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Robust Portfolio Modeling for the \textit{ex post} Analysis
on Innovation Programme

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1 Introduction

Many countries offer selective policy measures which seek to foster the commercial exploitation of results from science-based research. These measures are typically supported by systematic processes of data collection where the \textit{ex ante} and \textit{ex post} evaluations are recorded. These evaluations have not although been exploited effectively for further analysis. Traditional approaches to evaluate the success potential of new technology-based firms are often too technical and suffer their inflexibility (see, e.g., Lõfsten and Lindelöf, 2002).

There are several factors that complicate the use of data collected from \textit{ex ante} and \textit{ex post} evaluations. Such factors are e.g. lack of unequivocal performance indicators, exigency of quantitative data and weak causal relationships. Qualitative data collected with questionnaires is typically open to various interpretations. There might even be excessive amount of lacking responses. Thus, it is questionable to construct definite decision models or conclusions based on such data. There is a need for new ideas and approaches to analyze the success potential of research based business ideas.

Typical approaches for seeking differences between projects and the causal factors for these differences include regression analysis and examination of the correlations between performance measures and \textit{ex ante} evaluations. These approaches, however, lack the viewpoint of multiattribute value theory, i.e. they examine connections between single variables. It is more convenient to think that success arises from many small factors - not from any single variable. In this paper we describe a new methodology, based on recently developed robust portfolio modeling (RPM) (see Liesiö, Mild and Salo, 2005). It is often interesting to distinguish the best projects from the set of all projects and compare them to the “worst” projects. This kind of separation is done in a robust manner with this methodology and then these subsets are compared to each other. This kind of approach is novel, and has an obvious novelty value related to it.
For the examination used in this methodology we need variables derived from *ex post* evaluations that in some sense describe the success of a project. We also need background variables from *ex ante* evaluations, whose relation to the success is examined. The objective is to search differences among these background variables between the “best” and the “worst” subset. This methodology also makes it possible to form hypothesis about the relationships between *ex ante* and *ex post* evaluations and to test these hypothesis’ statistical significance. This methodology should not be used to construct strict decision rules or conclusions, but instead to support critical debate, discussion and reasoning. This approach might also point out new ideas of causal relations and this way give a fresh, creative nuance to the debate. This paper suggests that this methodology can assist in understanding determinants of exceptional (non)performance, thus also in that way serving as a useful complement to other forms of analysis.

2 Multicriteria Portfolio Analysis

2.1 Multiattribute Value

In this context the term value refers to the strength of preference of an object. We assume that the DM is maximizing the “overall value” i.e. chooses the alternative with the greatest value. However, the alternatives typically have various attributes. With respect to a certain attribute, one alternative may have the highest attribute specific value, but with respect to another attribute, some other alternative might be the best. In these situations we need a framework, e.g. multiattribute value theory (MAVT), for constructing an aggregate value to describe the "overall value" of an alternative. The most common representation is the additive value function (see e.g. Keeney and Raiffa, 1976; Saaty, 1980; Clemen, 1996), which is widely used in the theory of multiple criteria decision making (MCDM). MCDM problems arise
in various areas, e.g., in hospitals and healthcare systems (Kleinmuntz and
Kleinmuntz, 2001), in risk management (Strauss and Stummer, 2002) and in
industrial portfolio analysis (Stummer and Heidenberg, 2003). Besides addi-
tive value representation, there are also other approaches to multiattribute
decision problems not studied more closely in this paper.

2.1.1 Additive Value Representation

Let a multiattribute project with $n$ attributes and a cost be denoted by
\( x^j = (c^j; x_1^j, x_2^j, \ldots, x_n^j) \), where \( c^j \) is the cost of the \( j \)-th project and \( x_i^j \) is
the \( j \)-th project’s performance with regard to attribute \( i \). Let the set of all
projects be denoted by \( X = \{ x^1, \ldots, x^m \} \). For each attribute \( i \) it is
possible to form an attribute specific value function \( v_i(x_i^j) \). There are several
methods for constructing attribute specific value functions (see e.g. French,
1986). Value function is typically normalized by positive affine transforma-
tion so that the worst possible outcome with regard to attribute \( i \) gets the
value 0 and the best possible gets the value 1.

From attribute specific scores \( v_i(x_i^j) \) we form the overall additive value:

\[
V(x^j) = \sum_{i=1}^{n} w_i v_i(x_i^j),
\]

where the \( w_i \) is the weight of the attribute \( i \). Without the loss of generality,
it can be assumed that (the weights are typically transformed by positive
affine transformation to the following form)

\[
w = (w_1, w_2, \ldots, w_n) \in S_0 := \{ w \mid w_i \geq 0, \; i = 1, \ldots, n \; \wedge \; \sum_{i=1}^{n} w_i = 1 \}
\]

When the scores and weights are normalized as above, we get a clear and
precise interpretation of the weights. The weights reflect the subjective rela-
tive importance of the attributes, with respect to the available ranges
of the attribute performances. By relative importance we mean that only
the weight ratios between attributes are of interest. Coefficient $w_i$ presents the value gained when alternative’s performance with regard to attribute $i$ changes from the worst possible to the best possible. If there exists such $x^j$ that it has the best possible performance with regard to all attributes, the overall value of this project $V(x^j) = 1$.

In the classical MAVT context, it is assumed that the weights are precise i.e. there exists a unique $w$ that “truly” describes the DM’s preferences of value trade-offs. In that case, we say that the weight information is complete. With complete weight information the most preferred project is the one with the greatest overall value (1).

2.1.2 Incomplete weight information

The classical approach assumes that the weight information is complete. It is, although, typically very hard to give complete weight information, and it can be argued that the DM is not able or willing to give this information (see, e.g., Weber, 1987). The DM may although be able to give incomplete preference statements, e.g. "attribute $i$ is more important than attribute $j$" (in the sense weights are interpreted, of course).

Methods for dealing with incomplete weight information have been studied widely (see, e.g., Hazen, 1986; Weber, 1987; Salo and Hämäläinen, 1992; 1995; 2001; Salo and Punkka, 2004). By incomplete weight information we mean that there is more than one weight vector that is consistent with the preference statements available. In the framework of incomplete weight information a set of feasible weights $S$ is generated from the preference statements available. As there is no unequivocal weight vector $w$, there is not necessarily a unique project that maximizes the overall value (1).

Instead of searching the best alternative by maximizing the aggregate value
(1) with single point estimate of $\mathbf{w}$, focus is here put on *dominance* relations or *potential optimality* (see, e.g., Hazen, 1986). The key significance in incomplete weight information framework is that the approach is “to deal with” the incomplete information, instead of trying to “extract” the incompleteness from the model. These kinds of approaches broaden the practical applicability of MAVT and give a new aspect to MCDM.

The set of feasible weights, *feasible weight region* $S$, is assumed to be a polytope in $S_0$. This means that $S \subseteq S_0$ is constrained by a set of linear inequalities and is closed.

### 2.2 Robust Portfolio Modeling

Capital budgeting is a problem of allocating scarce resources among the projects. The capital budgeting problem was first considered by Lorie and Savage (1955) and was formulated as a linear integer programming problem later. Luenberger (1998) distinguishes capital budgeting between general portfolio optimization so that capital budgeting refers to problem with fixed, discrete alternatives (projects) instead of a problem of collecting appropriate amounts of continuous investment instruments.

A portfolio $p$ is a subset of $X$, $p \in \mathcal{P}(X)$, where $X = \{x^1, \ldots, x^m\}$ is the set of all projects and $\mathcal{P}(X)$ is the power set of $X$. Let us define the value of a portfolio $p$ as the sum of its constituent project’s values. Because we include incomplete weight information, it is convenient to write the value $V$ also as a function of $\mathbf{w}$.

$$V(p, \mathbf{w}) = \sum_{x^j \in p} V(x^j, \mathbf{w}) = \sum_{x^j \in p} \sum_{i=1}^{n} w_i v_i(x^j_i).$$  \hspace{1cm} (2)

Similarly we define the cost of a portfolio $p$ as a sum of its project’s costs:

$$C(p) = \sum_{\{j| x^j \in p\}} c^j.$$  \hspace{1cm} (3)
The aim in the capital budgeting problem is to choose a feasible portfolio (the cost of a portfolio does not exceed the budget) with maximal value. Let the available budget be denoted by $B$, and define the set of feasible portfolios by $P_F = \{ p \mid C(p) \leq B \}$. The capital budgeting problem with given weights $w$ is to find the portfolio $p^* \in P_F$ that solves the maximization problem

$$
\max_{p \in P_F} V(p, w) = \sum_{x^j \in p} V(x^j, w) = \sum_{x^j \in p} \sum_{i=1}^n w_i v_i(x^j_i). 
$$

(4)

2.2.1 Non-dominated Portfolios

Since we have incomplete information in our value function, there does not necessarily exist a unique best portfolio, because different weights can lead to different solutions. We can, however, analyze the “goodness” of a portfolio by introducing the concept of dominance. By examining dominance relations, we can exclude “bad” portfolios and thereby reduce the set of relevant portfolios. We need the following definitions:

**Definition 1** Portfolio $p \in P_F$ dominates portfolio $p' \in P_F$, denoted by $p \succ p'$, iff

\begin{align*}
(i) & \quad \forall w \in S \quad V(p, w) \geq V(p', w) \\
and & \quad (ii) \quad \exists w \in S \quad V(p, w) > V(p', w)
\end{align*}

**Definition 2** Portfolio $p \in P_F$ is dominated, if there exists $p' \in P_F$ such that $p' \succ p$. Otherwise, $p$ is non-dominated.

Let the set of all non-dominated portfolios be denoted by $P_{ND}$

$$
P_{ND} = \{ p \in P_F \mid p' \not\succ p \ \forall p' \in P_F \}.
$$

A rational DM does not choose a dominated portfolio $p'$, because by definition there is another portfolio $p$ that yields greater or equal value with any feasible weights. Thus, dominated portfolios can be discarded from further analysis.
2.2.2 Core Index

The number of non-dominated portfolios may, however, be quite large and there is an obvious need for more decision support. Salo and Hämäläinen (2003) have synthesized preference elicitation and decision rules, which are used to find a unique solution. Recently, there have been new, fresh ideas that provide measures for portfolios and projects (see Mild, 2004; Liesiö, 2004). It is especially interesting to examine measures for individual projects, because ultimately the portfolio selection is m-tuple of “included/excluded” decisions. This approach also gives the DM more convenient and intuitive way to construct the final solution project by project, instead of choosing a complete portfolio at once.

Mild (2004) presents a project performance measure, the core index.

**Definition 3** The core index of project $x^j$ over the set of portfolios $P \subseteq P_F$, denoted by $CI_P(x^j)$, is the fraction of the number of those portfolios in $P$ that include the project $x^j$:

$$CI_P(x^j) = \frac{|\{p \in P \mid x^j \in p\}|}{|P|}.$$  \hspace{1cm} (5)

Specifically, we define the core index of project $x^j$: $CI(x^j) = CI_{P_{ND}}(x^j)$.

We assume that the DM chooses one of the non-dominated portfolios. Thus, if $CI(x^j) = 1$, then $x^j$ is a robust selection because every non-dominated portfolio includes it. Such project is called a core project. Correspondingly, if $CI(x^j) = 0$, $x^j$ is not included in any of the non-dominated portfolios. Such project is called an exterior project. We also define the core $C = \{x^j \mid CI(x^j) = 1\}$ and the exterior $E = \{x^j \mid CI(x^j) = 0\}$.

The core index can be used as a project specific decision rule, such that that all core projects should be selected and all exterior projects should be
discarded. Projects with $0 < CI(x^j) < 1$ are included in some of the non-dominated portfolios but not all. Core index can thus be seen as a novel measure for the projects.

However, core index can also be used for “screening” purposes (Liesiö 2004) i.e. that the core $C$ and the exterior $E$ can be discarded from further analysis. The problem reduces into a smaller one and further efforts can be focused in the projects with $0 < CI(x^j) < 1$.

3 Uses in \textit{ex post} Performance Assessment

3.1 Evaluating the Projects

Each project is \textit{ex post} evaluated with various attributes. The data collected is often too extensive and thus we need to choose relevant factors which characterize good or bad success. The \textit{ex post} evaluations with regard to these relevant factors can be scored to attribute specific values by using scoring methods (see, e.g., French, 1986). Let the evaluation scores of a project $x^j$ be denoted by $v_1(x^j_1), v_2(x^j_2), \ldots, v_n(x^j_n)$. We now formulate a total evaluation score of a project as an additive value function

$$V(x^j, w) = \sum_{i=1}^{n} w_i v_i(x^j_i).$$

(6)

The weight information is assumed to be incomplete, and therefore the projects cannot be ranked straightforwardly by total evaluation scores. Instead, we analyze the ensembles of projects by formulating a portfolio selection problem

$$\max_{C(p) \leq B} \sum_{x^j \in p} V(x^j, w) = \sum_{x^j \in p} \sum_{i=1}^{n} w_i v_i(x^j_i)$$

(7)
The budget constraint $B$ can be chosen rather freely. By choosing constraint $B$ such that problem (7) has a reasonable amount of non-dominated portfolios, RPM methodology presented below is applicable. For example by choosing $B = C(X)/2$, the number of non-dominated portfolios is typically adequate.

Now for given budget constraint $B$ and feasible weight region $S$, we can examine the portfolio selection problem with RPM methods (see section 2.2) readily available. Especially, by solving the set of non-dominated portfolios $P_{ND}$, core indexes can easily be obtained and used to measure project specific performance.

In order to compare the ex ante evaluations between the most successful and the most unsuccessful projects, the aim is to separate projects into these “good” and “bad” subsets and then compare them. This way it can be statistically tested, whether there is differences in background variables between successful and unsuccessful projects. The subsets must be small enough to be representative, but if they are too small, no statistical significance ever occurs. We suggest that a favorable size for these subsets is about one quarter of the number of all projects.

By using the core index we can rank the projects and choose the best and the worst quarter to represent “good” and “bad” subsets.

### 3.2 Examining Causal Factors

We also have ex ante evaluations, from which we obtain background variables. The key idea is to examine the differences in the background variables between the projects included in “good” and “bad” subsets, respectively.
We can use statistical testing to test the homogeneity of the background data between the two subsets. We want to conclude that there is a difference also in the *ex ante* measurements between the two subsets, pointing out causality between *ex ante* and *ex post* measurements. The differences in *ex ante* evaluations can point out causal factors for relative success and failure.

The statistical test to be used depends on the background data. If normality of a background data can be assumed, t-test would be the obvious choice for statistical testing. Although, usually the evaluations are gathered as multiple choice questionnaires. In this case the background variables are class scale and the association test to be used is the $\chi^2$ association test for homogeneity.

4. Illustrative Example

4.1 TULI-programme

In early 90's Finnish National Technology Agency (Tekes), whose core activity is to provide R&D funding for Finnish industry and research organizations, introduced a new funding programme, TULI (Research into Business). The TULI-programme is a national programme, which has two main activities. On the first hand, TULI has full-time commercialization experts who work in university campuses and who screen, motivate and evaluate research based new business ideas. Secondly, TULI offers pre-seed funding for eligible projects. Currently more than 500 research based business ideas are evaluated and more than 200 are funded annually.

In this case we consider such projects, which had resulted in a so called start-up company, i.e. a company was founded to exploit the corresponding business idea and earlier research. The number of projects in which adequate data was available was about 50. The data was collected through an electronic questionnaire in a web-based database in two points of time: first before the
project was granted funding (ex ante), and later after the project was executed the success of project was evaluated (ex post). The ex ante evaluation gathered background information about the project and included questions e.g. about the owner, earlier finance and co-operations of the project. The ex post evaluation questionnaire included questions e.g. about succeeding in execution of business plan and finance plan.

4.2 Analysis

The analysis of the projects was carried out with a decision support software PRO-OPTIMAL (Project-portfolio RObust OPTImization with Multi-Attribute vaLue), which provides a graphical interface for defining a RPM problem and solving it. There is a screen shot of this software in figure 1. PRO-OPTIMAL also supports RICH-method (Salo and Punkka 2004) used

![Figure 1: Screen shot of the PRO-OPTIMAL software](image-url)
in this analysis. This software also searches nondominated portfolios and calculates the core indexes, among many other features.

The number of projects included in the analysis was 48 \((m = 48)\), which were evaluated with regard to seven criteria \((n = 7)\) chosen by the programme executive. These criteria were derived from the *ex post* evaluation questions presented in table 1. These questions deal with e.g. execution of the business plan. These questions were analyzed to describe the performance in such matters that are set as objectives for a successful project.

Table 1: The *ex post* evaluation questions, criteria were derived from. All questions were answered with “yes/no/n.a.”.

<table>
<thead>
<tr>
<th>Criterion number</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Company’s financial situation corresponds to the business plan and the finance plan.</td>
</tr>
<tr>
<td>2</td>
<td>Company’s cash flow is according the business plan.</td>
</tr>
<tr>
<td>3</td>
<td>Company’s team corresponds the business plan.</td>
</tr>
<tr>
<td>4</td>
<td>Company’s sales and marketing realized according to the business plan.</td>
</tr>
<tr>
<td>5</td>
<td>There has been a business angel in the company.</td>
</tr>
<tr>
<td>6</td>
<td>There has been a major investment in the company.</td>
</tr>
<tr>
<td>7</td>
<td>The company is located in a technology center.</td>
</tr>
</tbody>
</table>

Each question was answered with “yes/no/n.a.”. In every question the programme executive considered that the answer “yes” indicates a successful project. Correspondingly the answer “no” was seen as an indicator of bad success and the inability to answer a certain question was considered to be more likely negative than positive indicator. Thus, the answer “yes” provided with an attribute specific value of 1 and the answer “no” with value of 0. Because “n.a.” was thought closer to “no” than “yes”, it provided with
an attribute specific value of 0.2. These values were determined jointly with the programme executive.

Also an incomplete rank order for the importance of these criteria was given by the programme executive. This incomplete rank order is illustrated in figure 2.

![PRO-OPTIMAL Criteria Weights](image)

Figure 2: The incomplete rank order for the importance of the criteria

Instead of obtaining an optimal portfolio, the objective here is to form the robust “good” and “bad” subset of projects. Because the projects could be funded with a floating amount of funding, the decision was considered merely a fund / not to fund -decision. Every project was associated with an equal cost of 1 and the budget constraint was set to $m/2 = 24$, so that half of the projects could be chosen in a portfolio.
The PRO-OPTIMAL software found six non-dominated portfolios to this problem, and it calculated the core indexes of every project. The core indexes of projects are presented in figure 3.

The core $C$ and the exterior $E$ are relatively large. These subsets form, however, a good separation of all projects and we choose these subsets to represent the “good” and the “bad” subsets mentioned above to be examined more closely. There are several qualitative measures gathered from the ex ante evaluations and we can compare their realizations in these subsets.

For example, there are some differences in the preceeding financing of the projects: only 10% of the core projects had earlier (before TULI) been given funding from the National Technology Agency, whereas 38% of the exterior projects had been given this funding. Secondly, only 30% of the projects in the core had been given support from a university’s or a research facility’s innovation services, whereas 48% of the projects in the exterior had the corresponding support. This information is presented also in diagrams in appendix A.

We can perform a statistical test (see, e.g., Milton and Arnold, 1995) ($\chi^2$-test for homogeneity) for these differences as follows. The null hypothesis $H_0$ is that the ex ante evaluations of core and exterior projects originate from the same probability distribution. The degree of freedom is 1, because there are two sets to be compared and only two possible answers. The test statistics for the first difference is $\chi_0^2 \approx 4.38$ and for the second difference $\chi_0^2 \approx 1.34$. The critical value of the test statistics $\chi_{1-\alpha}^2 \approx 3.84$ with the risk level $\alpha = 0.05$ [11]. Thus, the first difference is statistically significant, but the second one is not.
Figure 3: Core indexes for single projects
4.3 Results

There are several differences between core projects and exterior projects in ex ante evaluations. Despite the fact that not many of these differences are statistically significant there are, though, few interesting viewpoints. All cooperation, for instance with universities, seems to be more typical for core projects than exterior projects. Also, the use of team work is more typical in core projects, and controversially a project with just one innovator is more likely an exterior project. These differences are presented in appendix A.

5 Conclusions

In the illustrative example we could see that there were differences in the ex ante evaluations between the core projects and the exterior projects. The project performance measures were although constructed using only ex post evaluations. We also found out that the some of these differences were statistically significant. This strongly suggests that there is some kind of causality between the ex ante evaluations and the ex post performance.

The methodology does not, however, try to explain the causality itself. Neither should it be used to strictly determine if causality exists, but to find possible (unknown) causal relationships. This methodology can raise new hypotheses of causal relationships, and offers a creative perspective to the critical debate.

The methodology also enables the use of statistical tests. If the statistical tests can verify that the differences between the core and the exterior (in the ex ante evaluations) are statistically significant, it also provides with argumentation support for the existence of causal relationships.
The main weakness of this methodology is that it is moderately time consuming to construct an *ex post* evaluation scoring model. Typically these evaluations are purely qualitative, and constructing the additive value function needs some effort. Secondly, the methodology does not come up with decision rules or judgements, but it should be used as a supportive tool for decision analysis. The methodology has a novel approach and it has avoided becoming too stiff.
References


Appendix

A Case TULI: some differences between the core and the exterior

Before TULI, the project was funded by the Finnish National Technology Agency

Core C

90 %

10 %

Exterior E

62 %

38 %

Before TULI, the project had been given support by an university or a research center

Core C

70 %

30 %

Exterior E

52 %

48 %
The project was executed in co-operation with a technology center

Core C
- 5% (n.a.)
- 50% (Yes)
- 45% (No)

Exterior E
- 5% (n.a.)
- 62% (Yes)
- 33% (No)

The project originated by a single innovator

Core C
- 5% (Yes)
- 95% (No)

Exterior E
- 19% (Yes)
- 81% (No)