Optimal Prognostics and Health Management-driven inspection and maintenance strategies for industrial systems

A. Mancuso *[1,3], M. Compare[3,4], A. Salo[1] and E. Zio[2,3,4]

[1]Department of Mathematics and Systems Analysis, Aalto University, Finland

[2]MINES ParisTech, PSL Research University, CRC, Sophia Antipolis, France

[3]Dipartimento di Energia, Politecnico di Milano, Italy

[4]Aramis s.r.l., Milano, Italy

Abstract

The performance of the Prognostics and Health Management (PHM) depends both on the functioning of the measurement acquisition system and on the actual state of the system being monitored. The dependencies between these systems must be considered when developing optimal inspection and maintenance strategies. This paper develops a methodology to support maintenance decisions for industrial systems with PHM capabilities. The methodology employs influence diagrams when seeking to maximize the expected utility of system operation. The optimization problem is solved by mixed-integer linear programming, subject to budget and technical constraints. Chance constraints can be also included, for instance to curtail risks based on measures such as the Value at Risk (VaR) and the Conditional Value at Risk (CVaR) of system operation. The viability of the methodology is demonstrated by optimizing the inspection and maintenance strategy for a gas turbine equipped with PHM capabilities. The computation of the Value of Perfect Information (VoPI) provides additional insights on maintenance management.

Keywords: Predictive Maintenance, Prognostics and Health Management, Influence Diagrams, Decision Programming, Value of Perfect Information, Gas Turbine.

*Corresponding author. Tel.: +358 504084419. E-mail address: alessandro.mancuso@aalto.fi (A. Mancuso)

1 Introduction

Digitalization is a fundamental driver of Industry 4.0, a novel paradigm which enhances production efficiency through information and communication technologies [1, 2]. These technologies also provide a foundation for Predictive Maintenance (PM) for industrial components and systems, whereby condition monitoring data is employed to perform three tasks:

(i) **detection** of abnormal states, by identifying deviations from normal operating conditions in production processes, manufacturing equipment and products;

(ii) **diagnostics**, by classifying abnormal states;

(iii) **prognostics**, by predicting the evolution of abnormal states up to failure.

Detection, diagnostics and prognostics constitute the Prognostics and Health Management (PHM, [3, 4, 5, 6]). These tasks help implement efficient, just-in-time and just-right maintenance strategies by selecting the right action for the right component at the right time, thus maximizing production revenues and minimizing costs and losses, including assets [7]. Furthermore, PHM performance metrics have been introduced to characterize errors in detection, diagnostics and prognostics [8, 9]. Based on these metrics, several models have been developed to optimize Operations and Maintenance (O&M) decisions [7] and investments in PHM capabilities [10, 11, 12, 13].

A main limitation of earlier PM models is the assumption that sensors always work correctly, although in practice sensors may malfunction: freezing (or constant), noise, spike (or short), drift and quantization are the most common sensor malfunctions [14]. Faulty sensors may provide inaccurate measurements of the monitored physical parameters, affecting the performance of the PHM algorithms by conveying inaccurate or misleading information about the actual system state. This can cause missing alarms or unnecessary system downtimes, resulting in large financial losses. For example, the spillover effect (cross-sensitivity [15]) is known to propagate the anomalous monitoring data from a faulty sensor to other healthy signals, causing difficulties in choosing the correct maintenance action (i.e., fix or replace the sensor).

The detection of a sensor malfunction, which is often performed through *sensor data validation*, has been addressed by different methods, including Auto Associative Neural Network (AANN, [16]), Nonlinear Partial Least Squares Modeling (NLPLS, [17]), Principal Component Analysis (PCA, [18, 19]), Auto Associative Kernel Regression [20, 21] and Multivariate State Estimation Technique (MSET, [22]). However, algorithms for sensor validation too are affected by errors that depend on the health state of the monitored system. Specifically, if the monitored system does not work correctly, sensor validation is less effective in detecting incoherent deviations of the faulty sensor values with respect to data provided by other sensors. The main reason is that sensor validation algorithms are generally trained by signal data which is generated by healthy system operation only. Based on this training data, the algorithm learns to reconstruct the behaviors of the monitored signals when the system operates normally, which differs from those acquired when the system operates in a degraded state. Although the change in the signal behaviors is fundamental to the early detection of the system anomaly, it nonetheless lowers the performance of the sensor validation algorithms, because they have not been trained in the degraded setting.

When a system is equipped with PHM capabilities, maintenance decisions refer to two sequential actions: first, the inspection of either the sensors or the industrial system; second, the necessary repairs depending on the inspection outcomes. The costs of the sequential actions are very different, with different effects on the health state of the system and uncertainties about the performance of the PHM algorithms.

The above considerations suggest that optimal maintenance decision problems in a PM setting must be framed as a multi-stage decision problem, encoding the mutual dependence of the PHM algorithms tracking the system health state on those for sensor validation [23, 24]. On this topic, Driessen et al. [25] present a cost evaluation of maintenance policies for a single-component system, which is periodically subject to imperfect inspections. Do et al. [26] evaluate different maintenance policies for a deteriorating system in which the inspection policy is based on the residual useful life. Papakonstantinou and Shinozuka [27] employ Partially Observable Markov Decision Processes (POMDP) to optimize inspection and maintenance policies based on stochastic models and uncertain structural data in real time. Literature reviews (e.g., [28, 29]) call for increased attention to optimization models on condition-based maintenance, but they do not account for the imperfect performance of condition monitoring and inspections [30, 31, 32].

Influence diagrams [33] are one of the well-established techniques for structuring and solving multi-stage decision problems. They are commonly solved, for instance, through local transformations such as arc reversals and node removals in the diagram [34]. Tatman et al. [35] develop the equivalent decision tree representation, which is solved by dynamic programming [36]. Nonetheless, these standard techniques have limitations. First, they rely on the "no-forgetting" assumption, meaning that earlier decisions are known when making later ones. Although this may be not too limiting in practice, the information flow in industrial practice can at times be disrupted due to communication failures. Second, the use of dynamic programming is restrictive in that the objective function cannot include risk measures such as Value-at-Risk or semi-absolute deviation, which reflect the variability of consequences across all possible outcomes.

To overcome these limitations, we employ the Decision Programming approach proposed by Salo et al. [37], which employs Mixed Integer Linear Programming (MILP, [38, 39]) to solve multi-stage decision problems under uncertainty. Specifically, we employ Decision Programming to identify the optimal inspection and maintenance strategy for an industrial system with realistic PHM capabilities and sensor validation algorithms: each combination of states of the nodes of the influence diagram is mapped onto the two-stage decision maximizing the system utility. To the authors' best knowledge, this is the first time that a maintenance decision support model is developed in this practical setting, considering realistic PHM and sensor validation systems. In current industrial practice, the choice of maintenance strategies for industrial systems with PHM capabilities has been driven mainly by expert judgment [40].

The remainder of the paper is as follows. Section 2 introduces the influence diagrams and the problem formulation. Section 3 presents the optimization model and additional constraints. Section 4 proposes a case study from industry, concerning the optimization of inspection and maintenance strategies of a gas turbine. Section 5 discusses the potential and limitations of the proposed methodology. Finally, Section 6 concludes the paper and outlines extensions for future research.

2 Formulation of the influence diagram

An *influence diagram* is a directed acyclic graph that represents probabilistic causal dependencies between events and decisions [33]. Figure 1 shows an example of influence diagram which consists of three types of nodes:

(i) *chance nodes* C (indicated by circles) represent the random events of the scenarios;

(ii) *decision nodes* D (indicated by squares) represent possible choices of actions;

(iii) *value nodes* U (indicated by hexagons) represent the utility of system operation.

Causal dependencies in the set of nodes $N = C \cup D \cup U$ are represented by directed arcs $A \subseteq \{(i,j)|i,j \in N,\ i \neq j\}$. Specifically, arc $(i,j) \in A$ connects node i to node j to show that the state at node j is conditionally dependent on that at node i. The direct predecessors of node j belong to the *information set* $I(j) = \{i \in N|(i,j) \in A\}$. The arcs directed to chance nodes indicate probabilistic dependencies, whereas those directed to decision nodes denote the availability of information [34]. Because the network is acyclic, the nodes can be indexed with consecutive integers so that the indexes of nodes $i \in I(j)$ are lower than the index of node j.

Node $j \in C \cup D$ corresponds to the variable X_j, whose realization s_j assumes values in the discrete set of states S_j. The meaning of these variables is different for chance and decision nodes. Specifically, the states of decision nodes denote the choice of the risk mitigation actions that can be taken to reduce the probability of system failure. The decision X_j at node $j \in D$ depends on the *information state* $s_{I(j)}$

which belongs to the Cartesian product

$$S_{I(j)} = \bigtimes_{i \in I(j)} S_i, \tag{1}$$

defined by the combinations of states for all nodes in the information set. The information state determines what information is available when making the decision.

On the other hand, the states of chance nodes denote the health state of the system components [41]. These states represent mutually exclusive events, for which the uncertainty in the realization is described by the probability distribution on the states S_j. If the chance node does not depend on other nodes (no incoming arcs), there is an unconditional probability distribution $\mathbb{P}[X_j]$on the set S_j. For each chance node $j \in C$ which has a non-empty information set $I(j)$, the conditional probability of the state $s_j \in S_j$ is $\mathbb{P}[X_j = s_j | X_{I(j)} = s_{I(j)}]$, where $X_{I(j)} = s_{I(j)}$ denotes that the realizations of the variables X_i for nodes $i \in I(j)$ are the same as those in the information state $s_{I(j)}$.

A *policy* for decision node $j \in D$ is a function Z_j that maps information state to corresponding decisions $Z_j : S_{I(j)} \mapsto S_j$. The binary variables $z[s_j|s_{I(j)}]$ model the policy Z_j such that

$$Z_j[s_{I(j)}] = s_j \Longleftrightarrow z[s_j|s_{I(j)}] = 1. \tag{2}$$

Specifically, the policy of decision node $j \in D$ depends on the information state $s_{I(j)}$, meaning that the choices of actions depend on the information provided by sensors and inspections. The set of combinations of policies for all decision nodes D is a *strategy* Z.

A *scenario* s is a specific combination of states s_i of all chance and decision nodes. Thus, the set of all possible scenarios is $\mathbb{S} = \bigtimes_{i \in C \cup D} S_i$, each scenario defining a specific combination of random events and a respective strategy of actions. For a specific strategy Z, the probability of scenario s is

$$p(s) = \prod_{j \in C} \mathbb{P}[X_j = s_j | X_{I(j)} = s_{I(j)}], \tag{3}$$

if Z is such that it consists of policies Z_j such that $Z_j[s_{I(j)}] = s_j$, and 0 otherwise. In summary, the probability $\pi(s)$ of scenario s is

$$\pi(s) = \begin{cases} p(s), & \text{if } z[s_j|s_{I(j)}] = 1 \, \forall j \in D \\ 0, & \text{otherwise} \end{cases}. \tag{4}$$

Finally, each scenario s is associated with a consequence whose value $V(s)$ represents the utility of system operation discounted by the costs of deploying of the selected actions. The value nodes U encode the values $V(s)$ for all scenarios s. While it is possible to consider multiple value nodes, this paper focuses on a single objective optimization in which the aim is to maximize the utility of system operation, subject to possible resource and risk constraints.

3 Optimization model

The optimal strategy can be found through a mixed-integer linear programming model formulation, proposed by Salo et al. [37]. In this model, the probability $\pi(s)$ of scenario s is defined through the equations:

$$\sum_{s_j \in S_j} z[s_j|s_{I(j)}] = 1, \quad \forall j \in D, \forall s_{I(j)} \in S_{I(j)} \tag{5}$$

$$0 \leq \pi(s) \leq p(s), \quad \forall s \in \mathbb{S} \tag{6}$$

$$\pi(s) \geq p(s) + \sum_{j \in D} z[s_j|s_{I(j)}] - |D|, \quad \forall s \in \mathbb{S} \tag{7}$$

$$\pi(s) \leq z[s_j|s_{I(j)}], \quad \forall s \in \mathbb{S} \tag{8}$$

$$z[s_j|s_{I(j)}] \in \{0,1\}, \quad \forall j \in D, \forall s_j \in S_j, \forall s_{I(j)} \in S_{I(j)}. \tag{9}$$

If $z[s_j|s_{I(j)}] = 1$ for all s_j in scenario s, then probability $\pi(s)$ is the upper bound $p(s)$ because constraints (6) and (7) imply

$$\begin{cases} 0 \leq \pi(s) \leq p(s) \\ \pi(s) \geq p(s). \end{cases} \tag{10}$$

On the other hand, if any binary variable $z[s_j|s_{I(j)}] = 0$ for any s_j in scenario s, then probability $\pi(s) = 0$ because constraint (8) implies $\pi(s) \leq 0$.

The optimal strategy Z^* is the strategy that maximizes the expected utility of system operation so that

$$\mathbb{E}[V(Z)] = \sum_{s \in \mathbb{S}} \pi(s)\, V(s) \tag{11}$$

subject to constraints (5)-(9). Specifically, constraints (5) ensure that only one decision $s_j \in S_j$ is taken at each decision node $j \in D$ for every information state $s_{I(j)} \in S_{I(j)}$. Constraints (6) bound the probabilities $\pi(s)$ of scenarios $s \in \mathbb{S}$. Constraints (7) ensure that the scenario probabilities $\pi(s)$ cannot be smaller than their upper bounds $p(s)$ for scenario s such that $z[s_j|s_{I(j)}] = 1$, $j \in D$. Constraints (8) ensure that only those scenarios for which $z[s_j|s_{I(j)}] = 1$ for all $j \in D$ can have positive probabilities. Finally, constraints (9) specify the domain of all binary variables $z[s_j|s_{I(j)}]$.

In addition, the optimization model can include technical constraints that affect the deployment of risk mitigation actions. For instance, the constraint

$$z[s_j|s_{I(j)}] \leq z[s_\ell|s_{I(\ell)}] \quad \forall (s_{I(j)}, s_{I(\ell)}) \in S_{I(j)} \times S_{I(\ell)} \tag{12}$$

means that the action s_j cannot be deployed unless action s_ℓ is employed, regardless of the information states $s_{I(j)}$ and $s_{I(\ell)}$ of nodes j and ℓ.

3.1 Additional constraints

Let $Q(s_j|s_{I(j)})$ be the cost of risk mitigation action s_j at decision node $j \in D$ for the information state $s_{I(j)}$, then the total cost $Q(s)$ of implementing the actions for scenario $s \in \mathbb{S}$ is

$$Q(s) = \sum_{j \in D} Q(s_j|s_{I(j)})\, z[s_j|s_{I(j)}]. \tag{13}$$

For each scenario $s \in \mathbb{S}$, it is possible to require that the total cost of risk mitigation actions is lower than the budget B, so that $Q(s) \leq B$. If this constraint is too strict, chance constraints can be introduced, for instance to limit the probability of exceeding the budget to $\beta \in [0, 1)$ as

$$\sum_{\{s \in \mathbb{S} | Q(s) > B\}} \pi(s) \leq \beta. \tag{14}$$

One can also consider constraints on risk measures, for instance to bound the Value at Risk (VaR) and the Conditional Value at Risk (CVaR) of system operation [42]. At probability level $\alpha > 0$, the Value at Risk of strategy Z is

$$\mathrm{VaR}_\alpha(Z) = \sup \{t \in \mathbb{R} \mid \sum_{\{s \in \mathbb{S} | V(s) \leq t\}} \pi(s) < \alpha\}, \tag{15}$$

where the sum of probabilities considers only the scenarios for which the value $V(s)$ meets or exceeds the target level $t \in \mathbb{R}$.

In addition to the VaR, constraints on the Conditional Value at Risk (CVaR) limit the expected shortfall in the worst performing scenarios [43]. Thus, the Conditional Value at Risk of strategy Z is the expected value of the α-tail distribution of the utility function so that

$$\mathrm{CVaR}_\alpha(Z) = \mathrm{VaR}_\alpha(Z)\,[\alpha - \sum_{\{s\in\mathbb{S}|V(s)<\mathrm{VaR}_\alpha(Z)\}} \pi(s)] + \sum_{\{s\in\mathbb{S}|V(s)<\mathrm{VaR}_\alpha(Z)\}} \pi(s)\,V(s). \tag{16}$$

Conditional Value at Risk is a coherent risk measure: unlike VaR, it also reflects the shape of the distribution tail. For this reason, it is commonly considered a more informative risk measure than VaR [44].

3.2 Value of Perfect Information

The Value of Perfect Information (VoPI) refers to the additional value that can be gained by obtaining perfect information about the system state, based on which the operations are optimized. Thus, VoPI quantifies the willingness to pay for the transition from the current PHM system to the perfect one [45]. As mentioned in Section 1, PHM monitoring and system inspections provide imperfect information about the state of the industrial system. In this framework, it is possible to compute VoPI as the difference between the optimal expected value for two situations: (i) when the system state is correctly observed and (ii) when the system state is observed with possible errors. The first situation corresponds to perfect information on the system state, whereas the second situation to imperfect information. Consequently, the VoPI can be computed as

$$\mathrm{VoPI} = \mathbb{E}[V(Z^{**}|\text{Perfect Information})] - \mathbb{E}[V(Z^{*})]. \tag{17}$$

In the case of inspection and maintenance decisions, perfect information refers to a situation in which sensors and inspections correctly indicate the state of the industrial system [46]. Specifically, the system state is reported correctly by the monitoring system with probability one if and only if the monitored state equals the actual system state, and 0 otherwise. Perfect information makes it possible to select the optimal strategy Z^{**}, which may differ from the optimal strategy Z^{*} with imperfect information.

The VoPI represents the increase in expected value when the maintenance strategy can be decided based on perfect information about the system state [47]. This provides insights into the value of investing in improving the PHM capabilities. Note that this analysis can be performed before any additional information, by assuming that perfect measurement information is obtained.

4 Case study

The case study presents a maintenance decision framework for a Gas Turbine (GT) equipped with PHM and sensor validation capabilities, which provide imperfect information on the current state of the GT and its sensors. In industrial practice, the PHM of a GT relies on hundreds of sensors tracking the health states of a large number of components with different impacts on GT operation. For illustrative purposes, we assume that global indicators on the states of the GT and PHM system are available.

The GT undergoes periodic inspection and maintenance actions on which decisions are taken every 4000 working hours. Figure 1 represents the influence diagram for planning the GT inspections and maintenance, composed of the set of chance nodes (circles), the set of decision nodes (squares) and the value node (hexagon). In particular, node H refers to the working hours of the GT, which are technically referred to as *fired hours*. The realizations are discrete states with time interval of 4000 hours so that

$$s_H \in \{0, 4000, 8000, 12000, 16000, 20000, ...\}. \tag{18}$$

Node H is deterministic, which can be considered as a degenerate chance node with probability one for the current state only. This representation is useful for modelling the causal dependence between (i) the GT states and (ii) sensor states from the working hours of the GT, allowing the optimization problem can be solved for each of the states of working hours.

The fired hours affect the *sensor* state s_{SS} and the *turbine* state s_{TS}, which implies $I(SS) = I(TS) = \{H\}$. The sensor and the GT health states are qualitative evaluations included in the sets

$$s_{SS}, s_{TS} \in \{Excellent, Good, Fair, Poor, Failing\}, \tag{19}$$

Figures 2 and 3 illustrate the probability distributions of turbine and sensor states, respectively. Specifically, the probability of these states depend on the fired hours H (horizontal line), in keeping with the conditional probabilities $\mathbb{P}[X_{SS} = s_{SS}|X_H = s_H]$ and $\mathbb{P}[X_{TS} = s_{TS}|X_H = s_H]$. These values can be inferred from the inspection outcomes, when the multi-state degradation setting is adopted in the GT maintenance practice [31, 48].

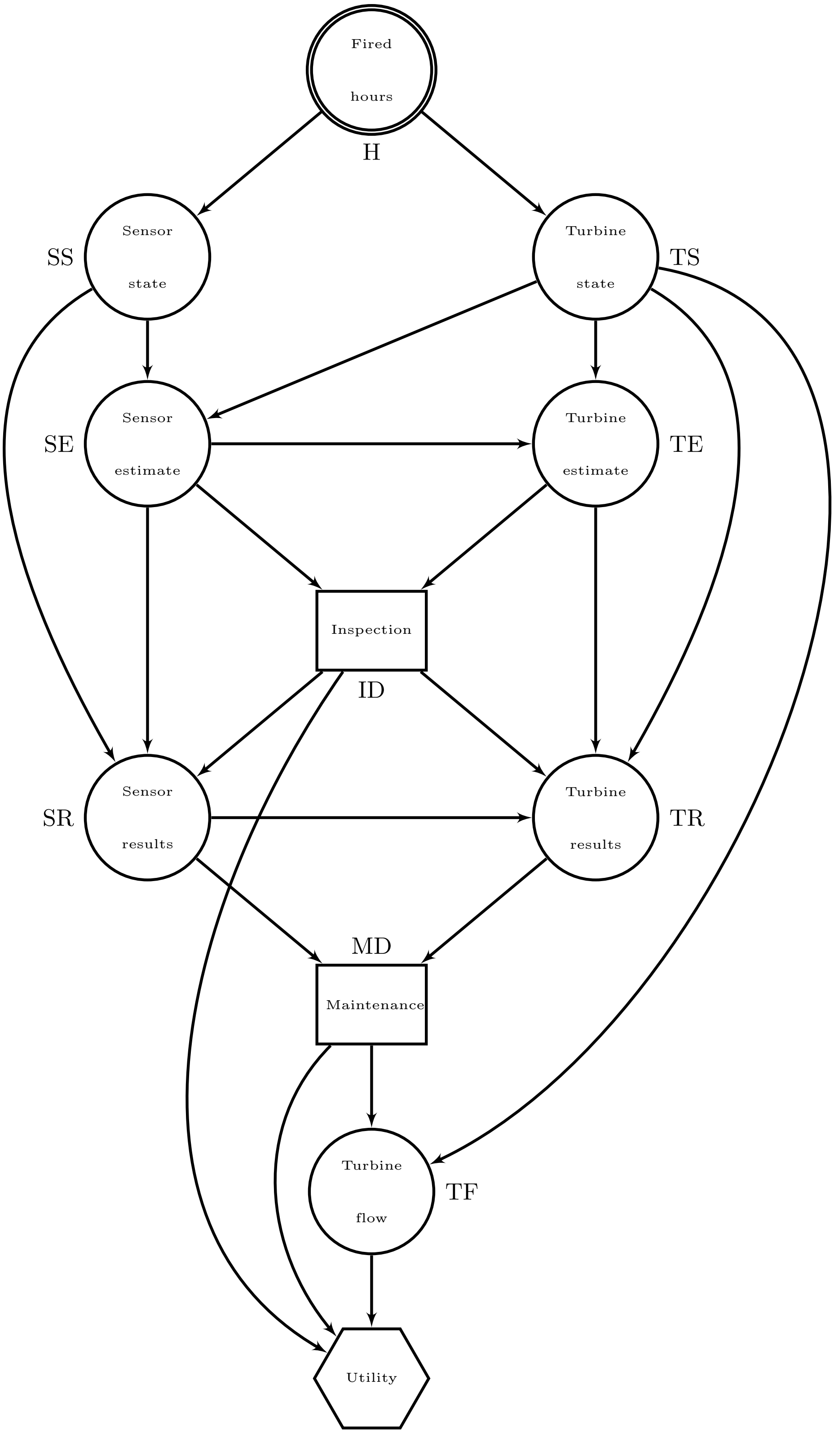

Figure 1: Influence diagram for programming inspections and maintenance of a turbine.

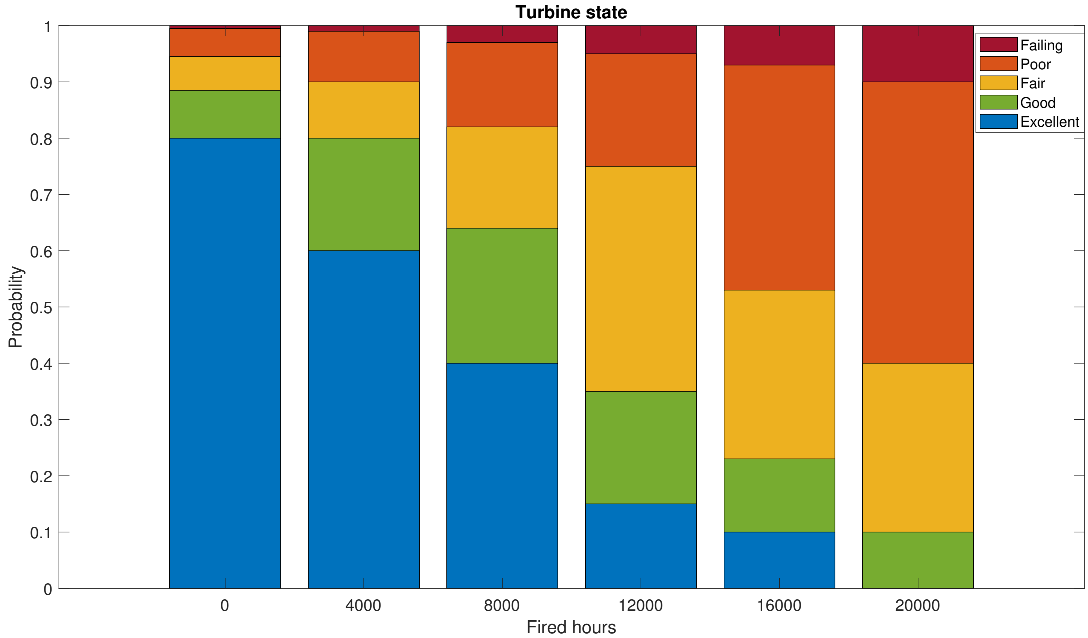

Figure 2: Probability distribution for *Turbine state*.

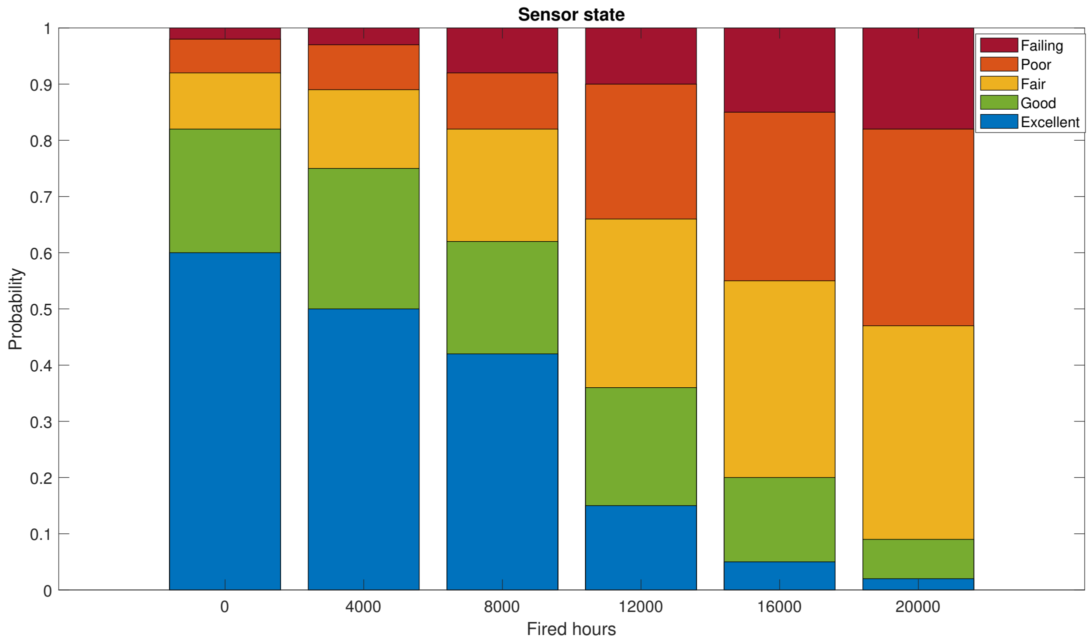

Figure 3: Probability distribution for *Sensor state*.

Based on machine-learning models [49], the PHM algorithms provide following *estimates* of the sensor state and the turbine state

$$s_{SE}, s_{TE} \in \{Excellent, Good, Fair, Poor, Failing\}, \tag{20}$$

The estimate s_{SE} of the sensor state depends on both the actual state of the sensor and the actual state of the GT. The estimate s_{TE} of the GT state depends on both its actual state and the estimate of the sensor state, because the sensor validation affects the PHM performance.

The PHM estimates on sensors and turbine provide information for the *Inspection Decision*, in the set

$$s_{ID} \in \{None, Sensor\,Check, Condition\,Monitoring\}, \tag{21}$$

where $Sensor\,Check$ indicates an analysis of the signal data and $Condition\,Monitoring$ refers to a maintenance action on the sensor acquisition chain. Neither action requires the GT to stop, and the *result* s_{SR} on the sensor state is in the set

$$s_{SR} \in \{Excellent, Good, Fair, Poor, Failing\}. \tag{22}$$

Without inspection, the results s_{SR} on the sensor state correspond to the sensor estimate s_{SE} provided by the PHM. If any inspection is performed, the results s_{SR} on the sensor state report the actual sensor state s_{SS} correctly with 95% probability. The observation is erroneous with 5% probability, which has been equally distributed across the two incorrect states close to the actual state.

The sensor inspection provides relevant information on the accuracy of the estimate s_{TE}. Specifically, it supports the definition of an updated estimate s_{TR} of the turbine state such that

$$s_{TR} \in \{Excellent, Good, Fair, Poor, Failing\}. \tag{23}$$

The results s_{TR} on the turbine state depend also on the results s_{SR} on sensor state and on the actual and estimated turbine states (i.e, s_{TS} and s_{TE}, respectively). Without inspection, the results s_{TR} on the turbine state correspond to the estimate s_{TE} of the turbine state, provided by the PHM. If the inspection is $Sensor\,Check$, the results s_{TR} on the turbine state correspond to the actual turbine state s_{TS} with a probability which depends on the sensor results s_{SR}. Finally, if the inspection is $Condition\,Monitoring$, the results s_{TR} on the turbine state report the actual turbine state s_{TS} correctly with 98% probability. The observation is erroneous with 2% probability, which has been equally distributed across the two incorrect states close to the actual state.

The results s_{SR} and s_{TR} on the sensor state and the turbine state provide information for the *Maintenance Decision*, in the set

$$s_{MD} \in \{None, Level1, Level2\}, \tag{24}$$

where $Level1$ restores the turbine state to 4000 hours earlier and $Level2$ restores the turbine state effectively to 0 hours of operation, i.e. as good as new. In this respect, Figure 4 represents the model of degradation and restoration processes of the GT and the PHM. Arrows indicate probabilistic transitions between states during the next 4000 hours, based on the maintenance decision. For illustration, Table 1 reports the transition probabilities for the degradation and renovation of the GT. For example, if the turbine is currently in state $s_{TS} = Good$ and the maintenance decision $s_{MD} = Level2$ is deployed, the turbine is in state $s_{TS} = Excellent$ with probability 99% and in state $s_{TS} = Good$ with probability 1% during the next 4000 hours.

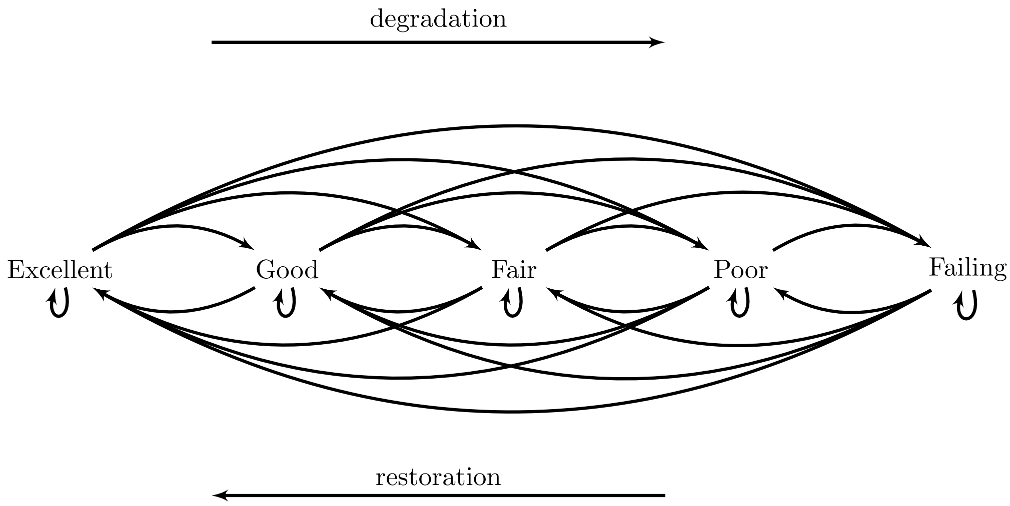

Figure 4: Probabilistic model of the degradation and restoration process of the turbine.

Table 1: Probabilistic transitions between turbine states.

		s_{TF}				
s_{TS}	s_{MD}	*Excellent*	*Good*	*Fair*	*Poor*	*Failing*
Excellent	None	0.80	0.10	0.05	0.03	0.02
	Level 1	0.90	0.05	0.03	0.02	0
	Level 2	0.99	0.01	0	0	0
Good	None	0	0.80	0.10	0.07	0.03
	Level 1	0.03	0.90	0.05	0.02	0
	Level 2	0.99	0.01	0	0	0
Fair	None	0	0	0.75	0.15	0.1
	Level 1	0	0.03	0.90	0.05	0.02
	Level 2	0.99	0.01	0	0	0
Poor	None	0	0	0	0.75	0.25
	Level 1	0	0.05	0.85	0.07	0.03
	Level 2	0.99	0.01	0	0	0
Failing	None	0	0	0	0	1
	Level 1	0	0	0.75	0.15	0.1
	Level 2	0.99	0.01	0	0	0

The turbine state s_{TS} and maintenance decision s_{MD} affect the *Turbine Flow*, which has the following discrete values

$$s_{TF} \in \{Excellent, Good, Fair, Poor, Failing\}. \tag{25}$$

The GT flow rate impacts the utility of the system operation, reduced by the costs of inspections s_{ID} and maintenance s_{MD}. Figure 5 shows illustrative utilities based on the state of the turbine flow, whereas Table 2 shows the costs for inspection and maintenance actions. The utilities and costs are illustrative units but have not been derived from an actual industrial system.

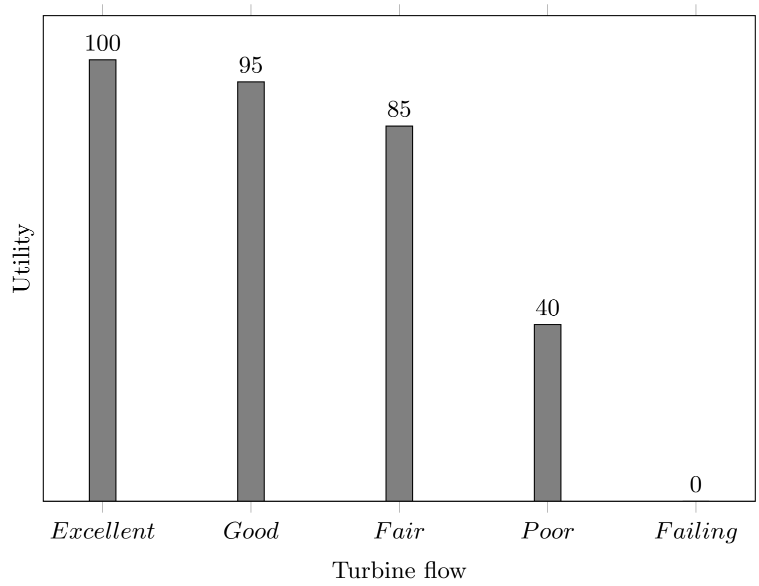

Figure 5: Illustrative utilities of the system operation.

Table 2: Costs for inspection and maintenance actions.

		s_{MD}		
		None	Level 1	Level 2
s_{ID}	None	0	25	50
	Sensor check	2	27	52
	Condition monitoring	8	33	58

The solution of the optimization model provides the optimal inspection and maintenance strategies for every level of *fired hours* and for each information state which is available when making these decisions. For illustration, Tables 3 and 4 present the optimal inspection and maintenance strategies of the GT at $H = 16000$ fired hours, respectively. Specifically, the inspection strategy depends on the estimates s_{SE} and s_{TE} of the sensor and turbine states, whereas the maintenance strategy depends on the results of the inspections s_{SR} and s_{TR}.

Table 3: Optimal inspection strategy at $H = 16000$ hours.

		s_{TE}				
		Excellent	*Good*	*Fair*	*Poor*	*Failing*
s_{SE}	*Excellent*	None	None	None	None	None
	Good	None	None	None	None	None
	Fair	None	None	None	None	None
	Poor	None	None	Monitoring	None	None
	Failing	Monitoring	Monitoring	Monitoring	None	None

Table 4: Optimal maintenance strategy at $H = 16000$ hours.

		s_{TR}				
		Excellent	*Good*	*Fair*	*Poor*	*Failing*
s_{SR}	*Excellent*	None	None	None	Level 1	Level 2
	Good	None	None	None	Level 1	Level 2
	Fair	None	None	None	Level 1	Level 1
	Poor	None	None	None	Level 1	Level 1
	Failing	None	None	None	Level 1	Level 1

The optimal inspection strategy is such that no inspection is performed if the sensor state estimate is *Excellent*, *Good* or *Fair* and if the turbine state estimate is *Poor* or *Failing*. On the other hand, Condition Monitoring needs to be performed if the sensor state estimate is *Poor* or *Failing* for specific circumstances of the turbine state estimate. Furthermore, the optimal maintenance strategy shows not to perform any maintenance if the turbine state estimate is *Excellent*, *Good* or *Fair*, but Level 1 maintenance should be performed if the turbine state is *Poor*. If the turbine state estimate is *Failing*, the optimal maintenance strategy depends on the sensor state estimate: Level 2 maintenance is necessary if the sensor state estimate is *Excellent* or *Good* and Level 1 maintenance otherwise.

The solutions in Tables 3 and 4 need to be examined together. Specifically, when the turbine is estimated to be in a degraded state (*Poor* or *Failing*), the optimal strategy is to proceed with maintenance actions, without improving the accuracy of the estimate of the turbine state through inspections. This choice depends on the high reliability of the monitoring sensors. On the other hand, when the estimate of the turbine is in a healthy state (*Excellent*, *Good* or *Fair*), the choice on the inspections depends on the estimated state of the monitoring sensors. Thus, the optimal strategy is to inspect the turbine when the sensors are expected to be degraded. Note that the optimization results are not generic in that they depend on the parameters of this case study.

Figure 6 illustrates the optimal expected utility of the system and the respective Value of Perfect Information, for each discrete state of the turbine fired hours. To model perfect information on the system state, the probabilities of the estimates s_{SE} and s_{TE} of the sensor and turbine states are defined as

$$\mathbb{P}[X_{SE} = s_{SE} | X_{SS} = s_{SS}, X_{TS} = s_{TS}] = \begin{cases} 1, & \text{if } s_{SE} = s_{SS} \\ 0, & \text{otherwise,} \end{cases} \tag{26}$$

$$\mathbb{P}[X_{TE} = s_{TE} | X_{SE} = s_{SE}, X_{TS} = s_{TS}] = \begin{cases} 1, & \text{if } s_{TE} = s_{TS} \\ 0, & \text{otherwise.} \end{cases} \tag{27}$$

By increasing the fired hours, the expected utility decreases due to increasing chances of degradation of the GT. For the same reason, the VoPI increases as information on the actual state of the GT yields more value for system operation [50]. Thus, investments in improving the PHM and sensor validation can be expected to yield higher returns when the GT has degraded more. The computation of the VoPI is obtained by assuming that the PHM provides perfect information on the system state.

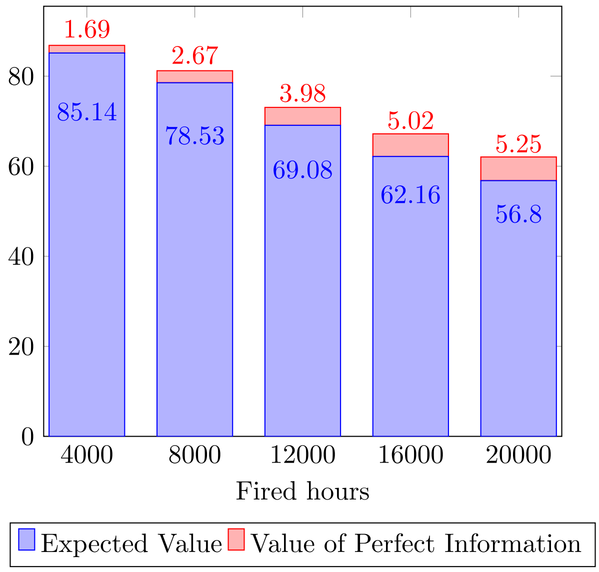

Figure 6: Expected Value and Value of Perfect Information of the turbine operation.

Besides employing risk measures as constraints of the optimization model, it is possible to analyse the VaR and CVaR of the optimization solutions to better understand the results. This analysis provides additional insights on these solutions without the need to specify the threshold probability α and target level t before the optimization. For some choices of the parameters α and t, these parameters could be so stringent that the optimization model has no feasible solutions. For this reason, an ex-post analysis of the results bypasses this issue, avoiding optimization runs with long computational times.

Figure 7 shows the cumulative probability of the utility for the optimal strategy Z^* at $H = 16000$

hours. From this probability distribution, it is possible to compute $\mathrm{VaR}_\alpha(Z^*)$ and $\mathrm{CVaR}_\alpha(Z^*)$ at probability level α, as defined in Eqs. (15) and (16). Table 5 lists the VaR and CVaR of system operation at different probability levels α: by increasing the α value, both the VaR and the CVaR also increase.

For the analysed probability levels α, the CVaR is higher than zero only for $\alpha > 0.1$. In view of these results, the optimal strategy for $H = 16000$ hours involves limited risks in that the expected utilities will be negative with probability of less than 10%.

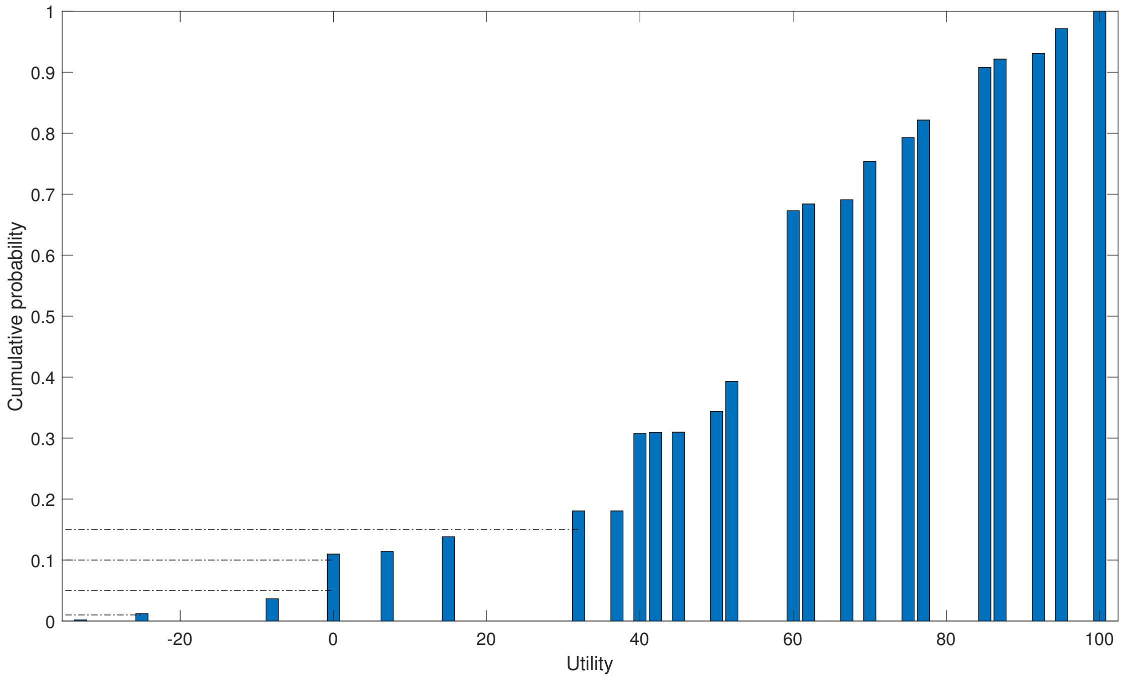

Figure 7: Cumulative probability distribution of system utility.

Table 5: VaR and CVaR for different α values.

α	VaR	CVaR
0.01	-25	-26.466
0.05	0	-10.3156
0.1	0	-5.1578
0.15	32	1.6781

5 Discussion

The computation of the VoPI makes it possible to assess investments which seek to improve the PHM capabilities. In this case, it is necessary to compare the VoPI with the costs for improving the PHM capabilities. Specifically, if the costs for improvement are lower than the VoPI, it is recommended to renovate the monitoring sensors, because the renovation leads to savings on system inspections.

The value of the PHM capabilities would increase if the company owns a fleet of industrial systems. In this case, the influence diagram represents the states of all the systems and the state of the monitoring sensors with a unique value node. The optimization model would then suggest the optimal combination of inspections for the fleet of systems, leading to additional savings for the company due to sharing fixed costs among several systems. In addition, the cost of the PHM capabilities would be shared among the fleet of systems with benefits on the failure detection, diagnostics and prognostics, due to the collection of a larger amount of data to build statistical analyses. However, the optimization may require more computational time for a large number of node states due to the curse of dimensionality. For this purpose, it is possible to decompose the large problem into a hierarchy of sub-problems to optimize the resource allocation across the fleet of industrial systems.

This framework can be extended to account for decisions in multiple time stages through Dynamic Bayesian Networks [51]. In this case, the chance nodes represent the random events of the state of the system components over the time stages and the decision nodes represent the decisions on inspections and maintenance at each time stage [52]. This gives rise to a model for long-term decisions on the industrial system in order to anticipate or postpone inspections and maintenance actions according to the predicted development of system failure. However, the number of decision variables will grow with the number of time stages, meaning that the computational time for the optimization solutions would increase.

Finally, the optimization results rely on the discretization of the probability distribution on the states of industrial systems and sensors [53]. Increasing the number of component states improves the accuracy of the model in the definition of the probability distribution, but it also increases the number of scenarios to be evaluated to define the optimal strategy. If the optimal strategy is the same for different information sets, it can be helpful to aggregate the information sets in order to limit the probability elicitation of the states and the computational time of the following runs of the optimization problem.

6 Conclusion

In this paper, we have developed a methodology for the optimal selection of inspection and maintenance strategies for industrial systems equipped with PHM. These strategies maximize the value for the company deriving from system operation, computed as the system utility discounted by the costs for inspection and maintenance actions.

The framework employs influence diagrams to model causal dependencies between system states and decisions on risk mitigation actions. Based on Decision Programming, the optimization model defines the optimal strategies for the system, accounting for budget and technical constraints. The solution is obtained through a mixed-integer linear problem, which considers all possible scenarios on the system states and decisions. The viability of the methodology has been illustrated with an example concerning a gas turbine equipped with PHM.

Overall, we have demonstrated that in the choice for inspection and maintenance strategies, there is a need to consider the unreliability of the PHM as well, given that the optimal strategies depend on both the healthy or degraded state of the industrial system and the state of the monitoring sensors.

This framework requires that the conditional probability tables can be specified, depending on the amount of health states of the system and the sensors. Possible extensions include the introduction of imprecise and uncertain information about the model parameters. For instance, an expert may provide imprecise values about state probabilities and impacts of risk mitigation actions. Such imprecision and uncertainty must be properly represented and propagated throughout the optimization model to obtain robust solutions.

Acknowledgements

The research has been supported by the Strategic Research Council of the Academy of Finland, specifically the research project Platform Value Now. The case study has been performed using Julia Programming language with the technical support of M.Sc. Juho Andelmin.

References

[1] M. Hermann, T. Pentek, and B. Otto. Design principles for industrie 4.0 scenarios. In *Proceedings of the 2016 49th Hawaii International Conference on System Sciences (HICSS)*, pages 3928–3937. IEEE, 2016.

[2] H. Lasi, P. Fettke, H.G. Kemper, T. Feld, and M. Hoffmann. Industry 4.0. *Business and Information Systems Engineering*, 6(4):239–242, 2014.

[3] E. Zio and M. Compare. Evaluating maintenance policies by quantitative modeling and analysis. *Reliability Engineering and System Safety*, 109:53–65, 2013.

[4] E. Zio. Some challenges and opportunities in reliability engineering. *IEEE Transactions on Reliability*, 65(4):1769–1782, 2016.

[5] M. Pecht. Prognostics and Health Management of Electronics. *Encyclopedia of Structural Health Monitoring*, 2009.

[6] J.M. Simões, C.F. Gomes, and M.M. Yasin. A literature review of maintenance performance measurement: A conceptual framework and directions for future research. *Journal of Quality in Maintenance Engineering*, 17(2):116–137, 2011.

[7] R Rocchetta, L Bellani, M Compare, E Zio, and E Patelli. A reinforcement learning framework for optimal operation and maintenance of power grids. *Applied Energy*, 241:291–301, 2019.

[8] A. Saxena, J. Celaya, B. Saha, S. Saha, and K. Goebel. Evaluating algorithm performance metrics tailored for prognostics. In *Proceedings of the 2009 IEEE Aerospace Conference*, pages 1–13. IEEE, 2009.

[9] A. Saxena, J. Celaya, B. Saha, S. Saha, and K. Goebel. Metrics for offline evaluation of prognostic performance. *International Journal of Prognostics and Health Management*, 1(1):4–23, 2010.

[10] K. Feldman, T. Jazouli, and P.A. Sandborn. A methodology for determining the return on investment associated with Prognostics and Health Management. *IEEE Transactions on Reliability*, 58(2):305–316, 2009.

[11] B.D. Youn, C. Hu, and P. Wang. Resilience-driven system design of complex engineered systems. *Journal of Mechanical Design*, 133(10):1–15, 2011.

[12] M. Compare, L. Bellani, and E. Zio. Reliability model of a component equipped with PHM capabilities. *Reliability Engineering and System Safety*, 168:4–11, 2017.

[13] M. Compare, L. Bellani, and E. Zio. Availability model of a PHM-equipped component. *IEEE Transactions on Reliability*, 66(2):487–501, 2017.

[14] A.B. Sharma, L. Golubchik, and R. Govindan. Sensor faults: Detection methods and prevalence in real-world datasets. *ACM Transactions on Sensor Networks (TOSN)*, 6(3):23, 2010.

[15] J. Coble, P. Ramuhalli, R. Meyer, and H. Hashemian. Online sensor calibration assessment in nuclear power systems. *IEEE Instrumentation & Measurement Magazine*, 16(3):32–37, 2013.

[16] J.W. Hines, D.J. Wrest, and R.E. Uhrig. Signal validation using an adaptive neural fuzzy inference system. *Nuclear Technology*, 119(2):181–193, 1997.

[17] B. Rasmussen, J.W. Hines, and R.E. Uhrig. A novel approach to process modeling for instrument surveillance and calibration verification. *Nuclear Technology*, 143(2):217–226, 2003.

[18] G. Kerschen, P. De Boe, J.C. Golinval, and K. Worden. Sensor validation using principal component analysis. *Smart Materials and Structures*, 14(1):36–42, 2005.

[19] P. Baraldi, G. Gola, E. Zio, D. Roverso, and M. Hoffmann. A randomized model ensemble approach for reconstructing signals from faulty sensors. *Expert Systems with Applications*, 38(8):9211–9224, 2011.

[20] P. Baraldi, F. Di Maio, P. Turati, and E. Zio. Robust signal reconstruction for condition monitoring of industrial components via a modified auto associative kernel regression method. *Mechanical Systems and Signal Processing*, 60:29–44, 2015.

[21] D. Garvey, J. Garvey, R. Seibert, J.W. Hines, and S.A. Arndt. Application of on-line monitoring techniques to nuclear plant data. In *Proceedings of the 5th International Topical Meeting on Nuclear Plant Instrumentation Controls, and Human Machine Interface Technology*, 2006.

[22] K.C. Gross, R.M. Singer, S.W. Wegerich, J.P. Herzog, R. VanAlstine, and F. Bockhorst. Application of a model-based fault detection system to nuclear plant signals. In *Proceedings of the International Conference on Intelligent Systems Applications to Power Systems*, 1997.

[23] F. Khan and M. Haddara. Risk-based maintenance (RBM): a quantitative approach for maintenance/inspection scheduling and planning. *Journal of Loss Prevention in the Process Industries*, 16(6):561–573, 2003.

[24] L. Krishnasamy, F. Khan, and M. Haddara. Development of a risk-based maintenance (RBM) strategy for a power-generating plant. *Journal of Loss Prevention in the Process Industries*, 18(2):69–81, 2005.

[25] J.P.C. Driessen, H. Peng, and G.J. Van Houtum. Maintenance optimization under non-constant probabilities of imperfect inspections. *Reliability Engineering & System Safety*, 165:115–123, 2017.

[26] P. Do, A. Voisin, E. Levrat, and B. Iung. A proactive condition-based maintenance strategy with both perfect and imperfect maintenance actions. *Reliability Engineering & System Safety*, 133:22–32, 2015.

[27] K.G. Papakonstantinou and M. Shinozuka. Planning structural inspection and maintenance policies via dynamic programming and Markov processes. Part I: Theory. *Reliability Engineering & System Safety*, 130:202–213, 2014.

[28] S. Alaswad and Y. Xiang. A review on condition-based maintenance optimization models for stochastically deteriorating system. *Reliability Engineering & System Safety*, 157:54–63, 2017.

[29] M. Keizer, S.D.P. Flapper, and R.H. Teunter. Condition-based maintenance policies for systems with multiple dependent components: A review. *European Journal of Operational Research*, 261(2):405–420, 2017.

[30] S. Panagiotidou and G. Tagaras. Optimal integrated process control and maintenance under general deterioration. *Reliability Engineering & System Safety*, 104:58–70, 2012.

[31] M. Compare, F. Martini, and E. Zio. Genetic algorithms for condition-based maintenance optimization under uncertainty. *European Journal of Operational Research*, 244(2):611–623, 2015.

[32] N. Rasmekomen and A.K. Parlikad. Condition-based maintenance of multi-component systems with degradation state-rate interactions. *Reliability Engineering & System Safety*, 148:1–10, 2016.

[33] T.D. Nielsen and F.V. Jensen. *Bayesian Networks and Decision Graphs.* Springer Science & Business Media, 2009.

[34] R.D. Shachter. Evaluating influence diagrams. *Operations Research*, 34(6):871–882, 1986.

[35] J.A. Tatman and R.D. Shachter. Dynamic programming and influence diagrams. *IEEE Transactions on Systems, Man, and Cybernetics*, 20(2):365–379, 1990.

[36] D.P. Bertsekas. *Dynamic Programming and Optimal Control*, volume 1. Athena scientific Belmont, MA, 1995.

[37] A. Salo, J. Andelmin, and F. Oliveira. Decision programming for multi-stage optimization under uncertainty. `https://arxiv.org/pdf/1910.09196.pdf`, 2019. [Online: accessed 11-December-2019].

[38] D. Bertsimas and J.N. Tsitsiklis. *Introduction to Linear Optimization*, volume 6. Athena Scientific Belmont, MA, 1997.

[39] C.A. Floudas and X. Lin. Mixed integer linear programming in process scheduling: Modeling, algorithms, and applications. *Annals of Operations Research*, 139(1):131–162, 2005.

[40] A.J. Guillén, A. Crespo, M. Macchi, and J. Gómez. On the role of Prognostics and Health Management in advanced maintenance systems. *Production Planning & Control*, 27(12):991–1004, 2016.

[41] G. Levitin, A. Lisnianski, and I. Ushakov. Reliability of multi-state systems: a historical overview. In *Mathematical and Statistical Methods in Reliability*, pages 123–137. World Scientific, 2003.

[42] P. Artzner, F. Delbaen, J.M. Eber, and D. Heath. Coherent measures of risk. *Mathematical Finance*, 9(3):203–228, 1999.

[43] Juuso Liesiö and Ahti Salo. Scenario-based portfolio selection of investment projects with incomplete probability and utility information. *European Journal of Operational Research*, 217(1):162–172, 2012.

[44] R.T. Rockafellar and S. Uryasev. Conditional Value-at-Risk for general loss distributions. *Journal of Banking & Finance*, 26(7):1443–1471, 2002.

[45] M. Memarzadeh and M. Pozzi. Value of information in sequential decision making: Component inspection, permanent monitoring and system-level scheduling. *Reliability Engineering & System Safety*, 154:137–151, 2016.

[46] Ronald A Howard. Information value theory. *IEEE Transactions on Systems Science and Cybernetics*, 2(1):22–26, 1966.

[47] E. Vilkkumaa, J. Liesiö, and A. Salo. Optimal strategies for selecting project portfolios using uncertain value estimates. *European Journal of Operational Research*, 233(3):772–783, 2014.

[48] M. Compare, F. Martini, S. Mattafirri, F. Carlevaro, and E. Zio. Semi-Markov model for the oxidation degradation mechanism in gas turbine nozzles. *IEEE Transactions on Reliability*, 65:574–581, 2016.

[49] Chunsheng Yang, Takayuki Ito, Yubin Yang, and Jie Liu. Developing machine learning-based models to estimate time to failure for PHM. In *Proceedings of the 2016 IEEE International Conference on Prognostics and Health Management (ICPHM)*, pages 1–6. IEEE, 2016.

[50] J. Keisler. Value of information in portfolio decision analysis. *Decision Analysis*, 1(3):177–189, 2004.

[51] K.P. Murphy and S. Russell. *Dynamic Bayesian Networks: Representation, Inference and Learning.* University of California, Berkeley Dissertation, 2002.

[52] A. Mancuso, M. Compare, A. Salo, and E. Zio. Portfolio optimization of safety measures for the prevention of time-dependent accident scenarios. *Reliability Engineering & System Safety*, 190(106500):1–9, 2019.

[53] C. Li and A. Der Kiureghian. Optimal discretization of random fields. *Journal of Engineering Mechanics*, 119(6):1136–1154, 1993.