Timo Pekkala, 63016P

Value at Risk in Bank Risk Management

Mat-2.4108 Independent Research Projects in Applied Mathematics

Helsinki, June 6, 2010
## Contents

1 Introduction  

2 Value at Risk Methodology  
   2.1 A Case for Holistic Risk Management  
   2.2 Quantifying Tail Events  
   2.3 Practical Implementations of Value at Risk  
   2.4 Common Criticism  
   2.5 Improving on the RiskMetrics Value at Risk  
      2.5.1 Accurate Market Models  
      2.5.2 Sophisticated Metrics  
      2.5.3 Computational Efficiency  

3 Performance and Role of Value at Risk in Banks  
   3.1 Goodness of the Value at Risk Figures  
   3.2 Benchmarking Value at Risk  
   3.3 Value at Risk Deficiencies  

4 Conclusions and Future Outlook
1 Introduction

This research project discusses a popular risk measurement methodology known as Value at Risk and its use in bank risk management. Value at Risk is an approach to risk management that gained popularity rapidly as the method was introduced and formalized by RiskMetrics in the middle of the 1990’s. The methodology is introduced here in the extent that is necessary in order to assess its the suitability and performance in risk management.

An emphasis is placed on assessing the method’s suitability for bank risk management. Its strengths are introduced and a fair account of publicly presented criticism is given. We also take note of how VaR methods have withstood the recent financial crisis.

2 Value at Risk Methodology

Value at Risk (VaR) methodology aims to quantify the level of the worst case outcomes in a situation where the future is uncertain. VaR is defined as a threshold value that the losses should not exceed in a given time period and a given confidence level. This principle is laid out in figure 1. The figure represents schematically the distribution of possible outcomes of a stochastic process. The VaR threshold value limits the left tail of the distribution. The outcome of the process hits the grayed out portion only in 5% of cases. This general approach can be applied to uncertain returns of financial assets as well as to different physical processes whose risks need to be quantified.

The definition of the VaR measure is theoretical in that VaR is agnostic to the underlying process. One can calculate a VaR value for many different things: a manufacturing company might assess its operations by estimating VaR of the yield of an important key process, for instance. Employing VaR requires a certain degree of stochastic modeling regarding the underlying process. Modeling complex real-world phenomena can therefore be a daunting task. In finance, however, the processes are often simpler, data is readily available and it is relatively easy to analyze. It is therefore natural that VaR methods
have found wide adoption in the banking and finance industry, and in risk management departments of those organizations in particular.

As for using VaR in finance, many approaches exist. In this study we concentrate on methods used to determine VaR for the trading book of a commercial bank. Use of VaR in the financial industry has its roots in the developments of the 20th century. VaR can be seen as a logical continuation of the growing complexity of financial instruments and the tightening regulatory environment of the last century.

Damodaran (2008) gives a short history of VaR noting that “the impetus for the use of VaR measures [...] came from the crises that beset financial service firms over time and the regulatory responses to these crises”. Damodaran has an emphasis on the US regulatory environment. The first of the crises is said to be the Great Depression, during which bank failures led to regulations limiting the level of borrowing in terms of the banks equity capital. The next step of the development of the current regulatory environment came through in the 1970’s, when exchange rates were allowed to float and the derivatives market started to operate in a significant scale. This led to a refinement of capital requirements by dividing financial assets into different classes, each with different capital requirements. Different asset classes would from then on require different risk management practices.

Damodaran places the first VaR-like regulatory measure to be 1980’s, when “the [Securities Exchange Commission (SEC)] tied the capital requirements...
of financial service firms to the losses that would be incurred [] with 95% confidence over a thirty-day interval”. According to the author, this approach spurred the development of bank internal measurement techniques that resembled that introduced by the SEC. The term Value at Risk was not used to describe these techniques. Not until 1995, when J.P. Morgan published their internal results of analyses “across various security and asset classes”. They coined the term Value at Risk to describe the new metric. The method was widely accepted, in part because of the collapse of the British investment bank Barings (Damodaran, 2008; Wikipedia, 2010a). The failure of Barings made the need for more effective risk management measures even more apparent.

2.1 A Case for Holistic Risk Management

The growing trading book of banks and the ever-growing use of derivative products in both commercial banks and investment banks has been the prominent trend in the 1990’s and in the 21st century. The case of Long Term Capital Management (LTCM) is indicative of this trend in that it tells the tale of a hedge fund’s use of aggressive trading strategies in an environment that lacks sufficient regulation. The consequences shook the whole industry and were a catalyst for even wider penetration of the VaR model in the financial industry.

LTCM was a hedge fund founded in 1993 by John Meriwether and a team of prestigious partners. The funds goal was to seek funds from few high net worth individuals and institutions, which enabled it to operate without much financial regulation and to use very aggressive trading strategies to achieve high returns.

The fund did amass capital quickly. Its trading strategies included different sorts of arbitrage trading and investment in developing regions. Specifically the fund sought to take advantage of arbitrage opportunities between certain equity and fixed income pairs by taking both long and short positions and betting on the future development of the correlation of that pair.
Such strategies did indeed generate a hefty profit in the first years of the funds operations. In the late 1990’s the fund started making losses, which was aggravated by the Russian Government bonds defaulting in 1998. In order to survive these initial losses LTCM had to liquidate important positions, thus further worsening the situation. This liquidation spree demolished several of the betting strategies the fund had built on certain asset pairs.

The situation LTCM had gotten itself into was severe from the standpoint of the total industry. Therefore the fund was bailed out by other prominent industry participants and the Federal Reserve Bank of New York in order to avoid a total melt down of the market possibly induced by the fund liquidating all of its assets. At the end, the new shareholders of the fund were successful at liquidating the fund over a longer period of time with minimal losses.

The case of LTCM shows the potentially detrimental effect of betting on the dynamics of the market — and on the correlations of assets in particular. The expected returns from these kinds of strategies are minuscule in relation to the potential losses. Quantifying the risks associated with such strategies properly could have made the situation more apparent to all stakeholders involved with LTCM, and the market at whole. And indeed, VaR models were seen as an improvement after the demise of LTCM and they gained in popularity quickly (Damodaran, 2008).

The problems LTCM caused were severe on a industry level scale. The losses would not have harmed the shareholders alone, but could have potentially caused havoc in Wall Street. That, although no regulation was imposed on a fund operating with the clientele LTCM was operating with. The failure of the fund led eventually to SEC mandating banks to use their own VaR models in assessing the banks’ capital requirements.

### 2.2 Quantifying Tail Events

As noted in the previous section, VaR gives a value for a certain quantile of the distribution that is being investigated. This section specifies some
methods that are commonly used to achieve this.

2.3 Practical Implementations of Value at Risk

The definition of VaR is indifferent in terms of the underlying market model or means of calculation. There are many different approaches to calculating the threshold value that defines the chosen quantile. These methods differ in terms of whether an empirical or a theoretical distribution is used, and on how the underlying measure is calculated.

Let us now concentrate on the value of portfolio of financial assets for the remainder of this study. More accurately, let the return of that portfolio be the value of interest. The most basic approach is to analyze the historical values of the returns. Say, take the 1,000 most recent daily returns and determine the 99% VaR over a one day period. This task is taken care by simply putting the daily return observations in order and choosing the 990th value.

This approach completely discards the possibility of the underlying portfolio changing as well as assumes the market to remain unchanged over the the whole time period. To improve this measure VaR calculations are usually brought to the level of the different market factors affecting the value of the portfolio. The analysis is then separated into two parts: One part involves defining and measuring the value distribution of the underlying market factors. The second part of the analysis comprises of valuing the portfolio in terms of those varying market factors.

All practical VaR methods that are in use today rely on the work of J.P. Morgan and their RiskMetrics initiative of 1993-1995 and the associated publicly available documentation (Morgan, 1996). The most useful part of RiskMetrics' work is the set of cross asset class variances and covariances (Damodaran, 2008). In conjunction with the described methods these covariances allow any interested party to implement VaR in any organization. The original RiskMetrics VaR has led to different variations in terms of how the risk figure is calculated. Three widely user VaR methodologies and their use in the indus-
try are described in (Morgan, 1996) and their use in the industry is analyzed by Pearson and Smithson (2002), for example. These three categories are Historical Simulation, Monte Carlo Simulation and Analytic Delta-Gamma method. These methods along with their key characteristics are summarized in the upper part of table 1.

<table>
<thead>
<tr>
<th></th>
<th>Value change estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Analytical</td>
</tr>
<tr>
<td>Full VaR</td>
<td>RiskMetrics Covariance matrices and instrument mappings.</td>
</tr>
<tr>
<td></td>
<td>Historical simulation Not applicable.</td>
</tr>
<tr>
<td></td>
<td>Monte Carlo Not applicable.</td>
</tr>
<tr>
<td>Partial VaR model</td>
<td>Implied Volatilities Covariance matrices and instrument mappings.</td>
</tr>
<tr>
<td></td>
<td>User defined Sensitivity analysis on single instruments.</td>
</tr>
</tbody>
</table>

Table 1: RiskMetrics analysis of different VaR methodologies (Morgan, 1996).

The name of the analytical Delta-Gamma method comes from the valuation method used in the analysis. Here each position is characterized by sensitivities to different market factors. These sensitivities are denoted with Greek letters, out of which delta and gamma represent first and second order derivatives in respect to the underlying. In conjunction with the sensitivity characterization of the portfolio a covariance matrix of the market factors is used. The variance-covariance structure incorporates all necessary information about the market dynamics. This method was formally introduced by RiskMetrics in 1994 with extensive analysis on how the covariances should be estimated. Thus, this approach is often associated with RiskMetrics.

In historical simulation the distribution of returns is estimated by revaluing the current portfolio with historical changes in the market factors. The time
window of the historical returns is the guiding parameter when using this model. Full valuation is often used in conjunction with historical simulation (Morgan, 1996), but using a delta-gamma approach is also possible.

The Monte Carlo simulation model is the most accurate method, though it is also the most computationally expensive one. It relies on an accurate model of the underlying market factors. The market model is often estimated in very much the same way as in the original RiskMetrics covariance approach. The original RiskMetrics method involves estimating the variances and covariances of the factors by using an exponentially weighted moving average (EWMA) with the decay factor of $\lambda = 0.94$. The market model is then used to create market scenarios for the Monte Carlo simulation.

The next step in the Monte Carlo approach is to value the portfolio in the different market scenarios. Typically a full valuation is preferred, as it leads to very accurate results with option positions that can be valued analytically. A full valuation is also very easy to execute given that suitable valuation formulas are available. Instruments that require simulation for accurate valuation are not an ideal subject, as the computational burden becomes obtrusively high.

Jorion (2007) discusses the advantages of the Monte Carlo approaches. First of all, the market model can be defined in a very flexible fashion, incorporating “time variation in volatility or in expected returns, fat tails, and extreme scenarios”. The passage of time can also be taken into account.

2.4 Common Criticism

The RiskMetrics VaR was faced with an eager crowd of risk managers when it was introduced. The method found rapid adoption in many applications relatively quickly. The ready made concept and the easily understandable interpretation of the results made adoption easy. The apparent simplicity of VaR makes it easy for managers and executives grow reliant on the method and form overly optimistic expectations for it. The popular concept of Value at Risk has not always faced up to the task, and it has therefore drawn lots
One of the most obvious points of criticism is directed at the choice of the market model. Market variables are often modeled using some well known parametric distribution, such as the normal distribution or the t-distribution. Often these distributions fit quite well. There is however a common problem with fitting these distributions. Typically the observed return distributions exhibit what is called a ‘fat tail’. That is, there are more extreme observations than the distribution assumptions would allow for. This problem is often accentuated by the fact that there are relatively few observations in the tails, thus making it difficult to fit suitable distribution candidates.

The problem of fat tails is depicted in figure 2. The figure shows a histogram of the daily returns of the FTSE100 index in 2005-2009. A maximum-likelihood fit is made for both a normal distribution and a t-distribution with 5 degrees of freedom. The normal distribution is visually a poor fit. The t-distribution assumption allows for a better fit, but it does not tackle the fat tails properly either.
The notion of ‘fat tails’ introduced above is a very practical one. Taleb is a well known critic against the over-reliance on VaR and he has discussed ‘fat tails’ more comprehensively in (Taleb, 2007) and puts the problem in a wider context. He recognizes that the most dangerous outcomes are those that have never been experienced and that can not be modeled using common knowledge. In his view modeling the market using distributional assumptions, such as normality, exposes us to even larger risks. He calls such unexpected events Black Swans.

One line of criticism concentrates on the use of VaR as a tool for decision making and as a central guiding framework. VaR concentrates inherently to the lower tail of the return distribution, and thus conveys only a small part of the information about the process being looked at. For a trader, for instance, one figure that describes an arbitrary quantile of the tail of the distribution holds little value when making investment decisions. The effect VaR has in that situation is that the dealer is encouraged to seek high returns by taking risks that fall beneath the VaR threshold but could have catastrophic consequences in the rare cases the tail events do happen. Einhorn (2008) argues this and states provocatively: “This is like an airbag that works all the time, except when you have a car accident.”

More criticism against the current VaR risk management regime is articulated by Nocera (2.1.2009) of The New York Times. Nocera discusses the role of VaR in the financial crisis that began in 2007 in the US. He states that one central failing point of bank internal VaR models is the skewed way they communicate risks to management. At the same time bank were allowed to set their capital requirements according to their own VaR models. Due to the nature of VaR, these models were not able to quantify the effect of an industry wide meltdown. The false feeling of certainty coupled with the blessing of the regulatory bodies of extensive usage of VaR in assessing the required level of capital were in large part responsible for the crisis.
2.5 Improving on the RiskMetrics Value at Risk

The basic approaches to VaR calculation presented in the previous section represent the most important branches of VaR methodologies. They are applicable in many general cases. There are however more sophisticated methods that for the most part rely on the basic ideas of historical simulation, Monte Carlo simulation and analytical delta-gamma approximations.

2.5.1 Accurate Market Models

A large part of VaR criticism is directed at the assumptions made in constructing the market model. These problems are especially pronounced when VaR is calculated using parametric models such as Monte Carlo simulation and delta-gamma VaR. It is therefore beneficial to fine tune the model of the dynamics of the market variables as much as is possible given the challenges in estimating necessary parameters and processing capacity.

Extreme value theory (EVT) is a theory of statistics that deals with estimating the characteristics of rare events in the tail of a distribution (Wikipedia, 2010b). EVT “describes the limiting behavior of the maximum of a sequence of random variables” and gives the form of the extreme tail distributions (Pearson and Smithson, 2002). The method has been used in modeling univariate credit risk, where the outcome is dependent on a single variable and tail events are extremely rare. There is little experience on using EVT on portfolios that involve multiple risk factors.

One possibility to improve the accuracy of the market model is to incorporate time series models into the market model. Modeling the time dependency of risk factor volatilities can improve results especially when VaR is calculated over a longer holding period. This is a difficult task, though. When using a parametric VaR method, managing the time varying covariance matrix may prove difficult.
2.5.2 Sophisticated Metrics

The VaR measure is a simple threshold value specifying a certain quantile of the return distribution. It specifies the level of losses that is exceeded only in a small proportion of cases. However, it says nothing about the level of losses that occur in those few cases the threshold level is exceeded. One metric designed to characterize the nature of the distribution tail is the expected shortfall or expected tail loss (ETL) method. In essence, ETL gives the expected level of losses conditional to exceeding the VaR level. That is, ETL is the average of losses in the tail beyond the VaR level.

This measure reveals more information about the tail events, thus tackling in part the problem of fat tails. ETL is also a more convenient measure in that ETL is subadditive whereas VaR is not. Subadditivity means that ETL always gives a lower risk for a combined portfolio of two separate portfolios than what would be the sum of the risks of those portfolios. VaR might give a larger risk for the combined portfolio. Since VaR is a popular method of risk aggregation, it is natural to extend the method to analysis of the components of the total aggregate risk. Common extensions to VaR results include the component VaR (cVaR) and the marginal VaR. cVaR is defined as the change of the VaR value due to removing a certain sub-portfolio from the total portfolio. As VaR is not an additive measure, cVaR gives valuable information about diversification effects in the portfolio. Marginal VaR is in essence the first order derivative of the portfolio in respect to a certain sub-portfolio.

2.5.3 Computational Efficiency

Pearson and Smithson (2002) present commonly used methods for speeding up the Value at Risk calculations.

First of all, the Monte Carlo method can be sped up considerably by using sensitivity approximations to calculate value changes instead of full valuation. The accuracy of this method is often improved by taking into account
the price sensitivity in respect to time to maturity — the theta. The delta-
gamma-theta method permits for an accurate simulation of the market dy-
namics, but suffers from the obvious shortcomings of using an approximate
valuation model.

Another way to improve the efficiency of the Monte Carlo method is to reduce
the amount of valuations. One could divide the task into two sets in terms of
the market factors. One set comprises of those values of the market factors
that are critical for valuation. Valuation in that range of market factors is
done properly in full. For the other set of factor values a delta-gamma-like
approximation will suffice.

Another approach introduced is that of Frye (1998). Frye uses principal
component analysis (PCA) to reduce the dynamics of the market factors to
a smaller set of factors. For the relatively low number of artificial factors a set
of typical values is chosen. The portfolio is then revalued at these nodes of the
grid of chosen factor values. In the simulation step portfolio values between
these predefined states get interpolated using the pre-calculated values. A
similar method involves estimating the distributions of the market factors
and reducing their number by PCA to build market scenarios in accordance
with their distribution characteristics. Portfolio values are then calculated
for these scenarios. Monte Carlo sampling is then performed on this set of
predefined portfolio values.

3 Performance and Role of Value at Risk in Banks

The performance of bank VaR risk management tools has not been studied
very thoroughly. The technical details of different implementation ap-
proaches have been researched more eagerly. This is probably due to the
fact that bank internal figures are hard to come by. The subject is however
studied by Berkowitz and O’Brien (2002). They analyze the performance of
VaR estimates of six large multinational US banks against their historical
trading profit and loss (P&L). All six banks “meet the Basle ‘large trader’
criterion” and maintain P&L and VaR time series “to assess compliance with
the Basle market risk capital requirements”. The measure under inspection
is the 99% Value at Risk for a one-day holding period.

3.1 Goodness of the Value at Risk Figures

Berkowitz and O’Brien use several methods in their analysis. First, the VaR
forecast is scrutinized by testing whether the 99% coverage rate is a valid
hypothesis. Second, the performance of the VaR forecast is compared with
a simpler reduced model of P&L. A combination of GARCH and a simple
ARMA model is used to model the P&L time series of each bank. A 99%
VaR estimate is derived using that model and the resulting figure is compared
with the bank VaR models.

By investigating the observed P&Ls and the VaR estimates the authors con-
clude that the bank VaR estimates are conservative, in that the VaR thresh-
old lies considerably lower than the 99th percentile of the historical P&L
distribution. However, performance varies between different banks and some
banks’ VaR measures. For some banks the VaR figure trails more closely
the corresponding percentile of the empirical P&L distribution. It is also ob-
served that the losses exceeding the VaR threshold exhibit abnormally large
values. This is an indication of fat tails of the return distribution, and as
the authors point out, is in part explained by an abnormally volatile market
period that is included in the estimation period.

The authors point out a fairly obvious character of the P&Ls of different
banks during the volatile market periods: They find a clear correlation be-
tween the losses of different banks, and note that this could be of concern
for the regulators overseeing the overall stability of the banking system. The
strength of this effect is however affected by the choice of the estimation
period as well as by whether the correlations are calculated over a one-day
or a 5-day holding period. Moreover, when looking at the VaR figures, no
significant correlation patterns can be found.
Further on Berkowitz and O’Brien use statistical methods to “compare the targeted violation rate [...] to the observation rate”. They find that four out of the six banks produce VaR figures that comply with the hypothesis that the coverage rate be 1%, that is the VaR threshold is exceeded in only 1% of cases. The two rejections are explained are caused by the frequency of violations being less than 1%. This indicates that the VaR measures of these banks are overly conservative.

3.2 Benchmarking Value at Risk

Looking at the bank VaR figures in isolation is not sufficient to assess their performance. Benchmarking the numbers against separate models will lead to better results. Berkowitz and O’Brien introduce a benchmark model with the notion that of a possible deficiency of the bank VaR models. Two banks show a statistically significant level of autocorrelation. That is, “given a violation on one day there is a high probability of a violation the next day”. So these models show signs of clustering of the VaR threshold violations. To investigate this further Berkowitz and O’Brien compare the bank VaR figures to a combined model of an ARMA(1,1) process and a GARCH(1,1) process. The objective is to try to capture time dependencies in the volatility of the return series.

The time-series model of the P&Ls performs very well. It tracks the 99th percentile of the empirical P&L series better than the bank VaR models, primarily because the model takes directly into account the changes in volatility. The autocorrelation observed in the VaR time series is not present The extent to which the bank VaR models implement this feature is unknown, but clearly their performance in this respect is not up to par. The time series models perform also better in that they produced VaR levels are lower than those of the bank VaR models, that is they do not seem to be overly conservative. At the same time they “achieve the targeted violation rate and a 99th percentile VaR coverage”. This makes the time series model very attractive, because the lower VaR levels lead to lower capital requirements while the quality of the model remains at an acceptable level. The authors do point
out one phenomenon the time series model can not adjust itself to: the results for the model vary to some extent depending on the chosen estimation period. It seems the market experiences a regime shift of some sort, which the model is not able to take into account.

3.3 Value at Risk Deficiencies

The differences between the bank VaR results in (Berkowitz and O’Brien, 2002) and the reduced-form time series model lead the authors to lay out possible explanations for this disparity. First, the banks examined are large corporations with huge trading portfolios. The size of the portfolio forces the VaR models to rely on many approximations considering the market factors and the valuation of the portfolio. Second, the authors recognize the difficulty of taking into account the changing nature of the market factor volatilities. “[...] Accounting for effects in the covariance matrix of a large set of market risks may be an insurmountable task”. Third, bank VaR models do not typically take into account client fees. Those fees are however incorporated in the analysis of the empirical P&Ls. This does make the VaR figures seem more conservative, as the client fees improve the 99th percentile of the empirical P&L distribution.

Fourth, banks do not necessarily employ the VaR model in a consistent manner across all the different positions and asset classes. VaR of different assets can be calculated using different models, and the figures would then be aggregated using over-simplistic assumptions such as correlations of one or zero across asset classes. According to the authors, some regulatory identities in fact require that sub-group VaR figures be aggregated by directly summing the sub-VaR figures.

Finally, the level of sophistication of the VaR implementations within the different banks seems to play a key role. The comparison of the six chosen banks seems to support this claim. “To date, the experience of examiners has been that building a satisfactory market risk model on a large scale is a long and arduous process.”
4 Conclusions and Future Outlook

We have now covered the basics of the Value at Risk methodology and introduced some aspects of its applicability in bank risk management. VaR is an intuitive and appealing way to aggregate risks in a portfolio or a whole organization and it does reveal important information about the nature of the position. It performs fairly well in stable conditions and gives a reasonably good picture of the risks assuming the world is not going to change too much. VaR critics point out, however, that the model is inherently unable to foresee the most catastrophic events. VaR does not take into account the rarest of events, although it is supposed to convey information about exactly that low end tail. This is a serious flaw of the model, which is sometimes countered by incorporating separate stress test in risk analyses.

However, regulatory requirements are not the only reason VaR has gained such popularity in the industry. Accurate measurement of risk is a key component of allocating capital within a bank, as Pyle (1997) notes. “Managers need reliable risk measures to direct capital to activities with the best risk/reward ratios.” This is crucial to managing the the bank efficiently for the benefit of the shareholders and other stakeholders.

It remains to be seen, whether regulation on the use of bank internal VaR models in determining the capital requirements will tighten. Before the recent financial crisis big commercial bank were able to use their own tailor made models for capital allocation provided that they opened their books to regulators in a certain extent. Political pressure in the US will most likely drive changes in the legislation and lead to tighter regulation.
REFERENCES

References


