The development of a mid-term planning model of a hydropower system

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References
1. Introduction

The purpose of this study is to develop a mid-term planning model of a hydropower system. The new model will be regarded as an alternative for the model used currently. Several additional features are implemented to the model and their effects are studied, with the intention of finding the value-adding ones. The work was done as a case study, using a specific hydropower system as an example. However, the model can be adapted to any hydropower system.

Chapter 2 is about electricity as a product and electricity markets; of which Nord Pool, the pan-Nordic electricity market, is described closer. In Chapter 3 the main principles of hydropower are discussed, and certain issues concerning the production planning of such system are described. In Chapter 4 a basic model of a hydropower system is discussed and formulated. The model is developed further in Chapter 5, with first penalty variables added. Chapter 6 includes the addition of ramp penalties and of total production deviation penalty. In Chapter 7 an essential feature, the effect of the turbine head, is studied and implemented. Finally, conclusions are drawn and the work is summarized in chapter 8.

2. Electricity

2.1. Electricity as a product

Electricity differs from many other goods because it cannot be stored. Therefore, electric power has to be produced at the same moment in time as it is consumed. [1] When one switches on electric equipment, power demand increases and the increase has to be met by producing more power. Thus, the production of electricity has to be equal to the demand at all times. The demand is the sum of the customer loads, the transmission and distribution losses and transactions in electricity markets.

The total power demand can be divided into three parts: the base load, the cycling load and the peaking load. The base load is the demand which exists all the time, 24 hours a day 365 days a year. The cycling load refers to a periodically existing demand. In the daytime the load is usually higher than in the night time, on weekdays higher than at weekends, and the load during the winter varies from the load during the summer. For example in Finland the need of electricity is higher during the cold winter period due to augmented need for heating. The peaking load, for its part, occurs irregularly. [2]
2.2. Changing electricity markets

An electricity market is an exchange or a pool, through which electricity is bought and sold, and spot prices are determined. In addition to a spot market, most deregulated markets have or will develop, markets for contracts for future delivery of electricity - partly due to high volatility in electricity spot market prices. These contracts can be both physical, for consumption purposes or financial, for risk sharing purposes. [6]

Electricity markets have undergone significant changes lately. Fairly recently, the markets were still strictly regulated all around the world. Most electricity companies were state-owned, and consumers were forced to buy the required electricity from a specific company.

The first step towards globalisation of the electricity markets, was internal deregulation. The deregulation process began in some countries in the early 1990’s. The second step was, or rather will be, the establishment of multinational electricity markets. Currently, Nord Pool is one of a kind, but will certainly have successors.

2.3. Nord Pool

The Nordic electric power market features direct trading among players (bilateral trade) and trading via the Nordic Power Exchange, Nord Pool. A steadily increasing proportion of power trade takes place via Nord Pool.

Nord Pool is the world’s only multinational exchange for trading electricity. Nord Pool was established is 1993, and its ownership is divided between two national grid companies, Statnett SF in Norway (50%) and Affärsverket Svenska Kraftnät in Sweden (50%). [4]

Electricity and related financial instruments such as futures and forwards are traded in Nord Pool, in the physical and in the financial markets respectively. Currently Nord Pool has around 350 participants. Price for electricity, the spot price, is formed hourly in Nord Pool on the basis of supply and demand offers made by its members. [5]

2.3.1. History of Nord Pool

In 1991, the Norwegian parliament decided to deregulate the national market for power trading. Statnett Marked AS (later Nord Pool ASA) was established as an independent company two years later. Total volume of energy traded in the first operating year was 18.4 TWh of energy. Sweden joined the market in 1996, and the first multinational exchange for trade in power contracts was
established. Next enlargement took place in 1998 as Finland joined the Nordic market. A truly pan-Nordic power exchange was realised when Eastern Denmark, the Western part of the country already having been involved from 1999, was fully integrated into the market in October 2000 and all Nordic countries operated in a joint market.

2.3.2. The countries involved

Nord Pool’s founder country Norway, with a domestic production almost exclusively based on hydropower, is highly dependent on hydrological conditions. Norway is a net exporter of electricity during a normal hydrological year, but becomes a net importer in a dry year with low inflows [5].

The production in Sweden is divided mostly between hydropower and nuclear power, of which the share of the nuclear power is growing. Sweden is still quite dependent on the hydrological conditions, but not on such a scale as Norway.

Electricity production in Finland is quite diverse, consisting of hydropower, nuclear power, and fossil-fuelled power, including both conventional and combined district heating and power. Another salient characteristic of Finland is industrial power generation, which covers more than half of the demand for electric power in the industrial sectors. Finland is a net importer of electricity. [4]

The smallest Nordic country in terms of electricity production and consumption, Denmark, has a production structure different from the other Nordic countries, with neither usable rivers nor nuclear reactorst. Denmark’s domestic supply for electricity is produced by burning gas and coal and by wind power. Table 1 represents the total energy consumption and production in the Nordic countries, as well as the distribution of the production to different forms.

Table 1. Energy production and consumption in Nordic countries in 2003 (source: Nordpool)

<table>
<thead>
<tr>
<th></th>
<th>Total energy consumption (TWh)</th>
<th>Total energy production (TWh)</th>
<th>of which</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Hydro power</td>
</tr>
<tr>
<td>Norway</td>
<td>115</td>
<td>107</td>
<td>99 %</td>
</tr>
<tr>
<td>Sweden</td>
<td>145</td>
<td>132</td>
<td>40 %</td>
</tr>
<tr>
<td>Finland</td>
<td>85</td>
<td>80</td>
<td>12 %</td>
</tr>
<tr>
<td>Denmark</td>
<td>35</td>
<td>44</td>
<td>0 %</td>
</tr>
<tr>
<td>Total</td>
<td>380</td>
<td>363</td>
<td></td>
</tr>
</tbody>
</table>


2.3.3. Benefits and prospects of Nord Pool

Before the development of Nord Pool and the liberalization of electricity markets, there were many existing monopoly situations in practice; i.e. consumers, especially private residential customers, were not able to choose their electricity suppliers. In industry, due to the regulated situation, many energy-intensive industries were economically forced to own part of their electricity supply. In other words, there was much vertical integration. [5]

The purpose of deregulation, which opens as much as possible of the system to competition, is to make the electric power sector less expensive and more efficient. Power sector investments and operation should be as efficient, as environmentally sound and as economical as possible. The electric power market is gradually being opened to competition in many European countries, and cross-border power trading is rapidly increasing. Objective for Nord Pool is to serve as an example for, as well as actively participate in, developing European power markets.

3. Hydro power

3.1. Overview of a hydro power system

A hydropower system consists of reservoirs and power plants which form a specific network. The system often consist of a river and its tributaries, and reservoirs, such as natural or artificial lakes. Figure 1 represents a small hydropower system.

Figure 1. Example of a hydropower system
The hydropower system represented in Figure 1 consists of five reservoirs and seven plants. In addition to the water coming from higher up in the system, there is a certain local inflow, consisting of rainwater, into each reservoir. For example, the total inflow into reservoir 5 is composed of the water going through plants 3 and 4 and of the local inflow of the reservoir itself. On the other hand, the inflow into reservoirs 1, 2 and 4 consists solely of corresponding local inflows. In a hydropower system, the "same" water can be used to produce energy more than once. For example, a local inflow into reservoir 1 is run through plants 1, 3, 5, 6 and 7.

A seasonal reservoir is a reservoir, which can store a significant amount of the annual inflow. When a hydropower plant is situated between two seasonal reservoirs, it is fully controllable, the only constraints being the maximum discharge of the plant and the annual availability of water in the upper reservoir. If situated in a river with practically no reservoirs above, the plant is called a run-of-river plant. Plants 6 and 7 belong to the former category. All inflows to these plants have to be immediately discharged or passed by (spilled). Spilling the water means that its potential energy is lost, and the water cannot be taken advantage of. Most small hydropower plants in Finland are of the latter type [1].

3.2. From water to energy

Hydro energy is based on the potential energy of water, which is converted into mechanical energy by a hydro turbine. This mechanical energy is converted further into electrical energy by a generator. The electrical energy is then distributed to customers via a transmission or a distribution network.

The mechanical power of the hydro turbine can be presented as a function of turbine head, discharge and the efficiency factor [2]. The head is the vertical distance between the plant reservoir (water storage before the plant) and the tail water (water storage after the plant). The discharge is the water flow going through the turbine. The higher the head and the bigger the discharge, the higher is the amount of mechanical energy produced.

3.3. Hydro power production planning

The planning of power systems is under extensive study all around the world. Privatisation, competition and deregulation of the electricity sector, as well as the tightening of environmental constraints increase the need for optimal planning. [1] The planning can be divided into different
time horizons. In hourly planning, the power plants in operation are scheduled optimally. Short-term planning covers the production planning from one week to six months forward, whereas midterm planning deals with time horizons from six months to three years. Decisions for longer periods of time are made as part of long-term planning.

A power system has to serve many critical needs simultaneously. The production needs to be efficient and thus profitable. Environmental aspects are to be taken seriously; the efficient use of energy sources leads to environmentally better production. Fulfilment of the demand at any time can be guaranteed by solid technology, over-capacity and reserves. However, over-capacity and reserves cause extra costs. Thus, there is a trade-off between economy, and having a guaranteed supply. [2]

As mentioned earlier, electricity is non-storable, and thus seasonal variations in supply and demand cause seasonality into electricity spot prices. However, the production capacity in a hydro reservoir is storable and therefore a hydropower producer decides at each moment whether to use the reservoirs with the current spot price or to wait for higher prices in the future [3]. The spot price is not the only factor affecting the production decisions; the reservoir water levels and expected inflows are to be taken into consideration as well.

Once the reservoir is full, all new inflow is spilled. Thus, if the water level and/or the expected future inflow into the reservoir is high, the owner of the plant is more eager to produce energy than with a low water level and/or low estimated future inflow. Due to the uncertainty in future rainfall and temperature, the inflow into the reservoirs is stochastic. When the water level is high, an increase in the inflow uncertainty increases the risk of spillage making earlier production more favourable. [3]

Hydropower production is a dynamic problem, in which decisions made today influence future production decisions [3].

3.4. Mid-term planning decisions

As mentioned, mid-term planning covers a time horizon from 6 months to 3 years. The objective is to maximize the profit gained from the hydropower system over the time horizon. The income in each period is calculated by multiplying the amount of energy produced by the corresponding spot price. The company needs to develop an accurate spot price forecast covering the whole time horizon to be able to profit from the higher selling prices.
The amount of water inflow into the system is another stochastic issue, which needs to be forecast. Historical inflow profiles can be used to form individual inflow predictions for each reservoir and plant.

There are many constraints to be taken into account when making decisions in mid-term planning. The amount of water in a reservoir is usually limited, due to security and environmental requirements. Both the acceptable maximum and the minimum level of water are given separately for each period of the time horizon. E.g. during the period of spring flood the maximum limits are lower than at the end of summer. The characteristics of a power plant define its maximum discharge. In addition, minimum discharges are introduced at times. A certain minimum limit of production for the whole system at any given time period can be defined, as well as the maximum spillage accepted for a plant, and the maximum difference between reservoir water levels in consecutive time periods, to mention some additional constraints.

Once the appropriate forecasts for the stochastic factors have been introduced, and all the constraints to be satisfied have been defined, the decisions concerning the production system can be made. The key decisions are; how much water to keep in each reservoir, and how much to discharge and to spill from each plant in each period. Since the system can be characterised as a network, all individual decisions have an effect on the whole system.

In the next chapter, a particular hydropower system including basic functionalities is modelled. However, the model can be generalised to represent any hydropower system.
4. Basic model of the hydro power system

The hydropower system studied consists of 9 reservoirs and 15 plants. The time horizon in which the study is conducted is 1 year, divided into 52 periods of one week each. Due to significant differences in demand of electricity between daytime and nighttime, and between weekdays and weekends, the spot price varies significantly as well. Therefore, each week is divided into three so-called price-band periods: day period, night period and weekend period. Some variables and parameters, such as the spot price and plant discharges, are defined and considered on price-band levels, while others, such as reservoir contents and local inflows, are considered on weekly basis.

This chapter presents a basic model of the system. The basic model is a simple model without any additional features. The model is developed further starting from Chapter 5.
4.1. **Model notation**

The following notations are used further on this work.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>Time</td>
</tr>
<tr>
<td>$T$</td>
<td>Number of periods in the time horizon</td>
</tr>
<tr>
<td>$R$</td>
<td>Number of reservoirs</td>
</tr>
<tr>
<td>$P$</td>
<td>Number of plants</td>
</tr>
<tr>
<td>$DU$</td>
<td>Day unit</td>
</tr>
<tr>
<td>$SS(t)$</td>
<td>Weekly level total net spot sales in period $t$ (MWh)</td>
</tr>
<tr>
<td>$SS _ pb(t)$</td>
<td>Price-band level total net spot sales in period $t$ (MWh), $SS _ pb(t) = [SS _ d(t) \ SS _ n(t) \ SS _ we(t)]$</td>
</tr>
<tr>
<td>$R(t)$</td>
<td>Weekly revenue in period $t$ (€)</td>
</tr>
<tr>
<td>$s(t)$</td>
<td>Price-band level spot price in period $t$ (€/MWh), $s(t) = [s _ d(t) \ s _ n(t) \ s _ we(t)]$</td>
</tr>
<tr>
<td>$pb _ w$</td>
<td>Weight for price-band periods (h), $pb _ w(t) = [pb _ d(t) \ pb _ n(t) \ pb _ we(t)]$</td>
</tr>
<tr>
<td>$x(t)$</td>
<td>Reservoir content in period $t$ (DU)</td>
</tr>
<tr>
<td>$q _ res(t)$</td>
<td>Reservoir discharge in period $t$ (DU)</td>
</tr>
<tr>
<td>$v _ res(t)$</td>
<td>Reservoir local inflow in period $t$ (DU)</td>
</tr>
<tr>
<td>$x _ safe _ high(t)$</td>
<td>Reservoir content safety margin upper limit in period $t$ (DU)</td>
</tr>
<tr>
<td>$x _ safe _ low(t)$</td>
<td>Reservoir content safety margin lower limit in period $t$ (DU)</td>
</tr>
<tr>
<td>$x _ start$</td>
<td>Reservoir content start point value (DU)</td>
</tr>
<tr>
<td>$x _ end$</td>
<td>Reservoir content end limit (DU)</td>
</tr>
<tr>
<td>$G _ res$</td>
<td>Reservoir identity matrix [size $R \times R$]</td>
</tr>
<tr>
<td>$H _ res$</td>
<td>Matrix defining plants preceding the reservoirs [size $R \times P$]</td>
</tr>
<tr>
<td>$q(t)$</td>
<td>Plant discharge in period $t$ (DU)</td>
</tr>
<tr>
<td>$q _ pb(t)$</td>
<td>Price-band level plant discharge in period $t$ (m$^3$/s), $q _ pb(t) = [q _ d(t) \ q _ n(t) \ q _ we(t)]$</td>
</tr>
<tr>
<td>$w(t)$</td>
<td>Plant spillage in period $t$ (DU)</td>
</tr>
<tr>
<td>$v _ plant(t)$</td>
<td>Plant local inflow in period $t$ (DU)</td>
</tr>
</tbody>
</table>
4.2. Model formulation

The basic model consists of water balance equations, reservoir and plant restrictions and of formulas converting the discharge to energy.

4.2.1. Water balances

Water balances for reservoirs and plants need to be defined to ensure proper functioning of the model. Reservoir water level at the end of a period depends on the reservoir water level at the beginning of the period (end of the previous period), inflow during the period and discharge during the period [7].

For the first period, the reservoir water balance can be written

$$x(1) = x_{\text{start}} - G_{\text{res}} \ast q_{\text{res}(1)} + H_{\text{res}} \ast (q(1) + w(1)) + v_{\text{res}(1)}$$

And for the other periods, t=2 to T:

$$x(t) = x(t-1) - G_{\text{res}} \ast q_{\text{res}(t)} + H_{\text{res}} \ast (q(t) + w(t)) + v_{\text{res}(t)}$$

The amount of water discharged through, and spilled by a plant in any given period, must be equal to the amount of water coming from upstream combined with the plant’s local inflow. Hence, the plant water balance is formulated

$$q(t) + w(t) = G_{\text{plant}} \ast q_{\text{res}(t)} + H_{\text{plant}} \ast (q(t) + w(t)) + v_{\text{plant}(t)}$$

4.2.2. Reservoir and plant restrictions

In the basic model all restrictions are so-called hard restrictions, which cannot be ignored at any point. Reservoir content and plant discharge restrictions are the basic restrictions implemented to the basic model. The reservoir content is forced to stay between the limits in each period:

$$x_{\text{safe low}}(t) \leq x(t) \leq x_{\text{safe high}}(t)$$
Plant discharge is a variable defined also on a price-band level. Therefore, the discharges in day, night, and weekend periods of each week need to be defined to stay between given discharge limits.

\[ q_{\text{min}}(t) \leq q_{\text{pb}}(t) \leq q_{\text{max}}(t) \] (5)

The reservoir contents at the end of the last period (period T) need to be equal to the defined reservoir content end limits

\[ x(T) = x_{\text{end}} \] (6)

The plant discharges and spillages are defined as non-negative: \( q(t), w(t) \geq 0 \).

### 4.2.3. Energy production

A logical plant is modeled by means of its energy generation characteristic, describing the conversion of potential energy of water into electrical energy. The energy generation characteristics are assumed to be piecewise linear and concave as shown in Figure 2. [8]

**Figure 2. Example of a power generation curve**

Points forming the curve can be presented as \( (q_i, u_i) \), where \( i = 0 \) to \( n \). The power generated from a certain discharge \( q(t) \) can be written

\[ u(t) = f \times \min\{u_i(q(t)) \}, i = 1..n \] , (7)

where \( f \) is the share of the plant owned by the company and

\[ u_i(q(t)) = a_i + b_i \times q(t) \] (8)

The parameters \( a_i \) and \( b_i \) are solved using the curve points as follows.

\[
\begin{aligned}
    b_i &= \frac{u_i - u_{i-1}}{q_i - q_{i-1}}, \quad i = 1 \text{ to } n \\
    a_i &= u_i - b_i \times q_i
\end{aligned}
\] (9)
The power is calculated on price-band level from price-band level discharges.

\[ u_{pb}(t) = f \star \min[u_i(q_{pb}(t))], i = 1..n \]  

(10)

Weekly level plant discharges and generated powers can be calculated as the weighted average of price-band level discharges and powers respectively.

\[ q(t) = \frac{q_{pb}(t) * pb \_ w^T}{pb \_ d + pb \_ n + pb \_ we} \]  

(11)

\[ u(t) = \frac{u_{pb}(t) * pb \_ w^T}{pb \_ d + pb \_ n + pb \_ we} \]  

(12)

Total (all plants) price-band level spot sales to the market, are calculated by multiplying the price-band level powers by the price-band weights.

\[ SS \_ pb(t) = [1 \ldots 1] * u_{pb}(t) * pb \_ w^T \]  

(13)

The total revenue over the time horizon, the variable to be optimised, can thus be calculated by multiplying the price-band level spot sales by the corresponding spot prices.

\[ R(t) = SS \_ pb(t) * s(t)^T \]  

(14)

### 4.3. Model solution

The model can be regarded as a network model, and therefore, it is calculated using a certain network algorithm with dual simplex cleanup. As the main purpose of this study is modelling, the choosing of the optimization algorithm will not be discussed further.

The objective of the model is to maximize the total revenue over the time horizon. Optimization variables include the weekly level reservoir contents and discharges, weekly level plant discharges and spillages and price-band level plant discharges. In addition to the optimal values of the variables, model output provides the powers generated in each plant and total spot sales to market. All, the needed price-band level, weekly level and total figures, can be derived from the above.

In addition, an important source of information for the company is the marginal water value. The values can be derived from optimal values of dual variables (mwv *) connected to the reservoir water balance equations. The values are converted from €/DU to €/MWh using the following formula:
\[ mwv(t) = \frac{mwv^*(t)}{p_{TOT}} = \frac{mwv^*(t)}{\sum p_p} \] (15),

where \( p_p \) is the total production of plant \( p \) over the time horizon, divided by the total discharge over the time horizon, and \( p_{TOT} \) is the sum of the production/discharge ratios of all plants downstream. E.g., to calculate the marginal water values for plant 2 of Figure 1, one would need to first calculate the production/discharge ratios of plants 2, 3, 5, 6 and 7.

### 4.4. Short review of the model results

The total production and the total sales to market are assumed to follow the price curve. Thus, the production is assumed to be low during low-price periods and high during price-peaks. The model was solved using certain spot price and inflow forecasts. Total production and spot price curves are sketched in Figure 3.

![Figure 3. Total production and spot prices](image)

It is easy to see, that the optimal amount produced in each period is highly dependent on the corresponding spot price. The production is at its highest around period 40, when the spot price is at its highest as well. Similarly, between periods 11 and 15 the prices are low, therefore there is no use in producing huge amounts of electricity.

The water is stored in reservoirs during periods with low prices, and only some minimal discharge is run through the plants. When the price begins to increase, the reservoirs are emptied in order to gain maximal profit by running the plants on maximal capacity.
No surprises turned up while studying the model results; thus, the model seems to function properly.

5. Implementation of first penalty variables to the model

5.1. The use of penalties

At times, it might pay off not to obey certain limits or restrictions. In case of e.g. a surprising increase in spot price it might be profitable to reserve less water in the reservoirs than the modelled minimum limits (which typically include a safety margin from the real-life hard restrictions due to uncertain future inflow and deterministic model) would allow, and produce more energy to be sold at higher prices. Nevertheless, the restrictions cannot be violated without sanctions. Therefore, certain penalty variables (units of violation) are defined to illustrate the violations. Certain penalty prices (€/unit of violation) are then connected to the corresponding penalty variables and the sum of all penalties is taken into account in the objective function.

The penalty function of violating a certain soft limit can be represented

![Penalty function](image)

**Figure 4. Penalty function (source: [8])**

In Figure 4 $X_H$ and $X^H$ are the minimum and maximum hard limits for variable $X$, and $X_S$ and $X^S$ the soft limits respectively. The value of the variable $X$ being between the soft limits, it is not penalized. Once the value goes below the minimum soft limit or over the maximum soft limit, the penalty is activated. The penalty function in this example is linear meaning that the penalty price per unit of violation is the same regardless of the amount of violation. The variable is, however, forced to stay between the hard limits.
5.2. The penalties added to the model

In this chapter, the reservoir content and plant discharge limits are converted from hard to soft limits. The penalizing of the violation of the reservoir content limits is discussed in chapter 5.2.1. and of the plant discharge in chapter 5.2.2.

5.2.1. Reservoir content limit violation penalty

Two penalty variables for each reservoir for each period are needed, one for the violation of the minimum content limit and other one for the violation of the maximum content limit. The penalty variables are non-negative and are activated (get a positive, non-zero value) when the corresponding limits are violated. Reservoir content restrictions are re-formulated

\[
\begin{align*}
\{ x(t) - P_{x_{\text{max}}}(t) &\leq x_{\text{Max}}(t) \\
- x(t) - P_{x_{\text{min}}}(t) &\leq -x_{\text{Min}}(t) \}
\end{align*}
\]

where \(x_{\text{Max}}(t)\) and \(x_{\text{Min}}(t)\) are the upper and lower safety margin limits and \(P_{x_{\text{max}}}(t)\) and \(P_{x_{\text{min}}}(t)\) the violations of the limits in question.

For instance, the upper safety margin limit for the reservoir content is 5 day units (DU), the minimum limit 3 DU and the reservoir content 7 DU. The corresponding penalty variable for the violation of the upper limit is activated, getting a value of 2 DU, while the variable for the violation of the lower limit stays non-active and gets a value of 0 DU.

Reservoir content penalty variables are first multiplied by the corresponding penalty prices and the product, defining the total penalty for the violation, is subtracted from the objective function value. All activated penalty variables therefore decrease the objective value. For example let us suppose that the corresponding penalty price for the upper limit violation above is 30 €/DU. This leads to a total penalty of 60 € (2 DU * 30 €/DU).

5.2.2. Plant discharge limit violation penalty

The penalty variables for plant discharge limit violations are added to price-band level discharge restrictions. This means, that there are two penalty variables (one for violation of the maximum limit, another one for violation of the minimum limit) connected to each price-band level discharge of each period for each plant.

The discharge restrictions are re-formulated similarly to reservoir content restrictions:
\[
\begin{align*}
q(t) - Pq_{\text{max}}(t) &\leq q_{\text{Max}}(t) \\
-q(t) - Pq_{\text{min}}(t) &\leq -q_{\text{Min}}(t)
\end{align*}
\]
\[\text{(17)}\]
where \(q_{\text{Max}}(t)\) and \(q_{\text{Min}}(t)\) are the upper and lower safety margin limits and \(Pq_{\text{max}}(t)\) and \(Pq_{\text{min}}(t)\) the violations of the limits in question.

For instance, the maximum discharge limit is 10 m\(^3\)/s, the minimum limit 5 m\(^3\)/s, and the realized discharge 4 m\(^3\)/s. The penalty variable for the violation of the maximum limit stays at 0 m\(^3\)/s while the penalty variable for the violation of the minimum limit gets a value of 1 m\(^3\)/s. The penalizations of the violations occurred during day, night and weekend periods are weighted respecting to their shares of weekly total hours. The total penalty is the product of the penalty variable and the corresponding penalty price, weighted by the number of hours. The total penalty is subtracted from the objective function value.

### 5.3. Validation of the model

Certain verifications can be made to ensure the model rationality. The basic verifications are represented in chapter 5.3.1. and a special verification in chapter 5.3.2.

#### 5.3.1. Basic validations

Two fundamental verifications are first made to ensure that the addition of the penalty variables functions as presumed. The first verification is done by setting all penalty prices to almost zero. The other verification is done in the opposite direction; all penalty prices are set extremely high. The conclusions of the verifications are discussed below.

All reservoir content and discharge penalty prices were first set to 0.001 €/unit of violation. The presumption is that since the violations of the restrictions are penalized in a very gentle way (in fact almost not at all), the optimal solution would ignore the restrictions totally and concentrate the production to the periods with the highest spot prices. Figure 5. illustrates the weekly revenues obtained with this model.
It is easy to see that the production is concentrated on a single period, here week 3, with the highest spot price. Oversized amounts of water (far over maximum limits) are kept in the reservoirs up to week 3 and then discharged all at once. For the last weeks of the time-scale, the contents lay far beyond the minimum limits. Figure 6. illustrates the content of a certain reservoir in the system.

As presumed, with the penalty prices set close to zero, the model is fully ignorant of all restrictions. The model is thus functioning properly.

In order to conduct the second verification, all reservoir content and discharge penalty prices were set to one million euros per unit of violation (1000000 €/unit). The presumption was that because of the extremely strict penalization, no violations would occur and the optimal solution would be similar to the one obtained with the basic model (not including penalties).
The objective function values in both cases were equal, and despite some small differences between certain values, the results could be regarded alike. Hence, it can be stated that the second verification carries out a desired result as well, and the model is ready for use.

5.3.2. Activation of selected penalties

To illustrate how the model exploits the possibility of using penalty variables, the penalty prices for violating certain minimum reservoir content limits are set to almost zero in some periods with low spot prices, while all the other penalty prices are on normal levels. The assumption is that the model would discharge the water from the reservoirs before the low-price period, and therefore keep the contents of the reservoirs in question below the minimum limits.

The results are consistent; undershoots of minimum limits occur for the reservoirs in question in certain periods. Figure 7. represents reservoir y’s content and the maximum and the minimum safety margin limits.

As can be seen, low penalty price makes taking penalties profitable.

6. Addition of ramp penalties and of total production penalty

6.1. Ramp penalties

While examining reservoir content and plant discharge curves, it can be noted that certain variables are constantly on the limits of the allowed range, and there are large differences between values of successive periods. Figure 8., which represents the content of a certain reservoir, serves as an example of such behaviour.
One solution to avoid such behaviour is to add new penalty variables to illustrate the difference (ramp) in reservoir content or in plant discharge between two consecutive periods. It is not reasonable to penalize the whole difference, but only the exceeding of a predefined ramp limit. Therefore, we use two variables to illustrate each difference, as shown in figure 9.

In figure 9, the penalty is zero as long as the difference between two consecutive values is smaller than the maximum ramp value. If the difference is higher than the maximum value, the exceeding part (ramp_b) is penalized using a penalty coefficient (price). E.g. if the maximum ramp value is 10 DU and the difference between two consecutive periods is 15 DU, ramp_a gets a value of 10 DU and the variable to be penalized, ramp_b, a value of 5 DU.

It has to be taken into consideration that the difference can also be "negative". It depends on which of the compared values is greater. Therefore, we need two inequalities to illustrate each difference. The inequalities for reservoir content ramp are represented below:

\[
x(t) - x(t-1) \leq xRU_{up}a(t) + xRU_{up}b(t)
\]

\[
0 \leq xRU_{up}a(t) \leq xRampMax(t) , \quad xRU_{up}b(t) \geq 0
\]

\[
x(t-1) - x(t) \leq xRD_{down}a(t) + xRD_{down}b(t)
\]

\[
0 \leq xRD_{down}a(t) \leq xRampMax(t) , \quad xRD_{down}b(t) \geq 0
\]
where  
\( \text{xRUp}_a(t) = \text{non-penalized positive ramp} \)  
\( \text{xRUp}_b(t) = \text{penalized positive ramp} \)  
\( \text{xRDown}_a(t) = \text{non-penalized negative ramp} \)  
\( \text{xRDown}_b(t) = \text{penalized negative ramp} \)  
\( \text{xRampMax}(t) = \text{ramp limit, defines the non-penalized area} \)

### 6.2. Addition of ramp penalties to the model

Ramp penalties for the reservoir content and the weekly level plant discharge are added to the model.

Reservoir content ramp control is worked out as shown in equations (18) and (19). A ramp limit (\( \text{xRampMax} \)) is to be determined to define the allowed or non-penalized difference between two consecutive reservoir contents. A penalty price (€/DU) for a ramp exceeding the limit is to be set as well. The part of the ramp exceeding the limit is multiplied by the corresponding penalty price and the product is subtracted from the objective function value. Total penalty for reservoir content ramp in period \( t \) is thus \( \text{xRampPrice}(t) \ast (\text{xRUp}_b(t) + \text{xRDown}_b(t)) \).

For instance, the reservoir content in period \( t \) is 10 DU and in period \( t+1 \) 15 DU. The ramp limit is 3 DU and the penalty price for the limit exceeding part of the ramp is 10 €/DU. The reservoir content has increased, i.e. the ramp is positive. \( \text{xRUp}_a \) gets a value of 3 DU (maximum limit) and \( \text{xRUp}_b \) a value of 2 DU. \( \text{xRDown}_a \) and \( \text{xRDown}_b \) are both equal to zero. Total penalty for the ramp is thus \( 10€/DU \ast (2 DU + 0 DU) = 20€ \).

The procedure regarding the plant discharge ramp control is similar to the one of the reservoir content ramp. Two inequalities are formed for each ramp to illustrate both positive and negative differences between two consecutive discharges. The allowed ramp limit is to be determined to define the non-penalized difference between two consecutive discharges. A penalty price (€/DU) for the limit exceeding part of the ramp is to be set as well. The limit exceeding part of the ramp is then multiplied by the corresponding penalty price and the product is subtracted from the objective function value. Total penalty for plant discharge ramp in period \( t \) is therefore \( q\text{RampPrice}(t) \ast (q\text{RUp}_b(t) + q\text{RDown}_b(t)) \).

The results given by the “ramp model” are compared to the results of the “old model”. Figure 10 represents the content of reservoir y, and figure 11 the discharge of plant p.
Differences are remarkable. Due to high ramp penalty costs the reservoirs and plants are controlled a lot smoother in the ramp model, and high peaks are avoided. The model functions as presumed.

6.3. **Total production penalty**

At times there might be a certain level of total production we would like to attain, and the deviation from the predefined level should be as small as possible. The solution is to add penalty variables similar to the ones used in reservoir content and plant discharge ramp control.

A certain deviation limit is set, and the deviation exceeding the limit will be penalized. The deviation can be either negative or positive; hence, we need two inequalities to illustrate the penalization of each deviation, as in the case of the reservoir content or the plant discharge ramp control.

The part of the deviation exceeding the limit is penalized by multiplying it by the corresponding penalty price. E.g., total spot sales for a certain week are 50000 MWh, and the reference value was set to 55000 MWh. The allowed limit of deviation is 2000 MWh, and the corresponding penalty price is 10 €/MWh. Spot sales being lower than the reference value, the deviation is negative. Variables illustrating positive deviation are equal to zero, while corresponding variables for negative deviation get truly positive values; the non-penalized deviation is 2000 MWh and the deviation to be penalized is 3000 MWh. Hence, the total penalty cost is 10 €/MWh * 3000 MWh = 30000 €.

7. **Modelling of the turbine head**

The power generated with certain discharge depends on the turbine head. The more there is water in the preceding reservoir, the more power will be generated with the same marginal discharge amount (say 10 m3/s). So far, it has been assumed that power generation curves are the same regardless of
reservoir content. The purpose of the procedure discussed in this chapter is to optimise the model iteratively by updating the generation curves, thus obtaining results that are more realistic.

7.1. **Implementation of the turbine head effect in the model**

The update of the generation curves is done reservoir and period-wise by using specific scaling coefficients. The scaling coefficients are updated after each iteration based on the reservoir contents of the previous iteration. The updating method is based on principles of the concept of successive linearisation. The initial scaling coefficients are set equal to one for all reservoirs in all periods. The model is solved and the optimal reservoir contents (oldResCont(r,t)) are saved. New scaling coefficients c(r,t) are calculated from

\[
c(r,t) = a_r + b_r \times \left( \frac{old \text{ Re sCont}(r,t)}{max \text{ Re sCont}(r)} \right)
\]

where \(a_r\) and \(b_r\) are reservoir wise constants depending on the characteristics of the reservoir, and \(max\text{ResCont}(r)\) the over-time maximum content for reservoir \(r\).

The next task is to rebuild and to resolve the model using the new scaling coefficients. New optimal reservoir contents (newResCont(r,t)) are saved, and compared to the previous ones. Total absolute difference (Diff) over reservoirs and periods is calculated

\[
Diff = \sum_{r=1}^{R} \sum_{t=1}^{T} |new \text{ Re sCont}(r,t) - old \text{ Re sCont}(r,t)|
\]

The total absolute difference is the ratio of primary interest. The goal of the optimisation is to obtain a convergent result. It is reached when the ratio has decreased beyond a predefined limit (e.g. 0.5% of the ratio obtained with the first iteration).

To speed up (or even guarantee, the model is now actually non-convex!) the convergence, after a certain number of iterations the reservoir contents are forced to stay within given bounds, for example

\[
\begin{align*}
max \text{ Bound}(r,t) &= old \text{ Re sCont}(r,t) + 0.5 \times max \text{ Cont}(r) = old \text{ Re sCont}(r,t) + a \\
min \text{ Bound}(r,t) &= old \text{ Re sCont}(r,t) - 0.5 \times max \text{ Cont}(r) = old \text{ Re sCont}(r,t) - a
\end{align*}
\]

\[
\Rightarrow old \text{ Re sCont}(r,t) - a \leq new \text{ Re sCont}(r,t) \leq old \text{ Re sCont}(r,t) + a
\]
In the following iteration rounds the feasible area of the reservoir content is constantly decreased by multiplying the range by a factor \( f (0<f<1) \):

\[
a(\text{iteration } i) = f \cdot a(\text{iteration } i - 1)
\]  

(23)

The procedure will converge, but not necessarily to the global optimum. It might as well have ended up in a local maximum, or possibly not reached an optimum at all! This is because the model is non-convex due to turbine head dependency of the generation curve. Nevertheless, we assume that the post-optimisation has more benefits than disadvantages, and should therefore be at least considered to be utilized within the decision-making process. One clear benefit is that the production values and reservoir content values are consistent with each other, when considering the real-life dynamics of the power system. The use of the model leads to gradually improved accuracy and the optimality of the result in each iteration. It has to be noted that the result from each iteration is a feasible and more accurate solution, than the result obtained using the basic model, where the turbine head is ignored.

### 7.2. Results of the implementation

The model was run using the allowed range-decreasing factor of 0.8. Up to forty iterations were calculated to obtain a better view of the model functionality.

To make the results of the models comparable, the results of the model without the turbine head effect were converted using the power generation scaling coefficients. The coefficients were calculated (reservoir and period wise) based on optimal reservoir content values as in equation (20). Optimal price-band level powers were multiplied by the corresponding scaling coefficients, and the river level total sales were counted by means of scaled price-band level powers. The scaled revenue was obtained by multiplying the scaled river level total sales by the corresponding spot prices.

The following figure represents the iteration ratio by function of the iteration number:
Figure 12. Iteration ratio

On the first iteration the ratio is 25000 DU. It decreases rapidly being less than 10% of the initial value on the 13th iteration, less than 1% on the 32nd iteration, and on the 40th iteration around 0.13% of the first iteration (36 DU). The result is certainly converged.

An important issue is the comparison of the total revenues and the total productions (sales) obtained with the two models. The total revenue is slightly higher with the turbine head model, thus there is small improvement in the objective value. The total production is also higher with the turbine head model, although, the difference is again quite small. On the other hand, the average selling price is higher with the old model. Too much attention should not be directed to it, since the objective of the model is to maximize the total revenue.

8. Summary and conclusions

Electricity is a product with special features. Its most significant characteristic is, that it is not storable. Therefore, the demand for electricity has to be met by a simultaneous supply of the same amount.

The electricity market has faced large changes in the recent years. The deregulation has opened new markets to electricity companies and the competition has intensified. The Nordic countries were the first to introduce an international exchange of electricity, Nord Pool. Member companies buy and sell electricity and related financial instruments through the pool. The development of the market has brought many benefits, of which the improvement of the efficiency of the whole sector is not the least important.

At the same time, the importance of the production planning of energy has increased significantly. The production form discussed in this work is hydro power. A hydro power system can be regarded
as a network, and it consists of reservoirs and power plants. The main decisions to be made are:
how much water to discharge through the plant turbines (the amount of production), and how much
water to keep in the reservoirs (to be used later). The decisions depend strongly on the electricity
spot price forecast and on the predicted local inflow.

The purpose of this study was to implement a hydro power planning model for a time horizon of 6
months to three years. First, it was ensured to be functioning properly by comparing its results to
the ones obtained with an existing model. The next step was to include some additional features to
the model. These additional features included penalty variables for the exceeding of reservoir
content and plant discharge limits, for the deviation of a predefined level of production, and the
modeling of the turbine head effect.

All features were successively modeled, and their additional value was studied. It is certain, that the
new model with the new features is more versatile compared to the old model. The new model is
easier to be adjusted to changes in electricity spot price and water inflow forecasts.
References:


