Optimal Exploitation of Mineral Resources with Contingent Portfolio Programming

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1 Introduction

Stochastic programming is an approach to the modelling of optimization problems involving uncertainties (Birge and Louveaux, 1997). It accounts for several uncertainties and provides a framework for risk management. Multi-periodic settings are also supported, which allows for the optimization of dynamic decision rules.

In Contingent Portfolio Programming (CPP, Gustafsson and Salo, 2005), uncertainties are modelled by decision trees in the spirit of stochastic programming. The decision maker (DM) seeks to maximize the value of a portfolio of risky assets over a planning horizon with regard to a set of scenarios. The problem can be reduced to a Mixed Integer Linear Program (MILP) so that large scale problems can be solved.

When analysing ventures (mining, oil drilling or research funding, for instance) through stochastic programming, scenario generation is essential. Ho et al. (1995) present a method for creating multivariate binomial processes. The processes can therefore depend on several variables and time-varying correlations among the variables may be modelled. Kettunen et al. (2007) develop a framework for dynamic risk management for electricity contracts using CPP with a load and price uncertainty accounting for the correlation between these variables.

Kamrad and Ernst (2001) provide an approach to the evaluation of mining ventures. They formulate an arbitrage valuation based capital budgeting problem for a single mine with output yield and market uncertainty. However, their approach lacks the portfolio perspective over several periods, which could be important for long term decision making. These features can be accounted for in a CPP model. Also, Kamrad and Ernst assume a continuous operating strategy, which can be deemed as unrealistic, as operating strategies are often discrete by nature. For instance, contracts with the miners may be periodic, forcing commitment to longer periods. Considerations of the liquidity of the mine are also of interest.

In modern (financial) portfolio theory, many important benefits can be
gained through portfolio effects. The key results are that (i) diversification reduces risk and optimising a portfolio yields better results than optimising the units of a portfolio an adding them up and (ii) inclusion of a risk free asset enables the creation of portfolios that outperform all but the market portfolio. Thus, when considering decision makers (DM) that are not risk neutral, this is a relevant aspect.

Clearly, the CPP approach makes it necessary to model the properties of mines, such as the total deposit and the ease of exploit. Also, the market conditions for the underlying asset are of interest. In this study, we provide a simple, yet extendable, framework for modeling the mine and the market. A thorough introduction can be found in Harris (1990) and Luenberger (1998).

The goal of this study is to demonstrate some of the CPP model features through an example, which most importantly features multiperiodicity and risk management. The example represent a mining problem, which is formulated as a CPP model with one uncertainty and two resources, namely money and gold.

The rest of this study is organised as follows. Section 2 formulates the CPP model for a mining venture. Section 3 presents a risk measure suitable for this context and a few remarks about the meaning of different risk measures. Section 4 describes the scenario generation process and the method of parameter estimation. Section 5 presents the numerical results of applying this framework to a simple mining venture. Section 6 summarises the main results of this study.

2 Contingent Portfolio Programming

Let us apply a CPP approach to the mining problem defined by quantities in Table 1 and variables in Table 2. These parameters are sufficient for the analysis of simple mining investment decisions.

In CPP, uncertainties are modelled as a scenario tree that extends over the planning horizon \( t = 0, \ldots, T \). The set of possible scenarios in period
Table 1: Fixed parameters in the contingent portfolio programming model.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>Terminal time</td>
</tr>
<tr>
<td>$t = 0, \ldots, T$</td>
<td>Planning horizon</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>The base scenario</td>
</tr>
<tr>
<td>$\omega_t$</td>
<td>A scenario at time $t$</td>
</tr>
<tr>
<td>$\Omega_t$</td>
<td>The set of possible scenarios at time $t$</td>
</tr>
<tr>
<td>$B : \Omega_t \rightarrow \Omega_{t-1}, t &gt; 0$</td>
<td>Backward operator</td>
</tr>
<tr>
<td>$i = 1, \ldots, N$</td>
<td>Mines</td>
</tr>
<tr>
<td>$W_i(\omega_0)$</td>
<td>Initial deposit in the base scenario</td>
</tr>
<tr>
<td>$C_0(\omega_0)$</td>
<td>Initial endowment (budget)</td>
</tr>
</tbody>
</table>

Constant quantities over one period

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_i$</td>
<td>Maximum proportion excavated by mine $i$</td>
</tr>
<tr>
<td>$c^i_E$</td>
<td>Fixed cost of excavating mine $i$</td>
</tr>
<tr>
<td>$c_L$</td>
<td>Licensing fee</td>
</tr>
<tr>
<td>$r_f$</td>
<td>Risk-free interest rate</td>
</tr>
<tr>
<td>$r_L$</td>
<td>Liquidation rate</td>
</tr>
</tbody>
</table>

Quantities over one period at time $t$

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p(\omega_t)$</td>
<td>Price of metal in scenario $\omega_t$</td>
</tr>
<tr>
<td>$P_{\omega_t}$</td>
<td>Unconditional probability of scenario $\omega_t$</td>
</tr>
</tbody>
</table>
Table 2: State specific decision variables in the CPP model.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_i^e(\omega_t)$</td>
<td>Binary variable indicating if mine $i$ is excavated in scenario $\omega_t$</td>
</tr>
<tr>
<td>$X_i^l(\omega_t)$</td>
<td>Binary variable indicating if license to mine $i$ in scenario $\omega_t$ is kept</td>
</tr>
<tr>
<td>$X_t$</td>
<td>Project portfolio management strategy for mining activities</td>
</tr>
<tr>
<td>$W_i(\omega_t)$</td>
<td>Deposit in mine $i$ in scenario $\omega_t$</td>
</tr>
<tr>
<td>$R_i(\omega_t)$</td>
<td>Excavated resources from mine $i$ in scenario $\omega_t$</td>
</tr>
<tr>
<td>$Z_i(\omega_t)$</td>
<td>Investments in raw material in scenario $\omega_t$</td>
</tr>
<tr>
<td>$W_a(\omega_t)$</td>
<td>Total amount of investments in raw material in scenario $\omega_t$</td>
</tr>
<tr>
<td>$Z_t$</td>
<td>Project portfolio management strategy for ore</td>
</tr>
<tr>
<td>$Z_t$</td>
<td>Raw material investment strategy until period $t$</td>
</tr>
</tbody>
</table>

$t$ is denoted by $\Omega_t$ and we assume that there is a single base scenario $\omega_0$. Thus the set of all scenarios is $\Omega = \{\omega_0\} \bigcup_{t=1}^T \Omega_t$. The scenarios in the last period $T$, i.e. scenarios in $\Omega_T$, are called terminal scenarios. Every scenario $\omega \in \Omega_t$, $t > 0$ has a unique predecessor in period $t-1$ given by the backward operator $B : \Omega_t \rightarrow \Omega_{t-1}, t > 0$.

We have $i = 1, \ldots, N$ mines with an initial deposit $W_i(\omega_0)$ in the base scenario $\omega_0$. Let $0 < \rho_i < 1$ be the proportion of resources that can be excavated in a given period and $c_i^E$ the fixed cost of excavation from mine $i$, and $c_L$ the licensing fee. The price of metal in scenario $\omega_t$ is $p(\omega_t)$.

Let $X_i^e(\omega_t)$ be a binary variable indicating if mine $i$ is excavated in scenario $\omega_t$ (1 if excavated, otherwise 0). Let $X_i^l(\omega_t)$ be a binary variable indicating if the rights to mine $i$ are retained in scenario $\omega_t$ (1 if retained, otherwise 0). This leads to the constraint $X_i^l(\omega_t) \geq X_i^e(\omega_t)$, and $X_i^l(\omega_t) \leq X_i^l(B(\omega_t))$ because rights are permanently lost if not retained. The license fee constraint could be modified to extend over several periods, that is paying the fee would yield mining rights for two or more periods. This could be accomplished by replacing the above constraint with $X_i^l(\omega_\tau) \geq X_i^e(\omega_t)$, where $\tau$ is the period...
in which the license starts.

The project portfolio management strategy $\mathbf{X}$ is defined by $X_i^e(\omega_t)$ and $X_i^l(\omega_t)$, $i = 0, \ldots, T$, $\omega_t \in \Omega$. In the sequel, $\mathbf{X}_t$ denotes the portfolio management strategy which extends from the base scenario up until period $t$, i.e. it is defined by $X_i^e(\omega)$ and $X_i^l(\omega)$, $i = 0, \ldots, t$, $\omega \in \{\omega_0\} \cup_{r=1}^T \Omega_r$.

Assume a project portfolio strategy $\mathbf{X}$. Let $W_i(\omega_t)$ be the amount of metal deposit in mine $i$ at the outset of scenario $\omega_t$. Thus the amount of excavated resources is $R_i(\omega_t) := W_i(B(\omega_t)) - W_i(\omega_t)$ subject to $0 \leq R_i(\omega_t) \leq \rho_i W_i(B(\omega_t))$ (negative amounts cannot be excavated, nor can we extract more than the largest proportion possible). The amount of resources can change only if the mine is excavated, thus $R_i(\omega_t) \leq M X_i^e(\omega_t)$, where $M$ is a large constant.

Let $Z_a(\omega_t) \in \mathbb{R}$ denote the investments made in the raw material in scenario $\omega_t$. This may be negative (indicating disinvestment) or positive (indicating investments). Then the total amount invested in raw material in scenario $\omega_t$ is $W_a(\omega_t) = W_a(B(\omega_t)) + Z_a(\omega_t)$ (assuming no trading costs). As with the mines, assume a project portfolio strategy $\mathbf{Z}$ defined by $Z_a(\omega_t)$, $\omega_t \in \Omega$. Again, $\mathbf{Z}_t$ denotes the portfolio management strategy that extends from the base scenario up until period $t$.

Let $r_L$ be the liquidation rate at which net present value is accounted for in the computation of the asset balance and the risk-free interest rate $r_f$.

The initial endowment (budget) in the base scenario is $C_0(\omega_t)$ and assume that there are no initial endowments in other scenarios ($C_0(\omega_t) = 0, t > 0$). This assumption could be modified to account for proceeds from previous investments.

The cash flow from mine $i$ in scenario $\omega_t$, $t > 0$ is

$$CF_i(\omega_t) = \underbrace{R_i(\omega_t)p(\omega_t)}_{\text{Mineral sold}} - \underbrace{X_i^l(\omega_t)c_L}_{\text{Licence costs}} - \underbrace{X_i^e(\omega_t)c_i^E}_{\text{Excavation costs}}$$

\[1\text{Actually } W_i(\omega_t) \text{ depends on the strategy until period } t, \text{ i.e. } W_i(\mathbf{X}_t, \omega_t), \text{ but when assuming a fixed strategy it can be described as a function of only } \omega_t\]
and in the base scenario it is

\[ \text{CF}_i(\omega_0) = X^e_i(\omega_0)\rho W_i(\omega_0)p(\omega_0) - X^l_i(\omega_0)cL - X^e_i(\omega_0)c^F. \]

The cash flow from asset trading in scenario \( \omega_t \) is

\[ \text{CF}_a(\omega_t) = -p(\omega_t)Z_a(\omega_t). \]

In scenario \( \omega_t \), the total cash flow is

\[ \text{CF}(\omega_t) = \text{CF}_a(\omega_t) + \sum_{i=1}^{N} \text{CF}_i(\omega_t), \]

and the cash surplus is

\[ \text{CS}(\omega_t) = \begin{cases} 
C_0(\omega_0) + \text{CF}(\omega_0), & t = 0 \\
\text{CS}(B(\omega_t))(1 + r_f) + \text{CF}(\omega_t), & t > 0. 
\end{cases} \]

The liquidation value of mine \( i \) in scenario \( \omega_t \) is

\[ \text{LV}_i(\omega_t) = r_Lp(\omega_t)W_i(\omega_t), \]

if the licence is kept \( (X^l_i(\omega_t) = 1) \) and zero otherwise \( (X^l_i(\omega_t) = 0) \). This value describes the future value of the mine. In other words, the mine has value only if the license is kept, as otherwise it cannot be excavated. The liquidation value of assets in scenario \( \omega_t \) is

\[ \text{LV}_a(\omega_t) = p(\omega_t)W_a(\omega_t) \]

and liquidation value of all assets in scenario \( \omega_t \) is

\[ \text{LV}(\omega_t) = \text{LV}_a(\omega_t) + \sum_{i=1}^{N} \text{LV}_i(\omega_t). \]

Based on this, the DM’s aggregate cash position in scenario \( \omega_t \) becomes

\[ \text{CP}(\omega_t) = \text{CS}(\omega_t) + \text{LV}(\omega_t). \]

A reference cash position \( C_{ref}(\omega_t) \) is the cash position available in scenario \( \omega_t \) if the initial budget would be invested at the risk free rate, that is

\[ C_{ref}(\omega_t) = C_0(\omega_0)(1 + r_f)^t, t = 1, \ldots, T. \]
Figure 1: Illustration of the relationship between \( VAR \) and \( CVAR \) with confidence level \( \beta \).

### 3 Risk Constraints

A risk measure describes the amount of risk associated with a particular course of action. The choice of risk measure is nontrivial. Value-at-risk (VAR), that is the maximum loss at a given confidence level, has been proposed as a risk measure, but it is neither convex nor coherent, and hence it is not very suitable (Szegö, 2002). However, a solution could be the use of conditional-VAR (CVAR), that is the expected loss in case of a tail event:

\[
CVAR = E[\text{Loss} \mid \text{Loss is greater than VAR}] .
\]

The relation between \( VAR \) and \( CVAR \) is shown in Figure 3.

When dealing with cash flows, it may be convenient to consider cash flows instead of cash. Kettunen and Salo (2006) introduced conditional-cash-flow-at-risk (CCFAR) as a risk measure and risk constraints are imposed. CCFAR is defined as the expected loss if a tail event in the worst \( 1 - \beta \) cases. It can be calculated from the linear programming problem:

\[
CCFAR_\beta(X, Z, \alpha) = \min_{\alpha, \kappa} \left( \alpha + \frac{1}{1 - \beta} \sum_{\omega \in \Omega_T} \kappa_\omega \right)
\]

\[
s.t. \kappa_\omega \geq P_\omega(C_{ref} - CP_\omega - \alpha), \ \kappa_\omega \geq 0
\]

This formulation may be extended to allow for several risk constraints over different periods and confidence levels.
Now, the optimal risk constrained mining and investment strategy can be found by solving with regard to $X$, $Z$, $CS$, $\alpha$ and $\kappa_\omega$ the linear program (LP)

$$\max \sum_{\omega \in \Omega_T} p_\omega CP_\omega - \xi CCFAR$$

$$\text{s.t. } CCFAR \leq R$$

with all earlier constraints as well. The small constant $\xi > 0$ is introduced in the objective function to minimize the problem with regard to $CCFAR$, because then solving the LP also yields the cash-flow-at-risk (CFAR, see Rockafeller and Uryasev, 2000), that is the cash flow which covers all but the $\beta$ worst cash flows. Specifically, at optimum we have by $CFAR = \alpha$.

4 The Price Process

In our model, the market is the sole source of uncertainties. Uncertainty from multiple sources with correlations, which could be associated with the initial mine deposit, for instance, could be accounted for using techniques presented in Ho et al. (1995). Market uncertainties are modelled through a multiplicative price process, which can be described through a binomial lattice (Luenberger, 1998). Therefore we assume a price process $S(0), S(1), \ldots, S(N)$ generated by a multiplicative model

$$S(k+1) = u(k)S(k), \quad k = 0, \ldots, N-1,$$

where $u(k)$ are independently random distributed variables describing the relative price change. The model becomes additive by taking a logarithm

$$\ln S(k+1) = \ln S(k) + \ln u(k).$$

4.1 Construction of a binomial lattice

A binomial (recombining) lattice is constructed by defining a base scenario. This scenario and all its descendents have two different scenarios possible to
Figure 2: A three period binomial lattice with base scenario $\omega_0$. The probability of moving with up a branch is $p$ and moving down is $1-p$. By definition, the scenario $\omega_t$ is an element of $\Omega_t$ for all $t > 0$.

follow. The result of this process is illustrated in Figure 4.1. Let $S(0)$ be the price at the time $t_0$, $p$ the probability of the price going up, $0 < d < 1$ the relative price change when the price is going down and $1 < u < \infty$ the relative price change when the price is going up. These parameters define a binomial lattice.

Let the planning horizon be $t = 0, \ldots, T$. Let $x(t) = d$ if the price goes down and $x(t) = u$ if the price goes up at time $t$. Then the state of the system at time $t$ is described by the vector $\mathbf{x}^t = (x(1), \ldots, x(t))$. Let $I = I(\mathbf{x}^t) = |\{i|x(i) = u\}|$ and $J = J(\mathbf{x}^t) = |\{j|x(j) = d\}|$, where $|.|$ denotes the number of elements in a set. The probability of a state can be calculated as

$$P(\mathbf{x}^t) = p^I(1-p)^J$$

and the price as

$$S(\mathbf{x}^t) = u^I d^J S(0).$$
At time $t$ there can be $t+1$ different prices. The probability of the states $X_{ij} = \{x^t | I = i, J = j \}$ can be calculated with

$$P(X_{ij}) = p^i(1-p)^j \frac{t!}{I!J!}.$$ 

These probabilities coincide with scenario probabilities, that is, if in a scenario $\omega_t$ the price has gone up $i$ times and down $j$ times ($i + j = t$), then probability of this scenario is $P(\omega_t) = P(X_{ij})$. The price in that scenario is

$$S(\omega_t) = u^i d^j S(0).$$

Thus, the prices in scenarios are path independent. However, the strategies involved with a scenario are path dependent, because the gained resources depend on the scenario path. For instance, in a scenario $\omega_2$ if the price went first up in $\omega_1$ and the mine owner sold all of the gold, he would have more money than if the price were to decline. The wealth level in scenario $\omega_2$ influences the future actions, because having a lower wealth level increases the risk of having a wealth level lower than required by the $CCFAR$-limit. Due to the risk limit, the owner may choose to operate differently depending on the price of gold in the previous scenarios.

### 4.2 Parameter Estimation

The model parameters (cf. Table 1) need to be determined or estimated. The terminal time reflects the time period that the parameters of the model may be regarded stable enough. It may also reflect the time the investor is willing to bind her resources to the project.

The risk free interest rate defines the time value of money and a return requirement for capital invested. The rate may be approximated from security interest rates, such as U.S. treasury bills, or rate indices, such as EURIBOR. In case of a large company, also measures such as Weighted Average Cost of Capital (WACC) may be used.

The parameters for the price process, which imitate the market, is defined by a binomial lattice. Its key properties are the expected market growth $\nu$
and the variance of the growth $\sigma_2$, that (with certain assumptions) define the prices in the lattice. For the estimation of these parameters, we refer to Luenberger (1998).
Table 3: Information on the gold mines of a mining company in the example case

5 Illustrative Example

A gold mining company possesses three mines according to Table 3. The current price for gold is 10.8 €/g and the price is assumed to evolve as described by the binomial lattice $B$, where $p = 0.55$, $u = 1.17$ and $d = 1/u$. The volatility for the price of gold implied by these parameters is large compared to recent observations (which is around 13 percent of annualised daily standard deviation, Chicago Board of Trade, 2007), but is used for demonstration purposes. The expected price of gold at the end of the period is 12.41 €. These high volatility values are used here for demonstration purposes. The license fee is $c_L = 130000$ €, initial cash is $C_0 = 200000$ € and the risk free interest rate is $r_f = 0.03$.

The company faces future liabilities in five years to the amount of 150000 € (thus giving a planning horizon $t = 0, \ldots, 5$). Despite the expected upward price development, the company wishes to be able to pay with a 95 % confidence interval. Therefore a $CCFAR_{0.90}$ limit is imposed such that if the reference index is $C_{ref} = C_0(\omega_0)(1 + r_f)$, then $C_{ref} - CCFAR \geq 150000 \Rightarrow CCFAR \leq C_{ref} - 150000$. In this case $CCFAR \leq 82000$.

The mines have a liquidation value described by the factor $r_L = 0.45$ of the amount of unexploited gold in the mine. This value describes the future value of the mine to the owner, as otherwise the model would have to suggest the relinquishing of the license at the end of the planning horizon. In a more sophisticated evaluation, the liquidation value should be calculated in greater detail. If the license expires at the end of the planning horizon,
the liquidation value should be zero.

Based on these parameters the optimal strategy yields\(^2\) an expected cash position of 2.714 M\(\text{€}\) and an expected cash of 588000\(\text{€}\). *CCFAR* is 72000\(\text{€}\) and expected downside risk is 2.1 M\(\text{€}\). Thus after the liabilities the company is expected to have 438000\(\text{€}\) for future investments and it is expected to be able to handle it’s liabilities in case of a tail event. That is, the company’s expected income in case of a tail event matches the company’s liabilities and given the 90 % \(\beta\)-confidence level, an ability to pay in 95 % of all possible outcomes is implied.

The optimal operating strategy for the mines can be found in Figure 3 through Figure 5. Clearly, mine 3 is the best one as it is operated in most scenarios. This is due to the low excavation cost combined with a large initial deposit, which contributes to the total value in form of liquidation value. Mine 2 is slightly better than mine 1, since due to the lower excavation cost mine 2 is profitable even when prices are low.

The optimal investment strategy in raw material can be found in Figure 6. Mining activities are profitable under these settings, as investments to the market are low. Only when the price drops enough is the operating and license cost enough to account for the risk premium and making investing beneficial.

Note, that *CCFAR* is below the required 82000\(\text{€}\) indicating, that the risk constraint is not active in this setting. Because there is an upward trend in gold prices the company decides to evade risks even at high prices. Therefore it sets it confidence level to \(\beta = 0.99\). This results in a lower expected terminal cash position of 2.706 M\(\text{€}\) as the *CCFAR* constraint becomes active. On the other hand, as indicated by the small loss in expected terminal cash position, imposing a stricter risk constraint does not have a large impact on the optimal strategy. This is due to the fact, that the circumstance is very beneficial for the mine owner, as prices are expected to rise and the

\(^2\)The computation was performed with Xpress by Dashoptimization. The model was modified from an outline made by Janne Kettunen.
mining activity is profitable to start with. At the level of individual mines, the only changes occur to the management strategy for mine 2 (see Figure 7). However, with other initial parameters the impact could be more significant.

6 Conclusions

This study presents an illustrative example of modeling a mining venture through a stochastic programming model, namely CPP. The model is given a context in an illustrative example, where some of the model features are presented. These features include portfolio approach, strategy path dependence and risk management through the risk measure CCFAR.

CPP delivers a flexible framework for large scale portfolio choosing problems including several uncertainties with time-varying correlations. The model is flexible in that it could easily model more different types of uncertainties and properties of a mine. A potential use could be found in the mining industry, with a possible implementation presented in this study. The risk measure CCFAR was used in order to capture the DM’s risk attitude in a portfolio choosing problem.

This study has not considered issues of computational efficiency. Models including tree structures grow rapidly, that is the computation time grows exponentially with regard to the number of periods and parameters. This could be a limiting factor in choosing of a model. However, with few uncertainties and relatively few periods, the framework will produce computationally feasible problems even for hundreds of mines.

This study has relevance in exploring the combination of stochastic programming and modern portfolio theory. It was, however, harder than anticipated to juxtapose these aspects. A more insightful comparison would result from treating these alternatives (mines, market trading and risk free rent) as separate stocks than define a market line in the sense of CAPM. Then, the \( \alpha \) and \( \beta \) of each alternative could describe the investment and the effect of the CVAR constraint in terms of CAPM could be described.
Figure 3: Strategy X for mine 1. Up branches correspond to prices going up and down branches to prices going down. Underlining indicates an active license. Time $t = 0, \ldots, 5$ is column-wise left to right.
Figure 4: Strategy $X$ for mine 2. Up branches correspond to prices going up and down branches to prices going down. Underlining indicates an active license. Time $t = 0, \ldots, 5$ is column-wise left to right.
Figure 5: Strategy $X$ for mine 3. Up branches correspond to prices going up and down branches to prices going down. Underlining indicates an active license. Time $t = 0, \ldots, 5$ is column-wise left to right.
Figure 6: Raw material investment plan $Z$. Up branches correspond to prices going up and down branches to prices going down. Time $t = 0, \ldots, 5$ is column-wise left to right.
Figure 7: Strategy X for mine 2 with extreme risk aversion. Up branches correspond to prices going up and down branches to prices going down. Underlining indicates an active license. Time \( t = 0, \ldots, 5 \) is column-wise left to right. Triangles indicate changes due to extreme risk aversion.
References


