the other' and 'attribute C is twice as important as attribute B' logically imply that 'attribute C is more important than attribute A, but no more than twice as important'. If the decision maker is willing to provide a new statement that is within these consistency bounds, the new augmented constraint set will be consistent, too; however, if she wishes to enter a statement that is *not* within these bounds, some of the earlier statements would have be to revisited and revised. As a result, the management of inconsistencies has broader implications for preference elicitation: should the DM be encouraged to provide relatively 'narrow' statements (which tend to support more conclusive dominance results, but are more prone to inconsistencies) or 'broad' statements (which entail a lower risk of inconsistencies, but are likely to produce less conclusive dominance results)? (see also [44]). Related issues of consistency preservation arise also in group decision contexts: for example, when synthesizing individual statements, the correct interpretation of criterion weights may have to be ensured through the explicit consideration of trade-offs [28, 26].

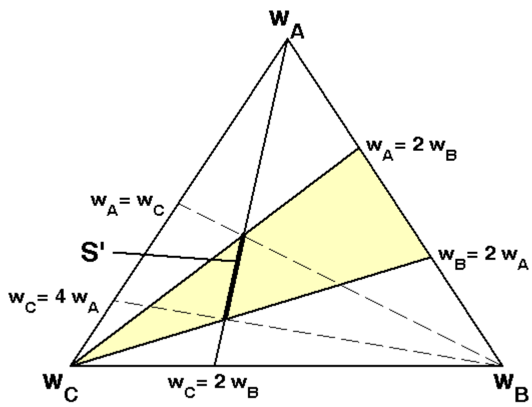


Figure 2. Consistency bounds implied by two ratio-based comparisons of attributes (A and B, B and C) for the third pairwise comparison (A and C).

### 1.2.6 Dominance Structures, Decision Rules and Rank Reversals

A much contested topic in the literature on hierarchical weighting models is the phenomenon of *rank reversals* which, in brief terms, means that the introduction or removal of an alternative changes the relative ranks of *other* alternatives. Such changes would suggest that the DM's preferences for the alternatives that are being compared depend not only on these alternatives, but also on what other alternatives may be included in or excluded from the analysis. Rank reversals have been a source of considerable controversy, and many researchers have regarded them as a major flaw in decision support methodologies such as the Analytic Hierarchy Process (see, e.g., [7, 10, 15, 63]).

It is therefore pertinent to ask which decision rules may exhibit rank reversals. To begin with, we may conclude that rank reversal cannot occur in decision rules which attach a single performance measure to an alternative, based on the weighted aggregation of numerical parameters that are not affected by the other alternatives, for instance through the use of normalization rules (this is not the case in the AHP where the introduction of an additional alternative typically affects the local priorities of *all* alternatives due to normalization [7, 15, 63]). Clearly, no decision rules based on such *unitary* performance measures will exhibit rank reversals, even if additional alternatives are introduced or existing alternatives are removed. This observation implies that several of the decision rules discussed above (e.g., maximax, maximin, central values, central weights) are immune to rank reversals.

However, if the scores can be impacted by other alternatives, or if the decision rule is based on the comparison of several alternatives, rank reversals may be possible. This is the case, for instance when using the minimization of maximum loss of value decision rule. To demonstrate this, assume that there are two attributes and two alternatives with scores $v^1 = (v^1_1, v^1_2) = (0.4, 0.6)$ and $v^2 = (v^2_1, v^2_2) = (0.6, 0.3)$, and that no weight information is available (i.e., the set of feasible weights is $W = \{(w_1, w_2) \mid w_1 + w_2 = 1, w_i \geq 0, i = 1, 2\}$). Then, the first of these alternatives $x^1$ has the smaller maximum loss value, because $\max_{w \in W}[(6-4)w_1 + 0w_2] = 2$ while the corresponding maximum for $x^2$ is 3. Now, if a third alternative with scores $v^3 = (v^3_1, v^3_2) = (0.8, 0.1)$ is added, the maximum value losses for the alternatives become 4,3 and 5, respectively, indicating that $x^2$ now becomes the recommended alternative, although $x^1$ was the recommended alternative before the introduction of $x^3$. Proceeding in much the same way, one can show that even other measures based on the comparison of value differences among two or more alternatives (e.g, aggregated net intensity; [33, 1]) may exhibit rank reversals.

Another example is the Stochastic Multiobjective Acceptability Analysis (SMAA; Lahdelma et al. [36]) where the decision recommendation is based on the comparison of the relative sizes over which a given alternative is optimal. Technically, this means that the set $W(x^j) = \{w \in W \mid \sum_{i=1}^n w_i v_i^j \geq \sum_{i=1}^n w_i v_i^k \ \forall\, x_k \neq x_j\}$

(where $W = \{(w_1,\ldots,w_n) \mid \sum_{i=1}^{n} w_i = 1, w_i \geq 0, i = 1,\ldots,n\}$) contains those attribute weights for which alternative $x^j$ will have the highest aggregate overall value. This weight set is used to establish a performance measure–called the *acceptability index* $AI(x^j)$–which is defined as the ratio between the volumes of $W(x^j)$ and $W$. The larger the acceptability index, the more support it will receive relative to the other alternatives.

To demonstrate that SMAA, too, exhibits rank reversals, assume that there are two attributes and two alternatives $x^1, x^2$ characterized by the score information $v^1 = (v_1^1, v_2^1) = (0.7, 0)$ and $(v_1^2, v_2^2) = (0, 0.6)$. Because the corresponding weight sets are $W(x^1) = \{w \in W \mid w_1 \geq \frac{6}{13}\}$ and $W(x^2) = \{w \in W \mid w_1 \leq \frac{6}{13}\}$, alternative $x^1$ has the larger acceptability index in SMAA and is therefore the recommended alternative. Next, assume that a third alternative $x^3$ with scores $v^3 = (v_1^3, v_2^3) = (0.6, 0.225)$ is introduced. The weight sets then become $W(x^1) = \{w \in W \mid w_1 \geq \frac{9}{13}\}, W(x^3) = \{w \in W \mid \frac{5}{13} \leq w_1 \leq \frac{9}{13}\}$ and $W(x^2) = \{w \in W \mid w_1 \leq \frac{5}{13}\}$. Thus, the second alternative $x^2$ now obtains the highest acceptability index and becomes the recommended alternative, although its acceptability index was smaller than that of alternative $x^1$ before the third alternative was introduced: a rank reversal has occurred.

Positive reports from applications (see, e.g., [37]) suggest that DMs may feel comfortable with the SMAA method. Yet the possibility of rank reversals casts some doubt on its validity as a decision support methodology. Another source of potential concern is that the early variants of SMAA do not encourage the DM to learn about her preferences by making explicit preference statements. Thus, although the weight set $W(x^j)$ that yields support for alternative $x^j$ may be larger than the other weight sets for the alternatives $W(x^k), k \neq j$, this set may consist of weights that are not aligned with the DM's (unstated) preferences. Thus, further validity checks may be needed to ensure that the weights on which is the acceptability index is based are compatible with the DM's preferences (cf. [71]).

If rank reversals are deemed unacceptable, the above discussion suggests that unitary performance measures should be given precedence over other decision rules in the derivation of decision recommendations. The other decision rules may still be useful for other purposes: for example, the computation of the maximum loss of value for alternatives highlights just how much aggregate value the decision maker may forego by choosing an alternative when dominance results do not hold. This measure also gives an upper bound on how much more additional value could be, at best, attained by continuing preference elicitation efforts.

## 1.3 Case Studies and Decision Support Tools

The literature on preference programming has gradually matured from purely methodological contributions towards the deployment of these methods in high-impact applications. In an early case study, Anandaligam [4] describes how incomplete preference information can be harnessed in the comparison of strategies for mitigating the harmful consequences of acid raid. Hämäläinen et al. [28] consider the use of preference programming in assisting groups of decision makers in the comparison of energy policy options. Hämäläinen and Pöyhönen [26] report experiences from the development of policies for traffic management, highlighting the impacts of preference programming on the decision outcome and the decision support process. Hämäläinen et al. [27] describe a multi-stakeholder participatory process where incomplete preference information was employed to support the comparison of alternative countermeasures in nuclear emergency planning.

Cristiano et al. [12] provide support for the optimal design of a surgical product by accommodating incomplete preference information in quality function deployment. Gustafsson et al. [19] apply the PRIME method to the valuation of a high technology company and illustrate how preference programming can be used in scenario-based forecasting problems. Salo and Liesiö [65] describe a series of workshops where the RICH method [66] was employed to help Scandinavian research managers establish priorities for research and technology development activities in an international research program. Ojanen et al. [48] prioritize alternative risk management measures by developing a hierarchical representation of relevant criteria and by soliciting ordinal preference statements from two groups of decision makers (i.e., client perspective, utility perspective). Alanne et al. [3] assess building technologies by using the PAIRS method [61] to account for economic and technological uncertainties.

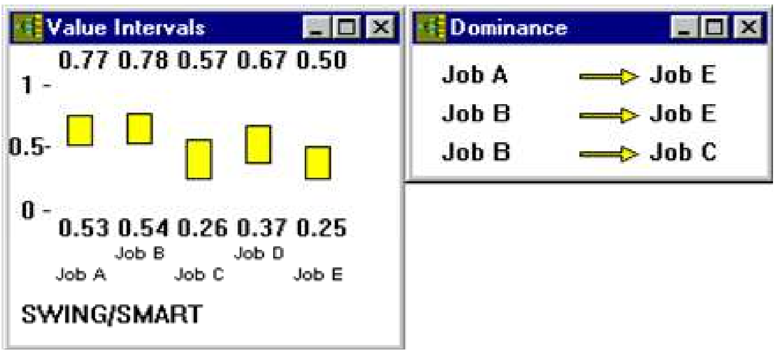


Figure 3. A screenshot from WinPRE with overall value intervals and dominance structures.

Because preference programming methods involve more computations than conventional approaches based on the elicitation of point estimates, the availability of adequate software tools is an essential precondition for their deployment. At present, there are several software tools for preference programming. WINPRE (Workbench for INteractive PREference Programming)[1] provides computerized support for both PAIRS [61] and the use of interval valued-statements in the AHP [62] (see Figure 3 for examples of the user interface). The RINGS system [32] allows the DMs to analyze range-based information in multi-attribute group decision making. PRIME Decisions [19] features *elicitation tours* which assist the DM in the specification of interval-valued ratio statements. VIP Analysis and its extensions [13, 14] support groups of decision makers who seek to reach a consensus.

RICH Decisions [2] is a web-based decision support tool which admits and processes incomplete ordinal preference information in accordance with the RICH method [67, 66]. A web-front end is also offered by RPM Decisions[3] which provides support for the Robust Portfolio Modeling methodology [40, 41], designed for the project portfolio selection in the presence multiple attributes and possibly incomplete information about attribute weights and project scores. Interactive web-functionalities are also provided by the Even Swaps-software which applies preference programming to develop suggestions for what swaps the DM should consider next when using the Even Swaps-method [45, 47].

## 1.4 Experiences from Applications

Preference programming methods have already been applied across very different domains. Indeed, experiences from these studies warrant some remarks about the benefits of these methods and preconditions for their successful deployment:

- Preference programming methods make it possible to check the implications of incomplete information even at the earliest phases of the analysis. Thus, the preferred alternative(s) can be possibly identified more easily and quickly while the resulting recommendations are still robust and methodological sound. Another important benefit is that these methods enable *iterative decision support processes* where tentative results can be provided early on, which is useful because such results help engage the DMs into the decision support process. Moreover, the subsequent phases of information elicitation can be focused on those attributes and alternatives about which the additional information is likely to contribute most to the development of more conclusive results.

[1] See http://www.sal.tkk.fi/English/Downloadables/winpre.html
[2] See http://www.rich.hut.fi/
[3] See http://http.rpm.tkk.fi/

- The numerous ways in which incomplete information can be elicited from the DM makes it necessary to plan and implement *elicitation processes* through which this information is acquired. Without such planning, the elicitation process may appear unstructured, or it may fail to ensure that all alternatives are treated with the same degree of thoroughness. Sufficient prior planning is also motivated by effectiveness, because the number of numerical estimates elicited from the DM may be larger than in conventional approaches: for instance, when stating interval-valued score information, the DM needs to state two crisp numbers as opposed to a single point estimate.

- If the preference model remains incomplete after the initial preference elicitation efforts (in the sense that the sets of feasible parameter remain large; see [57] for measures of incompleteness), dominance structures are unlikely to identify a single dominating alternative, particularly if there are many alternatives whose scores are correlated. In such situations, it is pertinent to examine different decision rules in order to gain complementary insights in facilitated decision workshops, for instance (e.g., [26]).

Overall, preference programming holds considerable potential in decision problems where reliable information about the DMs' preferences or the alternatives' impacts cannot be readily obtained. Such settings include, for instance, the evaluation of risk mitigation strategies in highly uncertain domains where the application of the *precautionary principle* is warranted (see, e.g., [27, 60, 70]). Here, instead of recommending seemingly 'optimal' alternatives, the limitations of information in such settings need to be recognized, for instance by giving precedence to *robust* alternatives that perform satisfactorily across the full range of plausible parameter values.

The recognition that the costs of information elicitation can be significant makes it possible to characterize novel uses of preference programming. For instance, these methods can be employed for the purpose of *screening* a large number of alternatives to a smaller set of non-dominated alternatives. Such a screening process can start with an initial phase where some information about all alternatives is first analyzed before proceeding to a more detailed analysis of the remaining non-dominated alternatives. Specifically, if the number of initial alternatives is large and the costs of information elicitation are relatively high, a phased analysis if this kind is likely to be more cost-effective than a process which seeks to acquire complete information about all alternatives at the outset.

## 1.5 Guidelines for Applications

The fundamental philosophy in the preference programming methods is quite simple. First, make an effort to elicit as much information as is reasonably possible.

Second, check if the elicited information makes it possible to identify a dominating alternative, or an alternative that can be selected with a reasonable degree of confidence (as measured, say, by the maximum possible loss of value). If this is the case, present this alternative as the recommended decision. Otherwise, seek possibilities for eliciting additional score and weight information by adopting elicitation strategies that are likely to reduce the set of non-dominated alternatives.

The phases in the above process highlight that there are close links between the steps of (i) eliciting preference information, (ii) computing dominance structures and decision rules and (iii) terminating the decision support process. For instance, recommendations based on decision rules will be contingent on how many or few statements have been elicited from the DM up until the point where these rules are applied. As an example, assume that the set of available information is not balanced (meaning that some parameters are almost exactly specified while there is hardly any information about others). Then, there is a possibility that one alternative may be (dis)favorably evaluated in comparison with others, only because the information elicitation process has not yet progressed to the point where statements about its parameters are elicited. In consequence, attention must be paid to questions of how elicitation questions are posed to the DM, and when and how intermediate results are presented.

- An attempt should be made to obtain equally 'complete' score information about all decision alternatives, in the sense that the DM is equally confident in that their preference specifications contain the 'true' scores. Such an interpretation can be encouraged by interpreting the lower and upper bounds in terms of symmetric confidence intervals to the preference statements, for example. One may also apply fuzzy mathematics in the aggregation of such confidence intervals (see, e.g., [58]).

- The same level of 'thoroughness' should be pursued also when assessing the relative importance of attributes. Otherwise, for instance, there is a possibility that an alternative will appear weak if it has its highest scores (relative to those of other alternatives) on attributes about which less weight information has been provided. In this case, the computations in applying the maximin decision rule would assign little weight to these attributes so that the alternative would have a small maximin value–even if its standing might improve when more information is elicited about these attributes.

- From the viewpoint of transparency, it may be advisable *not* to mix different types of preference elicitation questions in weight elicitation, because this may define a feasible region whose geometric structure is less symmetric than what would be obtained by using questions of the same type (e.g., interval-valued ratio statements). Another benefit of restricting the number of question types is that this may results in an elicitation process that can be more readily understood by the DMs (see, e.g.,

## 1.6 Outstanding Research Questions

While the methodological contributions and reported case studies offer important insights into the uses of preference programming, there are nevertheless several areas that call for further research:

- **Development of elicitation approaches:** Further work is needed on how incomplete preference information can be best elicited from the DMs. Some advances have been made by organizing the elicitation process into structured subsequences (see, e.g., [44]). The PRIME Decisions tool encourages the DM to complete several elicitation tours which consist of sequences of elicitation tasks [19]. In effect, effective and defensible elicitation strategies can be built from such these kinds of 'building blocks' with the help of which the decision support process can be aligned with the above guidelines. Related research should also address to what extent preference programming approaches may mitigate behavioral biases in the elicitation of attribute weights or possibly even create new ones (see, e.g., [24, 74]).

- **Impacts in different contexts and uses:** An explicit *ante* consideration of how much better decisions (say, as measured by the expected aggregate value or proportion of optimal choices) can be reached through preference programming. Advances in this area can be supported, among others, through simulation studies that analyze which elicitation approaches and decision rules perform best, subject to varying assumptions about the number of attributes and alternatives, distribution of attribute weights, correlations among alternatives, and costs of preference elicitation, among others. The results in [6, 60, 66] exemplify results from this kind of research, even though they focus mostly on ratio statements and ordinal preference information. Quite importantly, preference programming methods also enable various *ex post* sensitivity analyses with regard to all model parameters [44]).

- **Approaches for group decision support:** Many authors have argued that preference programming methods are particularly suitable for group decision making (see, e.g., [26, 28, 33]). There is, however, call for empirical evidence on how these methods can be best deployed in group settings. Interestingly enough, the group context also makes possible to introduce entirely new decision making principles. For instance, the group members may agree that each member shall acquire at least one third of the total value of his or her personally preferred alternative. After the introduction of such cross-cutting requirements, the aggregate group value can then be maximized, in the assurance that the resulting recommendation will comply with the principles that the group has set for itself.

- **Development of software tools and reflective case studies:** The computations and visualizations in preference programming methods typically require dedicated software tools. Although several such tools exist (e.g., WinPRE, PRIME

Decisions, RICH Decisions; [22]), further attention must be given to tool development. The integration of such tools with other IT systems may lead to new applications: for instance, one could envisage computerized search agents that would use preference programming methods to identify items that would be of most interest to potential buyers. Furthermore, because the benefits of preference programming techniques are ultimately realized in the context of applications, there is a need for reflective case studies. Among other things, such studies need to address the qualitative impacts that the application of preference programming methods may have on the decision support process (e.g., satisfaction with the process; commitment to the decision).

## 1.7 Conclusion

We have reviewed preference programming methods which accommodate incomplete preference information in hierarchical weighting models and synthesize such information into well-founded decision recommendations. By building on experiences from reported case studies, we have also developed guidelines for the deployment of these methods. These guidelines help ensure, among other things, that the consecutive phases of preference elicitation and preference synthesis are properly interlinked, and that all alternatives are treated equally during the development of decision recommendations.

More specifically, preference programming methods seem particularly suitable in decision problems where the elicitation of complete information is either impossible or involves prohibitively high costs, or where the DMs are simply more prepared to characterize their preferences through interval statements rather than through exact point estimates. These methods also offer possibilities for carrying global sensitivity analyses [46], and in group decision making they help incorporate the preferences of all group members who can thus be engaged in an interactive decision support process [28]. Moreover, the use of preference programming during the early phases of the analysis can be motivated by an attempt to reduce the set of relevant decision alternatives so that subsequent elicitation efforts can be focused on the remaining non-dominated alternatives. At best, such *screening* processes may offer much better overall cost-benefit characteristics than conventional approaches. We expect that these advantages, together with the improved availability of decision support tools, will contribute to the wider use of preference programming methods.

**Acknowledgements** This research has been supported by the Academy of Finland. We are grateful to Juuso Liesiö, Pekka Mild and Antti Punkka for their comments on this paper.

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