Robust Portfolio Decision Analysis: An Application to the Energy Research and Development Portfolio Problem

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Abstract: Inspired by challenges in designing energy technology policy in the face of climate change, we address the problem of decision making under "deep uncertainty." We introduce an approach we call Robust Portfolio Decision Analysis, building on Belief Dominance as a prescriptive operationalization of a concept that has appeared in the literature under a number of names. The Belief Dominance concept synthesizes multiple conflicting sources of information to uncover alternatives that are intelligent responses in the presence of many beliefs. We use this concept to determine the set of non-dominated portfolios and to identify corresponding robust individual alternatives, thereby uncovering viable alternatives that may not be revealed otherwise. Our approach is particularly appropriate with multiple stakeholders, as it helps identify common ground while leaving flexibility for negotiation. We develop a proof-of-concept application aimed at informing decisions over investments into clean energy technology R&D portfolios in the context of climate change and illustrate how Robust Portfolio Decision Analysis helps identify robust individual investments.

I. Introduction

In this paper, we develop a prescriptive approach to decision making under "deep uncertainty" and apply it to the design of government-funded energy technology Research and Development portfolios in the face of climate change. We refer specifically to what Cox (2012) has defined as *Level 3 Deep Uncertainty,* in which there are sets of plausible beliefs, such as multiple priors due to disagreement between experts, which give rise to ambiguity. In contrast, we do not address what Cox has defined as *Level 4 Deep Uncertainty*, in which there is a complete lack of information about futures, models, outcomes, and weights.

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Public and private investments in R&D will play a major role in making decarbonization strategies feasible, both technically and economically (Arrow et al. 2009). Because R&D budgets are limited and there are many technological options competing for these funds, the investment portfolio must be designed efficiently, given the relevant uncertainties. While both models and experts' judgments have been employed to quantify these uncertainties, there have been frequent disagreements on the "correct" probability distribution over the future costs of energy technologies, or on how such distributions may be affected by R&D investments. Past work has shown significant disagreements among expert elicitation studies forecasting the impact of R&D expenditures on technological progress (Verdolini et al. 2018; Anadon et al. 2016), leading to diverging recommendations for R&D funding (Anadón et al. 2017; Baker et al. 2015a).

This leads to a question of how to synthesize the conflicting experts' views to support decision making; and whether subjective expected utility is the right approach. Recent work on climate change (Heal and Millner 2014; Millner et al. 2013) has argued for including ambiguity aversion into the objective function, but there is no consensus on which non-expected utility decision rules should be used (Borgonovo and Marinacci 2015); see Millner et al. (2013), Loulou and Kanudia (1999), Woodward and Bishop (1997), Athanassoglou and Bosetti (2015) for various applications to climate change. This is not a minor theoretical point: in the broad context of climate change policy-making Drouet et al. (2015) show that accounting for ambiguity aversion can lead to much more stringent mitigation strategies. In the context of choosing the optimal energy R&D portfolio, Baker et al. (2015a) show that the decision rule can significantly affect the optimal investment mix.

In this paper, we introduce Robust Portfolio Decision Analysis to address this problem. At the center of this approach is a dominance concept we call Belief Dominance, which is compatible with most robustness and ambiguity-aversion methods in the literature in the sense that Belief Dominance retains at least one portfolio that is optimal under *any* decision rule. However, it reduces the need for stakeholders to agree on a particular rule upfront, leaving flexibility for decision makers to incorporate other concerns. It has three characteristics that constitute improvements with respect to any individual rule. First, it focuses on the analysis of *portfolios* consisting of many individual alternatives, as opposed to evaluating individual alternatives one by one. In instances such as our application, thinking at the level of individual alternatives does not make sense strategically, as energy technologies interact in determining the flexibility, reliability and affordability of the power sector. Second, it identifies the set of all portfolios that are *non-dominated* across relevant beliefs ("beliefs" is used here in the Bayesian sense as referring

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to probability distributions) (Savage 1954; Etner et al. 2012). This set includes all portfolios that are optimal under some weighting of the beliefs, but may contain others as well, thus offering a reasonable starting point for any negotiation process. Third, it uses the set of non-dominated portfolios to identify *individual alternatives* that belong to all the non-dominated portfolios; and those that are not found in any of them. We argue that the alternatives contained in all non-dominated portfolios are robustly good and should be selected, because *not* selecting them would lead to a dominated portfolio. Conversely, individual alternatives that are not in any non-dominated portfolio should be rejected, because selecting them would lead to a dominated portfolio. The robust alternatives represent "common ground" among beliefs, which improves the outcomes of negotiation and deliberation (Mansbridge and Martin 2013). In the context of climate change, where the window of opportunity for mitigation is rapidly closing, the ability to move forward on at least some individual alternatives can be extremely important.

Our contributions include: (1) redefining the concept we call "Belief Dominance" for the prescriptive literature, with clear distinctions between alternatives, information/uncertainty, and preferences; and illustrating how the concept is compatible with many robustness concepts in the literature; and (2) extending this dominance concept to portfolios, synthesizing concepts from portfolio decision analysis and applying it to the important and difficult problem of identifying publicly-funded energy technology R&D portfolios.

The remainder of this section reviews relevant concepts and approaches and introduces the energy technology R&D problem. In Section 2, we develop the theoretical framework and relate Belief Dominance to other robustness concepts. Section 3 discusses the solution of the R&D problem, and Section 4 concludes.

I.1   Literature review

The question of how to make decisions under deep uncertainty can be restated as a question of how to synthesize multiple conflicting beliefs, which has been approached in many ways (Brockhoff 1983). The most traditional approach is to aggregate beliefs to produce a single, portable probability distribution (see CHora et al. 2013; Lichtendahl et al. 2013, for discussions of aggregation methods). The resulting distribution can then be used in a Subjective Expected Utility (SEU) framework (Baker and Solak 2011). The SEU framework satisfies a set of axioms laid out by Von Neumann and Morgenstern (2007) and Savage (1954), and has long been held up as an example of rationality.

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In recent years, a second set of approaches has been gaining interest, arguing that SEU is not "externally consistent" (Gilboa et al. 2009). These approaches allow for ambiguity aversion and apply non-expected utility decision rules to the set of beliefs, thus synthesizing them in the specific decision context. This set of approaches includes Maxmin (Ribas et al. 2010), α-Maxmin, and Minimax Regret, which, as shown by Stoye (2011), can be derived by relaxing some SEU axioms while adding others. Other approaches include Smooth Ambiguity Aversion (Klibanoff et al. 2005) and Soft Robustness (Ben-Tal et al. 2010). For example, Hassanzadeh et al. (2014) combine robust optimization (using a method that balances the worst case and the nominal case) with multi-objective methods in an application to an R&D problem.

In this paper, we build on a concept from the literature that relaxes the axiom of completeness in deriving a dominance concept. We translate this concept from the descriptive and normative literature into an operational prescriptive concept that we call Belief Dominance. The aim of the normative literature is to provide a set of axioms that result in dominance relations. Examples include Bewley (2002)'s "Knightian decision making", which refers to Aumann (1962) and requires strict dominance under every prior; Stoye (2012), which refers back to an older literature on "admissibility" (Arrow 1951), and assumes the largest possible set of priors; and Gilboa et al. (2010)'s "objectively rational", which requires that the priors belong to a closed convex set. Our concept is closest to the concept in Gilboa et al. (2010).

Danan et al. (2016) move this concept, which they call "Unambiguous Preferences", farther toward a normative application in defining robust social decisions. They focus on a social decision maker that has to synthetize multiple preferences and multiple beliefs. Most relevant to our work is the second part of their paper, which focuses on the case of stakeholders with common preferences. They prove that a social preference satisfies a kind of Pareto principle if and only if it can be represented as the concept we call Belief Dominance over the union of all stakeholders' beliefs.

Stoye (2012) shows that "admissibility", a version of Belief Dominance that does not limit the set of distributions, in a sense encompasses many other decision rules. It can be derived from a set of axioms that, relaxed one at a time and replaced by completeness, lead to Subjective Expected Utility, Minimax, α-Maxmin, or Minimax Regret, respectively. Similarly, Danan et al. (2016) show that if a preference relation is a completion of Belief Dominance, then it can be represented by a general decision rule they call a variable caution rule. This decision rule can be parameterized to represent SEU or Minimax. They note that the converse is true (any decision rule that can be represented as a variable caution rule is a completion of Belief Dominance) under the condition that the variable caution rule is limited to satisfy Belief Dominance. In Danan et al. (2016), following much of the economics literature, individuals have

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direct preferences over probability distributions, rather than over outcomes and risk. Thus, one of our contributions is to re-state the concept of Belief Dominance in a general framework with clear distinctions between alternatives, beliefs, and preferences (Howard 1988) so that it can be operationalized as a prescriptive decision rule (Bell et al. 1988).

Some versions of the Belief Dominance concept have appeared in the prescriptive literature, albeit in limited form and specific contexts. For example, Robust Portfolio Modeling (RPM) (Liesiö et al. 2007, 2008) is an approach for selecting a portfolio (i.e., a subset) from a *discrete* set of individual alternatives (such as R&D projects) in the framework of multi-attribute value theory subject to resource and logical constraints. Thus, the alternatives are discrete, while the constraints can have continuous parameters (e.g., size of the budget). In RPM, there can be incomplete information about criteria weights and the alternatives' scores (performance of alternatives with regard to the criteria (i.e., score), which in both cases is captured through set inclusion (i.e., convex sets of feasible attribute weights or feasible score intervals). Grushka-Cockayne et al. (2008) apply the model in Liesiö et al. (2007, 2008) to an air traffic management problem.

In contrast to RPM, the RPDA approach proposed in the present paper considers multiple beliefs, which are modelled through sets of feasible probabilities distributions $f(z|x)$ over the outcomes z that are associated with the selection of the portfolio x. Thus, the structure of the underlying decision model differs from that in RPM, even if the concept of Core Index we define below is analogous to that of RPM in guiding the selection of alternatives.

Iancu and Trichakis (2013) apply robust optimization for linear optimization problems, calling it Pareto Efficiency, which we argue is not precise. Specifically, Pareto Dominance has traditionally referred to comparing alternatives in the space of objectives or criteria (Varian 1992)[1]. Belief dominance, in contrast, dominance is employed to compare alternatives in the space of beliefs. Thus, it has parallels to Pareto Dominance, but reflects the realities of policy advice where the role of experts tends to be limited to making statements about what outcomes the decisions would lead to, while the articulation of preferences over these outcomes remains the responsibility of policy makers.

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[1] In some literature the term Pareto Dominance is used in an abstract sense to refer to a relationship between any two vectors, as in Voorneveld (2003).

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Weber (1987) has introduced a broader concept, called Bernoulli Dominance, in which the range of model parameters about which uncertainties or disagreements are modelled through incomplete information is extended to consider different utility functions and different objectives. However, this leads to non-trivial questions about the presence of, and the need to, model interrelationships between these parameters. For instance, the preferences could, to some extent, depend on the beliefs concerning outcomes. Yet the interpretation of results concerning Weber's Bernoulli dominance could prove challenging as the dimensions of ascertaining non-dominance could vary from one pair of portfolios to another (e.g., in some cases beliefs, in some others utilities). The set of non-dominated portfolios could also become so large that it would permit far less conclusive decision recommendations. For these reasons and the sake of ensuring transparency, we start by addressing one dimension of disagreement, the disagreement over beliefs.

Aven (2016) takes another approach, suggesting integrating numerical probabilities with more qualitative approaches. In fact, the idea of providing and evaluating multiple alternatives rather than a single best decision has been applied in various bottom-up exploratory approaches including Robust Decision Making (RDM) (Rosenhead et al. 1972; Lempert and Collins 2007), Decision Scaling (Brown et al. 2012), and Info Gap (Ben-Haim 2004); see Kalra et al. (2014) for a discussion of how these models can foster to agreement over decisions. These methods typically analyze a small set of pre-defined alternatives for robustness and then suggest possible new alternatives (see Herman et al. 2015 for a review). These approaches synthesize a range of beliefs and models within a decision context by visually communicating the corresponding range of outcomes. Our approach complements these approaches in that we use available probabilistic information to derive a good set of alternatives that can be analyzed with the above methods.

An approach somewhat parallel to ours is Many Objective Robust Decision Making, MORDM (Kasprzyk et al. 2013; Hadka et al. 2015) which, like our method, uses optimization techniques to identify a set of good alternatives for subsequent analysis. MORDM, however, as indicated by its name, focuses on cases with multiple objectives, using Pareto Satisficing as its criteria for identifying a set of good alternatives for subsequent analysis. In future work, our method could be combined with MORDM to produce a set of alternatives that are non-dominated in terms of both objectives and beliefs.

The approaches of soft operational research (see, e.g., Ackermann 2012) are also relevant in that it can be beneficial to employ systematic problem structuring methods to build shared frameworks for assessing alternatives in resource allocation. Such frameworks can be particularly powerful in how the

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multiple actors, their interests and perspectives, as well as alternatives and associated uncertainties are related (for a review, see Mingers and Rosenhead 2004). Here, methods of group decision making and multi-criteria decision analysis can be employed to synthesize preferences, to illustrate how alternatives differ from the perspective of different actors, and to identify which portfolios of alternatives are likely to be accepted by them (Vilkkumaa et al. 2014). In the structuring of problems for portfolio decisions, there are specific behavioural biases one needs to be aware of (Fasolo et al. 2010).

I.2   Energy Technology R&D Portfolio in Response to Climate Change

An important question being addressed by many nations is how to allocate research funds across a wide variety of energy technologies with varying improvement potential and differing impacts on the economy and environment ("EU Science Hub - European Commission" 2009). This complex research question has been approached through different avenues, including (i) the development of a broad range of energy-climate-economy models (Integrated assessment models or IAMs (see Clarke et al. 2014 for a complete review) and (ii) multiple studies of expert judgments on the potential for technological change (Anadón et al. 2014; Anadón et al. 2012; Baker et al. 2008, 2009a, 2009b; Baker and Keisler 2011; Bosetti et al. 2012; Catenacci et al. 2013; Chan et al. 2011; Fiorese et al. 2014, 2013). Studies of expert judgments have quantified key uncertainties and they have been extensively used to inform decisions affecting technological change. However, there are a number of independent and disparate studies, leading to multiple distributions for the parameters under investigation. In this paper, we explore how to best bring together these multiple distribution with these integrated assessment models using a framework that derives robust model-based conclusions while recognizing disagreements in the uncertainties expressed by multiple stakeholders (see **Figure 1** for an influence diagram of the decision process).

The perspective in our proof-of-concept application is that of social welfare, not considering a single decision maker, but rather a policy process that responds to many stakeholders, including citizens, industry, and interest groups, often from multiple regions. The key decision we focus on is how much government-funded R&D investment to allocate to which technologies. The key uncertainty is how the R&D investment will impact the performance of the technologies, mainly in terms of costs and efficiencies, according to the expert elicitations. While this problem can be modeled in different ways, we follow the most natural formulation, which is that the R&D investments effect the probability distribution over the costs and efficiencies of the technologies. We focus on social welfare related to the costs of controlling climate change and the costs of investing in R&D. To estimate the costs of controlling climate change, we

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turn to an IAM. The IAM takes as input a vector of technology costs and efficiencies and provides as output the cost of attaining a particular climate goal. Combined with the probability distributions over the costs and efficiencies, this provides a probability distribution over the ultimate cost of achieving a climate goal.

Figure 1: An influence diagram of the decision problem. The square node represents the decision of how much to invest in which technology; oval node represents uncertainty about technological outcomes of R&D investment; the arrow into the uncertainty node means that the R&D investment effects the probability distribution over technology performance; other arrows represent information; rounded square represents model calculations; and the diamond node is the objective value: to maximize utility of an R&D investment portfolio.

If we had a single, agreed-upon belief for how R&D impacts the probability distribution over technical change, this would be a problem of decision making under uncertainty. Yet there is not one, agreed-upon set of beliefs about the impact of R&D on technology performance, which turns this problem into one of deep uncertainty. Specifically, we bring together probability distributions about the future of five low carbon technologies from three large multi-technology expert elicitation projects carried out independently over the course of 5 years by researchers at three institutions: UMass Amherst (Baker et al. 2008, 2009a, 2009b; Baker and Keisler 2011), Harvard (Anadón et al. 2012; Anadón et al. 2014; Chan et al. 2011), and FEEM (Bosetti et al. 2012; Catenacci et al. 2013; Fiorese et al. 2013, 2014). Each elicitation project gathered multiple expert opinions about similar technologies, but using different methods. All experts' beliefs from the three projects were aggregated into three alternative probability distributions over the future costs of each of the five technologies, conditioned on three levels of spending in dedicated R&D. See Baker et al. (2015b) for a summary of the data.

The beliefs about the future of these five low carbon technologies are integrated with information about how these technologies impact the cost of mitigation. This information comes from running the integrated assessment model to cover the full space of future technological costs. Finally, the framework developed in this paper brings these two sources of information together to define non-dominated portfolios of R&D investments.

In the next section we present the general theoretical framework of Robust Portfolio Decision Analysis, referring back to this specific application to provide intuition. In Section 3 we apply the theoretical framework to derive insights about robust portfolios and robust individual R&D investments.

II. Theoretical Framework of Robust Portfolio Decision Analysis

There are two key pieces to the RPDA framework. The first is the concept of Belief Dominance, defined so that a portfolio *A* dominates *B* if *A* is preferred to *B* for all probability distributions that represent plausible beliefs concerning the outcomes of these portfolios. In our application, a portfolio of R&D investments in energy technologies dominates another if it is preferred across the full set of experts' beliefs concerning the impact of R&D on the cost and performance of energy technologies. The second piece is to analyze the set of all non-dominated portfolios to derive implications about individual alternatives. In our application, this shifts the focus to specific R&D technology investments in order to find, for example, those investments that are present in all non-dominated portfolios or those that are never present.

II.1   Eliminating Bad Portfolios: Belief Dominance

We present the concept of Belief Dominance following the modeling paradigm used in, for example, Athey (2002), Baker (2006), Epstein (1980), and Rothschild and Stiglitz (1971), remaining consistent with Bertsimas et al. (2011) and Hadar and Russell (1969). There is another strand of literature with a different, but wholly consistent, modeling paradigm, including Klibanoff et al. (2005) in which the central concept is that of an "act". Following Howard (1988), we focus on the three key elements of a decision problem: preferences, alternatives, and beliefs.

Consider the generic decision model

$$\max_{\mathbf{x}} V\left(\mathbf{x}, f\right) \tag{1}$$

where *V* is the expected value of an objective function U given belief *f*

$$V\left(\mathbf{x}, f\right) \equiv \int_{z \in Z} U\left(\mathbf{z}\right) f\left(\mathbf{z} \mid \mathbf{x}\right) d\mathbf{z} \,, \tag{2}$$

where $\mathbf{x} \in X \subseteq \mathbb{R}^n$ is an *n*-dimensional vector of decision variables so that each vector $\mathbf{x}$ representing an *alternative* in the classic Decision Analysis nomenclature. The set of alternatives X is assumed to be finite. For the rest of the paper we will refer to a *portfolio* when we are talking about a multi-dimensional decision vector $\mathbf{x}$. We refer to the components of the vector, x_i as *individual*

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alternatives. In our application, portfolios $\mathbf{x}$ are portfolios of R&D investments and individual alternatives, x_i represent investment decisions into specific technologies.

The vector $\mathbf{z} \subseteq \mathbb{R}^m$ is a realization of the *m*-dimensional random variable Z that contains all the parameters relevant for the objective*.* This formulation is general in that the vector $\mathbf{z}$ can be partitioned into parameters that are deterministic or stochastic, and parameters for which the distribution depends on the portfolio **x** and those for which the distribution is independent of the portfolio **x** (that is endogenous and exogenous uncertainties, respectively). In our application, $\mathbf{z}$ is a vector of implementation costs and efficiency parameters for the individual technologies, plus the cost of R&D for the technologies in our decision problem. The first part of the vector – the implementation cost and efficiency – depends on the portfolio **x** and is random. The second part of the vector – the R&D cost – depends on the portfolio **x** but is deterministic. In our application, we do not have any uncertainties that do not depend on the portfolio **x**. However, it would be possible to include parameters, such as commodity costs in the economy, that were independent of the portfolio **x**. Belief $f(\mathbf{z}\,|\,\mathbf{x})$ is a probability distribution defined on Z, indicating the probability of the outcome $\mathbf{z}$ when the portfolio is **x.**

The objective function *U* represents *preferences*

$$U : Z \to \mathbb{R}$$

Depending on the problem, the objective function may contain calculations on how the outcomes of random variables combine into outcomes of interest. It may contain what is sometimes called a value function, providing weightings over different types of outcomes of interest; or it may contain what is called a utility function, representing preferences over risk. In our application, the objective function represents calculations, translating technology parameters into the cost of controlling climate change and the cost of investing in R&D.

Belief dominance compares portfolios over sets of beliefs, for a given utility function. Define the set Φ as a compact set of beliefs. We define *Belief Dominance* as follows: a portfolio *x belief dominates* portfolio *x'* over a set Φ of beliefs, denoted by $\mathbf{x} \succ \mathbf{x'}$, if and only if

$$V(\mathbf{x}, f) \geq V(\mathbf{x'}, f) \;\; \forall f \in \Phi \tag{3}$$

where the inequality is strict for at least one *f*. This definition is specific to the decision problem as defined by *U* which represents the mapping of the outcomes $\mathbf{z}$ to metrics of interest (such as costs or benefits) and includes the decision maker's preferences, such as weightings over different attributes and attitudes

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towards risk. This is a prescriptive version of the dominance concepts discussed in Bewley (2002), Stoye (2012), Gilboa et al. (2010), and Danan et al. (2016).

Here, we briefly discuss the relation between Belief Dominance and the two most common dominance concepts in the literature, Stochastic Dominance and Pareto Dominance. Each concept ranks different elements of decision problems: Belief dominance and Pareto dominance rank alternatives; stochastic dominance ranks beliefs or probability distributions. Both Pareto and stochastic dominance reflect disagreement/uncertainty over preferences: in Pareto dominance the disagreement/uncertainty is over how multiple criteria are ranked (Varian 1992; Alvarez-Benitez et al. 2005; Farina and Amato 2002; Keeney and Raiffa 1993); and in stochastic dominance the disagreement/uncertainty is over risk attitude. Our concept differs from the others in that the disagreement/uncertainty is not over *preferences*, but over *beliefs* about outcomes, represented by sets of probability distributions. Thus, neither dominance concept is *conceptually* equivalent to Belief Dominance, as beliefs and preferences are two distinct elements of a decision problem. Still, Belief Dominance and Pareto Dominance are *mathematically* equivalent, in the sense that algorithms developed for Pareto Dominance can be applied to Belief Dominance.

II.1.1 Non-Dominated Sets

When there are multiple beliefs (such as statements by experts who disagree), we suggest that analysis should yield a set of non-dominated portfolios, just as in the cases of stochastic and Pareto dominance; and furthermore, a broader disagreement over beliefs should lead to a larger set of non-dominated portfolios.

A portfolio x is non-dominated if there is no other portfolio x' that belief-dominates it. Define X_{ND} to be the set of non-dominated portfolios. Figure 2 provides a visual illustration of this concept. In our application, the set of non-dominated portfolios would include all R&D portfolios that are not dominated across the three beliefs represented by UMass, FEEM, and Harvard.
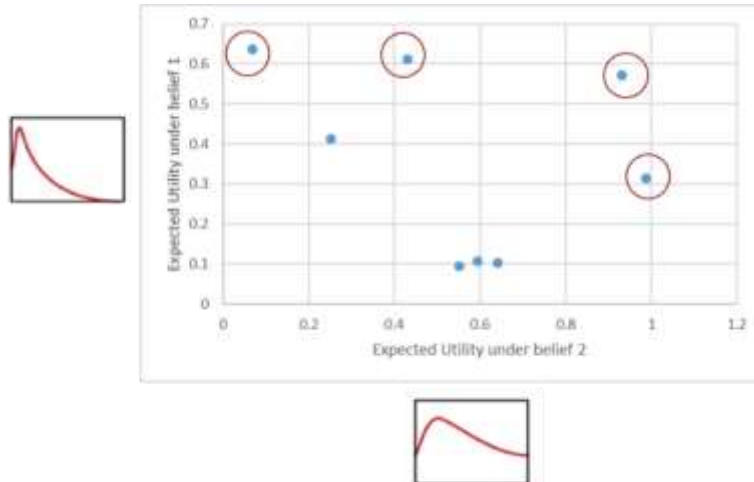
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Figure 2 Blue points represent the expected values of alternatives under each of the two beliefs, shown as probability densities near the relevant axes. Belief-non-dominated set includes circled points.

By the linearity of the integral, if one portfolio is dominated by another over a finite set of beliefs, it is also dominated over the convex combination of these beliefs. This implies that if the presence of dominance is established for all individual beliefs, then dominance also holds for all *combinations* of such beliefs. Thus, from now on, we assume that the set Φ is the convex hull of all relevant beliefs.

This simple result illustrates the power of this method with respect to traditional parametric sensitivity analysis. It has long been understood that sensitivity analysis – in this case finding the optimal solution under a number of candidate probability distributions – is not guaranteed to reveal the optimal solution (see Wallace 2000 for seminal paper). That is, the optimal solution is not guaranteed to be contained in the space spanned by the deterministic solutions (or the solutions of individual probability distributions). The *set* of non-dominated solutions does not have this problem: the optimal solution for any convex combination of the candidate distributions is guaranteed to be in the non-dominated set. Any solution that is optimal for any probability distribution that is a convex combination of the candidate distributions will be in the non-dominated set.

Identifying the non-dominated set helps decision makers eliminate portfolios that are not rational choices under any combination of beliefs. However, because it typically results in a set containing several portfolios, further analysis may be needed to reach a decision. In Section 3 we present such analyses and visualizations to help differentiate between the non-dominated portfolios.

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II.1.2   Comparison with Decision Rules

Stoye (2012) argues that the concept of admissibility (Belief Dominance less the requirement for a closed set of priors) "exhausts the overlap between many reasonable decision rules in a precise axiomatic sense." Specifically, Stoye (2012) shows that the Expected Utility, Maxmin Utility, and MiniMax Regret rules are each characterized by different subsets of the axioms defining admissibility. In this paper, as the focus is shifted to the prescriptive usage of these rules, we focus on how the sets of optimal alternatives that result from different decision rules relate to the belief-non-dominated set; and we expand beyond the decision rules considered in Stoye (2012).

We consider decision rules that allow for *probability distributions over outcomes* but consider *multiple possible beliefs*. For example, the Maxmin Expected Utility concept by Gilboa and Schmeidler (1989) chooses alternatives under the worst belief, rather than the simple Maxmin, which considers the worst possible outcome. For conciseness, we drop the reference to Expected Utility in each robustness concept. We define each concept precisely in Appendix A.I. We explore the concepts of Maxmin Expected Utility (Gilboa and Schmeidler 1989); Maximax Expected Utility; α-Maxmin Expected Utility (Ghirardato et al. 2002), where the decision-maker considers the weighted average of the worst expected payoff and the best expected payoff; Minmax Regret with multiple priors (Hayashi 2008); and the Smooth Ambiguity Aversion framework in (**K**libanoff, **M**arinacci, **M**ukerji 2005; KMM from now on), which is parallel to Expected Utility, incorporating an ambiguity aversion function in a similar role as a risk aversion function. We note that Subjective Expected Utility using averaged probabilities (SEUa from now on, Cerreia-Vioglio et al. (2013) can be regarded as a special case of KMM (Borgonovo and Marinacci 2015).

In what follows, we show that the Belief Dominance concept is compatible with all of these robustness concepts, in the sense that at least one optimal solution under each of these other concepts is in the belief-non-dominated set. We point out that any optimal solution to a robustness concept that is not in the belief-non-dominated set is (1) no better than those optimal solutions that are in the belief-non-dominated set under the robustness concept; and (2) strictly worse than the solutions in the belief-non-dominated set under at least one plausible probability distribution.

The Robustness concepts fall into two classes. Concepts in the first class, which includes Maxmin, Maximax, α-Maxmin, and Minmax Regret, choose the optimal solutions based on a subset of the beliefs in Φ. This implies that there may be multiple optimal solutions of which some may be dominated when considering the full range of distributions. Iancu and Trichakis (2013) were the first to point out this characteristic in the special case of Maxmin and a linear problem. They define and characterize the set of

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optimal solutions under Maxmin that are also belief-non-dominated, which they refer to as Pareto Robustly Optimal solutions.

The second class includes KMM and SEUa. If all distributions in Φ have a strictly positive weight or second order probability, all optimal solutions to these Robustness concepts are belief-non-dominated.

Let us define some terminology. Let C be a robustness concept, where:

Definition $C \in \{\text{maxmin, maximax}, \alpha\text{-maxmin, minmax regret, KMM, SEUa}\}$

Define X^C as the set of solutions $x^C \in X^C \subseteq X$ that are optimal under the robustness concept C. See the Appendices for formal definitions of the C-optimal sets and for all proofs. Applying Lemma 1 (in Appendix A.II), which establishes that belief non-dominance is transitive, implies that if a solution belief dominates a C-optimal solution, then that solution itself must be C-optimal, in Lemma 2.

Lemma 2: If $x \in X^C$ and $x' \succ x$ then $x' \in X^C$

For each robustness concept C, there is at least one optimal solution which belongs to the belief-non-dominated set.

Theorem 1: If robustness concept C satisfies Lemma 2 then $X^C \cap X_{ND} \neq \varnothing$

Proof: Define the set X_{ND}^C as the C-optimal solutions which are non-dominated by any other C-optimal solution $x \in X^C$: $X_{ND}^C = \{x \in X^C \mid \text{there does not exist } x' \in X^C \text{ such that } x' \succ x\}$

Note that X^C is non-empty since the set of alternatives is finite. The set X_{ND}^C can be built by examining the elements of X^C one by one and removing those *x* that are dominated by some other $x' \in X^C$. Since Belief Dominance is transitive, this set is not empty. Furthermore, the elements of X^C are non-dominated in the entire set *X*, by Lemma 2. **QED**

Theorem 2: Under the assumption that all beliefs in Φ have a strictly positive weight (for SEUa) or Second Order Probability (for KMM), all optimal solutions under SEUa and KMM are in the belief-non-dominated set: $X^C \subseteq X_{ND}$ for $C = [KMM, SEUa]$

Theorem 2 does not hold for robustness concepts Maxmin, Maximax, α-Maxmin and Minmax regret. Each of these concepts uses only a subset of the beliefs in Φ; therefore, some of the C-optimal solutions may be dominated by other C-optimal solutions. This is discussed in the case of Maxmin in Iancu and Trichakis (2013).

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There may be solutions in the non-dominated set that are not solutions to any of the robustness concepts. This highlights a key difference between belief-non-dominance and the other robustness concepts. All other concepts present the decision maker with fully ordered sets of solutions, causing decision makers to narrow their consideration based on the choice of robustness concept. As suggested by the profusion of robustness concepts in the literature, there is no agreement in the literature on which concept is best. Therefore, the non-dominated set gives decision makers the option to choose a solution to a particular robustness concept, but also to go beyond these concepts, perhaps incorporating qualitative concerns that may be quite difficult to model.

II.2 Exploring individual alternatives

In Section II.1 we presented a concept for narrowing down the set of acceptable portfolios to those that are not dominated across the full set of beliefs. Here, we discuss methods for exploring these portfolios to gain insights into the individual alternatives that make up the portfolios.

Our approach builds on the ideas of Robust Portfolio Modeling (Liesiö et al. 2007, 2008), which supports the selection of a portfolio of individual alternatives (such as individual R&D projects) from a large discrete set of candidates. The extension of RPM to scenario analysis (Liesiö and Salo 2012) employs set inclusion to capture uncertainties about the decision maker's risk preferences and beliefs by accommodating (1) sets of feasible utility functions over outcomes and (2) sets of feasible probability distributions over distinct scenarios. Results are obtained by determining which portfolios are non-dominated, in the sense that there does not exist any other portfolio that would be at least as good for all feasible combinations of utility functions and probabilities, and strictly better for some such combination.

The conceptual breakthrough in RPM is to analyze the set of non-dominated portfolios to inform choices among individual alternatives by dividing them into three categories. First, those individual alternatives that are contained in all non-dominated portfolios belong to the *core*. Second, individual alternatives that are not contained in any non-dominated portfolios are *exterior*. Finally, the *borderline* consists of individual alternatives that are included in some, but not all, non-dominated portfolios. To define this mathematically, let **x** be a vector including projects indexed by $i = 1..I$ and define $x_i = 1$ if project *i* is invested in and 0 otherwise. Recall that X_{ND} is the set of non-dominated portfolios. We define the three sets (illustrated in Table 1) as follows:

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$$core \equiv \{i \mid x_i = 1 \; \forall \mathbf{x} \in X_{ND}\}$$

$$ext \equiv \{i \mid x_i = 0 \; \forall \mathbf{x} \in X_{ND}\} \qquad (4)$$

$$bord \equiv \{i \mid i \notin core \text{ and } i \notin ext\}$$

Individual Projects:	a	b	c	d	e	f
Portfolios						
1	1	0	0	1	1	0
2	1	0	1	1	1	0
3	1	0	0	1	1	0
4	0	0	1	1	0	1
5	0	0	0	1	0	1
6	0	0	1	1	0	1

↑ exterior   ↑ core

Table 1: Six non-dominated portfolios (shown in rows) are composed of individual projects a, b,..,f. Project d is in the core; and project b is in the exterior.

An important theoretical result is that when uncertainties are reduced–in the sense that the set of feasible probability distributions becomes smaller–all core and exterior alternatives stay in their respective sets (see Theorem 2 in Liesiö and Salo, 2012). As a result, recommendations concerning the selection of core alternatives and the rejection of exterior alternatives are robust to learning, because these recommendations stay valid as additional information is obtained. For example, a technology investment that is in the core over a finite set of probability distributions will remain in the core for combinations of feasible probability distributions, including any subset of these distributions. Thus, research aimed at deriving recommendations that are more conclusive should be focused on the alternatives in the borderline set. For instance, it is possible to analyze if these borderline alternatives can be enhanced to make them equally attractive as some core alternatives (Gregory and Keeney 1994); or if gathering more information about the borderline alternatives allows them to be moved into the core or the exterior. However, when additional perspectives are added, the feasible set of probability distributions can become larger and the core and exterior sets may become smaller. In the beliefs are very different, all individual alternatives could belong to the borderline, providing little useful information.

III. Application implementation and results

In this section, we apply the RPDA framework to a proof-of-concept problem of choosing an energy technology R&D portfolio in the face of multiple beliefs over the prospects for technological change. Section III.1 discusses the computational implementation; Section III.2.1 discusses the results of applying belief dominance to eliminate portfolios; and Section III.2.2. discusses the resulting insights about individual alternatives.

III.1 Computational implementation

The decision problem is to choose a portfolio *x* of investments to maximize social welfare given multiple beliefs about the effectiveness of R&D in lowering the cost of key energy technologies. We simplify the problem by assuming that the societal goal is to reach the 2°C target set by the Paris Agreement. Thus, rather than maximizing social welfare, we consider the social planner's objective of meeting this target while minimizing the sum of the expected total emission abatement costs (C) and the R&D investment costs. Thus, U here represents costs and is to be minimized. As common in the R&D literature, we model this as a problem with endogenous uncertainty (Solak et al. 2010; Baker and Solak 2014). Bolded characters represent vectors so that *x* is the vector of investments in the different technologies:

$$\min_{\mathbf{x}} \int U(\mathbf{z}) f(\mathbf{z}|\mathbf{x}) d\mathbf{z} \tag{5}$$

Where $\mathbf{z} = (\mathbf{z}^P, \mathbf{z}^r)$ is the vector of technology characteristics, partitioned into performance characteristics, $\mathbf{z}^p$, and R&D investment cost, $\mathbf{z}^r$. The realization of this vector depends on the investment *x* through the conditional probability distribution $f(\mathbf{z}|\mathbf{x})$. For a given realization of $\mathbf{z}$, the objective function is

$$U(\mathbf{z}) = C(\mathbf{z}^P) + \kappa B(\mathbf{z}^r) \tag{6}$$

where $C(\cdot)$ is the total net present value of abatement costs (in trillions of dollars, using a discount rate of 3%) associated with the technological performance characteristics $\mathbf{z}^p$; abatement is defined as a reduction in emissions below a Business-as-usual baseline. $B(\mathbf{z}^r)$ is the total publicly-funded R&D investment in the portfolio, and κ is the opportunity cost multiplier. In our case, $\mathbf{z}^p$ is uncertain while the investment costs $\mathbf{z}^r$ are deterministic in the sense that the realization of $\mathbf{z}^r$ equals the cost of the investment portfolio *x,* i.e., there is a one-to-one mapping from *x* to respective investment costs $\mathbf{z}^r(\mathbf{x})$.

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In order to estimate the costs of achieving a climate goal, we use a specific IAM, GCAM (Kim et al. 2006). GCAM has been extensively used to explore the potential role of emerging energy supply technologies and the greenhouse gas consequences of specific policy measures or energy technology adoption. It provides insights into the interactions of energy technologies with each other and with the wider economy and the environment. The cost C, calculated using GCAM, is with reference to a Business-as-Usual baseline. We concentrate on a specific climate policy aiming at stabilizing global average temperature at roughly 2°C by the end of the century. This is implemented through a constraint on CO_2-equivalent concentration in the atmosphere set at 450 ppm-equivalent (i.e. including other greenhouse gasses using the global warming potential concept), which translates into a likely probability of maintaining the temperature below the 2°C target.

Each portfolio **x** specifies investments into five key energy technologies: Solar Photovoltaics (PV), Nuclear fission, Carbon Capture and Storage (CCS), electricity from Biomass (BE), and liquid Biofuels (BF). Each vector of R&D costs, $\mathbf{z}^r$ has five elements, an R&D investment cost for each technology. To calculate the total social cost of investing in a specific R&D portfolio, we sum the individual investment costs and multiply the amount of the R&D budget by κ, which is the opportunity cost multiplier. Theory suggests that the cost to society of R&D investment may be higher than the actual dollars spent. We use a value of $\kappa=4$; see (Nordhaus 2002; Popp 2006) for details. Previous work (Baker et al. 2015a; Baker and Solak 2014) has not shown strong sensitivity to this assumption.

The cost of investment for each individual project is the net present value of the annual cost over 20 years using a discount rate of 3%. Table 2, based on reported (Baker et al. 2015a) data on R&D cost assumptions for different levels of investments.

Solar			Nuclear			Biofuels			Bio-electricity			CCS		
Low	Mid	High	Low	Mid	High	Low	Mid	High	Low	Mid	High	Low	Mid	High
1.7	4.0	33.0	6.2	19.2	178.3	1.4	3.7	20.3	1.4	3.0	16.9	5.3	17.1	168.1

Table 2: Annual R&D expenditures cost of each project, in millions of dollars, assumed constant over a 20 year period.

There exist several ways of implementing the general problem presented in equation (5). In our case, the portfolios **x** are vectors of binary variables such that $x_i = 1$ if project i is invested in, and 0 otherwise. Each project i corresponds to one of the three investment levels, i.e. low, medium, or high, into one of the five technologies. Thus, for each technology there are three binary variables, of which exactly one will

be equal to 1. Each portfolio x specifies one of the three investment levels for each of the five technologies, and hence there is a total of $3^5 = 243$ possible portfolios.

The existence of multiple expert surveys describing the future probabilistic evolution of technological performance makes this a problem under deep uncertainty. To represent the deep uncertainty over z as a function of R&D, we use the index $\tau \in \{1, 2, 3\}$ for the three elicitation teams; beliefs f are indexed by τ so that $f_\tau(\mathbf{z}|\mathbf{x})$ represents the conditional beliefs over z as derived from team τ.

The vector of realizations $\mathbf{z}^p$ contains eight components, which consist of costs related to implementing each of the five technologies (as opposed to R&D cost) and an efficiency for CCS, biofuels, and bio-electricity. The costs for CCS and bio-technologies are capital costs; efficiencies are used to estimate operating costs. A complication is that the Harvard probability distributions do not distinguish between biofuels and electricity from biomass; here, we assume that the investment is evenly divided between the two technologies. To make the set of simulations with GCAM computationally feasible, we use the technique of importance sampling in a new way. Using a distribution resulting from the average of the three elicitations studies and the three levels of investments, we randomly draw 1000 points of the random vector $\mathbf{z}^p$ so that each outcome is represented by the 8-dimensional vector $\mathbf{z}_l^p$, $l = \{1, 2, ..., 1000\}$. Each of these vectors is evaluated using GCAM, resulting in 1000 values of $C\left(\mathbf{z}_l^p\right)$. We then apply importance sampling to re-calculate the probability of each point depending on the investment portfolio and the team. Baker et al. (2015a) used a set of diagnostics based on Owen (2015) and found that the samples performed in the acceptable range, with the possible exception of the biofuels and CCS efficiency parameters for the UMass and Combined distribution. See Baker et al. (2015a) for more details.

Thus, we have a set of technology values, z_l^p, $l = \{1, 2, ..., 1000\}$ and the (discrete) probability of a particular technology value realization, $f_\tau\left(z_l^p\big|x\right)$, which depends on the elicitation study τ and the portfolio, x. The "beliefs" over the R&D budgets are deterministic in this case as they depend only on the portfolio x but not on the elicitation team. We define $H(\mathbf{x};\tau)$, the discrete version of the objective function in equation (5), for a given set of beliefs τ, so that

$$H(\mathbf{x};\tau) \equiv \left[\sum_{l=1}^{1000} f_\tau\left(z_l^p\big|\mathbf{x}\right)C\left(z_l^p\right)\right] + \kappa B\left(z^r(\mathbf{x})\right). \tag{7}$$

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We say that a portfolio x *belief dominates* x' if $H(x; \tau) \leq H(x'; \tau)$ for all beliefs τ, with a strict inequality for at least one of the beliefs. A portfolio **x** is *non-dominated* if it is not dominated by any other feasible portfolio.

As the number of portfolios is small, we first calculate the total expected cost, H, for each of the 243 portfolios, using equation (7), then identify non-dominated sets using the simple cull algorithm introduced by Yukish (2004).

III.2 Results

We illustrate the RPDA framework, presenting the non-dominated energy R&D portfolios in the face of climate change, accounting for (i) uncertainty about the effectiveness of R&D in lowering the cost of the five low carbon technologies (measured through the experts' beliefs) and (ii) their relative potential to affect the final mitigation costs (measured through the integrated assessment model).

As we discussed, Belief Dominance retains at least one portfolio that is optimal under *any* decision rule (see Table 4 below) but also unpacks others that would otherwise be discarded as suboptimal. The identified energy R&D portfolios can be considered as a comprehensive and, at the same time, reasonably concise subset to kick start a discussion among agents and policymakers such as the allocation team within the EU initiative "Innovation Union". Without having to choose *a priori* a specific decision rule, which could pose challenges before any analysis begins, policymakers are shown only portfolios that are sensible in view of information from the experts.

We also show the range of the performances of non-dominated portfolios, thus providing decision makers with information that can be easily explored (see for example Figure 3 below). In addition, we uncover individual alternatives, such as investing aggressively in bioenergy power, that represent a common ground across all portfolios and beliefs. Any such alternative can thus be pointed out to the decision makers, independently of which side they are on or which elicitation they trust the most.

III.2.1 Applying Belief Dominance to Portfolios

Out of the 243 possible portfolios, only thirteen are non-dominated across the three probability distributions. Table 3 shows all non-dominated portfolios. They are listed in ascending order of the R&D expenditure they entail. Columns 2-6 provide the definition of the portfolios by showing the investment level in each technology. The last three columns show the objective value under the three different probability distributions. The objective values are color coded, with the highest cost in each column the darkest red.

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Portfolio	Technologies					Total R&D (millions of $2005)	ENPV (Cost, billions of $2005)		
	Solar	Nuc	BF	BE	CCS		Harvard	FEEM	UMass
1	Low	Mid	Mid	High	Low	47	22671	25442	15142
2	Low	Mid	Mid	High	Mid	59	21806	24434	15213
3	Mid	Mid	Mid	High	Mid	61	21659	24379	15528
4	Low	Mid	High	High	Mid	75	21654	24188	15720
5	Mid	Mid	High	High	Mid	78	21513	24163	16162
6	Low	High	Mid	High	Low	206	22744	25468	15153
7	Low	High	Low	High	Mid	215	21417	24307	20029
8	Low	High	Mid	High	Mid	218	21929	24525	15301
9	Mid	High	Mid	High	Mid	220	21741	24548	15509
10	Low	High	High	High	Mid	234	21770	24327	15509
11	Mid	High	High	High	Mid	237	21588	24345	15813
12	High	Mid	Low	High	High	239	21325	22747	20003
13	High	High	Low	High	High	398	21581	22901	19324

Table 3 Non-dominated portfolios. Columns 2-6 report the R&D investment level for each technology, Low, Mid or High. Column 7 is annual R&D investment. The last 3 columns report the Expected NPV of total abatement costs plus investment cost associated with each of the portfolios under the four sets of beliefs. Higher costs are emphasized by darker red colors.

Table 4 summarizes the results for a range of decision rules and parameterizations. As an example, Portfolio 1 is optimal under UMass distribution. Portfolio 12 is optimal under both Harvard and FEEM distributions and is also the Maxmin solution. If we give equal weight to FEEM, Harvard, and UMass, the optimal portfolio is 2. Portfolio 5 is the MiniMax Regret solution. Letting α vary between 0 and 1, we find the α-Maxmin optimal portfolios are 1, 2, 4, 13, 12, progressively increasing the ambiguity aversion. We performed a KMM analysis, using an exponential ambiguity aversion function, which has an ambiguity tolerance parameter similar to a risk tolerance parameter in an exponential utility function. Specifically, our objective is to maximize the difference between 26 000 billion and the portfolio's cost from the last three columns in table 3. The optimal portfolios under KMM are 2, 4, 5, 13, 12 with progressively higher ambiguity aversion.

Note that even considering a wide range of Robustness concepts and variations within those, the non-dominated set contains a number of portfolios (i.e., 3, 6 – 11) that were not uncovered by any of these other methods.

Portfolios	Robustness Concepts			
	SEUa	*a-Maxmin*	*Minmax Regret*	*KMM (equal weights)*
1	*UMass*	$\alpha = 0$		
2	Equal weight	$\alpha = 0.1...0.6$		Above 32.4
3				
4		$\alpha = 0.7$		6.3 - 32.4
5			Minmax Regret	6.0 - 6.2
6				
7				
8				
9				
10				
11				
12	*FEEM, Harvard*	$\alpha = 0.9, 1$ (Maxmin)		Below 4.7
13		$\alpha = 0.8$		4.7 - 6.0

Table 4. Non-dominated portfolios and solutions to robustness concepts. KMM uses an exponential ambiguity aversion function, maximizing the difference between 26,000 and the portfolio cost; the ambiguity tolerance in the last column are in trillions of dollars. Shaded rows are not solutions to any of the robustness concepts considered.

Figure 3 provides a visualization of the results, showing the distribution of how each portfolio ranks over the 5150 feasible weightings (in steps of 0.01) of the three beliefs. For example, Portfolio 2 ranked first or second among the 13 portfolios 50% of the time; but sometimes ranked as low as tenth. Portfolio 3 and 4 are ranked among the top four portfolios in 50% of the weightings, and they rarely ranked worse than sixth. Portfolio 12, the maxmin solution, is among the three worst portfolios 50% of the time.
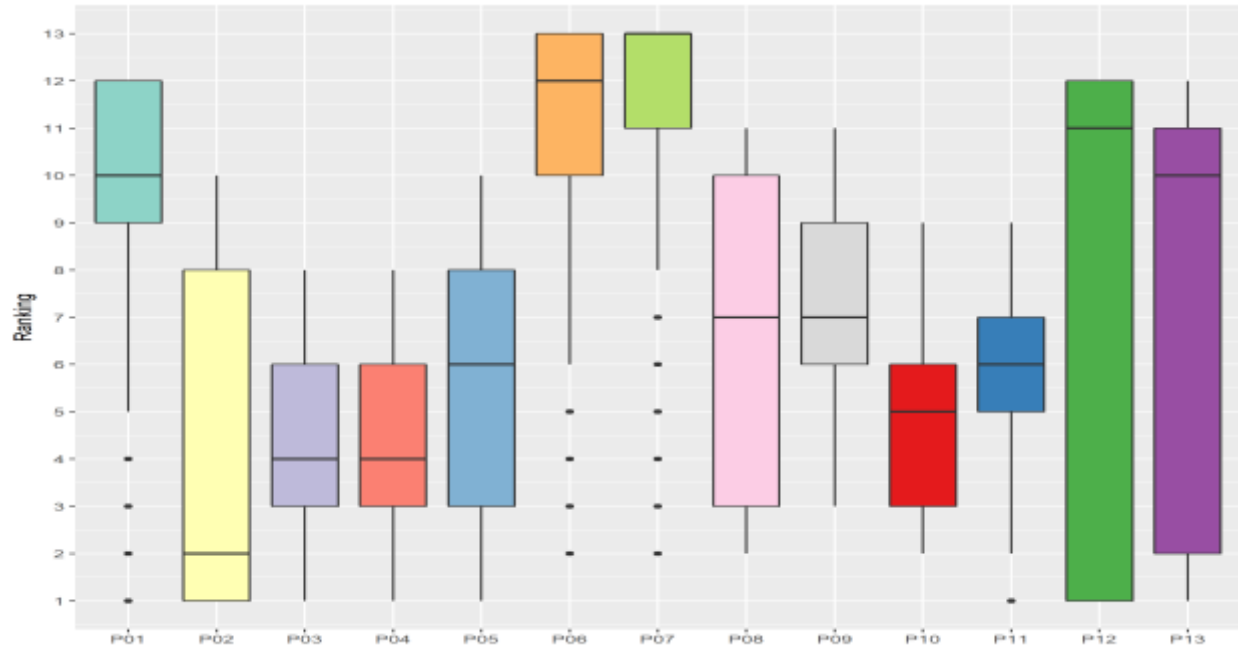
Figure 3. Distributions of the ranking positions for the 13 non-dominated portfolios over possible combinations of weights for the three elicitation studies. The lower and upper hinges correspond to the first and third quartiles (the 25th and 75th percentiles). The upper (lower) whisker extends from the hinge to the largest (smallest) value no further than 1.5 * inter-quartile range. Data beyond the end of the whiskers are called "outlying" points and are plotted individually.

The results can also be presented using a simplex. Each point on the simplex represents a weighting over the three beliefs, with the center representing equal weights. Figure 4 shows which portfolio is ranked first at each particular weighting. Portfolio 1 is only optimal if almost all weight is given to UMass; Portfolio 12 is optimal whenever the weight on UMass is low. Portfolio 2 is optimal for the widest range of weightings. Five portfolios (6-10), while non dominated, are never ranked first under any weight.

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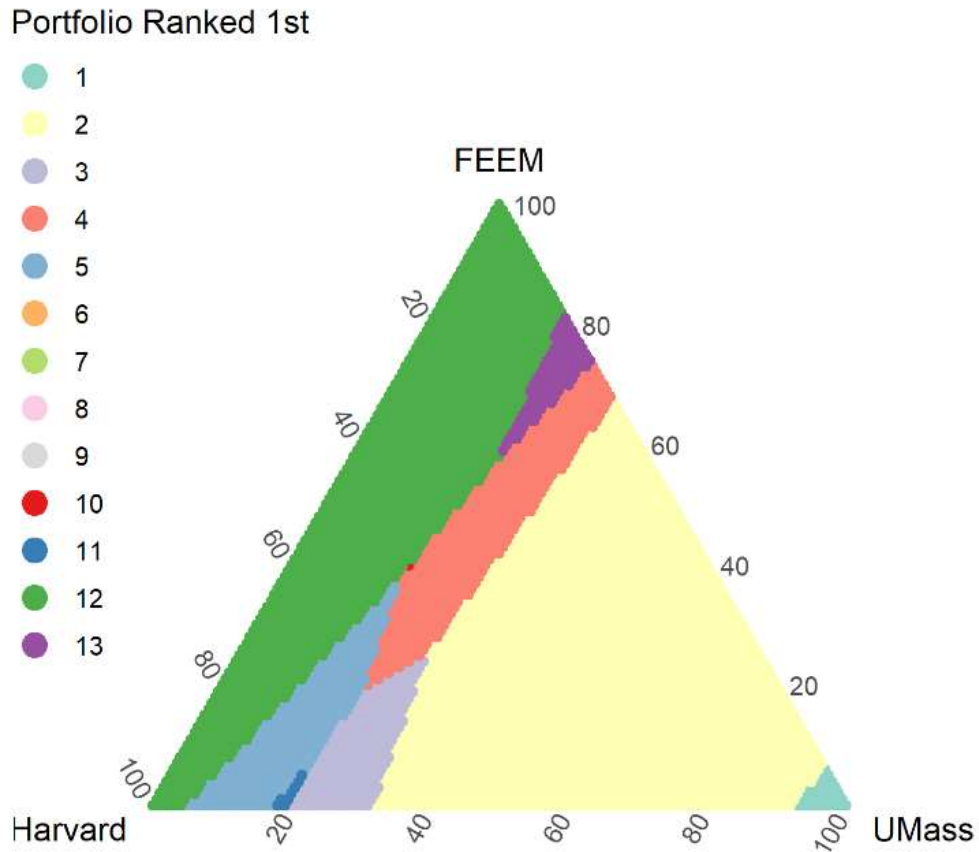
Figure 4. Over the space of possible weights given to the three elicitations, this simplex shows which portfolio is ranked first. Note that Portfolios 6-10 are never ranked first and do not appear in the Figure.

Figure 5 illustrates how these comparisons change if we look at the weightings over which a portfolio is in the top two, or the top three. In Figure 5, we show two examples. Portfolio 2, which had the largest area in Figure 4, does not change much as we consider the top 2 or top 3 (comparing the yellow area in top and bottom panels of Figure 5). When instead we look at Portfolio 10, which was not present in Figure 4, we find that the area of the simplex in which it ranks in the top 3 is approaching that of Portfolio 2.
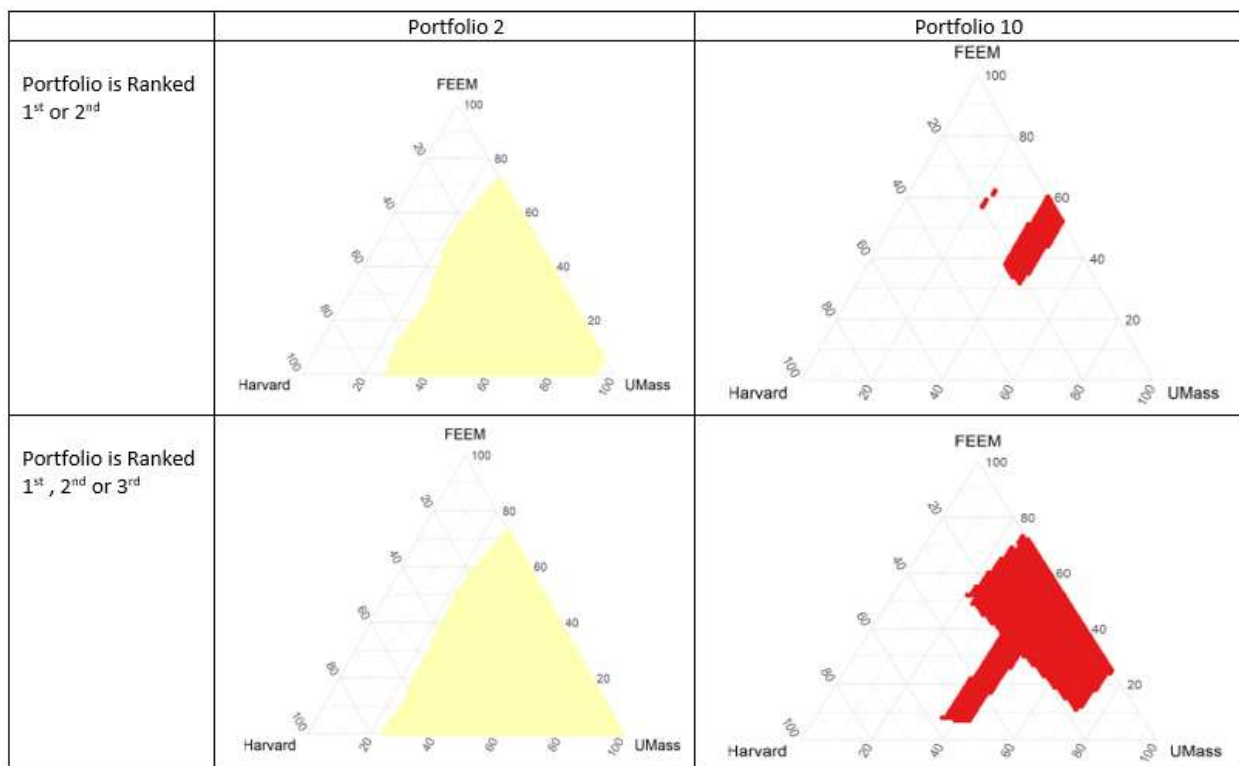
Figure 5. Over the space of possible weights given to the three elicitations, these simplexes show, for Portfolios 2 and 10, the set of weights for which the portfolio is ranked first or second (top panel); or first, second or third (bottom panel). See Figure A1 in Appendix for the same representation for all 13 Portfolios.

III.2.2   Insights into Individual Alternatives

We can use the belief-non-dominated portfolios to obtain robust results for individual technologies. In Table 3, there are two technologies with robust results across all portfolios. First, bio-electricity has a high investment in every non-dominated portfolio, so Bio-Electricity-High is in the core. This technology appears to be good regardless of which probability distribution is used to evaluate it. Second, nuclear has either a Mid or High investment in every non-dominated portfolio, so Nuclear-Low is excluded from the core. Thus, given the use of the GCAM model and the choice of the 2°C climate target, it is robust to invest in nuclear at least at mid- level, regardless of the probability distribution used. Given these insights, decision makers could incorporate other concerns to identify an overall portfolio investment.

To get further insights from RPDA, we define a Robustness Index, RI_j defined as the percentage of portfolios ranked jth or above in which the technology is present. For example, Solar Low is present in 59% of the top-ranked portfolios; and in 77% of the portfolios in the top three. Table 5 reports robustness indices for each technology investment level. Note, for example, that Biofuels High is only optimal in 16%

of the weightings; but it ranks in the top three 79% of the time, more than the other two biofuels options. This index is a generalization of strict robustness according to which a solution is robust if it "performs reasonably well in every possible future scenario" (Klamroth et al. 2017). The RI quantifies "reasonably well" using an ordinal measure; and reports *how often* an alternative performs this well, rather than requiring it to perform well for every scenario. Stakeholders and decision makers can then think about explicit tradeoffs

Technology		j=1	j=2	j=3
Solar	Low	59%	64%	77%
	Mid	11%	32%	62%
	High	30%	32%	32%
	Low	0%	0.43%	2%
	Mid	69%	73%	**100%**
CCS	High	30%	32%	32%
Nuclear	High	2%	60%	86%
	Mid	98%	**100%**	**100%**
Biofuels	Low	30%	32%	32%
	Mid	54%	61%	66%
	High	16%	32%	79%
Bioelectricity	BE - High	**100%**	**100%**	**100%**

Table 5, The Robustness Index RI$_j$ for each individual alternative, for j=1,2, or 3. Individual alternatives never present in a non-dominated portfolio are not reported.

Again, it might be relevant for the policymakers to see how much weight each elicitation studies has, as it could be politically unsuitable to give too little credit to any one study. We again turn to the simplex, with Figure 6 showing for each technology which of the three investment levels is ranked first for each combination of weights. While both mid and high investments in Nuclear are in non-dominated portfolios, the vast majority of weightings result in an optimal portfolio including a mid-investment in Nuclear. BioFuels (BF) reveals a non-linear pattern: the optimal investment is higher with equal weights than it is with heavier weight on any one belief. The figures highlight that solar and biofuels are subject to the most disagreement.

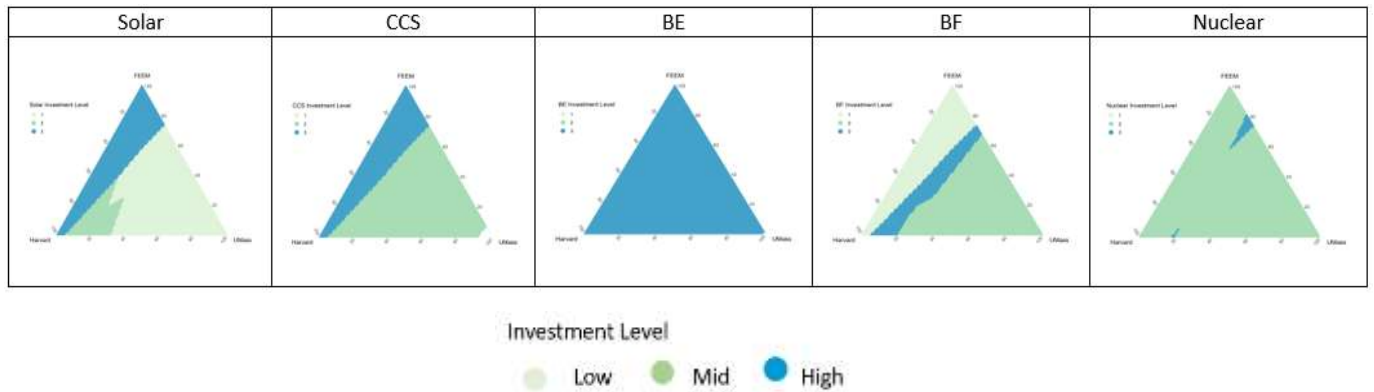| Solar | CCS | BE | BF | Nuclear |

Investment Level

Low — Mid — High

Figure 6. Over the space of possible weights given to the three elicitations, these simplexes show, for each of the five technologies, which level of investment is present in the portfolio ranked first.

Figure 7 illustrates the robustness of biofuel investments. The simplexes in the upper row illustrate the area of the simplex in which each level of investment is among the top two ranked portfolios; in the lower row, for the top three ranked portfolios. This visual representation of the robustness helps stakeholders see how close to individual beliefs a particular investment is. For example, it shows that a low investment in biofuels is not in the top three portfolios if UMass has a very low weighting; and a mid investment is not among the top three unless UMass has a very low weighting. Biofuels high, on the other hand appears in the top three portfolios for the widest range of rankings.
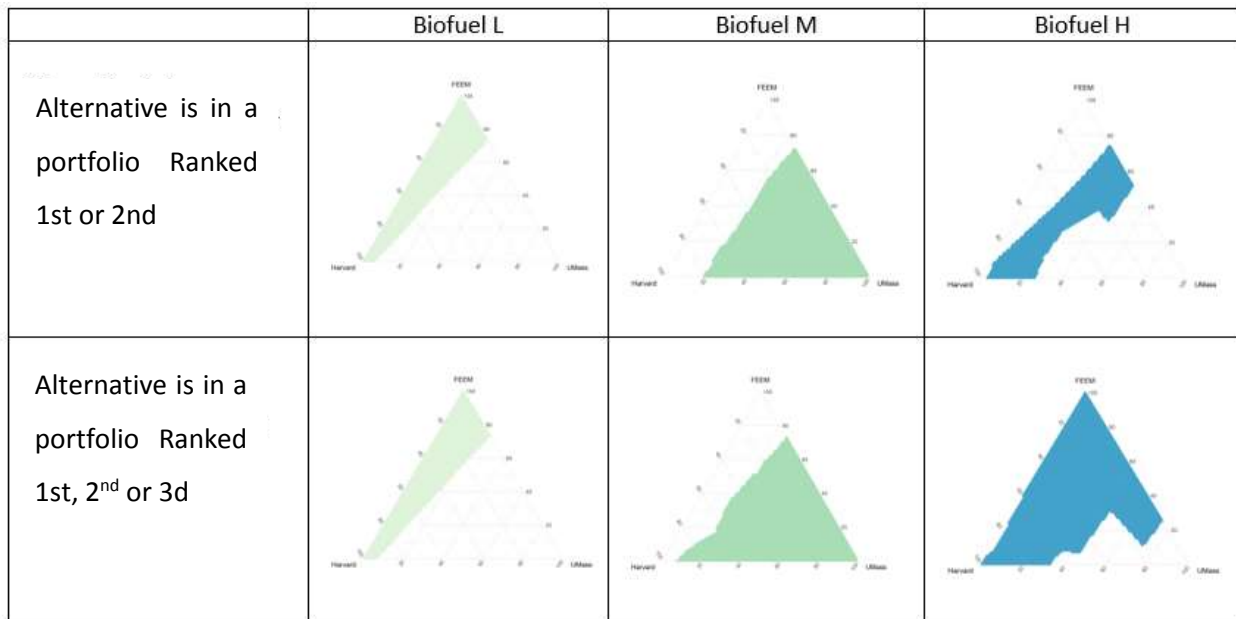

	Biofuel L	Biofuel M	Biofuel H
Alternative is in a portfolio Ranked 1st or 2nd			
Alternative is in a portfolio Ranked 1st, 2nd or 3d			

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As Table 2 shows, the amounts of R&D investments vary considerably from technology to technology. For example, the "high" investment amounts for bio-electricity and biofuels are similar to the "mid" amounts for nuclear and CCS. For context, the lowest total investment among the 243 portfolios (with a low investment in each) is $16 million per year; the highest total investment is $417 million per year. This compares to a range between $47-398 million per year among the non-dominated portfolios. The key driver of the size of the budgets among the non-dominated portfolios is the difference between a medium and a high investment in nuclear. The lowest budget non-dominated portfolio has a very low investment in solar and CCS, while the largest non-dominated portfolio has larger investments in CCS and Nuclear.

IV. Conclusions

We present Robust Portfolio Decision Analysis as a promising approach to decision making problems that are characterized by deep uncertainty and conflicting sources of information. The two key aspects of this approach are (1) the identification of all non-dominated portfolios of strategies when there are multiple, conflicting beliefs over relevant outcomes; and (2) the use of portfolio-level results in generating insights into individual strategies and their implications. We also show that our method is compatible with and generalizes many existing robustness concepts.

We apply our approach to a proof-of-concept problem: the design of a portfolio of publicly-funded research and development investments in future energy technologies, with an emphasis on social welfare. Our approach helps uncover multiple portfolios in addition to those that are suggested by commonly used robustness concepts, which helps avoid somewhat arbitrary rules for resolving disagreements and allows the decision maker to explore trade-offs that are difficult to model. This approach also shows which individual strategies are in all portfolios. In our application, we find common ground among the divergent expert beliefs in that a high investment in bioelectricity and at least a mid-investment in nuclear are robust to all beliefs, given the specific climate goal and the chosen integrated assessment model. Policy negotiators could build on this common ground when addressing non-quantifiable criteria, or by commissioning additional studies into solar and biofuels, resulting in information that would most likely impact the decision.

There are many different types and sources of deep uncertainty in general and most notably about climate change. In this paper, we have addressed multiple beliefs about one specific type of uncertainty: uncertainty over well-defined parameters (such as technology costs) represented by probability distributions. Another important source of uncertainty is the type of model used to calculate abatement costs (Revesz et al. 2014). In our analysis we employed a single IAM, the GCAM model, to translate technology parameters into societal costs and benefits; but other IAMs could be employed to provide the same analysis, which would likely result in different portfolio rankings.

The proposed Belief Dominance framework is flexible in that *different models* can be considered as sources of *different beliefs*. Nor is it limited to traditional portfolio problems such as technology R&D but, rather, can be applied to a wide range of applications, including a broader interpretation of the instruments of climate change policy. Apart from investments into energy technologies, individual alternatives can include other technology policies (e.g., standards or subsidies) as well as other climate change policies (e.g., carbon taxes, carbon caps, international trade agreements, near-term adaptation decisions), whereby uncertainties would be extended from technological progress to damage uncertainty, socio-economic uncertainties, and model uncertainty.

Although we have focused on well-defined objectives (e.g., total costs in our example), many problems involving deep uncertainty involve multiple stakeholders with conflicting objectives; such problems are often called "Wicked" (Churchman 1967). To address both of these aspects of wicked problems, our framework would need to be extended to include multiple objectives. The concepts we introduce here may inform the MORDM framework, allowing for the visualization of trade-offs in both objectives and beliefs. Alternatively, the Robust Portfolio Decision Analysis framework could be extended to include methods from MORDM to identify Pareto Satisficing alternatives.

Acknowledgments

References

Ackermann, F. 2012. "Problem Structuring Methods 'In the Dock': Arguing for the Case of Soft OR." *European Journal of Operational Research* 219 (3): 652–658.

Alvarez-Benitez, J. E., R. M. Everson, and J. E. Fieldsend. 2005. "A MOPSO Algorithm Based Exclusively on Pareto Dominance Concepts." In *International Conference on Evolutionary Multi-Criterion Optimization*, 459–473. Springer.

Anadón, L. D., V. Bosetti, M. Bunn, M. Catenacci, and A. Lee. 2012. "Expert Judgments about RD&D and the Future of Nuclear Energy." *Environmental Science & Technology* 46 (21): 11497–504.

Anadón, L. D., G. Chan, and A. Lee. 2014. "Expanding and Better Targeting US Investment in Energy Innovation: An Analytical Approach." *Transforming US Energy Innovation,* Eds: Anadon, L. D., M. Bunn, and V., Narayanamurti, Cambridge University Press.

Anadón, L. Díaz, E. Baker, and V. Bosetti. 2017. "Integrating Uncertainty into Public Energy Research and Development Decisions." *Nature Energy* 2 (5): 17071. https://doi.org/10.1038/nenergy.2017.71.

Anadón, L. D., E. Baker, V. Bosetti, and L. A. Reis. 2016. "Expert Views - and Disagreements - about the Potential of Energy Technology R&D." *Climatic Change* 136 (3–4): 677–91.

Arrow, K. J. 1951. "Alternative Approaches to the Theory of Choice in Risk-Taking Situations." *Econometrica* 19 (4): 404–37. https://doi.org/10.2307/1907465.

Arrow, K. J., L. Cohen, P. A. David, R. W Hahn, C. Kolstad, L. Lane, W. D. Montgomery, R. R. Nelson, R. Noll, and A. E. Smith. 2009. "A Statement on the Appropriate Role for Research and Development in Climate Policy." *The Economists' Voice* 6 (1).

Athanassoglou, S., and V. Bosetti. 2015. "Setting Environmental Policy When Experts Disagree." *Environmental and Resource Economics* 61 (4): 497–516. https://doi.org/10.1007/s10640-014-9804-x.

Athey, Su. 2002. "Monotone Comparative Statics under Uncertainty." *Quarterly Journal of Economics*, 187–223.

Aumann, R. J. 1962. "Utility Theory without the Completeness Axiom." *Econometrica: Journal of the Econometric Society*, 445–462.

Baker, E. 2006. "Increasing Risk and Increasing Informativeness: Equivalence Theorems." *Operations Research* 54 (1): 26–36.

Baker, E., V. Bosetti, L. D. Anadón, M. Henrion, and L. A. Reis. 2015b. "Future Costs of Key Low-Carbon Energy Technologies: Harmonization and Aggregation of Energy Technology Expert Elicitation Data." *Energy Policy* 80 (May): 219–32. https://doi.org/10.1016/j.enpol.2014.10.008.

Baker, E., H. Chon, and J. Keisler. 2009a. "Advanced Solar R&D: Combining Economic Analysis with Expert Elicitations to Inform Climate Policy." *Energy Economics* 31: S37–49.

Baker, E., H. Chon, and J. Keisler. 2009b. "Carbon Capture and Storage: Combining Economic Analysis with Expert Elicitations to Inform Climate Policy." *Climatic Change* 96 (3): 379–408.

Baker, E., H. Chon, and J. M. Keisler. 2008. "Advanced Nuclear Power: Combining Economic Analysis with Expert Elicitations to Inform Climate Policy." *Available at SSRN 1407048*.

Baker, E., and J. M. Keisler. 2011. "Cellulosic Biofuels: Expert Views on Prospects for Advancement." *Energy* 36 (1): 595–605.

Baker, E., O. Olaleye, and L. A. Reis. 2015a. "Decision Frameworks and the Investment in R&D." *Energy Policy* 80 (May): 275–85. https://doi.org/10.1016/j.enpol.2015.01.027.

———. 2015b. "Decision Frameworks and the Investment in R&D." *Energy Policy* 80 (May): 275–85..

Baker, E., and S. Solak. 2011. "Climate Change and Optimal Energy Technology R&D Policy." *European Journal of Operational Research* 213 (2): 442–54. https://doi.org/10.1016/j.ejor.2011.03.046.

———. 2014. "Management of Energy Technology for Sustainability: How to Fund Energy Technology Research and Development." *Production and Operations Management* 23 (3): 348–65.

Bell, D. E., H. Raiffa, and A. Tversky. 1988. *Decision Making: Descriptive, Normative, and Prescriptive Interactions*. Cambridge University Press.

Ben-Haim, Y. 2004. "Uncertainty, Probability and Information-Gaps." *Reliability Engineering & System Safety* 85 (1–3): 249–66.

Ben-Tal, A., D. Bertsimas, and D. B. Brown. 2010. "A Soft Robust Model for Optimization under Ambiguity." *Operations Research* 58 (4): 1220–34.

Bertsimas, D., D. B. Brown, and C. Caramanis. 2011. "Theory and Applications of Robust Optimization." *SIAM Review* 53 (3): 464–501.

Bewley, T. F. 2002. "Knightian Decision Theory. Part I." *Decisions in Economics and Finance* 25 (2): 79–110.

Borgonovo, E., and M. Marinacci. 2015. "Decision Analysis under Ambiguity." *European Journal of Operational Research* 244 (3): 823–836.

Bosetti, V., M. Catenacci, G. Fiorese, and E. Verdolini. 2012. "The Future Prospect of PV and CSP Solar Technologies: An Expert Elicitation Survey." *Energy Policy* 49: 308–17.

Brockhoff, K. 1983. "Group Processes for Forecasting." *European Journal of Operational Research* 13 (2): 115–127.

Brown, C., Y. Ghile, M. Laverty, and K. Li. 2012. "Decision Scaling: Linking Bottom-up Vulnerability Analysis with Climate Projections in the Water Sector." *Water Resources Research* 48 (W09537).

Catenacci, M. V. E., V. Bosetti, and G. Fiorese. 2013. "Going Electric: Expert Survey on the Future of Battery Technologies for Electric Vehicles." *Energy Policy* 61: 403–13.

Cerreia-Vioglio, S., F. Maccheroni, M. Marinacci, and L. Montrucchio. 2013. "Classical Subjective Expected Utility." *Proceedings of the National Academy of Sciences* 110 (17): 6754–6759.

Chan, Gabriel, L. Anadón, M. Chan, and A. Lee. 2011. "Expert Elicitation of Cost, Performance, and RD&D Budgets for Coal Power with CCS." *Energy Procedia*, 2685–92.

Churchman, C. W. 1967. *Guest Editorial: Wicked Problems*. JSTOR.

Clarke, L., K. Jiang, K. Akimoto, M. Babiker, G. Blanford, K. Fisher-Vanden, J.-C. Hourcade, et al. 2014. "Assessing Transformation Pathways." In *Climate Change 2014: Mitigation of Climate Change. IPCC Working Group III Contribution to AR5*. Cambridge University Press.

Cox Jr, L. A.. 2012. "Confronting Deep Uncertainties in Risk Analysis." *Risk Analysis: An International Journal* 32 (10): 1607–1629.

Danan, E., T. Gajdos, B. Hill, and J.-M. Tallon. 2016. "Robust Social Decisions." *American Economic Review* 106 (9): 2407–25. https://doi.org/10.1257/aer.20150678.

Epstein, L.G. 1980. "Decision Making and the Temporal Resolution of Uncertainty." *International Economic Review*, 269–283.

Etner, J., M. Jeleva, and J.-M. Tallon. 2012. "Decision Theory under Ambiguity." *Journal of Economic Surveys* 26 (2): 234–270.

"EU Science Hub - European Commission." 2009. EU Science Hub. October 7, 2009. https://ec.europa.eu/jrc/en/publication/reference-reports/rd-investment-priority-technologies-european-strategic-energy-technology-plan.

Farina, M., and P. Amato. 2002. "On the Optimal Solution Definition for Many-Criteria Optimization Problems." In *2002 Annual Meeting of the North American Fuzzy Information Processing Society Proceedings. NAFIPS-FLINT 2002 (Cat. No. 02TH8622)*, 233–238. IEEE.

Fasolo, B., A. Morton, and D. von Winterfeldt. 2011. "Behavioural Issues in Portfolio Decision Analysis." In: A. Salo, J. Keisler, and A. Morton (Eds.), *Portfolio Decision Analysis: Improved Methods for Resource Allocation*, Springer, New York.

Fiorese, G., M. Catenacci, E. Verdolini, and V. Bosetti. 2013. "Advanced Biofuels: Future Perspectives from an Expert Elicitation Survey." *Energy Policy* 56: 293–311.

Fiorese, G., M. Catenacci, V. Bosetti, and E. Verdolini. 2014. "The Power of Biomass: Experts Disclose the Potential for Success of Bioenergy Technologies." *Energy Policy* 65 (February): 94–114.

Gabrel, V., C. Murat, and A. Thiele. 2014. "Recent Advacnes in Robust Optimization: An Overview. " *European Journal of Operational Research* 235: 471–83.

Ghirardato, P., F. Maccheroni, and M. Marinacci. 2002. "Ambiguity from the Differential Viewpoint." Social Science Working Paper 1130, California Institute of Technology, Pasadena, CA.

Gilboa, I., A. Postlewaite, and D. Schmeidler. 2009. "Is It Always Rational to Satisfy Savage's Axioms?" *Economics and Philosophy* 25 (3): 285–96.

Gilboa, I., and D. Schmeidler. 1989. "Maxmin Expected Utility with Nonunique Prior." *Journal of Mathematics and Economics* 18 (2): 141–53.

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Gilboa, I., F. Maccheroni, M. Marinacci, and D. Schmeidler. 2010. "Objective and Subjective Rationality in a Multiple Prior Model." *Econometrica* 78 (2): 755–770.

Gregory, R., and R. L.. Keeney. 1994. "Creating Policy Alternatives Using Stakeholder Values." *Management Science* 40 (8): 1035–48.

Grushka-Cockayne, Y., B. De Reyck, and Z. Degraeve. 2008. "An Integrated Decision-Making Approach for Improving European Air Traffic Management." *Management Science* 54 (8): 1395–1409.

Hadar, J., and W. R. Russell. 1969. "Rules for Ordering Uncertain Prospects." *The American Economic Review* 59 (1): 25–34.

Hadka, D., J. Herman, P. M. Reed, and K. Keller. 2015. "An Open Source Framework for Many-Objective Robust Decision Making." *Environ Model. Softw.* 74 (December): 114–129.

Hassanzadeh, F., H. Nemati, and M. Sun. 2014. "Robust Optimization for Interactive Multiobjective Programming with Imprecise Information Applied to R&D Project Portfolio Selection." *European Journal of Operational Research* 238 (1): 41–53.

Hayashi, T. 2008. "Regret Aversion and Opportunity Dependence." *Journal of Economic Theory* 139 (1): 242–268.

Heal, G., and A. Millner. 2014. "Reflections: Uncertainty and Decision Making in Climate Change Economics." *Review of Environmental Economics and Policy* 8 (1): 120–37.

Herman, J. D., P. M. Reed, H. B. Zeff, and G. W. Characklis. 2015. "How Should Robustness Be Defined for Water Systems Planning under Change?" *Journal of Water Resources and Management* 140 (10): 04015012 .

Hora, S. C., Benjamin R. Fransen, N. Hawkins, and I. Susel. 2013. "Median Aggregation of Distribution Functions." *Decision Analysis* 10 (4): 279–91.

Howard, R. A. 1988. "Decision Analysis: Practice and Promise." *Management Science* 34 (6): 679–95.

Iancu, D. A., and N. Trichakis. 2013. "Pareto Efficiency in Robust Optimization." *Management Science* 60 (1): 130–147.

Kalra, N. S. Hallegatte, R. Lempert, C. Brown, A. Fozzard, S. Gill, and A. Shah. 2014. "Agreeing on Robust Decisions : New Processes for Decision Making under Deep Uncertainty." Policy Research Working Paper WPS 6906, The World Bank.

Kasprzyk, J. R., S. Nataraj, P. M. Reed, and R. J. Lempert. 2013. "Many Objective Robust Decision Making 897 for Complex Environmental Systems Undergoing Change." *Environmental Modelling & Software* 42: 55–71.

Keeney, R. L., and H. Raiffa. 1993. *Decisions with Multiple Objectives: Preferences and Value Trade-Offs*. Cambridge University Press.

Keller, K., B. M. Bolker, and D. F. Bradford. 2004. "Uncertain Climate Thresholds and Optimal Economic Growth." *Journal of Environmental Economics and Management* 48 (1): 723–41.

Kelly, D. L., and C. D. Kolstad. 1999. "Bayesian Learning, Growth, and Pollution." *Journal of Economic Dynamics and Control* 23 (4): 491–518.

Kim, S. H., J. Edmonds, J. Lurz, S. J. Smith, and M. Wise. 2006. "The O [bj] ECTS Framework for Integrated Assessment: Hybrid Modeling of Transportation." *The Energy Journal* 27: 63–91.

Klamroth, K., E. Köbis, A. Schöbel, and C. Tammer. 2017. "A Unified Approach to Uncertain Optimization." *European Journal of Operational Research* 260 (2): 403–420.

Klibanoff, P., M. Marinacci, and S. Mukerji. 2005. "A Smooth Model of Decision Making under Ambiguity." *Econometrica* 73 (6): 1849–92. https://doi.org/10.1111/j.1468-0262.2005.00640.x.

Lempert, R., and M. Collins. 2007. "Managing the Risk of Uncertain Threshold Responses: Comparison of Robust, Optimum, and Precautionary Approaches." *Risk Analysis* 27/4: 1009–26.

Lichtendahl, K. C. Jr, Y. Grushka-Cockayne, and R. L. Winkler. 2013. "Is It Better to Average Probabilities or Quantiles?" *Management Science* 59 (7): 1594–1611.

Liesiö, J., P. Mild, and A. Salo. 2007. "Preference Programming for Robust Portfolio Modeling and ProjectSelection." *European Journal of Operational Research* 181/3: 1488–1505.

———. 2008. "Robust Portfolio Modeling with Incomplete Cost Information and Project_Interdependencies." *European Journal of Operational Research* 190/3: 679–95.

Liesiö, J., and A. Salo. 2012. "Scenario-Based Portfolio Selection of Investment Projects with Incomplete Probability and Utility Information." *European Journal of Operational Research* 217/1: 162–72.

Loulou, R. and A. Kanudia. 1999. "Minimax Regret Strategies for Greenhouse Gas Abatement: Methodology and Application." *Operations Research Letters* 25 (5): 219–30.

Mansbridge, J., and C. J. Martin. 2013. "Negotiating Agreement in Politics." *Washington DC: American Political Science Association*.

Millner, A., S. Dietz, and G. Heal. 2013. "Scientific Ambiguity and Climate Policy." *Environmental and Resource Economics* 55 (1): 21–46.

Mingers, J., and J. Rosenhead. 2004. "Problem Structuring Methods in Action." *European Journal of Operational Research* 153 (3): 530–54.

Moret, S., F. Babonneau, M. Bierlaire, and F. Maréchal. 2020. "Decision Support for Strategic Energy Planning: A Robust Optimization Framework." European Journal of Operational Reseach 280: 539–54.

Nordhaus, W. D. 2002. "Modeling Induced Innovation in Climate-Change Policy." *Technological Change and the Environment* 9: 259–90.

Owen, A. B. 2015. "Monte Carlo Theory, Methods and Examples (Book Draft)."

Popp, D. 2006. "Innovation in Climate Policy Models: Implementing Lessons from the Economics of R&D." *Energy Economics* 28 (5): 596–609.

Revesz, R. L., P. H. Howard, K. Arrow, L. H. Goulder, R. E. Kopp, M. A. Livermore, M. Oppenheimer, and T. Sterner. 2014. "Global Warming: Improve Economic Models of Climate Change." *Nature* 508 (7495): 173–75.

Ribas, G. P., S. Hamacher, and A. Street. 2010. "Optimization under Uncertainty of the Integrated Oil Supply Chain Using Stochastic and Robust Programming." *International Transactions in Operational Research* 17: 777–96.

Rosenhead, J., M. Elton, and S. K. Gupta. 1972. "Robustness and Optimality as Criteria for Strategic Decisions." *Operational Research Quarterly* 23 (4): 413–31.

Rothschild, M., and J. E. Stiglitz. 1971. "Increasing Risk II: Its Economic Consequences." *Journal of Economic Theory* 3 (1): 66–84.

Savage, L. J. 1954. *The Foundations of Statistics*. Courier Dover Publications.

Solak, S., J.-P. B. Clarke, E. L. Johnson, and E. R. Barnes. 2010. "Optimization of R&D Project Portfolios under Endogenous Uncertainty." *European Journal of Operational Research* 207 (1): 420–433.

Stoye, J. 2011. "Statistical Decisions under Ambiguity." *Theory and Decision* 70 (2): 129–48.

Stoye, J. 2012. "Dominance and Admissibility without Priors." *Economics Letters* 116 (1): 118–120.

Varian, H. R. 1992. *Microeconomic Analysis*. Vol. 3. New York: Norton.

Verdolini, E., L. D. Anadon, E. Baker, and V. Bosetti. 2018. "The Future of Energy Technologies: An Overview of Expert Elicitations." *Review of Environmental Economics and Policy* (forthcoming).

Vilkkumaa, E., A. Salo, and J. Liesiö. 2014. "Multicriteria Portfolio Modeling for the Development of Shared Action Agendas." *Group Decision and Negotiation* 23: 49–70.

Von Neumann, J., and O. Morgenstern. 2007. *Theory of Games and Economic Behavior*. Princeton University Press.

Voorneveld, M., 2003. "Characterization of Pareto dominance." *Operations Research Letters* 31 (1): 7–11.

Wallace, S. W. 2000. "Decision Making under Uncertainty: Is Sensitivity Analysis of Any Use?" *Operations Research* 48 (1): 20–25.

Weber, M. 1987. "Decision Making with Incomplete Information." *European Journal of Operational Research* 28 (1): 44–57.

Woodward, R. T., and R. C. Bishop. 1997. "How to Decide When Experts Disagree: Uncertainty-Based Choice Rules in Environmental Policy." *Land Economics* 73 (4): 492–507. https://doi.org/10.2307/3147241.

Yukish, M. A. 2004. "Algorithms to Identify Pareto Points in Multi-Dimensional Data Sets", Doctoral Dissertation, The Pennsylvania State University.