Preference Information
in Multicriteria Games

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Notations

\( I = \{1, \ldots, l\} \)  
Strategy set of player 1

\( J = \{1, \ldots, m\} \)  
Strategy set of player 2

\( i \in I \)  
Strategy of player 1

\( j \in J \)  
Strategy of player 2

\( X = \{(x_1, \ldots, x_l) | \sum x_k = 1, x_k \geq 0\} \)  
Set of mixed strategies of player 1

\( Y = \{(y_1, \ldots, y_m) | \sum y_k = 1, y_k \geq 0\} \)  
Set of mixed strategies of player 2

\( x \in X \)  
Mixed strategy of player 1

\( y \in Y \)  
Mixed strategy of player 2

\( v_1(x, y) \)  
Expected payoff for player 1

\( v_2(x, y) \)  
Expected payoff for player 2

\( F_k \in \mathbb{R}^{l \times m} \)  
Payoff matrix for player 1, criterion \( k \)

\( G_k \in \mathbb{R}^{l \times m} \)  
Payoff matrix for player 2, criterion \( k \)

\( W \)  
Weight set representing the incomplete preference information of player 1

\( U \)  
Weight set representing the incomplete preference information of player 2

\( w \in W \)  
Weight vector representing the preferences of player 1

\( u \in U \)  
Weight vector representing the preferences of player 2

\( b > a \)  
\( b \) is preferred to \( a \)

\( b \succeq_a a \)  
\( b \) is weakly preferred to \( a \)

\( b >_W a \)  
\( b \) is preferred to \( a \) in weight set \( W \)

\( b \succeq_W a \)  
\( b \) is weakly preferred to \( a \) in weight set \( W \)

\( i' >_W (W|j) i \)  
Strategy \( i' \) dominates strategy \( i \) conditional to strategy \( j \) in weight set \( W \)

\( i' \succeq_W (W|j) i \)  
Strategy \( i' \) is weakly preferred to strategy \( i \) conditional to strategy \( j \) in weight set \( W \)

\( i' >_W (W|J') i \)  
Strategy \( i' \) dominates strategy \( i \) conditional to strategy set \( J' \) in weight set \( W \)

\( i' \succeq_W (W|J') i \)  
Strategy \( i' \) weakly dominates strategy \( i \) conditional to strategy set \( J' \) in weight set \( W \)

\( I_R \)  
Set of rationalizable strategies of player 1

\( J_R \)  
Set of rationalizable strategies of player 2

\( I_{CR} \)  
Set of cautiously rationalizable strategies of player 1

\( J_{CR} \)  
Set of cautiously rationalizable strategies of player 2

\( I_{ND} \)  
Set of non-dominated strategies in the decision model
1 Introduction

Game theory [1, 2] analyzes strategic interactions between several decision makers each of whom maximizes their own payoff. In traditional game theory the payoff values are real numbers, so that any two payoff values can be compared in a straightforward manner. In multicriteria games [3, 4] the outcomes are evaluated with respect to several, possibly conflicting, criteria, and thus the payoffs must be represented as vectors. These vector valued payoffs cannot be generally compared without further specification of the preferences of the players.

Multicriteria decision analysis [5, 6, 7] is a field where problems with multiple conflicting objectives are analyzed. Modeling of the preferences of the decision makers is a significant topic in decision analysis. In preference programming [8], incomplete information about preferences is modeled by restricting sets of possible weights that represent the preferences. In contrast, existing work on multicriteria games does not explicitly consider preference modeling.

In this study, the multicriteria game between two players is interpreted as two multicriteria decision problems that the players face simultaneously. The main goal of this study is to develop a multicriteria game model that takes into account incomplete information about the preferences of the players. A decision analysis method consisting of two stages is presented. In the first stage, possible strategies are analyzed using common knowledge about the preferences. Alternatives that a decision maker satisfying certain rationality requirements never selects are excluded. If the first stage does not produce a unique solution, the decision maker proceeds to analyze the situation as a multicriteria decision problem using additional information about his own preferences. The information about what strategies the opponent may select based on the first stage reduces the dimension of this decision problem. The decision making stage is considered only briefly in this study. A general presentation over multicriteria decision analysis can be found, e.g., in [5, 6, 7].

This study is organized as follows. A survey of existing work is presented in Section 2. The multicriteria game model developed in this study is described in Section 3. In Section 4, the effect of additional preference information to the solution of the game model is analyzed. Solving the game model in practice is described in Section 5 and an illustrative example presented in Section 6. Conclusions are drawn in Section 7.

2 Multicriteria Games in Literature

Multicriteria game models can be used to analyze situations where there are multiple agents who each have multiple, possibly conflicting, objectives and the outcome of the game depends on actions of all agents. Multicriteria games were first studied by Shapley [3] and Blackwell [4]. In the general two-player
game model, both players select their strategies simultaneously from finite sets of possible strategies. Depending on the strategies selected by players, each player then receives a \( n \)-dimensional payoff vector, where \( k \)-th component of the vector describes his payoff value related to \( k \)-th criterion. Available strategies and payoffs associated to them are assumed to be common knowledge. Formally, the game is defined as follows.

- Sets of available strategies: \( I = \{1, \ldots, l\} \) for player 1 and \( J = \{1, \ldots, m\} \) for player 2
- \( l \times m \)-dimensional payoff matrices \( F_k \) and \( G_k \) associated to each criterion \( k \in \{1, \ldots, n\} \) so that element \((i, j)\) of matrix \( F_k \) is the payoff associated to criterion \( k \) for player 1 and element \((i, j)\) of matrix \( G_k \) for player 2. In a zero sum game \( \forall k \in \{1, \ldots, n\} : G_k = -F_k \), i.e., player 2 minimizes each criterion that player 1 maximizes.

As in scalar games, mixed strategies for multicriteria games are defined as probability distributions over the available strategies. A mixed strategy for player 1 is a vector \( x \in X = \{(x_1, \ldots, x_l)\mid \sum x_k = 1, x_k \geq 0\} \) and for player 2 \( y \in Y = \{(y_1, \ldots, y_m)\mid \sum y_k = 1, y_k \geq 0\} \), where \( x_i \) is the probability of player 1 selecting strategy \( i \in I \) and \( y_j \) the probability of player 2 selecting strategy \( j \in J \).

The random events of selecting actual strategies from the distributions implied by the mixed strategies are assumed to be independent, thus probability of strategy pair \((i, j)\) being selected is \( x_i \cdot y_j \). Expected criterion \( k \) payoff for player 1 is therefore \( \sum_i \sum_j x_i y_j F_k(i, j) \), or in vector form \( x^T F_k y \). Expected vector payoffs as a function of mixed strategies \( x, y \) are then \( v_1(x, y) = (x^T F_1 y, \ldots, x^T F_n y) \) for player 1 and \( v_2(x, y) = (x^T G_1 y, \ldots, x^T G_n y) \) for player 2. The objectives of the players are then to maximize the expected payoff values related to each of their own criteria. In conclusion, the game can be represented as follows.

\[
\begin{align*}
\text{Player 1:} & \quad \max v_1(x, y) = (x^T F_1 y, \ldots, x^T F_n y), \quad x \in X \\
\text{Player 2:} & \quad \max v_2(x, y) = (x^T G_1 y, \ldots, x^T G_n y), \quad y \in Y
\end{align*}
\]

where the vector maximization denotes that players want to maximize each component of the payoff vectors.

### 2.1 Modeling of Preferences

#### 2.1.1 Dominance Relations

In order to define solution concepts for the game, relations for comparing payoffs are introduced [9]. Let \( \mathbf{a} = (a_1, \ldots, a_n) \) and \( \mathbf{b} = (b_1, \ldots, b_n) \) be two \( n \)-dimensional vector payoffs. Payoff \( \mathbf{b} \) is weakly preferred to \( \mathbf{a} \), denoted \( \mathbf{b} \succeq \mathbf{a} \) if and only if

\[
\forall k \in \{1, \ldots, n\} : b_k \geq a_k.
\]
Then, payoff $b$ is weakly preferred to $a$ when $b$ is at least as good as $a$ in every criterion. Payoff $b$ is preferred to $a$, denoted $b \succ a$ if and only if

$$\begin{cases} \forall k \in \{1, \ldots, n\}: b_k \geq a_k \\ \exists k \in \{1, \ldots, n\}: b_k > a_k \end{cases}$$

That is, payoff $b$ is at least as good as $a$ with respect to every criterion and strictly better with respect to at least one criterion. If the above conditions do not hold, payoff $b$ is not preferred to $a$, denoted by $b \ntriangleright a$. Note that it is possible that payoff $b$ is better than $a$ with respect to some criterion and payoff $a$ is better than $b$ with respect to another criterion. In that case, the payoffs $a$ and $b$ are incomparable as both $b \ntriangleright a$ and $a \ntriangleright b$. Then, one cannot determine which payoff is preferred based on dominance only.

### 2.1.2 Modeling of Preferences with Weight Vectors

The dominance relations defined above do not take into account relative importance of the criteria. Weakness in an unimportant criterion may cause an otherwise clearly better alternative to be non-dominating. Borm et al. [10] and Corley [11] introduce weighting of criteria by defining weight vectors $w = (w_1, \ldots, w_n), \sum w_k = 1$ and $u = (u_1, \ldots, u_n), \sum u_k = 1$. Payoffs are compared with respect to a weight vector $w$ where weight $w_k$ describes the relative importance of criterion $k$ in comparison to other criteria. Total value of a vector $a$ is defined as $\sum_k w_k a_k$, the weighted sum of criterion values, denoted in vector form by $w^T a$. Vector $b$ is preferred to $a$ conditional to weights $w$ if $w^T b > w^T a$, i.e., the total value of $b$ is higher, $\sum_k w_k b_k > \sum_k w_k a_k$. A corresponding scalar game is formulated by defining the objectives of the players as maximizing the weighted sums of expected vector payoffs:

$$\begin{align*}
\text{Player 1: } & \max w^T v_1(x, y) = \sum_i \sum_j \sum_k x_i y_j w_k F_k(i, j), \ x \in X \\
\text{Player 2: } & \max u^T v_2(x, y) = \sum_i \sum_j \sum_k x_i y_j u_k G_k(i, j), \ y \in Y
\end{align*}$$

(4)

In the aforementioned articles, the weights are used mainly as technical means to prove the existence of Pareto equilibria, and no clear preference interpretation is given. Zeleny [12] uses the same weights for both players, which implies an assumption that the players value the criteria in similar manner. As far as the author knows, no existing work on multicriteria games consider presentation of incomplete preference information by restricting the sets of feasible weights.

### 2.2 Solution Concepts

In this section, Pareto equilibrium is presented as the solution of the multicriteria game. See [9] for other solution concepts. Pareto equilibria are defined as strategy pairs where the strategy of each player is non-dominated conditional to the strategy
of the other player. That is, a player has no clear incentive to deviate from his equilibrium strategy even if he knows the strategy of the opponent. If for a strategy pair \((x^*, y^*)\):

\[
\begin{align*}
\forall x \in X : v_1(x^*, y^*) &\not\succ v_1(x, y^*) \\
\forall y \in Y : v_2(x^*, y^*) &\not\succ v_2(x^*, y)
\end{align*}
\]

the strategy pair is said to be a Pareto equilibrium of the game. [11]

A Pareto equilibrium always exists [11], possibly consisting of mixed strategies. Each Pareto-equilibrium corresponds to a Nash equilibrium of the scalar game defined by Eq. 4 with some strictly positive weights \(w\) and \(u\) [10]. Conversely, each Nash-equilibrium of the scalar game with strictly positive weights is a Pareto equilibrium of the vector game.

### 3  Multicriteria Game Models in Decision Support

In this study, multicriteria games are intended to be used as support for decision making. The decision analysis method presented here consists of two stages:

1. *The game model.* Payoffs, rationality of opponents, and preference information are assumed to be common knowledge. The goal of this stage is to determine what strategies the opponent might employ. In an ideal situation, all but one alternative are excluded.

2. *The decision model.* Strategy selection is analyzed as a decision problem using private preference information. The strategy selection by the opponent is considered as an external uncertain event in the decision model, using the information obtained on stage 1. Finally, the strategy is selected using the methods of multicriteria decision analysis [5, 6, 7].

The preference information contained in the weighted sums of payoff values is assumed to imply only the preference order of the payoffs - a higher value is preferred to a lower value but the magnitude of the difference is not defined. These values do not contain information about how the decision maker should select between uncertain outcomes. Modeling preferences under uncertainty would require specification of *utilities*, i.e., real numbers associated to each possible outcome such that the decision makers prefer to maximize expected utility. Articles about utility theory and issues related to decision making under uncertainty can be found in [13]. However, in this study, utility information is assumed to be unavailable.

In decision-making context, the use of mixed strategies is questionable as real-life decision makers generally do not randomize their decisions. Also, in a mixed
strategy equilibrium, there is never a strict incentive to play a mixed strategy as all positive-probability pure strategies give the same expected payoff values [14]. For these reasons, mixed strategies are not included as feasible alternatives in this study. Thus, the strategy sets of the players consist only of the pure strategies.

It is assumed that the players play the game only once and do not communicate before playing. Thus, they can predict the actions of each other only based on common knowledge. Two different solution concepts to the game are presented: equilibrium and rationalizable strategies. Equilibria and equilibrium strategies are defined similarly to the Pareto equilibria presented in Section 3. In an equilibrium, neither player has a clear incentive to deviate from an equilibrium strategy if the opponent plays the corresponding equilibrium strategy. However, as mixed strategies are excluded, neither uniqueness nor existence of these equilibria is guaranteed and they are inadequate in predicting which strategies might be played. As the players do not communicate, they cannot select between multiple equilibria.

A rational player is assumed to maximize his own payoff using all information available. Rationalizable strategies are sets of strategies that such a rational player may justifiably play when payoffs, the preference information, and rationality of both players are common knowledge. Whereas an equilibrium is a pair of single strategies, rationalizable strategies are a pair of strategy sets. For any given game there is a unique maximal set of rationalizable strategies, that consists of exactly those strategies that a rational player could be expected to play when rationality and the specification of the game model are common knowledge. Any of these strategies is a non-dominated selection against some plausible belief about what strategies the opponent may select. Cautiously rationalizable strategies are derived from the sets of rationalizable strategies by invoking an additional assumption that the players will not select strategies that are weakly dominated.

3.1 Preferences

If preference order of payoffs for each criterion are independent of values obtained for other criteria, i.e., a higher payoff in criterion \( k \) is always better, the criteria are mutually preferentially independent [5]. When mutual preferential independence holds, the preferences can be represented by an additive value function, i.e., the value of an outcome is a product of a weight vector and the payoff vector [5]. For weight vector \( \mathbf{w} \) and outcome \( \mathbf{b} \) the value is \( \mathbf{w}^T \mathbf{b} = \sum_k w_k b_k \). In this study, weights of player 1 are denoted by \( \mathbf{w} \) and weights of player 2 by \( \mathbf{u} \).

Complete information about the preferences of the players would amount to knowing the exact values of weight vectors \( \mathbf{w} \) and \( \mathbf{u} \). This would lead to a scalar game where players just maximize their respective additive value functions. In this study, incomplete preference information [8] is assumed for both players, i.e., the weights are not known precisely but are restricted to sets \( \mathbf{w} \in W \) and \( \mathbf{u} \in U \). The weights are not modeled as random variables, that is, no probability distribution for the weight vectors over the sets \( W \) and \( U \) is assumed. Instead, the analysis
focuses on what conclusions can be made using information about what values of \( w \) and \( u \) are feasible. In the game model of the first stage, the sets \( W \) and \( U \) are assumed to be common knowledge. Thus, only preference statements known to both players are considered. In the latter stage, the decision maker uses his private preference information, a set \( W_{\text{private}} \subset W \), in addition to information obtained in the analysis of the first stage.

### 3.2 The Game Model

Formally, the multicriteria game model with incomplete preference information consists of the following components:

- Sets of available strategies: \( I = \{1, \ldots, l\} \) for player 1 and \( J = \{1, \ldots, m\} \) for player 2.
- \( l \times m \)-dimensional payoff matrices \( F_k \) and \( G_k \) associated to each criterion \( k \in \{1, \ldots, n\} \) so that element \((i, j)\) of matrix \( F_k \) is the payoff associated to criterion \( k \) for player 1 and element \((i, j)\) of matrix \( G_k \) for player 2.
- The weight set \( W \) of player 1 and the weight set \( U \) of player 2.

Mixed strategies are excluded, so the payoff vectors are a function of chosen strategies \((i, j)\): \( f(i, j) = (F_1(i, j), \ldots, F_n(i, j)) \) for player 1 and \( g(i, j) = (G_1(i, j), \ldots, G_n(i, j)) \) for player 2.

The differences of this model and the multicriteria game model of Section 3 are the introduction of the weight sets \((W, U)\) and exclusion of mixed strategies. In the view of the author, these elements present a more suitable model for decision making.

#### 3.2.1 Comparing Multidimensional Payoff Vectors with respect to Sets of Weights

When preferences are exactly defined, payoffs are compared with respect to a weight vector \( w \) where weight \( w_k \) describes the relative importance of criterion \( k \). Vector \( b \) is preferred to \( a \) conditional to weights \( w \) if \( w^T b > w^T a \).

With incomplete preference information, payoffs are compared with respect to the sets of feasible weights. Payoff \( b \) is weakly preferred to payoff \( a \) in a set of weights \( W \), denoted by \( b \succeq_W a \) if and only if

\[
\forall w \in W : w^T b \geq w^T a. \tag{6}
\]

Weak preference means that payoff \( b \) is at least as good as \( a \) conditional to any weight vector in the weight set. If for at least some weight vector in \( W \) payoff \( b \)
is better, \( b \) is preferred to payoff \( a \) in a set of weights \( W \). Preference is denoted by \( b \succ_W a \) and holds if and only if

\[
\begin{align*}
\forall w \in W : w^T b & \geq w^T a \\
\exists w \in W : w^T b & > w^T a
\end{align*}
\]  

(7)

### 3.2.2 Dominance Relations between Strategies

To define solution concepts that take into account incomplete preference information, relations for comparing strategies with respect to weight sets are defined. Strategy \( i' \) dominates strategy \( i \) conditional to opponent strategy \( j \) in weight set \( W \), denoted by \( i' \succ (W|j) i \), if and only if the payoff resulting from strategies \((i', j)\) is preferred to the payoff of strategies \((i, j)\) in weight set \( W \), i.e., \( f(i', j) \succ_W f(i, j) \).

In other words, when the opponent plays \( j \), the payoff resulting from playing \( i' \) is at least as good as the payoff received by playing \( i \) with respect to all weight vectors in \( W \) and strictly better with respect to at least one weight vector in \( W \).

Strategy \( i' \) dominates strategy \( i \) conditional to a set of strategies \( J' \), \( i' \succ (W|J') i \), if

\[
\forall j \in J' : i' \succ (W|j) i.
\]  

(8)

The dominating strategy \( i' \) is a better choice than \( i \) against any opponent strategy selected from set \( J' \). Strategy \( i' \) weakly dominates strategy \( i \) conditional to a set of strategies \( J' \) if

\[
\begin{align*}
\forall j \in J' : & \ i' \succeq (W|j) i \\
\exists j \in J' : & \ i' \succ (W|j) i
\end{align*}
\]  

(9)

The weakly dominating strategy \( i' \) is at least as good as strategy \( i \) against any opponent strategy in set \( J' \) and better for at least some opponent strategy in set \( J' \).

### 3.2.3 Equilibrium

Strategy pair \((i^*, j^*)\) is defined to be an equilibrium solution of the game with respect to preference information \((W, U)\) if

\[
\begin{align*}
\forall i \in I : & \ i \not\succ_{(W|j^*)} i^* \\
\forall j \in J : & \ j \not\succ_{(U|i^*)} j^*
\end{align*}
\]  

(10)

An equilibrium is a strategy pair such that the strategy selection of each player is non-dominated conditional to the equilibrium strategy of the opponent. Thus,
even if the players know the strategies chosen by their opponents, they have no clear incentive to deviate from the equilibrium based on the preference information. However, it is possible that with more accurate preference information a player would want to deviate from some equilibrium strategies. Denote the set of pairs \((i^*, j^*)\) satisfying Eq. 10 by \(E(W, U)\). If the sets of weights are unrestricted, this is equivalent to Pareto equilibrium restricted to pure strategies. When mixed strategies are excluded, existence and uniqueness of equilibria is not guaranteed [1].

### 3.2.4 Rationalizable Strategies

Pearce [15] defines rationalizable strategies as sets of strategies that are best responses given some belief of opponent’s strategy that assumes the opponent will play a rationalizable strategy. For example, suppose that strategy 1 is rationalizable for player 2. Then, player 1 may justifiably hold a belief that player 2 will play 1. Then, if strategy 1 is an optimal response for player 1 to strategy 1 of player 2, strategy 1 is rationalizable for player 1. Formally, sets of strategies \((I_R, J_R)\) are rationalizable if they satisfy the following property:

\[
\begin{align*}
\forall i \in I_R, i' \in I : & \ i' \not\in (W | J_R) i \\
\forall j \in J_R, j' \in J : & \ j' \not\in (U | I_R) j .
\end{align*}
\]

For any two pairs of sets satisfying Eq. 11, the unions of the sets also satisfy Eq. 11. Thus, the union of all sets satisfying Eq. 11 is also rationalizable. Therefore, there exist maximal sets \((I_R, J_R)\) for which Eq. 11 holds such that all other sets satisfying Eq. 11 are subsets of \((I_R, J_R)\). Rationalizable strategies are defined as the maximal sets \((I_R, J_R)\).

In two-player games, rationalizable strategies are equivalent to strategies that survive iterative elimination of dominated strategies [15].

1. Let \(I_{R(0)} = I\), \(J_{R(0)} = J\).
2. Repeat the following iteration:

\[
\begin{align*}
I_{R(k+1)} &= \{ i \in I | \forall i' \in I_{R(k)} : i' \not\in (W | J_{R(k)}) i \} \\
J_{R(k+1)} &= \{ j \in J | \forall j' \in J_{R(k)} : j' \not\in (U | I_{R(k)}) j \} .
\end{align*}
\]

First, the players know that neither player will play a dominated strategy, and thus eliminate them from the analysis, leading to sets \((I_{R(1)}, J_{R(1)})\). Then, at each step of the elimination process, it is known that the other player has performed the same analysis and rationalizable strategies are limited to \((I_{R(k)}, J_{R(k)})\). Therefore, both players conclude that they will not select strategies that are dominated conditional to opponent strategy sets \((J_{R(k)}, I_{R(k)})\). Elimination of these strategies leads to sets \((I_{R(k+1)}, J_{R(k+1)})\). For some finite \(k\) the iteration converges to the rationalizable strategies \((I_R, J_R)\). From these sets no strategies can be excluded.
based only on common knowledge of rationality. To emphasize the respective weight sets, the sets of rationalizable strategies are denoted as $I_R(W;U), J_R(W;U)$.

Pearce [15] argues that a reasonable further assumption is that the players will not take unnecessary risks among the rationalizable strategies. That is, if strategy $i'$ is at least as good as $i$ with respect to any strategy that the opponent might play, and better than $i'$ for some strategy that the opponent might play, there is no reason to play $i$ rather than $i'$, and thus $i$ can be eliminated. This leads to cautious rationalizability where weakly dominated strategies are eliminated.

Cautiously rationalizable strategies are solved iteratively by first solving the rationalizable strategies. Then, from these sets, weakly dominated strategies are removed. These two steps are repeated until the process converges to sets that contain neither dominated nor weakly dominated strategies.

1. Let $I_{CR(0)} = I_R$, $J_{CR(0)} = J_R$.

2. 
   \[
   \begin{align*}
   I_{CR(k+1)} &= \{i \in I | \forall i' \in I_{CR(k)} : i' \nsubseteq (W|J_{CR(k)} i) \} \\
   J_{CR(k+1)} &= \{j \in J | \forall j' \in J_{CR(k)} : j' \nsubseteq (U|I_{CR(k)} j) \}
   \end{align*}
   \] (13)

3. Let $I_{CR(k+1)}, J_{CR(k+1)}$ be the sets of rationalizable strategies that result from starting the iterative elimination described in Eq. 12 from sets $I_{CR(k+1)}, J_{CR(k+1)}$. Go to step 2.

This process converges in a finite number of steps to sets $I_{CR}$ and $J_{CR}$ that contain neither dominated nor weakly dominated strategies.

### 3.3 The Decision Model

In the second stage of the analysis, the decision maker takes into account private information about his own preferences. The preferences are restricted into a set $W_{private} \subset W$.\footnote{Notations in this section are written from the point of view of player 1. Player 2 may perform similar analysis using his own private preference information.} This information is not common knowledge. Therefore, the opponent is not able to perform the same analysis. Thus, the solutions for each player in this stage are not required to be consistent with each other, but the strategy selection of the opponent can be regarded as an external uncertain event. If the set of cautiously rationalizable strategies for the opponent is restricted to a single strategy $j^*$, the outcome achieved with each strategy is known and thus only uncertainty besides errors in the specification of the model is the incomplete preference information. On the other hand, if there are several possible strategies the opponent may play, the decision maker is facing a decision problem under external uncertainty.
3.3.1 Decision with Known Opponent Strategy

If strategy $i'$ dominates $i$ in the private weight set, the player will not prefer selecting it, no matter what the actual weights are. Therefore, the decision maker should select his strategy among the strategies that are non-dominated in $W_{private}$.

$$I_{ND} = \{ i \in I_R | \forall i' \in I_R : i' \not\succ W_{private} \cup i \}$$

If there is no unique non-dominated strategy, the decision maker can further restrict the weight set. Obviously, this restriction cannot be arbitrary, but is based on preferences of the decision maker. Methods for eliciting preference information are discussed, e.g., in [16] and references therein. If the weight set cannot be reasonably restricted to produce a unique non-dominated strategy, the decision maker may select among the strategies in $I_{ND}$ using a decision rule. Possible decision rules suggested in decision analysis literature are, e.g., using representative parameters, i.e., a weight vector $w_0 \in W_{private}$ that is near the center of $W_{private}$ and minimax regret, that is, choosing a strategy that has smallest maximum loss of value compared to the best option, maximized over all possible weight vectors [8].

3.3.2 Decision with Unknown Opponent Strategy

If strategy $i'$ is better than $i$ independent of the actual weights and the opponent strategy, the decision maker should not select $i$. Therefore, the decision maker should select his strategy among the strategies that are non-dominated in $W_{private}$ conditional to the rationalizable strategies $J'$ of the opponent.

$$I_{ND} = \{ i \in I_R | \forall i' \in I_R : i' \not\succ W_{private} \cup J \}$$

As in the case of decision making under known opponent strategy, if there is no unique non-dominated strategy, the decision maker may further restrict the weight set. However, even exact knowledge of the weights does not guarantee a unique non-dominated strategy as the payoffs depend on the strategy selected by the opponent. The decision maker can gain insight about possible outcomes of the situation by scenario analysis, i.e., considering the strategy selection separately with respect to each possible strategy of the opponent. This scenario analysis gives insight about how the situation may evolve. However, when likelihoods of the scenarios is not considered, the approach does not lead to a recommendation about what strategy should be selected. If the decision maker can state probabilities for possible opponent actions and associate utilities to the payoffs, the strategy can be selected by maximizing expected utility [13]. If such information is not available, the decision maker should use a decision rule such that it can be applied under multiple scenarios, for example minimax regret considering maximum loss of value over all possible weight vectors and rationalizable strategies of the opponent.
general presentation over multicriteria decision making under uncertainty can be found, e.g., in [5, 6].

4 The Effect of Additional Preference Information

In this section, the effect of additional preference information to the solutions of the game model presented in Section 3 is analyzed. Gaining additional information about preferences of the player is modeled in this study by restricting the weights in a set $W' \subset W$. It is assumed that the true weight vector is never contained in the boundary of $W$, so that some points of the relative interior of $W$ are contained in the new set $W'$, i.e., $\text{int}(W) \cap W' \neq \emptyset$.\footnote{The relative interior is defined as $\text{int}(W) = \{ w \in W : \forall w' \in W \exists \delta > 0 \text{ s.t. } w + \epsilon (w - w') \in W \forall \epsilon \in [0, \delta] \}$. See [17] for a further explanation of why this assumption is necessary.} With this assumption, it is shown that additional information does not add new strategies to the equilibria and the sets of rationalizable strategies. A similar result is shown in [17] for sets of non-dominated portfolios in robust portfolio modeling.

Lemma 1 states that any strategy that is dominated in set $W$ is also dominated in $W'$. Based on this, Theorem 1 shows that there are no new equilibria with $W'$. Theorem 2 shows that the sets of rationalizable strategies with respect to $(W', U)$ will not contain any new strategies. As the definitions are symmetric between players, these results directly generalize to restriction of the weights of both players to sets $W' \subset W$ and $U' \subset U$.

Therefore, additional preference information leads to a more accurate solution. Furthermore, incompleteness of preference information does not cause excluding any strategies that would be contained in the solutions with more complete preference information. However, a similar result does not hold for cautiously rationalizable strategies. A counterexample is given in Remark 1.

**Lemma 1.** Let $W' \subset W$ and $\text{int}(W) \cap W' \neq \emptyset$. $i, i' \in I$ and $j \in J$. If $i' \succ_{(W|j)} i$, then $i' \succ_{(W'|j)} i$.

**Proof.** By definition, strategy $i'$ dominates strategy $i$ conditional to strategy $j$, if the payoff of strategy pair $(i, j)$ is preferred to the payoff of strategy pair $(i', j)$ in weight set $W'$, i.e., $f(i', j) \succ_{W'} f(i, j)$.

First, it is shown that $f(i', j)$ is weakly preferred to $f(i, j)$.

\[
\begin{align*}
 i' \succ_{(W|j)} i & \implies f(i', j) \succ_{W} f(i, j) \\
 & \implies \forall w \in W : \text{ w}' f(i', j) \ge w' f(i, j)
\end{align*}
\]
Weight set $W'$ is a subset of $W$, hence this holds also for all $w \in W'$. Thus,
\[
\Rightarrow \quad \forall w \in W' : \ w^T f(i', j) \geq w^T f(i, j)
\]
\[
\Rightarrow \quad f(i', j) \succeq_{W'} f(i, j) \Rightarrow i' \succeq_{(W' \cup j)} i
\]

Next, it is shown that the preference is strict, i.e., the second condition in Eq. 7 holds: $\exists w \in W' : \ w^T f(i', j) > w^T f(i, j)$.

It was assumed that $f(i', j) \succ_w f(i, j)$, therefore $\exists w^* \in W : \ w^T f(i', j) > w^T f(i, j)$. There exists a $w^* \in \text{int}(W) \cap W'$ as it was assumed that $\text{int}(W) \cap W' \neq \emptyset$. Because $w' \in \text{int}(W)$, for some $\epsilon > 0 \exists w^0 \in W : w^0 = w' + \epsilon(w' - w^*)$. Now,
\[
\begin{align*}
\frac{w^0}{T}(f(i', j) - f(i, j)) & = \left( \frac{1}{1+\epsilon} w^0 + \frac{\epsilon}{1+\epsilon} w^* \right) \cdot (f(i', j) - f(i, j)) \\
& = \frac{1}{1+\epsilon} w^0 + \frac{\epsilon}{1+\epsilon} w^* + \frac{\epsilon}{1+\epsilon} w^* \cdot (f(i', j) - f(i, j))
\end{align*}
\]
The first term is non-negative since $w^0 \in W$ and the second term is positive by the definition of $w^*$. Thus,
\[
w^T f(i', j) > w^T f(i, j).
\]

Both conditions of Eq. 7 hold and thus it is shown that $f(i', j) \succ_{W'} f(i, j)$.

\(\square\)

**Theorem 1.** If $W' \subset W$ and $\text{int}(W) \cap W' \neq \emptyset$, then $E(W', U) \subseteq E(W, U)$.

*Proof.* Let $(i^*, j^*) \in E(W', U)$. By Eq. 10,
\[
\forall i \in I : i \not\succeq_{(W' \cup j^*)} i^* \\
\forall j \in J : j \not\succeq_{(W' \cup i^*)} j^*
\]

Lemma 1 states that $i \succ_{(W' \cup j^*)} i^* \Rightarrow i \succ_{(W' \cup i^*)} j^*$. This is equivalent to $i \not\succeq_{(W' \cup j^*)} i^* \Rightarrow i \not\succeq_{(W' \cup i^*)} j^*$. Therefore,
\[
\forall i \in I : i \not\succeq_{(W' \cup j^*)} i^* \\
\forall j \in J : j \not\succeq_{(W' \cup i^*)} j^*
\]
and $(i^*, j^*) \in E(W, U)$. Thus, $E(W', U) \subseteq E(W, U)$.

\(\square\)

**Theorem 2.** If $W' \subset W$ and $\text{int}(W) \cap W' \neq \emptyset$, then $I_R(W', U) \subseteq I_R(W, U)$ and $J_R(W', U) \subseteq J_R(W, U)$

*Proof.* Lemma 1 states that $i \not\succeq_{W' \cup J_R(W', U)} i' \Rightarrow i \not\succeq_{W' \cup J_R(W, U)} i'$. The second condition of Eq. 11 is invariant under changing the weight set of the first player. Thus, $I_R(W', U), J_R(W', U)$ satisfy Eq. 11 with respect to $(W, U)$. The sets of rationalizable strategies $I_R(W, U), J_R(W, U)$ are defined as the maximal sets satisfying Eqs. 11. Therefore, $I_R(W', U) \subseteq I_R(W, U)$ and $J_R(W', U) \subseteq J_R(W, U)$.

\(\square\)
Remark 1. There exists a game and weight sets $W, W', U$ so that $W' \subset W$ and $\text{int}(W) \cap W' \neq \emptyset$, but $J_{CR}(W', U) \notin J_{CR}(W, U)$.

Proof. Assume a two-criteria game where both players have two strategies available. Let the preferences of player 2 be known perfectly, $U = \{u_0\} = \{(1, 0)\}$. Define the payoff matrices as follows:

$$F_1 = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \quad F_2 = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$$

$$G_1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad G_2 \text{ arbitrary}$$

Let $W = \{w \mid w_1, w_2 \geq 0, w_1 + w_2 = 1\}$. There are no dominated strategies, thus $I_R = \{1, 2\}, J_R = \{1, 2\}$. For player 2 strategy 2 weakly dominates strategy 1, thus $I_{CR}(W, U) = \{1, 2\}, J_{CR}(W, U) = \{2\}$. Let $W' = \{w \in W \mid w_1 > \frac{1}{2}\} \subset W$. For all $w \in W'$ the payoff of strategy 1 is better than the payoff of strategy 2, thus $I_R = \{1\}, J_R = \{1, 2\}$. When strategy 1 of player 1 is eliminated, neither strategy of player 2 is weakly dominated and $I_{CR}(W', U) = \{1\}, J_{CR}(W', U) = \{1, 2\}$. Thus, $J_{CR}(W', U) \notin J_{CR}(W, U)$. 

5 Solving the Game Model

Solving the game model consists of determining the sets of rationalizable or cautiously rationalizable strategies. These are determined by iterative eliminating the dominated strategies using Eqs. 12. The additive value function is a linear function of $w$. Therefore, the function attains maximum and minimum in extreme points of the set of feasible weights [18]. Hence, strategy $i$ is preferred to strategy $i'$ conditional to strategy set $J'$ in weight set $W$ if and only if conditions of Eq. 7 are satisfied in the extreme points of $W$. Thus, in the iterative elimination of dominated strategies, domination needs to be checked only in extreme points. The payoffs in extreme points are calculated as a linear combination of the original payoff matrices, $\sum_{k=1}^{n} w_k F_k$, denoted by $w^T F$.

If the sets $W$ and $U$ are restricted by linear inequalities, there is a finite number of extremal points. Then, the iterative elimination can be performed simply by checking for each pair of strategies $(i, i')$ whether the payoff of $i'$ is better in all extremal points with respect to all remaining strategies of the opponent. It may be possible that the rationalizable strategies can be solved using a vector linear programming formulation [19], without performing the iterative elimination procedure. In this study such an approach is not considered as the number of available strategies, criteria, and extremal point are assumed to be small, so that dominance can be checked manually. If the weight sets are restricted by nonlinear inequalities, there may be infinitely many extremal points. In general case, nonlinear optimization algorithms [20] have to be used to determine whether a strategy is dominated.
6 An Illustrative Example

Assume an air combat scenario where player 2 is attacking an air base of player 1. The criteria for player 1 are

1 Protection of the air base
2 Number of surviving aircraft of player 1
3 Number of destroyed aircraft of player 2

Each player has three available strategies, \( I = \{1, 2, 3\} \), \( J = \{1, 2, 3\} \), where strategy 3 denotes the most aggressive defense (attack for player 2) and strategy 1 is the least aggressive option.

The payoff matrices for player 1 are as follows:

\[
F_1 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0.8 & 0.5 \\ 1 & 0.9 & 0.6 \end{pmatrix}, \quad F_2 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0.75 & 0.625 \\ 1 & 0.5 & 0.375 \end{pmatrix}, \quad F_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.25 & 0.5 \\ 0 & 0.375 & 0.625 \end{pmatrix}.
\]

Player 2 is assumed to minimize each criterion of player 1, thus the game is zero-sum and:

\[
G_1 = \begin{pmatrix} -1 & 0 & 0 \\ -1 & -0.8 & -0.5 \\ -1 & -0.9 & -0.6 \end{pmatrix}, \quad G_2 = \begin{pmatrix} -1 & -1 & -1 \\ -1 & -0.75 & -0.625 \\ -1 & -0.5 & -0.375 \end{pmatrix}, \quad G_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -0.25 & -0.5 \\ 0 & -0.375 & -0.625 \end{pmatrix}.
\]

6.1 The Game Model with no Preference Information

Let us analyze the game with no preference information, that is,

\[
W = \{(w_1, w_2, w_3)| \sum_k w_k = 1, w_1, w_2, w_3 \geq 0 \}
\]

\[
U = \{(u_1, u_2, u_3)| \sum_k u_k = 1, u_1, u_2, u_3 \geq 0 \}
\]

With respect to criterion 1, strategy 3 is better than strategy 2, which is better than strategy 1. On the other hand, with respect to criterion 2, strategy 1 is better than strategy 2, and strategy 2 better than strategy 3. Thus, there are no dominated strategies for player 1. There are also no dominated strategies for player 2. Hence, the sets of rationalizable strategies for each player are the original

---

3 The protection of the air base is measured using the probability of the air base remaining unharmed conditional to selected strategies.

4 The numbers of surviving and destroyed aircraft are scaled to interval \([0, 1]\). Both sides are assumed to have equal number of aircraft at the beginning.
strategy sets, \( I_R = I \) and \( J_R = J \). Without the use of additional preference information, no strategies can be excluded based on the game model. Intuitively, importance of survival of own aircraft leads to a passive strategy and importance of destroying the opponent’s aircraft or battling over the air base lead to a more aggressive strategy. Thus, it is natural that without any preference information it cannot be predicted what strategies the players will choose.

6.2 The Game Model with Additional Preference Information

It is a generally acceptable assumption that for each player the survival of his own aircraft is more important than destroying the aircraft of the opponent. The new weight sets based on this information are:

\[
W' = \{(w_1, w_2, w_3) | \sum_k w_k = 1, w_1 \geq 0, w_2 \geq w_3 \geq 0\}
\]

\[
U' = \{(u_1, u_2, u_3) | \sum_k u_k = 1, u_1 \geq 0, u_3 \geq u_2 \geq 0\}
\]

The extremal points of \( W' \) are \( w'_1 = (1, 0, 0) \), \( w'_2 = (0, 1, 0) \), and \( w'_3 = (0, 0.5, 0.5) \). Payoffs for player 1 at these points are

\[
w'_1 \cdot F = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0.8 & 0.5 \\ 1 & 0.9 & 0.6 \end{pmatrix}
\]

\[
w'_2 \cdot F = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0.75 & 0.625 \\ 1 & 0.5 & 0.375 \end{pmatrix}
\]

\[
w'_3 \cdot F = \begin{pmatrix} 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.625 \\ 0.5 & 0.4375 & 0.5 \end{pmatrix}
\]

The extremal points of \( U' \) are \( u'_1 = (1, 0, 0) \), \( u'_2 = (0, 0.5, 0.5) \), and \( u'_3 = (0, 0, 1) \) and the payoffs for player 2 at the points are

\[
u'_1 \cdot G = \begin{pmatrix} -1 & 0 & 0 \\ -1 & -0.8 & -0.5 \\ -1 & -0.9 & -0.6 \end{pmatrix}
\]

\[
u'_2 \cdot G = \begin{pmatrix} -0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & -0.5625 \\ -0.5 & -0.4375 & -0.5 \end{pmatrix}
\]

\[
u'_3 \cdot G = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -0.25 & -0.5 \\ 0 & -0.375 & -0.625 \end{pmatrix}
\]
There are still no dominated strategies for either player.

Now, assume further additional preference information. Both players know that it is strategically important for player 2 to destroy the air base, so that the weight of criterion 1 is at least 0.8. The new weight set is

$$U'' = \{(u_1, u_2, u_3) \mid \sum_k u_k = 1, u_1 \geq 0.8, u_3 \geq u_2 \geq 0\}$$

The extreme points of $U''$ are $u_1'' = (1 \ 0 \ 0), u_2'' = (0.8 \ 0.2), u_3'' = (0.8 \ 0.1 \ 0.1)$. Payoffs for player 2 at these points are

$$u_1'' \cdot G = \begin{pmatrix} -1 & 0 & 0 \\ -1 & -0.8 & -0.5 \\ -1 & -0.9 & -0.6 \end{pmatrix}$$

$$u_2'' \cdot G = \begin{pmatrix} -0.8 & 0 & 0 \\ -0.8 & -0.69 & -0.5 \\ -0.8 & -0.795 & -0.605 \end{pmatrix}$$

$$u_3'' \cdot G = \begin{pmatrix} -0.9 & -0.1 & -0.1 \\ -0.9 & -0.74 & -0.51 \\ -0.9 & -0.81 & -0.58 \end{pmatrix}$$

Now, in all extreme points, the payoff of strategy 2 is better than the payoff of strategy 1, regardless of which strategy player 1 selects. Therefore, strategy 2 dominates strategy 1, which is eliminated.

The new payoff matrices for player 1 (taking into account only strategies \{2, 3\} of player 2) are

$$\begin{pmatrix} 0 & 0 \\ 0.8 & 0.5 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0.75 & 0.625 \end{pmatrix}, \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.625 \end{pmatrix}$$

There are still no dominated strategies for player 1, so the elimination of dominated strategies has converged to the sets of rationalizable strategies: $I_R = \{1, 2, 3\}, J_R = \{2, 3\}$. Based on rationalizability and the preference information used here one can conclude that player 2 will not select strategy 1, whereas all other strategies are possible. As the destruction of the air base is important for player 2, it is known that he wants to attack at least moderately aggressively.

The payoffs of strategy 3 of player 2 are better than the payoffs of strategy 2, if player 1 selects strategy 2 or 3 and equal if player 1 selects strategy 1. The importance of destroying the air base calls for an aggressive attack. However, strategy 2 is rationalizable, too, as if player 1 does not defend, a moderate attack is sufficient to destroy the air base. Nevertheless, if there is even a slight possibility that player 1 might defend, player 2 should select strategy 3. The concept of
cautiously rationalizable strategies takes this phenomenon into account. To solve for cautiously rationalizable strategies, weakly dominated strategies are eliminated. Player 1 has no weakly dominated strategies. For player 2, strategy 3 weakly dominates strategy 2 conditional to \( \{1, 2, 3\} \), as the payoff is at least as good with any strategy selection of player 1, and better if player 1 selects strategy 2 or 3. Therefore, strategy 2 is eliminated from \( J_{CR} \).

The new payoff matrices for player 1 (taking into account only strategy 3 of player 2) are

\[
\begin{pmatrix}
0 \\
0.5 \\
0.6
\end{pmatrix}, \quad \begin{pmatrix}
1 \\
0.625 \\
0.375
\end{pmatrix}, \quad \begin{pmatrix}
0.5 \\
0.625 \\
0.5
\end{pmatrix}
\]

Player 1 still has no (weakly) dominated strategies, thus the elimination has converged to the sets of cautiously rationalizable strategies \( I_{CR} = \{1, 2, 3\} \), \( J_{CR} = \{3\} \). Therefore, based on cautious rationalizability player 1 concludes that player 2 will select strategy 3, whereas player 2 cannot draw any conclusions about the strategy selected by player 1.

6.3 The Decision Model

The game model produced a unique solution for player 2. For player 1, no strategy was eliminated. Therefore, player 1 proceeds to use his private preference information, and the knowledge that player 2 will select strategy 3. In addition to the commonly known preference information \( W' \), assume that player 1 is able to state that the weight of protecting the air base is at least 0.5, which limits the set of feasible weights to:

\[
W_{private} = \{(w_1, w_2, w_3) | \sum_k w_k = 1, w_1 \geq 0.5, w_2 \geq w_3, w_3 \geq 0\}
\]

The extreme points of \( W_{private} \) are \( w_1^{(pr)} = (1 \ 0 \ 0) \), \( w_2^{(pr)} = (0.5 \ 0.5 \ 0) \) and \( w_3^{(pr)} = (0.5 \ 0.25 \ 0.25) \). Payoffs at these points are:

\[
w_1^{(pr)} \cdot F = \begin{pmatrix}
0 \\
0.5 \\
0.6
\end{pmatrix}, \quad w_2^{(pr)} \cdot F = \begin{pmatrix}
0.5 \\
0.56 \\
0.49
\end{pmatrix}, \quad w_3^{(pr)} \cdot F = \begin{pmatrix}
0.25 \\
0.53 \\
0.55
\end{pmatrix}
\]
Table 1: Strategy selection of player 1 in the decision model by employing different decision rules.

<table>
<thead>
<tr>
<th>Decision rule</th>
<th>Value of strategy 2</th>
<th>Value of strategy 3</th>
<th>Selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximax</td>
<td>0.56</td>
<td>0.55</td>
<td>Strategy 2</td>
</tr>
<tr>
<td>Maximin</td>
<td>0.49</td>
<td>0.5</td>
<td>Strategy 2</td>
</tr>
<tr>
<td>Central weights</td>
<td>0.53</td>
<td>0.55</td>
<td>Strategy 3</td>
</tr>
</tbody>
</table>

Strategy 2 is better than strategy 1 in all extreme points, and thus strategy 1 is dominated. The non-dominated strategies are \( I_{ND} = \{2, 3\} \). Based on the private preference information, player 1 is not able to decide between strategies 2 and 3 and has to either elicit further preference information or employ a decision rule.

### 6.3.1 Complete Preference Information

Assume player 1 knows his own preferences to be \( w_0 = (0.6 \; 0.4 \; 0) \). In other words, destroying the aircraft of the opponent is insignificant and protecting the air base is slightly more important than saving own aircraft. The values associated with each strategy are then

- Strategy 1: \( w_0^T (0 \; 1 \; 0)^T = 0.4 \)
- Strategy 2: \( w_0^T (0.5 \; 0.625 \; 0.5)^T = 0.55 \)
- Strategy 3: \( w_0^T (0.6 \; 0.375 \; 0.625)^T = 0.51 \)

Player 1 should select the strategy with highest value, i.e., strategy 2.

### 6.3.2 Strategy Selection with Decision Rules

Assume player 1 is not able to state his preferences more accurately than by restricting them in the set \( W_{private} \). In this case, player 1 employs a decision rule to select between strategies 2 and 3. Decision rules considered in this example are maximax where alternatives are compared according to highest possible value, maximin where alternatives are compared according to smallest possible value, and central weights where alternatives are compared by selecting a representative weight vector near the center of the weight set \( W_{private} \). Decisions by these decision rules are presented in Table 6.3.2.

Using maximin or maximax leads to selecting strategy 2, whereas central weights would suggest selecting strategy 3. Generally, the decision recommendation may depend on selection of decision rule.

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\(^5\)Salo and Hämäläinen [16] define central weights such that the weight of kth criteria is proportional to the midpoint between largest and smallest possible weight. In \( W_{private} \), the midpoints are \((0.75 \; 0.25 \; 0.125)\) and thus the central weights are \( w_c = (0.67 \; 0.22 \; 0.11) \).
7 Conclusions

This study focuses on multicriteria game models used in decision making. The aim of this study is to develop a multicriteria game model that takes into account incomplete information about preferences of the players. Preferences are modeled by an additive value function, that is, the value of a payoff is a product of a weight vector and a payoff vector. The payoff vector describes the payoff values of each criteria and the weight vector describes the relative importance of criteria. Incomplete preference information is taken into account by considering sets of feasible weights instead of single weight vectors.

The model developed here consists of two stages. First, a multicriteria game model is used to derive conclusions based on common knowledge about preferences. The solution concept proposed for this game is sets of rationalizable strategies, i.e., sets that a player may play without contradicting common knowledge of rationality and the weight sets. Then, in the second stage a player analyzes the strategy selection as a multicriteria decision problem using private and more accurate information about his preferences. It was shown that when the sets of possible weights are restricted, no new strategies become part of the solution. Thus, more accurate preference information leads to more accurate solutions. On the other hand, incomplete preference information will not cause exclusion of strategies that might be played with more accurate information about preferences.

In this study, only static two-player games are considered. Natural extensions of the model developed here would be games with more players, and repeated games, where players would gain additional information about the preferences of the opponent based on strategies selected in previous rounds. Another possible extension could be probabilistic modeling of beliefs about the preferences of the opponent, such that the weight vectors are defined as random variables over the weight sets. This would lead to a Bayesian game [1] where the types of the players would be represented by the weight vectors.

Existing multicriteria game models rely on mixed strategies and do not take into account incomplete preference information. Also, as far as the author knows, the concept of rationalizable strategies [15] has not been applied to multicriteria games. The author believes that the framework developed in this study presents a more accurate view of actual decision making situations and thus improves applicability of multicriteria game theory as a support for decision making.

References


