Forward Contract Hedging with Contingent Portfolio Programming

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1 Introduction

The purpose of this paper is to demonstrate how Contingent Portfolio Programming (CPP) can be used in management of the forward contract portfolio when the underlining asset is a perishable commodity, such as electricity, which cannot be stored to be used later. The decision problem is to choose an optimal portfolio of forward contracts for the delivery of commodity that has two unknown variables, the spot price and the level of demand. The model computes the optimal levels of forward contracts to be bought in order to meet all the demand. The computed solution can also be used as a step by step guide for choosing the optimal amount of hedging for the remaining time periods.

Gustafsson and Salo (2005) introduced CPP which is applied in this paper in the forward contract management. CPP is a model that allows decision problems to be constructed into a state tree which can be solved with linear programming tools.

The paper presents the theoretical model and applies it on numerical examples. The numerical examples demonstrate how a problem of reasonable size can be solved with the model. From the examples we can also see how the model takes into account the decision maker’s risk preferences by penalizing for variability in the expected return. The model has some restrictions due to our assumptions. This paper tells some ways to enhance the model and provides some directions for future research. The paper highlights some important areas in order to develop applications based on the model.

The paper is structured as follows. Chapter 2 introduces the forward contracts. Chapter 3 introduces the CPP model in brief and presents the forward contract management problem and present the way how CPP can be applied for it. Chapter 4 consists of numerical examples and chapter 5 concludes with a discussion.
2 Forward Contracts

Forward contracts on commodity are defined as contracts to purchase or sell given amount of the commodity at specific time and at specific price (Luenberger, 1998). Forward contracts are specified by legal documents which bind the contractors on the transaction in future. The value of the forward contract in electricity market from one period to succeeding period is assumed to be the probability adjusted average of spot prices in the succeeding scenarios as electricity is non storable. Thus, forward contract value $P(t,t')$ in period $t$ for delivery in period $t'$ ($t'>t$) is

$$P(t,t') = \sum_{i=1}^{n} p_i \cdot S_i,$$

(1)

where $S_i$ is the spot price in scenario $i$ in period $t'$, $p_i$ is the probability of that spot price, and $n$ is number of possible scenarios in period $t'$. Forward contracts are used when the decision maker (DM) wants to minimize the risk either on the availability of a commodity or on the price of that commodity in the future. This paper focuses on the latter, as our model requires the assumption that all demand is met. If we had to consider the availability of the commodity, we might have to face a problem of not being able to meet the demand in all situations. Combining these two assumptions, that all demand must be met and that the availability of the commodity could be restricted, would mean that all demand must be hedged for the final period. This is because there might not be available commodity to buy at the spot price, and thus the optimization problem would be to optimize the hedging for the final period (without the possibility to increase the supply with buying at spot price). It might also lead to restrictions in available forward contracts.

Basics on hedging with futures/forwards can be learnt from Brealey and Myers (2003). However, they do not discuss the hedging of perishable commodities as they do not see simple link between electricity’s spot price and its future price (which is the case on many other commodities). But on some cases, as is in the Nordpool electricity market, expectations theory can be used to estimate the becoming spot price from existing future prices (see Luenberger, 1998, for more information on expectations theory).
This paper assumes that the forward price can be derived from possible future spot prices using the equation (1). We assume that at each moment the commodity can be purchased in any size of units both at spot price and at forward contract price. We also assume that we are talking only about perishable commodities that can not be purchased beforehand and then stored to meet later demand. Perishable commodities differ from normal commodities as they can not be stored, which means that these commodities must be bought at the moment they are needed or with a previously signed forward contract that delivers the commodity at that specified moment it is needed. Further we assume that if the commodity is not used or sold prior to expire period, it is wasted (the forward contracts can not be sold at their expiration period because there would be issues relating to finding buyer for it in instant and if periods would be longer then there would be issues on pricing the forward contract if some of it has already expired). We allow selling of forward contracts at any other period (except expiration) on the price that is equal to similar forward in that period. For example, if we have bought forwards in period 0 to be delivered in period 2, we can sell them in period 1 at a price that is equal to price of a forward from period 1 to period 2. If the price would be different, there would be an arbitrage.

3 Contingent Portfolio Programming and How to Use It in Hedging Forward Contracts

3.1 Contingent Portfolio Programming

The CPP model is defined by resource types, the state tree and project-specific decision trees (Gustafsson and Salo, 2005). Resources are inputs and outputs that are consumed or produced by the projects. They can be production factors (money, labour) or intangibles (intellectual property). State tree represents the time-state model. Time horizon consists of \([0,\ldots,T]\) periods and in each period there is a set of possible states. The state tree starts with only one state in the first period. The next period has already more states. The predecessor in each state can be defined recursively, thus if we know the beginning state and the end state we can define each state in the between. The state tree is formed based on uncertain events and their probabilities. Consequently, the conditional
probability of a certain state can be computed recursively using the conditional probabilities. Projects are integrated into the CPP via decision sequences. A decision sequence consists of decision opportunities. Thus can be formed a decision tree which consists of the decision points. In each decision point the DM has knowledge about the previous scenarios and the actions taken before with regard to project. It is assumed that the decision points form a consistent tree so that each decision point has a unique collection of preceding decision points.

A project management strategy is defined by action variables associated with decision points in certain project. A portfolio management strategy is the DMs complete plan of action for all projects in all states. There is resource flow of certain resources induced by the project management strategy. Actions influence the resource flow only in the current state (that the action is taken) and relevant future states. The aggregate resource flow in certain state is obtained by adding the resource flows of all projects. CPP consists of four set of constraints: decision consistency constraints, resource constraints, optional constraints and deviation constraints. The CPP framework assumes that the initial resources are known and that as actions are taken resources are either consumed or produced. The future amount of resources can be thus computed in CPP at each point as well as the resources in the terminal states. The CPP model also takes into account the risk profile of the DM and provides the optimization problem the path that the DM should follow at each decision point. With the use of CPP project portfolio selection problem can be presented as a linear programming model. The solution of the model suggests optimum decisions in each state. In Table 1 we have example probabilities for a scenario tree. The probabilities for the demand and price to move up or down independently is 0.5 for both and thus the probability for certain state to follow is 0.25.

Table 1. Example scenario

<table>
<thead>
<tr>
<th></th>
<th>Demand</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Up</td>
<td>Down</td>
</tr>
<tr>
<td>Price</td>
<td>Up</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>Down</td>
<td>0.25</td>
</tr>
</tbody>
</table>
Figure 1 demonstrates this scenario tree for three time periods $T = 0,1,2$. From figure 1 can easily be seen the uniqueness of each state and its proceeding states.

![Example state tree](image)

**Figure 1.** Example state tree

### 3.2 Forward contract hedging with CPP

In this section the paper introduces the principles which then formulate forward contract management with CPP. This section is based on a discussion paper by Salo (2005). The decision problem is to choose an optimal portfolio for forward contracts for the delivery of commodity, subject to uncertainty about (i) the spot price and (ii) the level of demand. The Planning horizon consists of $t = 0,\ldots,T$ periods. The spot price is assumed to follow a binomial lattice where the probability for going up is $p_i^+$ and the probability for going down $p_i^-$ subject to the normalization condition requirement $p_i^+ + p_i^- = 1$. The multiplicative upward change is $u_i$ and multiplicative downward change is $d_i$. The multiplicative changes at the different nodes are independently distributed. The initial spot price in the base scenario $s^0$ is $P(0)$ (which is assumed to be known).

The demand is assumed to follow a binomial lattice where the probability for upward shift
is $p^+_2$ and $p^-_2$ similarly subject to normalization condition requirement $p^+_2 + p^-_2 = 1$. The multiplicative changes are $u_2$ for upward change and $d_2$ for downward change. The initial level of demand is $D(0)$ (which also is assumed to be known). The set of scenarios describing the movement of spot prices and level of demand is defined as $S' = \{ s' \in R_{2,n} | s'_{ij} \in \{0,1\}, i = 1,2, j = 1,\ldots,T \}$. The elements of these matrices are interpreted so that if the spot price goes up in period $j$, then $s'_{ij} = 1$; otherwise the spot price goes down and we have $s'_{ij} = 0$. Let us assume that $s' \in S', t \geq 1$. Now we can define the number of spot price moves upwards until period $t$ as $q^+_t(s^t) = \sum_{j=1}^{t'} s'_{ij}$; conversely we can define $q^-_t(s^t) = \sum_{j=1}^{t'} (1-s'_{ij})$. By construction we have $q^+_t(s^t) + q^-_t(s^t) = t$. This means that spot price $P(s^t)$ in scenario $s^t$ can be written as $P(s^t) = u^{q^+_t(\cdot)}_1 d^{q^-_t(\cdot)}_1 P(0)$.

Likewise for the demand; if the demand in period $j$ goes up, then $s'_{2j} = 1$; otherwise the demand goes down and we have $s'_{2j} = 0$. The backward operation on scenarios is defined so that if $s' \in S'$, $t > 1$, then $b(s^t) = s' \in S^{t-1}$ such that $s'_{ij} = s'_{2j}, j = 1,\ldots,t-1$. All the predecessors of $s'$ are denoted by $B(s^t')$. The dynamics and corresponding variables for the demand are defined in the same way as for the spot price. This means that the number of times demand moves up until period $t$ is $q^+_2(s^t) = \sum_{j=1}^{t'} s'_{2j}$ and the number of times it has moved down is $q^-_2(s^t) = \sum_{j=1}^{t'} (1-s'_{2j})$. Similarly $q^+_2(s^t) + q^-_2(s^t) = t$. The level of demand can be written $D(s^t) = u^d_2 d^{q^-_2(\cdot)}_2 D(0)$.

Assuming that the spot price movements are independent from demand uncertainties implies that the probability of certain scenario is $s', t = 1,\ldots,T$ is $p(s^t) = \prod_{i=1}^2 \left[ (p^+_i)^{q^+_i(\cdot)} (p^-_i)^{q^-_i(\cdot)} \right]$. In each scenario $s'$, there exist a forward contract for the later delivery $t' > t$ at a price which is equivalent to the conditional expected spot price for that period, i.e.

\[
P(s', t') = \left[ \sum_{s' \in S'} p(s^t) P(s') \right] .
\]  \hfill (2)
The problem is to decide the amount of forward contracts \( x(s', t') \geq 0, t' \geq t \) such that the demand is satisfied and the exposure to risk is minimized (in the sense which is addressed below). If \( t' = t \), then \( x(s', t') = x(s', t) \) is the amount of commodity that is purchased at the spot price \( P(s') \). The key constraint stems from the requirement that all demand must be satisfied with the forward contracts or by purchasing with the spot price, i.e.,

\[
\sum_{s' \in B(t')} x(s', t') + x(s', t) = D(s').
\]

(3)

The cash flows can be examined with the help of resource surpluses (resource flows in CPP), derived from the initial stock \( M \gg 0 \). This means that the ‘resource surplus’ at the base scenario \( s^0 \) is \( RS(s^0) = M - x(s^0, 0)P(0) \) where the initial stock \( M \) is an artificial variable that can be employed to measure just how much demand has been met. If \( s^1 \in S^1 \), the corresponding surplus is

\[
RS(s^1) = RS(s^0) - x(s^1, 1)P(s^0, 1) - x(s^1, 1)P(s^1).
\]

(4)

In the same way this can be generalized into the expression

\[
RS(s') = RS(b(s')) - \sum_{s' \in B(s')} x(s', t)P(s', t) - x(s', t)P(s').
\]

(5)

The objective function for the maximization problem is

\[
EV = \sum_{s' \in S^r} p(s')RS(s').
\]

(6)

From the viewpoint of risk management, the question is how to account for the variability and how to penalize for it. As in CPP, for each terminal scenario \( s' \in S^T \) we define deviational variables \( \Delta^+(s') \) and \( \Delta^-(s') \) so that
\begin{equation}
RS(s') - EV - \Delta^+(s') + \Delta^-(s') = 0,
\end{equation}

Where the terms \( \Delta^+(s') \geq 0 \) and \( \Delta^-(s') \geq 0 \) indicate (if positive) whether the resource position in scenario \( s' \) is larger or smaller than on the average. Now the objective function can be augmented through an additional term which penalizes for variability from the expectation. The objective function becomes

\begin{equation}
\max_{s' \in S'} \sum p(s') [RS(s') - \rho(\Delta^+(s') + \Delta^-(s'))],
\end{equation}

where the risk aversion coefficient \( \rho > 0 \) penalizes for variability.

\section{An Application of Contingent Portfolio Programming in Portfolio Optimization}

\subsection{Problem formulation}

The paper approaches the forward contract management by focusing on three simple examples. We assume that the commodity can not be stored and that all the demand must be met and can be purchased from the market. This leads to a situation in which commodities are either purchased beforehand with the forward contracts, purchased at the concerning moment with spot price or purchased with some combination of these two previous alternatives.

Let us consider a problem with an initial need of perishable commodity \( D(0) = 100 \) and that amount varying by time depending on various sources of uncertainty. At different points in timeline we have consumption either increasing by \( u_2 = 1.1 \) or degreasing by \( d_2 = 0.9 \) at certain probabilities; \( p_2^+ = 0.6 \) for upward and \( p_2^- = 0.4 \) for downward movement. Similarly, initial spot price is \( P(0) = 100 \) of and the price either rises by \( u_1 = 1.1 \) at probability \( p_1^+ = 0.4 \) or decreases by
$d_i = 0.9$ at probability of $p_i = 0.6$. The time horizon consists of three steps $t = 0, 1, 2$, and the risk aversion coefficient $\rho = 1$. The initial cash amount is $M = 50000$.

The problem is to determine the optimal amount of commodity bought with forward contracts in order to maximize the risk-adjusted resource surplus in the terminal state. The paper examines also the sensitivity of the model in regards to the risk aversion coefficient $\rho$ by trying three different coefficient values, $\rho = 0$, $\rho = 1$ and $\rho = 5$.

The values for price and demand were calculated as well as the probabilities of each state in each period. The possible prices in the first period are $P(s^1) = (110 \ 90)$, while the possible demands are $D(s^1) = (110 \ 90)$. Now if we combine these, we get the following scenario matrix for the first period (see table 1. to see how the matrix is compiled)

$$D(s^1).P(s^1) = \begin{pmatrix} 110,110 & 90,110 \\ 110,90 & 90,90 \end{pmatrix},$$

with the following probabilities

$$p(s^1) = \begin{pmatrix} 0.24 & 0.16 \\ 0.36 & 0.24 \end{pmatrix}.$$

Similarly these can be calculated for the second state. Now the price matrix is $P(s^2) = (121 \ 99 \ 99 \ 81)$ and the demand is $D(s^2) = (121 \ 99 \ 99 \ 81)$. Combining these we get the following matrix

$$D(s^2).P(s^2) = \begin{pmatrix} 121,121 & 99,121 & 99,121 & 81,121 \\ 121,99 & 99,99 & 99,99 & 81,99 \\ 121,99 & 99,99 & 99,99 & 81,99 \\ 121,81 & 99,81 & 99,81 & 81,81 \end{pmatrix},$$
with the following probabilities

\[
p(s^2) = \begin{pmatrix}
0.0576 & 0.0384 & 0.0384 & 0.0256 \\
0.0864 & 0.0576 & 0.0576 & 0.0384 \\
0.0864 & 0.0576 & 0.0576 & 0.0384 \\
0.1296 & 0.0864 & 0.0864 & 0.0576
\end{pmatrix}
\]

Finally we need to calculate the future prices by using the equation (2). We use \(1+\) to describe the cases when price has gone up in the period \(t = 1\) and \(1-\) to describe cases when price has gone down in the period \(t = 1\).

\[
P(0,1) = 98
\]
\[
P(0,2) = 96.04
\]
\[
P(1+,2) = 107.8
\]
\[
P(1-,2) = 88.2
\]

In Figure 2, the whole example setup is presented. In each state box (marked with complete line borders), the P is for the spot price in that state and D is the demand. In probability boxes (marked with fragmentary line borders), p stands for probability. Arrows that go through probability boxes represent possible following states, and the probability box tells the probability for that state to occur. The probability boxes on the left side of state boxes in period 2 (marked with fragmentary line with dots borders) represent the probability to eventually end to that certain state. Finally, arrows with future price text on them represent the possible forward contracts and their prices.
Figure 2. Example scenario

Future price 98

Period T=0

P₀=100
D₀=100

Future price 107.8

Future price 96.04

Period T=1

P₀=110
D₀=90

Future price 88.2

Period T=2

P₀=100
D₀=100

P₁=110
D₁=110

P₁=90
D₁=110

P₁=90
D₁=90

Period T=0

p=0.24

p=0.16

p=0.36

p=0.24

p=0.24

p=0.16

p=0.36

p=0.24

p=0.24

p=0.16

p=0.36

p=0.24

p=0.24

p=0.16

p=0.36

p=0.24

p=0.24

p=0.16

p=0.36

p=0.24

p=0.24

P₁=121
D₁=121

P₁=99
D₁=99

P₁=99
D₁=99

P₁=81
D₁=81

P₁=81
D₁=81

P₁=81
D₁=81

p=0.0576

p=0.0576

p=0.0576

p=0.0576

p=0.0576

p=0.0576

p=0.0576

p=0.0576

p=0.0576

p=0.0576

p=0.0576

p=0.0576

p=0.0576
4.2 Case results and result analysis

This case was solved using normal Excel solver to maximize the target function in order to demonstrate the usability of the model. However, the model is not limited by the linear programming software that is used.

In the numerical case, the results were computed by using the equation (5), from where we get the $RS(s^t)$ for each state $t$. Notice that in period $t = 0$ we have one $RS(s^0)$, in period $t = 1$ we have four states, thus four $RS(s^1)$, and in period $t = 2$ we have 16 $RS(s^2)$. Now using the formula we get (by inserting the initial values presented in the chapter 4.1)

$$RS(s^0) = 50000 - 100 * 100 = 40000.$$  

For each state in period $t = 1$, the $RS(s^1)$ is

$$RS(s^1) = \frac{40000}{0 \text{ period resource surplus}} - \frac{P(s^0,1) \times x(s^0,1)}{futures 0-1} - \frac{P(s^1) \times x(s^1,1)}{spot},$$

where $P(s^1)$ is the price of that state in period $t = 1$ and $x(s^1,1) \geq 0$ is the spot demand of that state in that period, $P(s^0,1)$ is the future price in that state of the period $t = 0$ to period $t = 1$ and $x(s^0,1) \geq 0$ is the amount bought with these future contracts in this state of the period $t = 1$. We also have the demand restriction, which says that demand in each state in period $t = 1$ must be met. That is $x(s^0,1) + x(s^1,1) = D(s^1)$.

For period $t = 2$, the $RS(s^2)$ is similarly calculated by using the equation (3)

$$RS(s^2) = RS(s^1) - \frac{P(s^0,2) \times x(s^0,2)}{1 \text{ period resource surplus}} - \frac{P(s^1,2) \times x(s^1,2)}{futures 0-2} - \frac{P(s^2) \times x(s^2,2)}{futures 1-2} - \frac{P(s^2) \times x(s^2,2)}{spot},$$
where, $\text{RS}(s^1)$ is the resource surplus in the period $t = 1$ (depends of the state in that period), 
$P(s^0, 2)$ and $P(s^1, 2)$ are the future prices from corresponding periods to period $t = 2$, $x(s^0, 2) \geq 0$ is the amount futures bought in period 0, $x(s^1, 2) \geq 0$ is the amount bought in the period $t = 1$, $x(s^2, 2) \geq 0$ is the amount bought with spot price and finally $P(s^2)$ and $D(s^2)$ are the spot price and demand in period $t = 2$ (both these depend on the state in which the system is in period $t = 2$). Again we have demand restriction that says $x(s^0, 2) + x(s^1, 2) + x(s^2, 2) = D(s^2)$

Now by using the equation (6) and $\text{RS}(s^2)$ we calculated $EV$. With formula (6) and by using the $EV$, we calculated $\Delta^+(s^1)$ and $\Delta^-(s^1)$. Finally with equation (6) we formed the maximization problem.

Calculating the optimal hedging with the initial values presented in chapter 4.1 (and using $\rho = 1$) we got that the maximum is achieved when $x(s^0, 1) = 90.60$ and $x(s^0, 2) = 118.80$. This means that we should buy 90.60 forwards in the period $t = 0$ to be delivered in the period $t = 1$. Now, for the second period, if we are in state where price and demand have both gone up, we would buy $x(s^1, 2) = 2.20$, which would meant that we would hedge against maximum possible demand in period $t = 2$ as we already have bought 118.8 forwards for period $t = 2$ in period $t = 0$. For state in which price has gone up and demand has gone down, result is that 24.55 of previously bought forwards (bought in the period $t = 0$ to be delivered in period $t = 2$) would be sold and $x(s^1, 2) = 4.75$ of new forwards would be bought for the period $t = 2$. Again the hedged amount is the maximum possible demand in period $t = 2$. For state in which price has gone down and demand up, we would buy $x(s^1, 2) = 2.20$ and for price down demand down scenario the result is that 23.37 of previously bought forwards (bought in the period $t = 0$ to be delivered in period $t = 2$) would be sold and $x(s^1, 2) = 3.57$ would be bought. Result says that for the period $t = 1$ minimum possible demand would be hedged and for the period $t = 2$ maximum possible demand would be hedged. The hedged forwards in the period $t = 2$ is a combination of forwards bought in periods $t = 0$ and $t = 1$. 
Similarly the hedging can be calculated for all the coefficients. In the case \( \rho = 0 \), the amount of hedging for the first period can be anything between zero and the minimum demand in the possible following state, i.e. \( x(s^0,1) = 90 \). Also, in the period \( t = 0 \) we would hedge for the minimum possible demand in period \( t = 2 \) i.e. \( x(s^0,2) = 81 \). In the period \( t = 1 \) we could hedge any amount between zero and the minimum possible demand in possible following state in period \( t = 2 \), so \( x(s^1,2) = [0, \min D(s^1_{1,2})] \), where \( 1' \) is the state in which the decision is made in period \( t = 1 \). This is intuitive, as if we do not penalize for variability, the only thing that matters is the costs. And because the futures are probability weighted averages of possible prices, on average it would make no difference what kind of portfolio we compose of buying spot or buying futures, as long as we do not waste any excess capacity (which is why we would only hedge against the minimum possible demand in the next period and rest we would buy on spot.)

In the case \( \rho = 5 \), result is that we will buy forward contracts for the maximum possible amount of demand in the terminal period, i.e. \( x(s^0,1) = 110 \) and \( x(s^0,2) = 121 \). In the period \( t = 1 \) in states where demand had gone down, futures contracts are sold so that futures equal the maximum possible demand in the next possible state in the period \( t = 2 \), i.e. 22 forwards (bought in period \( t = 0 \) for delivery in period \( t = 2 \)) are sold. This is to adjust to the lower demand in the terminal period, and by selling the extra futures we secure not losing any resources. This is also an intuitive result, as if we are going to penalise greatly for the variability, then losing money with buying too much will be offset by not having any variability in the final resource surpluses.

As can be seen from the results, the model is very sensitive to changes in coefficient. Coefficient 0 means hedging to amount of minimum demand and coefficient 5 means that hedging will be to the maximum demand in the following periods. Furthermore, it can be noted that there is no unique solution in many scenarios; there can be found many combinations between the amount of hedging and buying at a spot price. This is because of the uniqueness of each path in the state tree.

It should be also noted, that in our example we had only three periods, and so only one forward that was over longer time period. If we had more time periods, the number of these over-a-periods forwards would increase, making the model slower to compute as well as harder to program. Of
course, using proper software with more efficient algorithms could be used to diminish the problem. But using Excel has an advantage, as optimization programs are not available to everyone and easy for everyone to use.

The results of the numerical examples are no more than illustrative as we had only three time steps. The computation time was short but as the number of time periods increase the computing time grows rapidly. Nevertheless, Excel can be used to solve small problems. Gustafsson and Salo (2005) also demonstrate that larger problems can be solved using an optimization program.

5 Summary

5.1 Model pros and cons

The model takes into account the Decision maker’s (DM) risk profile by allowing the DM to specify the risk aversion coefficient. However, the model penalized similarly excess capacity and lack of capacity. This could be enhanced so that only excess capacity is penalised. The model is scalable in the scale that it is possible to compute in a reasonable time. Together with the fact that time periods are not bound to any length of time intervals it is possible to use this model in any time scale that DM wants.

The linear equations and thus linear solving enable the model to be computed in a reasonable time in many cases. Linearity also means that there is always a solution for the problem and that it is possible to solve.

The model misses the cyclic character of the markets. The model does not take into account the time-series which of course affect the demand (which can be seen for example by observing the electricity demand in certain times of the year). However, we could construct the state tree by not using general formulas for prices and demands, but instead values provided by experts that take also the cyclical character into consideration. This would lead to the fact that we cannot
automatically generate the tree, unless we would first create software to do that.

The model also misses the production capability of the DM. If the DM has its own production capability it can meet the demand also through own production. This makes the decision making process much more complicated. This may render the model less useful for some markets.

Some limitations of the CPP model are also included in the approach presented in this paper. The major restriction is the size of the decision tree as it grows exponentially when time horizon is expanded. The state tree limits the usability of CPP. Even though CPP is based on linear programming the model size grows greatly as the number of states is increased or the time horizon grows longer. But this is related to all tree based approaches and as Gustafsson and Salo (2005) note, the CPP model does have only a slight percentage of variables and constraints compared to conventional decision tree. In addition, Gustafsson and Salo (2005) demonstrate that CPP can be used in realistic sized problems to solve them.

CPP model has discrete time periods, which restricts the usage of the model in practice. For example, the price of electricity on the markets constantly changes resulting in continuous updating of the calculations.

5.2 Model restrictions and usability

Forward contract hedging with Contingent Portfolio Programming with the model presented in this paper has three assumptions that restrict the generality of the model:

1. The spot price movements are considered independent of the demand uncertainties.
2. It is assumed that forward contracts exist for any size and for any time period.
3. The forward contracts of excess capacity are wasted in the expiration period

The first assumption is relevant as it gives the possibility to calculate the probability of certain scenarios. On the other hand it restricts the approach to smaller market players. This is because
significant market players influences on the market price and the availability of the commodity. The assumption also misses out the possibilities of exceptional changes, such as disasters. Together with the assumption that all the demand can be met these two assumptions limit the usability of the model. Again we could produce the state tree by manually placing probabilities, prices and demands on each state, so that we could tackle the issue.

The second assumption does not limit the model in most of the cases. Whenever a large number of market players produce commodity, there usually is possibility to negotiate any kind of forward contracts (assuming that this does not affect the price). But it might be that the forward contracts can only be sold and bought at regulated markets and thus it may be that this assumption will not hold in all cases.

The third assumption makes this model to describe the worst case scenario for the DM. If the DM could sell the forward contracts he would not lose all the investments in forward contracts and this would lead to a solution where the amount of demand secured by forward contracts could be larger. This assumption does not limit the use of the model in mathematical sense but it limits the use of the model in real world cases. In some cases, we could assume that forward contracts could be sold at their expiration date at the spot price. If the time periods are long, this is not a bad assumption, but it requires volatile markets. Allowing selling of futures at a spot price on expire date could create problems associated with the time it takes to sell the contract and on the possible value of the contract if part of it has already expired. However, the selling can be easily applied to the model by allowing negative buying on spot price in states to represent the selling of commodity on spot price.

The model presented in this paper can not be used to manage storable commodities. If commodity can be stored the DM can always buy commodity either using forward contracts or directly from the market with spot price and storage the goods to meet the future demand, depending which is cheaper. The model can be applied to any perishable commodity that otherwise follows the assumptions and expectations stated in the paper. The following are examples of on what the model can be used.

Electricity contracts can easily be approached with the model discussed in this paper. According to
Pirilä (2005), electricity futures in the Nordpool are only for certain time intervals. There are futures on daily average prices for the next 3-9 days, average prices of the next eight weeks, and then there are some forward contracts. With these futures we can use this model for short term scenarios using days as time periods. With longer scenarios we have to take into account that there is not always possibility to buy futures from any time period to all succeeding time periods. If we would like to use this model, we could use OTC markets to complement the official Nordpool futures. But according to Pirilä (2005), most of the OTC trading is done on official Nordpool futures and non official futures might have different costs than that can be derived using normal future price formula. Although this model assumes that DM does not have own production capacity, it could be possible to assume own production capacity as futures contracts for all the time periods (Pirilä 2003). They would differ from other futures because their cost would be based on the production cost and because we would not have to use the own production if electricity spot price would be less than the production cost. Here is considered mainly the application of the model with Nordpool data, but basically only things that differ in other exchanges are the set of available futures and their delivery periods (and of course their liquidity and possible OTC trading). In electricity trading, it should be noted that there actually is very little point in taking shorter than one day time periods. Balasko (2001) discusses the issues relating to different pricing for off-peak and peak periods, which in our formula would produce wrong futures prices if we would buy futures from off-peak to peak, or vice versa. On the other hand, according to Collins (2002), electricity futures for long time periods are not accurate either. Collins examined data from 10 futures contracts and spot price for 13 moths on NYSE and found out, that while the average variation of the daily settlement prices for the futures contracts was 0.03, the average variation for the spot price was 0.35. As our model penalizes for variability, this indicates that during longer periods, hedging would probably be intense and then probably some of it would be sold as the time period comes closer.

Other application of this model could be in the carbon dioxide (CO₂) business. There are two unknown factors, the price of CO₂ emission (the emission exchange has both the spot price and the forward contract price for it) and the amount that the company produces emission each year. This is of course dependant on the amount of demand and thus the revenues, but the approach is the same. A slight assumption in this model is that the CO₂ credits that the producer has is not taken into
account (to be more general there can be assumed that those initial credits assigned to that certain producer is so insignificant amount that it can be left out of consideration. This, anyhow, is not always the case). Another variation comes from penalizing the demand that is not met. According to Krugman and Pizer (2004), the US SO\textsubscript{2} cap-and-trade program has been the basis for EU Emissions Trading System, so before modifying this model to meet emission markets reader should first study the existing literature on US SO\textsubscript{2} trading program.

Even logistics can be modelled with this model. The model permits the DM to choose the amount of logistics bought before hand. Similä (2005) has used CPP to model vehicle routing problem, where the problem is to issue the fleet to certain nodes. Maybe combining these two models could produce a model to describe the size of the fleet as well as how to allocate it.

Finally, when considering future research to expand the possible applications of the model, we note that the three previously stated assumptions limit the usability of the model. So finding ways around these assumptions could produce a much more robust model. One extension for the model is not to have the demand constraints, instead penalize if the demand is not met. In a real world, in many instances, we always have the alternative cost. It is never advisable for anyone to do anything that has costs which exceed the costs of alternative options.

The model could be extended in the sense of not having the assumption of spot price being independent of the demand uncertainties and on how to construct logical state trees that are not automatically generated, i.e. how each state could have its properties independently chosen. This would make the model much more usable in real world related issues.

The most important future research area is to focus on the cases in which the DM has own production capacity as then the model could meet the demand of real DMs in the field of such markets where the model could prove to be usable. As presented above, the most obvious way to think this issue could be through the use of the futures contracts.
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