Estimating Project Value Distributions Using Expert Evaluations

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1 Introduction

Project selection is a recurrent resource allocation decision problem faced by funding organizations which have a large number of project proposals and scarce resources to finance them. In resource allocation, the funding decisions depend on each other, as the decision to allocate funds to one project leaves less resources for other projects. This problem, which is faced by the funding organization, is challenging hence they often do not have complete information about proposed projects and the time available for evaluation of each proposal is limited.

One approach to improve the decision quality is to gather expert evaluations of the proposed projects. These expert evaluations can be viewed as proxies to the true value of the project. However, the evaluations typically differ from each other, yielding ambiguous evaluations about the true value of the project. This problem can be addressed through decision analytic methods to construct estimates for the distribution of the true values of projects. With accurate value distribution estimates, the decision process can be made more effective by pruning those proposals that are most likely to be rejected by the evaluation panel. This lowers the total cost of project portfolio selection and frees resources for other decision making problems.

Due to the uncertainty of the actual outcomes in decision analysis, the decision maker may experience postdecision regret over the chosen alternative. Even though we may be able to produce conditionally unbiased estimates for the values of different alternatives, we should expect that the realized values are lower than the estimated values of the project chosen. This phenomenon is called optimizer’s curse by Smith and Winkler (2006). Further Harrison and March (1984) argue that the optimizer’s curse is a phenomenon that has impact on all analytically guided decision making, thus making it an important part of the decision analysis that should not be neglected.

This study uses a decision analytical method for estimating the distribution of the value of project proposals based on expert evaluations (Salo, 2008). The approach is simple. The values of the project proposal are assumed
to be normally distributed with unknown mean and variance. To model the uncertainty of expert evaluations, a normally distributed error term with zero mean is introduced. Using these assumptions, we construct a model to find the maximum likelihood estimates for the unknown distribution parameters. The model will be tested with two different data sets. First, we apply basic simulation methods and then the model is also evaluated with more sophisticated optimization techniques.

The problem is motivated by project proposal evaluation, but the approach is rather generic as it only assumes that the underlying value factors are normally distributed. Thus it can be widely used in many different evaluation situations which include utilization of evaluation information.

The remainder of this study is organized as follows. Section 2 presents the decision analytical model. The results and implications of the study are presented in Section 3. Section 4 concludes.

2 Evaluation information in portfolio decision analysis

Financial decision making is characterised by the need to identify the best investment projects available. Methods for these capital budgeting decisions are well developed. The most common metric for evaluating financial project performance is the net present value approach. However, in many project selection problems net present value is not a suitable evaluation criterion. This is true in the case of government and public sector investments which not only seek to generate positive profits but also other socially beneficial objectives. According to Kleinmuntz (2007), decision analytic methods are often used in resource allocation problems, i.e., problems where the decision maker has scarce resources and a large number of possible projects to finance.

The benefit of decision analytical methods in portfolio decisions can be substantial. Keisler (2004) argues that applying decision analytic methods to portfolio management decisions is beneficial and that some sort of a disci-
plined process should be used. Keisler also reports that increases in portfolio values due to decision analysis have ranged from 15% to over 100% of the value of the original portfolio that would have been selected without the analysis; this greatly exceeds the estimated costs of the analysis.

Value of information is a prevalent question in portfolio management. In their early papers Matheson (1968) and Watson and Brown (1978) use value-of-information techniques to study the sources of value in decision analysis.

Keisler (2004) compares the value added of two different approaches to portfolio selection. He notes that at its simplest portfolio decision analysis consists of applying decision analytical methods to portfolio candidates one at the time whereafter the projects are ranked according to different metrics and the alternatives with highest estimates are chosen. This basic approach was then compared to the approach of taking an additional step to construct more refined value estimates. In his paper, Keisler examines the benefits of a refined project value estimates and disciplined use of objective prioritization. He notes that unless the project proposals incorporate a substantial amount of uncertainty, improved value estimates are not nearly as important as the use of a disciplined process.

Clemen and Kwit (2001) apply decision analytical methods to Eastman Kodak portfolio and estimated the value added created by decision analysis. Sharpe and Keeling (1998) use an approach of comparing the efficient frontier portfolios to all possible portfolios to construct the highest-value portfolio based on the return-on-investment ranking.

### 3 Estimating the parameters of the value distribution

In resource allocation decisions the minimum objective is to construct a ranking order for different project proposals according to their characteristics. The ultimate objective is to find detailed and accurate estimates for project values. Yet accurate value estimates for project values (i.e. R&D projects)
can be hard to find due to the substantial uncertainty incorporated by the projects.

This section depicts a method for estimating value distribution parameters following the work of Salo (2008). The main idea behind the rationale is that by using expert evaluations, we can construct a probability based model that can be used to determine the actual parameters of an unobservable project value distribution.

The section is structured such that first we start by formulating the parameter estimation problem. This part sets the target that we try to reach in the analysis. Then we take a closer look at the expert evaluations and how they relate to the actual project values in our framework. The last part of the chapter is more technical in nature and it provides the formulas for calculating the probabilities needed to formulate the estimation problem.

3.1 Formulating the parameter estimation problem

Here, expert evaluations refer to independent subjective valuations of a project proposal by different analysts. The underlying assumption is that the evaluations are regarded as unbiased estimators of the true value of project proposals, meaning that, at the average, the experts estimate the project values correctly. Based on this unbiasedness of the estimators it is possible to construct an estimate of the \textit{ex post} estimate for the distribution of real project values based on the expert evaluations.

There are $i = 1, \ldots, n$ project proposals which are all evaluated on a five-point scale from 1 to $C$. Here we have $C = 5$. These evaluations are then scaled to belong to the interval $[0,1]$. The prior probability distribution of project values is modeled as a discrete probability distribution where $p^i_k$ denotes the probability that project $i$ has the value of $k$, where $k \in \{1, \ldots, C\}$. The prior probabilities are such that by construction $\sum_{k=0}^{C} p^i_k = 1, \ \forall i$.

We define the expert evaluations as follows. We assume that for each project proposal $i$ there is at least one evaluation, thus for project $i$ there
are \( n(i) \geq 1 \) evaluations. The evaluation data can be presented as \( b(i,j) \in \{1,\ldots,C\}, i = 1,\ldots,n, j = 1,\ldots,n(i) \). We assume that the expert evaluations are conditionally unbiased estimators of the true project values, i.e. the expected value of the expert evaluations equals the actual value of the project proposal. However, the evaluations incorporate a substantial amount of uncertainty which needs to be taken into account. Thus we note that there are positive probabilities such that the evaluation of the project proposal \( i \) with true value of \( k \) belongs to some other quality class, subject to the condition that only relevant classes are possible. For simplicity, hereafter in this study we use notation \( b^i \) to indicate a single statement given to the project \( i \).

To construct expressions for distribution parameter estimation, we denote the five subintervals where the project value or the expert evaluation might belong as follows. \( C_1 = [0,0.2], C_2 = [0.2,0.4], C_3 = [0.4,0.6], C_4 = [0.6,0.8], C_5 = [0.8,1.0] \). The underlying question is to find the distribution parameters that maximize the probability of obtaining evaluation results from different intervals, on condition that the actual value belongs to a specific interval. The probability of acquiring a single statement \( b^i \) that belongs to class \( C_k \) can be estimated by

\[
P(b^i \in C_k) = \sum_{l=1}^{5} P(b^i \in C_k|a_i \in C_l)P(a_i \in C_l). \tag{1}
\]

The probabilities in the above summation can be calculated by the expressions to be introduced in the equations below.

In reality, one may not able to observe the true value of the project, moreover there may be a need to consider multiple statements for each proposal. To deal with several evaluation statements on a single project proposal, we consider all possible permutations of the statements, which greatly complicates the probability expression. We assume that there are a total number of \( m_i \) statements given to proposal \( i \) and that \( m_{ik} \) is the number of evaluation statements that belong to class \( C_k \). Thus the statements on a single project can be defined as an evaluation vector of the form \( b^i_{m_i} = (m_{i1},\ldots,m_{i5}) \).
Specifically we have that $\sum_k m_{ik} = m_i$. The probability of acquiring the evidence vector $b_{m_i}$ is

$$P(b_{m_i}) = \left( \begin{array}{c} m_i \\ m_{i1} \end{array} \right) \left( \begin{array}{c} m_i - m_{i1} \\ m_{i2} \end{array} \right) \left( \begin{array}{c} m_i - m_{i1} - m_{i2} \\ m_{i3} \end{array} \right) \left( \begin{array}{c} m_i - m_{i1} - m_{i2} - m_{i3} \\ m_{i4} \end{array} \right).$$

(2)

$$\sum_{l=1}^{5} \prod_{k=1}^{5} P(b^l_i \in C_k | a_i \in C_l)^{m_{ik}} P(a_i \in C_l).$$

Now that we have the probabilities for each proposal under the given expert evaluation, the final step is to consider all $n$ project proposals together. We assume the project proposals are independent of each other, and thus the probability of obtaining the evaluation vectors $b_{m_1}, b_{m_2}, \ldots, b_{m_n}$ is given by

$$P(b_{m_1}, b_{m_2}, \ldots, b_{m_n}) = P(b_{m_1}) P(b_{m_2}) \cdots P(b_{m_n}).$$

(3)

The term in (3) can be linearized by taking logarithms, because the maximum is reached with the same parameter values in both expressions

$$\ln P(b_{m_1}, b_{m_2}, \ldots, b_{m_n}) = \ln P(b_{m_1}) + \ln P(b_{m_2}) + \cdots + \ln P(b_{m_n}).$$

(4)

The equations (3) and (4) are the likelihood functions of the parameter estimation problem. The optimal parameter values are obtained by maximizing the likelihood function, i.e., maximizing the probability of acquiring specific evaluation evidence. The parameter values $\mu_x$, $\sigma_x^2$ and $\sigma_e^2$ obtained through the optimization thus determine unique distributions to the actual values of the projects and to the error incorporated in the expert evaluations.

### 3.2 Project value distributions

Now that we have formulated our optimization problem, the next step is to define the probability distributions that govern the project values and their relationships to the expert evaluations. For simplicity, it is assumed that
the value of the project is governed by normal distribution. We introduce two random variables which are defined as follows \( x \sim N(\mu_x, \sigma^2_x) \) and \( \epsilon \sim N(0, \sigma^2_\epsilon) \). Using these variables we further assume that the actual value of the project is given by

\[
a = \frac{e^x}{1 + e^x} \in [0, 1],
\]

and the expert evaluation considering the same project can be obtained by

\[
b(i, j) = \frac{e^{x+\epsilon}}{1 + e^{x+\epsilon}} \in [0, 1].
\]

The basic idea in the above expressions (5) and (6) is that those enable us to map the normally distributed random variables to the interval \( [0, 1] \) keeping the project values still governed by normal distribution. The resulting project value function is shown in figure 1. The expert evaluation function has a similar shape. It can be seen that the error parameter "tilts" the project value function and the magnitude of the deviation from the actual project value distribution depends on the size of the assumed error in the expert evaluations.

![Figure 1: Project value function with different values of random variable x. The dotted lines depict the effect of the evaluation errors.](image)

3.3 Calculating probabilities

In this last part of third chapter, we conclude the project value model presented in equations (3) and (4). The final step is to calculate the probabilities
of acquiring different pieces of evidence conditionally to the actual project value. First, we introduce the probability that the actual quality \( a \) of the project proposal belongs to some interval \([t, t']\). This can be calculated by

\[
P(a \in [t, t']) = P \left( t \leq \frac{e^x}{1+e^x} < t' \right) = P \left( 
\frac{t}{1-t} \leq e^x < \frac{t'}{1-t'} \right) = P \left( \ln \frac{t}{1-t} \leq x < \ln \frac{t'}{1-t'} \right) = \Phi_{\mu_x,\sigma_x^2} \left( \ln \frac{t'}{1-t'} \right) - \Phi_{\mu_x,\sigma_x^2} \left( \ln \frac{t}{1-t} \right),
\]

where \( \Phi_{\mu_x,\sigma_x^2} \) is the cumulative density function of the normal distribution \( N(\mu_x,\sigma_x^2) \).

As mentioned before, we assume that the range of project values is divided into five predetermined intervals. Thus we only need to consider the probabilities for five different intervals which can be easily obtained using the above expression. The expressions for each bracket of actual quality are

\[
P(a \in [0, 0.2]) = \Phi \left( \ln \frac{0.2}{0.8} \right) - \Phi(-\infty) = \Phi(-\ln 4),
\]

\[
P(a \in [0.2, 0.4]) = \Phi \left( \ln \frac{0.4}{0.6} \right) - \Phi \left( \ln \frac{0.2}{0.8} \right),
\]

\[
P(a \in [0.4, 0.6]) = \Phi \left( \ln \frac{0.6}{0.4} \right) - \Phi \left( \ln \frac{0.4}{0.6} \right),
\]

\[
P(a \in [0.6, 0.8]) = \Phi \left( \ln \frac{0.8}{0.2} \right) - \Phi \left( \ln \frac{0.6}{0.4} \right),
\]

\[
P(a \in [0.8, 1.0]) = \Phi \left( \ln \frac{1.0}{0.2} \right) - \Phi \left( \ln \frac{0.8}{0.2} \right) = 1 - \Phi(-\ln 4).
\]

The values approaching infinity in the first and last interval are taken into account as their limiting values. It can be easily verified that the only parameters governing the actual project values are the mean and variance of the normal distribution of project values.

Next, we examine the probabilities of acquiring a piece of evidence from the expert evaluations. As the experts have some knowledge of the project they
are evaluating the probability of acquiring the evaluation result is conditioned by the actual project value. The following expression holds for the conditional probability of the event that the evaluation belongs to the interval \([s, s']\) with the filtering condition that the actual quality of the proposal is \(a = t\), i.e.,

\[
P(x \in [s, s'][a = t]) = P\left(s \leq \frac{e^{x+\epsilon}}{1 + e^{x+\epsilon}} < s' | e^x = \frac{t}{1-t}\right)
\]

\[
= P\left(\frac{s}{1-s} \leq e^{x+\epsilon} < \frac{s'}{1-s'} | e^x = \frac{t}{1-t}\right)
\]

\[
= P\left(\ln \frac{s}{1-s} \leq x + \epsilon < \ln \frac{s'}{1-s'} | x = \ln \frac{t}{1-t}\right) \quad (8)
\]

\[
= P\left(\ln \left[\frac{s}{1-s} - \frac{t}{1-t}\right] \leq \epsilon < \ln \left[\frac{s'}{1-s'} - \frac{t}{1-t}\right]\right)
\]

\[
= \Phi_{0,\sigma_2^2}\left(\ln \left[\frac{s'}{1-s'} - \frac{t}{1-t}\right]\right) - \Phi_{0,\sigma_2^2}\left(\ln \left[\frac{s}{1-s} - \frac{t}{1-t}\right]\right),
\]

where \(\Phi_{0,\sigma_2^2}\) is the cumulative density function of the normal distribution \(N(0,\sigma_2^2)\).

Finally, we need to find an expression for the probability of acquiring an evaluation statement belonging to some interval when the probability is filtered by the condition that the actual project value is in a specific quality bracket. With the above expressions, the probability can be determined as follows. We assume that the evaluation evidence belongs to interval \([s_3, s_4]\) and the actual project value belongs to the quality bracket \([t_1, t_2]\). In order to get an expression for the probability, we have to divide the interval of the actual project value \([t_1, t_2]\) into \(n\) sub-intervals which all have equal width \(\delta = (t_2 - t_1)/n\). Using this specification, we formulate an approximation for
the probability

\[
P(x \in [s_3, s_4] | a \in [t_1, t_2]) = P(x \in [s_3, s_4] | a \in \bigcup_{i=0}^{n-1}[t_1 + i\delta, t_1 + (i + 1)\delta]) \\
= \sum_{i=0}^{n-1} P(x \in [s_3, s_4] | a \in [t_1 + i\delta, t_1 + (i + 1)\delta])P(a \in [t_1 + i\delta, t_1 + (i + 1)\delta]) \\
\approx \sum_{i=0}^{n-1} P(x \in [s_3, s_4] | a \in t_1 + (i + \frac{1}{2})\delta)P(a \in [t_1 + (i + 1)\delta]).
\] (9)

Now, we have derived all the necessary tools to calculate the probability of a project value distribution. The final expression is obtained by plugging the above derived expressions into the equation (3) or (4).

4 Illustrative simulations

To test the Salo’s (2008) model, we use some ad hoc style simulation and optimization techniques in order to get some picture of the behaviour of the model when applied to a real life data set.

4.1 Data set

The model is tested against a data set of several project proposals which have all been evaluated in the scale of 1 to 5 as the model assumes. Two data sets are used in the empirical part; the total data set that consists of 105 expert evaluated project proposals which each of them were evaluated by up to four experts, and a data set that is a strict sub set of the total data set that contains 19 project proposals which were also evaluated by up to four experts. The second set of project proposals all bear similar characteristics such that the set is a deterministically drawn from the original set. The basic characteristics of the project evaluation data sets are presented in tables 1 and 2. Descriptive statistics reveal that the data set is relatively strongly skewed to the right as the mean is 3.66, meaning that the majority of the project proposals consider the projects to be above the average in general.
Table 1: Descriptive statistics of the evaluation sets

<table>
<thead>
<tr>
<th>Total # of evaluations</th>
<th>Mean</th>
<th>Variance</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>289</td>
<td>3.66</td>
<td>1.33</td>
</tr>
<tr>
<td>Sub set</td>
<td>52</td>
<td>3.40</td>
<td>1.89</td>
</tr>
</tbody>
</table>

Table 2: Number of different evaluations in the data sets

<table>
<thead>
<tr>
<th># of 1s</th>
<th># of 2s</th>
<th># of 3s</th>
<th># of 4s</th>
<th># of 5s</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>13</td>
<td>35</td>
<td>73</td>
<td>83</td>
</tr>
<tr>
<td>Sub set</td>
<td>6</td>
<td>8</td>
<td>13</td>
<td>9</td>
</tr>
</tbody>
</table>

Another notable characteristic in our data is that the two highest values include approximately half of all the evaluations, indicating that, if the experts evaluate correctly in average, the proposed projects are mostly good.

Descriptive statistics reveal that the total data set is relatively strongly skewed to the right as the mean is 3.66, meaning that the majority of the project proposals consider the projects to be above the average in general. Another notable characteristic in our data is that the two highest values include approximately half of all the evaluations, indicating that, if the experts evaluate correctly in average, the proposed projects are mostly good.

Our other set is also relatively strongly skewed to the right, and the mean of the observations is 3.40. However, it should be noted that this data set has two local maximum mean values, 3.0 and 5.0, which might cause some concerns when the optimization precuders are applied. Also, while this set has only approximately six times less observations than the total set, it has a significantly larger sample variance.

4.2 Likelihood simulations

In the empirical part, we first applied naive simulation techniques to see how the model behaves when applied to the data. The proposal data set was
evaluated by letting the three distribution parameters vary within plausible ranges to find the maximum likelihood estimates to the parameters. Due to the four dimensionality of the optimization problem, the results are illustrated by fixing the value of project mean and letting the variance parameters of the project value distribution and the error distribution vary. We ran five simulations in which we let the project mean $\mu_x$ take deterministic values 0.2, 0.4, 0.6, 0.8 and 1.0, respectively. This is equivalent to calculating probabilities of acquiring the given evaluation vectors when the inherent value of the project proposal is $\mu_x$. We used the linearized version, equation (4), of the likelihood function in the simulations to evaluate the goodness of fit of the model with different estimation parameters. The results of the simulations are presented in figures 2, 3 and 4.

![Figure 2](image1.png)

**Figure 2:** *Values of the total data set likelihood function (4) when project mean is fixed at 0.2 (left) and 0.4 (right)*

The figures imply that the value of the evidence vector (4) is maximized with high project values i.e. 0.8 and 1.0, which is in accordance with the data sets as 58% of the evaluations of the total data set and 48% of evaluations in the sub set are either 4 or 5, meaning that they belong to the two largest evaluation intervals. The values of the likelihood function in different scenarios reveal some basic characteristics of the data sets used. In both cases the likelihood function gets the largest values when the mean is fixed at 0.8 and 1.0 indicating that there are more project proposals with high expert value
Figure 3: Values of the total data set likelihood function (4) when project mean is fixed at 0.6 (left) and 0.8 (right)

Figure 4: Values of the total data set likelihood function (4) when project mean is fixed at 1.0
Figure 5: Values of the sub set likelihood function (4) when project mean is fixed at 0.2 (left) and 0.4 (right)

Figure 6: Values of the sub set likelihood function (4) when project mean is fixed at 0.6 (left) and 0.8 (right)
estimates than low. Also, the results show that the likelihood function gets the lowest values when the project mean is fixed at 0.4 in the case of the total data set and when the mean is fixed at 0.2 in the case of the sub set. This indicates that the lower project values have a more extreme distribution, meaning that if the project value is low it is more likely to be the lowest possible value than any of the higher estimates. However, this is at odds with the total evaluation data set which contains more than double of evidence belonging to the interval of [0, 0.4] than belonging to the interval of [0, 0.2].

The results presented in the above figures also implicate that there is a strong tendency towards maximizing the value of the likelihood function as the variance of the project values approaches zero. However this characteristic is not desirable as it implies that the project values are deterministic in nature and that the only source of uncertainty derives from the expert evaluations.

To further enhance this model it might be relevant to introduce a penalty term into the maximum likelihood function in order to capture the problem
of maximizing the likelihood function as the uncertainty of expert evaluations increases. Here, we use quadratic penalty terms in the likelihood function to penalize the high values of the error variance, $\sigma_\epsilon^2$, and the low values of the variance of project value distribution, $\sigma_x^2$. The penalized likelihood function is given by

$$L = \sum_{i=1}^{n} \ln P \left( b_{i_{m_i}} \right) - \sigma_\epsilon^2 M_\epsilon - \left( \alpha_x - \sigma_x^2 \right) M_x,$$

(10)

where $M_\epsilon$ and $M_x$ are penalty weights and $\alpha_x$ is the sample variance calculated from the expert evaluations, which is a prudent guess as the project variance as the expert evaluations are considered to be unbiased estimators of the true project value.

For the penalized likelihood function, we ran similar simulations as in the non-penalized case. The values of the penalty weights were set at $M_\epsilon^{Tot} = 100$ and $M_x^{Tot} = 500$ for the total data set and at $M_\epsilon^{Sub} = 30$ and $M_x^{Sub} = 50$ for the sub set. The sample variances were at $\alpha_x^{Tot} = 1.33$ and $\alpha_x^{Sub} = 1.89$, respectively. The results of the simulations for the total data set are presented in figures 8, 9 and 10 and for the sub data set in figures 11, 12 and 13.

Figure 8: Values of the total data set penalized likelihood function (10) when project mean is fixed at 0.2 (left) and 0.4 (right)

It appears that the addition of the penalty terms can significantly reduce the “extreme effect” in the results and especially, it reduces the value of the likelihood function as the project value variance decreases. This is a desirable
Figure 9: Values of the total data set penalized likelihood function (10) when project mean is fixed at 0.6 (left) and 0.8 (right)

Figure 10: Values of the total data set penalized likelihood function (10) when project mean is fixed at 1.0
Figure 11: Values of the sub set penalized likelihood function (10) when project mean is fixed at 0.2 (left) and 0.4 (right)

Figure 12: Values of the sub set penalized likelihood function (10) when project mean is fixed at 0.6 (left) and 0.8 (right)
characteristic in the model as it provides a good starting point to enhance the model such that the problem of maximizing the likelihood function when the project values are deterministic can be tackled.

It should be noted that in our model the penalty parameters cannot be arbitrarily large as is the case in some optimization problems, but rather they need to be adjusted by the decision maker. This implies that this kind of likelihood analysis always demands subjective view of the decision maker and the results also reflect decision maker’s preferences.

Although there appears to be some pitfalls in the results, the model has a sound theoretical basis and the likelihood surfaces seem to have unique maximum points. Also the simulations with the penalized and the simulations with the penalized likelihood function gave encouraging results. However, the specification of the penalty function is not completely theoretically justified and needs to addressed more thoroughly.

It can also be seen from the figures that a unique maximum point of the
likelihood function can possibly be attained by using line search optimization methods which is examined in more detail in the next section.

4.3 Optimization results

In addition to the likelihood simulations presented above, some optimization procedures were applied to the problem. We used constant nonlinear optimization techniques implemented in the Matlab function \textit{fmincon}. The \textit{fmincon} function requires the calculation of gradients which is difficult and possibly impossible to do analytically in our problem. This challenge was overcome by using Matlab’s in-built procedures to calculate gradients with methods of finite-differencing. However, the cost of this approach is that it increases the computational time required.

The optimization problem is of the following form
\[
\min_{\mu_x, \sigma^2_x, \sigma^2_\epsilon} \ln P(b^{1}_{m_1}, b^{2}_{m_2}, \ldots, b^{n}_{m_n})
\]
\[
s.t. \quad ub_a \geq a_i \geq lb_a
\]
\[
ub_{\sigma^2_x} \geq \sigma^2_x \geq lb_{\sigma^2_x}
\]
\[
ub_{\sigma^2_\epsilon} \geq \sigma^2_\epsilon \geq lb_{\sigma^2_\epsilon}
\]
\[
(11)
\]

where \(ub\)s and \(lb\)s are lower bounds for their respective variables. The optimization techniques were applied to penalized forms of likelihood functions, due to their possible unique maximums. Optimization was conducted five times for both data sets using discrete project mean values as initial values for optimization. The initial values for variances are kept constant over all the calculations at \(\overline{\sigma^2_x} = 1.5\) and \(\overline{\sigma^2_\epsilon} = 1.0\).

Above optimization results support the findings from the likelihood simulations in previous chapter. In tables 3 and 4 we can see that both data sets seen to have relatively stable maximums, and the optimization algorithm converges towards the same optimum from all the tested initial values. The results presented in table 3 are analogous to the likelihood simulations.
Table 3: *Optimization results when upper bound for the project variance is set at $\sigma_x^2 = 2.5$.*

<table>
<thead>
<tr>
<th>$\bar{\mu}_x$</th>
<th>Value</th>
<th>$\mu_x$</th>
<th>$\sigma_x^2$</th>
<th>$\sigma_\epsilon^2$</th>
<th>Value</th>
<th>$\mu_x$</th>
<th>$\sigma_x^2$</th>
<th>$\sigma_\epsilon^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>$-782.31$</td>
<td>0.9801</td>
<td>1.0990</td>
<td>0.8443</td>
<td>$-182.26$</td>
<td>1.0000</td>
<td>1.6790</td>
<td>0.8604</td>
</tr>
<tr>
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<td>$-782.31$</td>
<td>0.9820</td>
<td>1.0993</td>
<td>0.8448</td>
<td>$-182.26$</td>
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<td>$-182.26$</td>
<td>1.0000</td>
<td>1.6790</td>
<td>0.8601</td>
</tr>
</tbody>
</table>

Table 4: *Optimization results when upper bound for the project variance is set at $\sigma_x^2 = 1.0$.*

<table>
<thead>
<tr>
<th>$\bar{\mu}_x$</th>
<th>Value</th>
<th>$\mu_x$</th>
<th>$\sigma_x^2$</th>
<th>$\sigma_\epsilon^2$</th>
<th>Value</th>
<th>$\mu_x$</th>
<th>$\sigma_x^2$</th>
<th>$\sigma_\epsilon^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>$-785.40$</td>
<td>0.9032</td>
<td>1.0000</td>
<td>0.8296</td>
<td>$-198.27$</td>
<td>0.5968</td>
<td>1.0000</td>
<td>0.8068</td>
</tr>
<tr>
<td>0.4</td>
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<td>0.9031</td>
<td>1.0000</td>
<td>0.8299</td>
<td>$-198.27$</td>
<td>0.5968</td>
<td>1.0000</td>
<td>0.8075</td>
</tr>
<tr>
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<td>0.9032</td>
<td>1.0000</td>
<td>0.8297</td>
<td>$-198.27$</td>
<td>0.5979</td>
<td>1.0000</td>
<td>0.8080</td>
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<tr>
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<td>0.9022</td>
<td>1.0000</td>
<td>0.8302</td>
<td>$-198.27$</td>
<td>0.5965</td>
<td>1.0000</td>
<td>0.8075</td>
</tr>
<tr>
<td>1.0</td>
<td>$-785.40$</td>
<td>0.9030</td>
<td>1.0000</td>
<td>0.8298</td>
<td>$-198.27$</td>
<td>0.5975</td>
<td>1.0000</td>
<td>0.8075</td>
</tr>
</tbody>
</table>
with the penalized likelihood function and it can be seen that the algorithm converges to those maximums that can be depicted in the likelihood surface figures (8 - 13). Results presented in table 4 are obtained by restricting the project value variance to exceed value 1.0. It can be seen that the optimization proceeds as expected as it converges to the upper bound of the allowed project variance interval. However, it should be noted that the optimal mean values differ significantly from the optimal values relatively unrestricted other optimization problem. The difference being stronger with the sub set results. This effect is most likely heavily effected by the chose of penalty function and penalty parameters. One way to examine this in more detail would be to apply the above optimization procedures to the likelihood function with different penalty parameters and with different penalty function specifications. This is, however, omitted from this study. It should also be noted that the error variance parameters are fairly stable accross all the optimization with values around 0.8, which implies that the relationship between error variance and likelihood function is not affected by the choice of penalty parameters to same extend as is the case with project value variance.

5 Conclusions

In this study we have examined the possibility of enhancing project portfolio selection with decision analysis techniques to estimate the project value distributions. The estimation results base on the a priori collected expert statements about the qualities of the projects. Using these evaluation statements as unbiased estimators of the actual project value, we constructed a modeling framework to estimate the maximum likelihood parameters for the project value and error distributions, which maximize the probability of acquiring the percieved evaluation statements.

The model was empirically tested using a set of project proposals which were given evaluations in a five point scale. First, we calculated probabilities of acquiring the given evaluation vectors when the mean of the project value
distribution was fixed. Then we introduced a new likelihood function which had quadratic error terms to cope with the problem maximizing the value of the likelihood function at the extreme values of the variance parameters.

The results imply that the maximum likelihood parameters can be obtained through this kind of analysis. However, based on the results the likelihood function is maximized when the variance of project value distribution approaches to zero and also increasing the variance of expert evaluation distribution also increases the probability of acquiring the specific evaluation results. This phenomenon was tackled by using a penalized likelihood function and this enabled to make the likelihood function more suitable for this kind of analysis. The results obtained by using the penalized likelihood function however were very sensitive to changes in the magnitude of the penalty parameters. This implies that our analysis as it is requires inputs that are incorporate decision maker’s subjective information about the problem encountered.

The model is computationally onerous largely due to multiple permutations that need to be calculated as the number of given evaluations increase. This is not a major issue in this analysis as the model is designed to help in decision making problems that do not need immediate excercision. However, one possible direction for future development would be better optimization of the calculation procedures due to that when the number of evaluations are close to several hundreds or thousands, the calculation time increases significantly.

All in all, this study revealed that Salo’s (2008) model is capable of producing unique optimums for the project value maximization problems and the results were quite promising. However, the strong effect of the penalty function selection needs addressed more thoroughly in order to develop a model that can be used to as an objective proxy in the decision making problems. In addition, project value evaluation and selection incorporate several other questions yet to be tackled. One of the most significant pitfalls to overcome is to find an efficient way to cope with optimizer’s curse and postdecision
regret. One possible solution which was proposed by Smith and Winkler is to use Bayesian based approach to manage the postdecision regret. However, this among other open question will be left for future research.
References


