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Dr. Custos, Dr. Opponent, Ladies and Gentlemen,

Economic growth is largely due to dramatic advancement in natural science and technology since the 16th century. Scientific Revolution and Enlightment together led to welfare we are experiencing in the modern world. Innovations in political philosophy and in legal systems supported the rapid development in general welfare as well. However, economic growth as such largely stems from the growth in technology and thus science. Improved efficiency in production and innovation are thus essential. Nowadays production in factories is largely optimized and automated. Scientific management, systems analysis and optimization are common in organizations. In this Dissertation, one of my aims is to convince you that similar efficiency considerations apply also in the ways that the physical reality works – even more so than previously has been known.

Automation engineering and control theory have been at the nexus of this technological growth process. Starting from the late 18th century, early feedback control systems in steam engines contributed greatly to Industrial Revolution. Mathematical control theory started to develop during the 19th century by for example Maxwell, Airy, Hurwitz and others. Modern control engineering was developed however only by the 20th century. The two World Wars did not stop this scientific progress but even accelerated it. Technology can be used either for peace or for war.

In control engineering, a dynamical system is steered in order to achieve some pre-determined goal or objective. A thermostat in a heating system is good example of a simple feedback control system – temperature gives feedback on how to adjust the heating power. The system under control can also be for example a nuclear reactor; to control reactivity using the boron control rods, or the national economy using various economic policies. Indeed, in economic control processes, monetary policy is about controlling the monetary conditions and thus inflation, for example. The same philosophy is behind the aim of countercyclical fiscal policies and Keynesian stimulus policies, where public investment and expenditure is used to stimulate aggregate demand and thus aggregate production. Control engineering is all around us.

In optimal control, we have a precisely defined objective and we have identified the system dynamics we wish to steer in order to achieve the objective. Optimal control theory was developed in the 1950’s simultaneously in the United States and the USSR. It was the time of Cold War and there was a need to control space rockets, satellites, missiles and other military related systems. Stochastic optimal control was developed in the early 1960’s as a variation on the theme. Optimal control theory is a branch of applied mathematics and systems analysis. It is however staggering to find out how useful optimal control theory is as a modelling paradigm. In terms of engineering, on top of guiding aerospace systems, it is used for example in industrial process control and in computer science and artificial intelligence research.

The historical roots of optimal control are in Calculus of Variations developed already in 1744 by Leonhard Euler. Calculus of Variations can be used to build the framework and theory of classical mechanics. Pierre Louis Moreau de Maupertuis developed the principle of least action in 1745 and this principle has been a central paradigm in physics ever since. Maupertuis is known in Finland perhaps mostly due to his scientific expedition in Tornio valley in 1736-1737. He and his peers found out that Earth is flattened in terms of its poles. It confirmed the predictions of Newtonian mechanics.

In a random setting, we are trying to control a system, which is perturbed by some external noise, due to Brownian motion for example. Therefore, in stochastic optimal control, we are controlling systems, where we are trying to obtain best performance on average. Stochastic control systems occur in financial markets, economics and in natural science, as well as in artificial intelligence research and in engineering more generally. Typically, the cost functional measures some expenditure of energy or other natural or economic costs.

Brownian motion is the archetype of a continuous-time random process since it has very useful properties. First of all, it has no memory, and it is regular in the sense that a particle undergoing Brownian motion is localized on average (the martingale property); the best estimate of future is the current state. Moreover, Brownian motion is sufficiently ugly in the sense that it is nowhere differentiable. Brownian motion has independent normally distributed increments. Therefore, it is a perfect candidate for representing noise in various technical-natural-economic systems. That is why it is widely used in finance and engineering as well as in control theory.

In the present Dissertation, I will apply the principle of least action and stochastic optimal control theory to derive results in economics, finance and theoretical physics.

Let us first consider economics and fiscal policies.

The on-going Corona crisis has increased the levels of public debt everywhere. Therefore, questions regarding solvency of sovereigns has become more and more relevant. The current zero interest rate environment has perhaps made us a little bit careless in terms of the increasing public and private debt. In public finance, stochastic optimal control can be used to model a rational government who tries to drive down debt optimally, without hurting the economy and voters too much. Efficient and successful fiscal policy can then reward the politicians in the subsequent general elections.

My research shows that with minimal assumptions, debt dynamics can become nonlinear and thus rather unpredictable. The model includes the possibility that interest rates on public debt and economic growth depend partly on the fiscal policy chosen and on the indebtness of the government. In other words, there are strong fiscal multipliers present. This means that policy matters and that there are large risks present in public debt as economic dynamics can be nonlinear or even chaotic.

What about control theory and financial markets?

Investment science and efficiency of financial markets is another important theme in my research. Understanding financial markets is important as our pensions and personal investments depend partly on them, and the economic system is fully dependent on well-functioning financial markets. In 2008 we learned how important the financial market and banks are also to the general functioning of the world economy as a whole.

In my model, stochastic optimal control is used to make the efficient markets hypothesis operational. In efficient markets, all relevant new information is reflected in the prices of assets immediately. In other words, prices reflect all available relevant information in the markets. This assumption can be modelled using stochastic optimal control theory.

I assume that the financial market tries to minimize certain costs related to new information. In the dissertation, these requirements will predict that the average returns obey a certain model from turbulence modelling. In this way, there are unexpected links between finance and physics of turbulence. In addition, the linearized optimality condition is exactly the celebrated Black-Scholes equation for pricing of financial derivatives such as options. As is known, financial derivatives are important in international finance and trade to hedge against certain risks related to for example currencies and interest rates. The results could prove to be useful in measuring the efficiency of financial markets and for example in algorithmic trading or for hedge funds.

Where does the Schrödinger equation come from?

In the Dissertation, it is shown that relativistic equations of quantum mechanics can be derived from a stochastic optimal control model in Minkowski spacetime at Planck scales, when the background noise amplitude is very small. Ultimately, ***in a linearized setting***, the optimality equations lead to the re-discovery of the relativistic Stueckelberg wave equation and the more well-known Schrödinger wave equation. Ernst Stueckelberg was a Swiss physicist, who developed his theories in the turn of the 1930s and 1940s.

The key assumption in the model is that there is extremely small-amplitude background noise or fluctuations in the fabric of the spacetime at very small scales. The noise amplitude is to be of the same order as Planck constant, as quantum mechanics concerns small time-scales and small test particles or small masses of particles under consideration.

The novelty of the approach in the Dissertation is related to the relativistic formulation and the explanation for the imaginary or complex structure of quantum mechanics. The bridge between Stueckelberg equation, Telegraphers' equation and Schrödinger equation is also important, I think.

In the study, on top of the spatial coordinates, time itself is assumed to undergo a random walk at extremely small scales. In particular, time undergoes a random walk on the complex plane. The ontological and philosophical aspects of these assumptions are not clear, but the assumptions result in the well-known equations of quantum mechanics.

Random time might be related to nonlocalities encountered in quantum physics when particles are entangled. This is the much discussed spooky action at a distance. Einstein never accepted it, but there is empirical evidence nevertheless.

From these phenomenological assumptions, the Heisenberg Uncertainty Principle is derived. The results give an indication that quantum mechanics can be understood, at least in terms of its key equations, in terms of statistical mechanics as has been put forward by Ballentine and Sir Karl Popper. Mysteries remain for future research nevertheless, although the present approach support realistic interpretations of quantum mechanics.

The Nobel laureate Richard Feynman once said that it is safe to say that nobody understands quantum mechanics. Perhaps this is the case still, but I believe that the results can help to advance further investigations in terms of the various interpretations of quantum mechanics. Recent experimental work done for example here at Aalto give further support for our theoretical findings.

Finally, on electromagnetism and General Relativity

Finally, using the principle of least action the field equations of electromagnetism are derived from the Einstein field equations of general relativity. It turns out that the Maxwell's equations are a condition for the spacetime to be Ricci-flat, which means certain average geometric regularity of the spacetime fabric. This means that the partial differential equations of electrodynamics are found to be an optimality condition so that the integral of scalar curvature of the spacetime is stationary.

Moreover, it is proposed there is a direct link between electric currents, charges, fields and the Weyl curvature. The article continues the tradition of Gunnar Nordström, who was a professor here at Aalto University some 100 years ago. The results indicate that the Einstein field equations of General Relativity can be seen as a nonlinear generalization of Maxwell's equations in electrodynamics.

Conclusions

The common theme, which binds the various threads together in this Dissertation, is the paradigm that nature as well as society are systems, which try to achieve given objectives as efficiently as possible. In terms of physics, this approach could provide further insight on the relationship of the theory of computation and physics, as it seems that dynamic programming is indeed a powerful tool to model quantum mechanics. The results provide also input in terms of various interpretations of quantum mechanics.

In terms of economics, decision theory and artificial intelligence, I would suppose that stochastic optimal control as a tool will only increase its relevance, given that the decision problems in society are getting more complex and difficult as we move forward.

It indeed seems that physics is nature's economics and economics is society's physics.

Finally, I would like to present the key points in Finnish as well.

Arvoisa Kustos, Arvoisa Vastaväittäjä, Arvoisa Yleisö!

Käsittelen väitöskirjassani pienimmän vaikutuksen periaatetta ja sen laajennoksen, stokastisen optimisäätöteorian sovelluksia talouteen ja fysikaalisiin systeemeihin.

Stokastisessa optimisäätöteoriassa pyrkimyksenä on päästä odotusarvomielessä parhaaseen mahdolliseen lopputulokseen, kun dynaamiseen systeemiin kohdistuu ulkoista kohinaa. Väitöskirjassa kehitetyt mallit osoittavat, miten stokastinen optimisäätöteoria soveltuu taloudellisten ilmiöiden mallintamiseen ja ohjaukseen sekä fysikaalisten lainalaisuuksien kuvaamiseen.

Väitöskirjassa osoitetaan, miten julkisen velan dynamiikka käyttäytyy yksinkertaisillakin oletuksilla epälineaarisesti ja millaista talouspolitiikkaa poliittisen päätöksentekijän olisi optimaalista harjoittaa, kun korkotaso ja talouskasvu ovat epävarmoja. Tulokset ovat relevantteja valtionvelan riskienhallinnan kannalta ja ne voivat tukea järkevää talouspolitiikkaa.

Väitöskirjassa rakennetaan myös malli tehokkaiden markkinoiden hypoteesin operationalisoimiseksi. Tehokkailla markkinoilla informaatiokustannusta pyritään minimoimaan ajan yli. Stokastisena optimisäätötehtävänä tämä liittyy johdannaisinstrumenttien hinnoitteluun, sillä mallin linearisoitu optimaalisuusyhtälö on sama kuin Black-Scholes -osittaisdifferentiaaliyhtälö. Väitöskirjassa esitetään myös tuottovaadetta koskeva malli, josta voi olla hyötyä varainhoitajille ja algoritmisen kaupankäynnin harjoittajille.

Väitöskirjassa osoitetaan myös, miten kvanttimekaniikan peruslainalaisuuksia voidaan ymmärtää stokastisen optimisäätöteorian näkökulmasta. Osoittautuu, että suhteellisuusteoreettisia kvanttimekaniikan lainalaisuuksia voidaan ymmärtää tulkinnasta, jossa aika-avaruus heilahtelee Planckin mittakaavoilla. Heisenbergin epätarkkuusperiaate saa täten uuden tulkinnan ja se johdetaan mallin oletuksista. Lisäksi osoitetaan, miten pienimmän vaikutuksen periaate, klassinen sähkömagnetismi ja yleinen suhteellisuusteoria voidaan ymmärtää samassa viitekehyksessä. Näillä tuloksilla on merkitystä teoreettisen fysiikan perustutkimuksen ja teoreettisen fysiikan tieteenfilosofian kannalta.

Tulokset viittaavat siihen, että todella fysiikka voidaan ymmärtää luonnon taloustieteenä ja taloustiede yhteiskunnan fysiikkana.

I ask you, Professor Giorgios Pavliotis appointed as opponent by Aalto University School of Science to present the observations that you consider appropriate for this Dissertation.