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An Application of the Two-Period Newsvendor Problem

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ABSTRACT

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In this study, we review the literature regarding newsvendor problem and present an application with product that expires in two days. We provide simulation to give insight of the problem behaviour. Monte-Carlo simulation is applied to determine optimal ordering quantities of the problem and to define how changes in the problem parameters, such as price and demand function, effect the optimal solution. The simulation study is done by comparing low and high profit products with two different demand profiles and inventory holding costs. The results and their implications to the decision making are also discussed with the results of the simulation.

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1 Introduction

Research history of the newsvendor problem reaches far back to the end of the 19th century [5] when economist Edgeworth applied a variant of it to a bank cash-flow problem. The problem and its applications are surely appealing to a wide range of researchers who want to determine the optimal ordering quantity and inventory management strategies for their business. In newsvendor problems the order is placed before the actual realization of demand, which leads to possible shortages or excess commitments that are costly.

The newsvendor problem's structure is quite straightforward and not too complex. The traditional problem and its optimal solution provides a good starting point to many other extensions of the problem. Pricing, demand models or stochastic supply yield could be possible modifications to address the problems with inventory management.

In this study, the focus will be determining the optimal ordering quantities when the actual product lasts longer than one period. The study will also explore sensitivity of the optimal ordering quantity and how variations in the problem parameter effect the expected profits, optimal ordering quantity and inventory levels. We will also determine how changing the demand distribution changes the nature of the problem and its solution.

The outline of this study is as follows. In Section 2, we review existing literature regarding the newsvendor problem and present the traditional newsvendor problem. Section 3 introduces the extended version of the problem with product that lasts two days. In Section 4 we present the results and sensitivity analysis of the problem and finally Section 5 presents the conclusions of the study.

2 Literature review

In traditional newsvendor problems a costly decision must be made before the realization of uncertain demand occur. It is generally known that the newsvendor should choose his ordering amount according to the critical fractile formula, matching it to uncertain demand. In short this means ordering less than the expected demand when profit-margin is low, and more than the expected demand when profit-margin is high. When additional extensions to the problem are presented, the solution changes significantly. We present basic formulation of the newsvendor problem in Section 3.

Introducing different variants of the newsvendor model to inventory management systems can lead to potential benefits. The implementation of the newsvendor-like models along with constantly improving information technology offer promising opportunities for researchers to collaborate with the practice and real world situations.

Petruzzi and Dada [5] study the dynamics of the problem introducing demand, which was dependent on the price of the product. This extended further to models that were applicable for demand which was either multiplicative or additive. In their study they examine different scenarios with also involving existing markets for excess products. Petruzzi and Dada also extend their study further to problems of multiple periods, where units left from one period are available to meet demands in subsequent periods. They also share their thoughts in applicability of the optimal quantities and optimal selling prices given by single- or multiple period models.

Qin, et al. [6] have written a comprehensive literature review on applying the newsvendor model to inventory management. In the study they focus on reviewing prior work and developing extensions related to customer demand, supplier pricing policies, stocking quantity and buyer risk profile. Qin et al. analyze how the optimal quantity in the newsvendor setting is affected if the demand is function of market price, marketing effort or stocking quantity.

They proceed to study the impacts of different pricing policies and discounts related to the different stocking quantities. High fixed costs of the manufacturer justify and encourage larger ordering quantities, which help economies of scale. Also different pricing strategies help suppliers identify price-insensitive buyers, who are willing to pay more.

Qin et al. also call for future research in the subject of the newsvendor problem and its applications. In their paper they especially call for further analysis on supplier capacity constraints. Also other supplier related constraints or modifications are called for in the study, like deviations from zero supplier lead time or integration of both stochastic demand and stochastic supply in newsvendor ordering decisions. Burke, Carrillo and Vakharia [1] have already shown that if demand is assumed to be uniformly distributed, closed form solutions could be obtained regardless of the underlying distribution of supplier reliabilities.

Käki and Salo [3] have further studied the newsvendor problem under supply uncertainty with independent and interdependent uniformly distributed demand and supply. The study focuses on operational uncertainty of supply instead of disruptional uncertainty. They also provided an interesting experiment with human subjects on behavior of decision makers in newsvendor problem. In short, subjects follow something that is described as 'pull-to-center' behavior, where individuals tend to make quantity decisions that are closer to expected demand than would be optimal.

Stochastic yield can affect the newsvendor setting for both single and multi supplier cases, where the production capacity or logistics can be limiting factor for the supplier. Keren et al. [4] present effects of stochastic yield in supply chain coordination and Yang et al. [9] solve the newsvendor problem with multiple suppliers and multiple products.

Yano and Lee [10] review random yield models and discuss issues related to the modeling costs, yield uncertainty and performance in the context of systems with random yields. Assumptions how yield uncertainty is characterized is often made without deeper understanding of the actual production process, which provides certain amount of defect products.

Yang et al. [9] solve the newsvendor problem under both demand and multiple supplier stochasticity. The buyer was facing random demand and has to decide ordering quantities and suppliers with different yields and prices. This could be solved with nonlinear programming routines like active set-method. They found that optimum to newsvendor problem is a function of both supplier cost and reliability. In general lower costs and higher supplier reliability lead to higher buyer profits. These results could be also used for suppliers, optimizing their prices to maximize their profit. Especially lower reliability suppliers could improve their market share by lowering their ordering prices.

Burke et al. [1] studies this further by providing analytical results and showing that cost is the key supplier selection criterion for the newsvendor. In their research they investigate implications of uncertain supplier reliability on a firm's sourcing decisions under stochastic demand.

Sourcing from a single supplier is an optimal strategy in environments in which demand uncertainty or salvage value for the product is high. Also situation, where a supplier has a large cost and reliability advantage compared to other suppliers is a situation where the single supplier is preferred. When a penalty for unsatisfied demand is high buyer is driven to multiple supplier strategy. An interesting side remark in the study is that because cost is the key order qualifier, suppliers with low cost are always bound to have at least some share of the total orders.

For more information on supply disruption Tomlin [8] has written an extensive study investigating effects of supply disruptions and strategies for disruption management. In the study he found that the nature of the disruptions often dictates the optimal policies for optimal disruption management.

Keren and Pilskin [4] provide a closed form solution for a risk averse newsvendor who faces uniform demand when utility function is any increasing differentiable function. They also find that increase in the variability of demand can increase or decrease the optimal order quantity, depending on the initial conditions of the problem.

Chen et al. [2] studies behavior of the newsvendor model with a risk-averse newsvendor under stochastic, price dependent demand using CVaR as decision criterion. They found that risk-averse newsvendor often opts for a smaller ordering quantity than his risk-neutral counterpart. If the price is fixed, the optimal ordering quantity of a risk-averse newsvendor is less than the risk-neutral optimal ordering quantity. CVaR is largely used when accounting also decision maker's viewpoint since it combines both risk and reward affiliated with said ordering decisions. The newsvendor problem has also been studied with experiments, comparing professionals' choices to optimal decisions.

3 Traditional newsvendor problem

We assume that the newsvendor maximizes profit while facing a random demand D with mean μ_D and support $[\underline{D}, \overline{D}]$, while $\underline{D} = 0$ and $\overline{D} \in R^+$. The newsvendor will order q products with a cost c from a supplier without any uncertainty (this means, that the newsvendor will receive exactly q products). These products can be further sold with a price p, but the products will be valid for only one period (no inventory will be left for the next period). The profit function for the standard newsvendor problem is

$$\pi(q) = E[p \cdot \min(q, D)] - c \cdot q. \tag{1}$$

Optimal stocking quantity q which maximizes profit can be determined with derivate of profit function. We can write profit function

$$
\begin{aligned}
\pi(q) &= p \cdot E(D - \max(D - q, 0)) - c \cdot q \\
&= p \cdot (E[D] - E[|D - q| + D - q]\frac{1}{2}) - c \cdot q \\
&= p \cdot \mu_D - \frac{p}{2} \cdot E[|D - q|] - \frac{p}{2}(\mu_D - q) - c \cdot q
\end{aligned}
$$

Next we find the maximum value of profit function derivating with ordering quantity

$$\pi'(q) = -\frac{p}{2} \cdot (2 \cdot F_D(q) - 1) + \frac{p}{2} - c = 0$$

If $\pi'(q)$ is continuous the maximum can be found where derivate is 0.

$$p \cdot F_D(q) = p - c \tag{2}$$

$$q = F_D^{-1}(\frac{p - c}{p}), \tag{3}$$

which is also called the critical factile formula.

Usually the model also consists of salvage value of unsold products, resulting in the profit function

$$\pi(q) = E[p \cdot \min(q, D)] - c \cdot q + s \cdot \max(0, q - D), \tag{4}$$

where s resembles the salvage value of unsold units, for example recycled materials.

Similarly the critical fractile is,

$$q = F_D^{-1}(\frac{p - c}{p - s}). \tag{5}$$

The products in the traditional newsvendor problem are valid only for a day instead of several days where the products do not go to waste. This is usually not the case in real life situations, where products can be stored for a limited period of time. Application of the basic newsvendor problem can be extended to inventory management problems determining optimal prices, supply networks or ordering quantities with different relations and properties.

4 Application to two-period problem

We apply the newsvendor model for a two period problem, where the product would be valid up to two periods after ordering. Yield from order is deterministic, so the newsvendor will always receive the exact amount of products he ordered. Demand will follow a predetermined distribution. The profit function for period 1 is

$$\pi_1 = p \cdot \min\{x_1, D_1\} - c_f \cdot x_1 - h \cdot \max\{\min\{x_1 - D_1, x_1\}, 0\}, \tag{6}$$

where the first term is the profit from sold units, second term is the cost from ordering x_1 units and last term is the cost of transferring excess units to the next time period. Variables are presented as in the traditional newsvendor problem, except the ordering quantity is presented as variable x. Variable y_t represents inventory of 1 day old products at time t and variable z_t the inventory of 2 day old products at time t. These will originate directly from the last term, the cost of transferring excess units to the next time period. The formulas for these variables will be written in full later.

For the second period the profit function is

$$\begin{aligned}\pi_2 &= p \cdot \min\{y_2 + x_2, D_2\} - c_f \cdot x_2 \\ &\quad - h(\max\{\min\{y_2 + x_2 - D_2, x_2\}, 0\} + \max\{\min\{y_2 - D_2, y_2\}, 0\}). \end{aligned} \tag{7}$$

For period 3 the profit function is

$$\begin{aligned}\pi_3 &= p \cdot \min\{z_3 + y_3 + x_3, D_3\} - c_f \cdot x_3 \\ &- h(\max\{\min\{y_3 - (D_3 - z_3), y_3\}, 0\} + \max\{\min\{x_3 - (D_3 - z_3 - y_3), x_3\}, 0\}),\end{aligned} \tag{8}$$

which can be written more conveniently with variables z_t and y_t

$$\pi_3 = p \cdot \min\{z_3 + y_3 + x_3, D_3\} - c_f \cdot x_3 - h(y_4 + z_4), \tag{9}$$

where the new variable z_t represents the amount of 2 day old products in the inventory. Variable y_t can be written as

$$y_t = \max\{\min\{x_{t-1} - (D_{t-1} - z_{t-1} - y_{t-1}), x_{t-1}\}, 0\}, \tag{10}$$

and variable z_t as

$$z_t = \max\{\min\{y_{t-1} - (D_{t-1} - z_{t-1}), y_{t-1}\}, 0\}. \quad (11)$$

For convenience, we write inventory at time t

$$i_t = x_t + y_t + z_t, \quad (12)$$

which results to profit function being

$$\pi_t = p \cdot \min\{i_t, D_t\} - c_f \cdot x_t - h(y_{t+1} + z_{t+1}). \quad (13)$$

The more general form the profit function can be written

$$\pi_t = p \cdot \min\{\sum_j q_t^j, D_t\} - c_f \cdot q_t^1 - h(\sum_{k=2}^{n} q_{t+1}^k), \quad (14)$$

where

$$q_t^j = \max\{\min\{q_{t-1}^{j-1} - (D_{t-1} - (\sum_{k=j}^{n} q_{t-1}^k), q_{t-1}^{j-1}\}, 0\} \quad (15)$$

$$= \max\{\min\{\sum_{k=j-1}^{n} q_{t-1}^k - D_{t-1}, q_{t-1}^{j-1}\}, 0\}, \quad (16)$$

which can be applied to products of other expiration dates.

After applying the model to two differently distributed demand models and examining the results, we study the sensitivity of the profit function by varying the parameters. Most interesting changes of the optimum solution will be when we change the product from being low-profit to high-profit, and see how the change affects the optimum ordering quantity. There is no penalty added to unsatisfied demand, where ordering less than demand may be favored versus ordering larger quantity than expected demand.

We are using MATLAB to simulate the system and to plot the results to figures. We are using two different demand models: $D \backsim U[0, 300]$ and $D \backsim N[150, 40]$. Simulation will be done using Monte-Carlo simulation with

D	Profit	h	q^*	π	μ_y	μ_z
Unif	High	0	168	1265.8	143.5	83.8
Unif	High	2	138	965.8	76.7	28.4
Unif	Low	0	140	346.4	80.8	31.1
Unif	Low	2	104	211.5	31.1	6.7
Norm	High	0	153	1326.6	130.0	68.7
Norm	High	2	139	1155.7	38.7	3.8
Norm	Low	0	146	427.1	77.8	20.2
Norm	Low	2	128	347.5	17.6	0.5

Table 1: Results of the simulation summarized

randomly generated demand from pre-determined distributions. The MATLAB code used is presented in the Appendix.

First we determine the effects of no inventory transfer price versus low inventory transfer price for the high and low profit products. For the sake of continuity, we use the same parameters as Schweitzer and Cachon [7]: for the high profit product, selling price $p = 12$, order cost $c_f = 3$ and for the low profit product order cost $c_f = 9$. Inventory and stock holding cost $h = 0$ when studying the effects of no holding costs and $h = 2$ with inventory and stock holding costs.

Finally we study the effects of other variables in the simulation: Does the optimal ordering quantity behave similarly between the two demand profiles or are there any major differences?

5 Results

First we examined how leaving out the cost of holding inventory effect the optimum ordering quantity. When there is no cost of holding inventory, the excess inventory is not punished unless it is over 2 days old, which leads to investments that does not produce any profit. But since we are discussing a high-profit product, it is natural that the optimum ordering quantity should be slightly higher than the expected demand. Results of the simulation are summarized in Table 1.

Without inventory costs we get optimal ordering quantity $q^* = 168$ units with mean profit $\pi = 1265.8$, holding mean inventory of 1 day old products $\mu_y = 143.5$ and two day old products $\mu_z = 85.8$. With uniform distribution $D \backsim U[0, 300]$ and high profit product this is fairly reasonable result, the

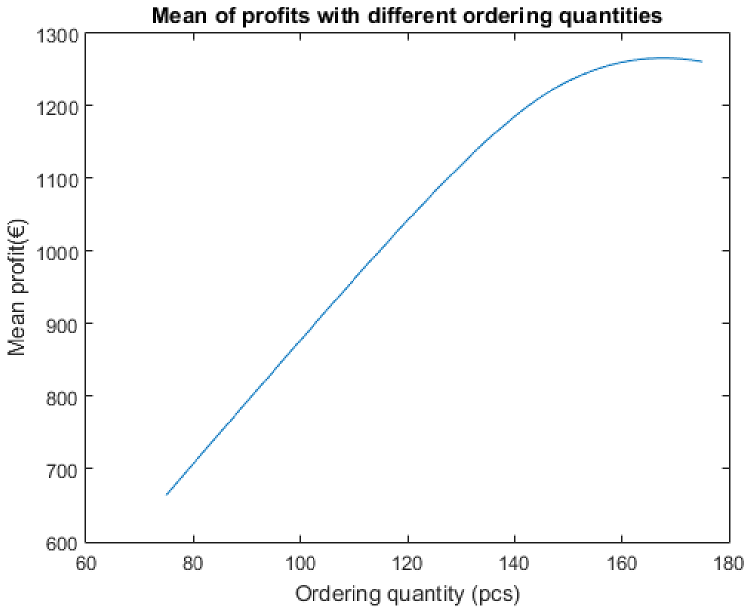

Figure 1: Mean profits with different ordering quantities, $D \backsim U[0, 300]$, $p = 12$, $c_f = 3$ and $h = 0$

optimal ordering amount being higher than the mean of demand. Example of the mean profits with different ordering quantities are presented in Figure 1.

When using an inventory holding cost $h = 2$, we notice an immediate drop in the optimal ordering quantity. We get the optimal ordering quantity $q^* = 138$ units with a mean profit $\pi = 965.8$, holding mean inventory of 1 day old products $\mu_y = 76.7$ and two day old products $\mu_z = 28.4$ units.

This implies that adding the inventory holding cost for each product decreases profits significantly and makes the newsvendor avoid holding greater amounts of inventory. Where there are no inventory costs, μ_y and μ_z were almost as high as q^*, with inventory costs they must be dropped significantly lower compared to the optimal ordering quantity.

When simulating the model with a lower profit product, it is clear that expected profits are bound to be lower. High variability of uniform distribution guides newsvendor to keep higher inventory levels to match the randomness of the demand. Without inventory costs optimal ordering quantity $q = 140$ units with mean profit $\pi = 346.4$ and inventory levels of $\mu_y = 80.8$ and $\mu_z = 31.1$. The optimal ordering quantity is now significantly lower than earlier, which is result of profit margin being lower. It is not profitable to order more units than it's expected, since the loss of profits when units going to waste is much higher in relation to the possible profit.

The profit $\pi = 221.5$ from the setup, where the inventory costs are added to the low profit situation, is the lowest of all simulations. This is justified by the fact that while low profit product does not encourage the newsvendor to keep excess inventory to fulfill unexpected demand. At the same time the variability of uniform distribution increases the need of backup inventory. This leaves the newsvendor in an unpleasant situation, where he must lower the ordering quantity $q^* = 104$ while still having relevant backup inventory.

When comparing the two demand distributions, it is worth noticing that using the normal distribution generates more situations where demand values are closer to the mean, compared to the uniform distribution. In figure 2 the two different demand profiles are presented with histograms.

Using normal distribution for demand, the high profit product and no inventory costs we get optimal ordering quantity of $q^* = 153$ and profit $\pi = 1326.6$ with inventory levels $\mu_y = 130.0$ and $\mu_z = 68.7$. Generally the inventory levels compared to uniform demand are significantly lower, since the normally distributed demand values are generally closer to the mean value than with the uniform distribution.

With inventory costs but high profit product the optimal ordering quantity $q^* = 139$ is not much different than with uniform distribution, but the mean profit of the simulations is really different $\pi = 1155.7$. Generally mean inventory $\mu_y = 38.7$ and $\mu_z = 3.8$ are much lower, since the demand values are closer to the expected demand.

The lower profit products with no inventory costs behave as expected, with the deviation from the mean demand being smaller than with the uniform distribution. The optimal ordering quantity $q^* = 146$ units is only few units lower than the mean of the distribution. Without inventory costs it is still profitable to keep relatively high inventory $\mu_y = 77,82$ and $\mu_z = 20,22$ even with the lower profit products, because the excess products can be sold up to two periods later.

When we add the costs of holding inventory, the optimal ordering quantity $q^* = 128$ units drops significantly lower, where the optimum is most likely achieved by keeping the holding inventory as small as possible $\mu_y = 17.6$ and $\mu_z = 0.5$ units. With significant portion of the demand values being near the mean value of distribution makes it possible to gain decent enough profit $\pi = 347.5$ with small inventory. When unsatisfied demand is not punished, the lower inventory strategies will become more attractive. An example of the

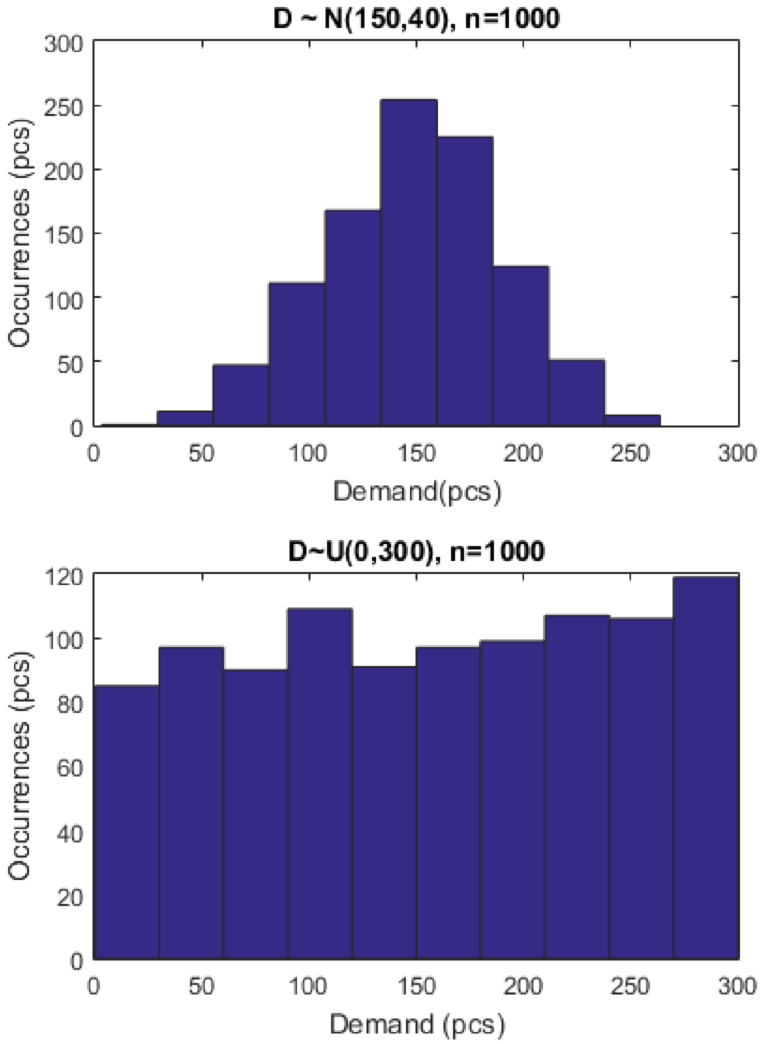

Figure 2: Histograms of demand profiles

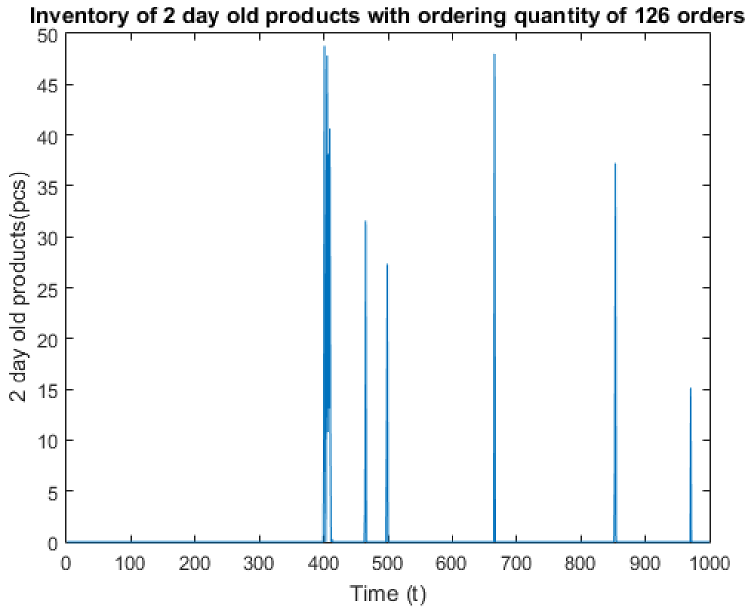

Figure 3: z_t during a low profit, inventory cost simulation with normally distributed demand.

inventory levels of the two day old products during simulation are presented in Figure 3

The results of the simulations are relatively in line with earlier studies like [3] Käki's and Salo's, with difference being that the optimal ordering quantities are closer to the mean demand because the inventory can even out the fluctuations in demand and therefore provide more products to use if needed.

6 Conclusions

We have presented and analyzed the newsvendor problem with a product that expires two days after ordering by using Monte Carlo simulation. We have also provided a sensitivity analysis on the problem parameters such as price, demand and inventory holding costs. We found that when the costs of holding inventory are present the optimal ordering quantity is always lower than the expected demand. The possibility of excess inventory decreases the profit enough to make settings with lower ordering quantities more desirable than those with higher ordering quantities. When simulating with no inventory holding costs the optimal ordering quantities are similar to the other studies such as Käki's and Salo's [3].

We also found that changing the demand distribution had significant effect on the optimal ordering quantity and gained profits. Normally distributed demand provided much higher mean profits versus uniform distribution which had higher variability during simulations. This also shows the importance of correct assumptions regarding the demand when simulating the problem.

Possible further areas for extending the study is to add stochasticity to the supplier yield and dependency between the product price and demand. The effect of inventory when supplier uncertainty is present will probably make holding decent amount of inventory a quite tempting option. It is still advised that when making decisions the results of this analysis should be used merely as guidelines more than the definite truth. The results can be also used as a base to justify higher or lower inventory levels depending on the actual set of variables on the problem.

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A Appendix

```
function [ prof,y,z ] = prof_func(q, d, p, c_f, h)
%porf_func function to calculate profits for given demand
%q is the ordering amount
%d is the demand array (length of array determines the amount of cycles)
%p is price
%c_f is ordering costs
%h is cost of transferring the products to next time period




y(1)=0;
z(1)=0;
prof(1) = p*min(q, d(1))-c_f*q-h*max(min(q-d(1), q),0);

y(2)=max(min(q-(d(1)-y(1)),q), 0);
z(2)=0;
prof(2)= p*min(q+y(2), d(2))-c_f*q-h*max(min(q-d(2), q),0);

y(3)=max(min(q - (d(2) - y(2) - z(2)), q), 0);
z(3)=max(min(y(2) - (d(2)-z(2)), y(2)), 0);
prof(3)= p*min(q+y(3)+z(3), d(3))-c_f*q-h*max(min(q-d(3), q),0);

d_c=4; %counter
y(d_c)=max(min(q - (d(d_c-1) - y(d_c-1) - z(d_c-1)), q), 0);
z(d_c)=max(min(y(d_c-1) - (d(d_c-1) - z(d_c-1)), y(d_c-1)), 0);

while d_c <= numel(d)
    %inventory on next round
    y(d_c+1)=max(min(q - (d(d_c) - y(d_c) - z(d_c)), q), 0);
    z(d_c+1)=max(min(y(d_c) - (d(d_c) - z(d_c)), y(d_c)), 0);
    prof(d_c)= p*min(q+y(d_c), d(d_c))-c_f*q-h*(y(d_c+1) + z(d_c+1));
    d_c=d_c+1;

end
y=y(1:end-1);
z=z(1:end-1);
end
```