Nordpool spot volatility modeling and option pricing

Mat-2.4108 Independent Research Project in Applied Mathematics

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1 Introduction

Electricity market, like other commodity markets as, is an important tool for power producers, consumers and as market speculators for maximizing their profits and hedging risks. From the perspective of an individual power producer it is an efficient tool for planning the profits and minimizing risks over horizons of various lengths. The investment cycle of buying or building a generation facility, the time scale associated with a fuel cycle of a condensing power plant and the time scales of weather patterns that largely determine the power price in the nordic markets differ greatly in length.

Buying fuels over a horizon the order of magnitude of years and making possible profits (or losses) on the electricity markets that change wildly overnight brings about the need for financial planning in order to hedge one’s banksheet against the price fluctuations.

Another factor that creates its own uncertainty to electricity markets is the unique production structure of electricity. There are various forms of power generation, each with their own physical characteristics, costs of production, ramp-up times, maximal capacities, but from the viewpoint of end user, the same product. From the viewpoint of the society, the free electricity trade serves as a tool for allocating resources efficiently. Assuming well functioning markets, at any given time the society should make use of the cheapest possible subset of all the capacity installed to generated the amount of electricity used at the time.\[^{9}\]

From the perspective of a market agent, this fact is one more complicating factor in the market. The differences in the cost structures of various forms of power generation make the marginal cost of producing one more megawatt of power highly dependent of how big a percentage of the total capacity is utilized at a given time. At time of high consumption the cheapest facilities not already in use are high cost per megawatt-hour power plants. As a result the marginal cost of producing electricity a highly non-linear function of the total consumption volume.\[^{1}\] \[^{2}\]

In this document a method for stochastically modeling the volatility of spot prices in the electricity markets is reviewed. We will focus on daily price changes, how this volatility can be modeled with a stochastic simulation and how it affects the price of derivative products whose underlying product is the electricity spot price.

In order to constrain the approach, we will refrain ourselves from a deep fundamental analysis of the underlying market. Fundamental effects that are not analysed but taken into account by stochastic uncertainty contain

- Reasons for the price fluctuations
- Weather patterns
- Effects of the holidays such as
Daylight saving changes

many of these are however taken into account by the price forecast that we will use as one part of our stochastic difference equation as such. We also try to give the reader a heuristic explanation on how the free parameters of the discussed model depend on the fundamental underlying phenomena.

The resolution on which we will approach the matter will be on daily level, since that is the most relevant time scale for a large scale production or consumption facility such as condensing power plants, pulp mills and the like. Furthermore, the market mechanism of Nord Pool allows bidding on such a resolution in a so-called block bid. A block bid guarantees that a quantity of electricity is sold on a all or nothing principle, guaranteeing that a large scale facility won’t be needed to be ramped up for only few hours of production.

Furthermore, we will not take into account effects of macroeconomics, possible changes in emission trading schemes and the like in a greater detail than to which they are already considered in the ready made forecast we are using. Also when considering the pricing of derivative products we will assume constant interest rate.

Since our primary interest lies in the spot price evolution, we will not consider exercise strategies of exotic options nor evolution of price forecasts and related forward curves. The archetype of options for which these factors are unnecessary are asian options, whose nominal profits to their holder depend directly on the spot price.

The methods described here have been used to analyze and forecast the evolution of the Nordpool system price on a daily resolution, but it is assumed that the approach is generalizable to other electricity markets and perhaps to other resolutions as well.
2 Black-Scholes-Merton approach

When considering option pricing, the simplest example to make is that of an European option. An European option allows the holder to buy a specified security at a specified price at a given time. The specified price is called the strike price and the specified time the expiry date. We will naturally make the assumption that the holder will not exercise the option unless it is favorable to him. That is, if the market price of the underlying security is higher than the strike price defined for the option. In addition to the so-called call options described before, there are put options that work exactly the same way, but they give the holder the option to sell the security instead of buying it.

Perhaps the best known and most established way of option pricing for European options is based on the seminal work of Black and Scholes [3]. Since the Geman-Roncoroni model, which is our ultimate tool, can be seen as an extension to the BSM approach, we give a brief introduction to the BSM framework as an introduction to the subject.

The assumptions underlying the BSM model (and the GR model as well) are [10]:

- Short selling is permitted
- Transaction costs are negligible
- Underlying assets pay no dividends over the life span of derivatives
- No arbitrage is allowed
- Security trading is continuous
- Risk free interest rate is constant
- The returns of a stock (or other underlying asset) have a certain expected value from which deviations to either side are equally likely in terms of a differential equation to be specified below.

The Black-Scholes stochastic differential equation for the price of an underlying asset, such as a stock that is assumed not to pay dividends for the sake of simplicity, is given as:

$$\frac{dP}{P} = \mu dt + \sigma dz. \quad (1)$$

In terms of the logarithm of the price $S$ one has

$$dS = \mu dt + \sigma dz. \quad (2)$$
Here $dz$ is assumed standard brownian motion. This SDE has the property that after a time $T$ the log-prices evolving according to this equation are normally distributed with mean at $\mu T$ and variance of $\sigma \sqrt{T}$.

In this idealized no-arbitrage framework where investors are risk neutral the expected profit is also the risk free interest rate $r$ giving that

$$E(S_T) = rT = \mu T.$$  \hspace{1cm} (3)

If one wishes to evaluate the value of a european option, one has to evaluate the expected profit the option. This can be divided into two cases, the option is either in the money or out of the money ie. worth exercising or not worth exercising. The value of the option is the profit of the option, should it be in the money, weighted by the appropriate probability distribution given by a gaussian variable, discounted to the present time. The present expected value of the profit must then equal the price of the option. In the case of a call option this means [4]:

$$c = dE(\max\left(P_T - K, 0\right),$$  \hspace{1cm} (4)

where $P_T$ is the price of the call option at expiry and $K$ is the strike price. $d$ is the discount factor over the investment horizon. In the case of the simple logarithmic brownian motion this value turns out to have an exact value in terms of the error function:

$$c(P_0, t) = N(d_1)K - N(d_2)Ke^{(r-T)}$$  \hspace{1cm} (5)

$$d_1 = \ln\left(P_0/K\right) + \left(r + \sigma^2/2\right)(T-t)/\sigma \sqrt{T-t}$$  \hspace{1cm} (6)

$$d_2 = d_1 - \sigma \sqrt{T-t},$$  \hspace{1cm} (7)

where $P_0$ is the current price of the underlying asset, $T$ is the maturity time and $t$ the current time.

Furthermore, the price of a put option calculated in a similar fashion or extracted from the call price using a mathematical identity called put-call parity [4].

In more complicated cases, one may generalize this approach and include more sophisticated terms in the stochastic process that generates the price trajectory. In essence the main structure of the evaluation remains the same, we are interested in the expected present value of the contract. However the distribution might be more complicated than the lognormal distribution for which closed-form solution can be found. Furthermore, the price of an asian option depends not only on the market price of the underlying security at a given instant of time, but the trajectory by which it proceeds there. In the case of non-exactly solvable probability distributions, we will simply estimate the integrals by equal-weight ensembles over randomly generated paths that obey the desired statistics.
3 Peculiarities of electricity markets

The main difference between electricity markets and commodity markets of other type, is the non-storable nature of electricity. On each moment of the day, each megawatt that is being used, has to be produced at the same instant. As a result the no-arbitrage pricing argument of Black and Scholes breaks down, since the risk-free portfolio can not be replicated by a portfolio that contains electricity, due to the non-storability of electricity. To some extent electricity can be stored for later use by using pumped-storage hydroelectricity systems and furthermore the flow of water can be adjusted to some extent to have more production at times of high consumption, but even with these two exceptions, electricity is hardly comparable to other commodities like minerals or oil for which the storage capacity is many orders of magnitude greater when compared to the average volume of consumption.

For a discussion on the theory of electricity option pricing and put-call parity in the case of non-storable commodity, see [5]

The purpose of the freely traded electricity market is to make sure that on any given moment the cheapest possible production methods are used to produce the amount of electricity needed. Since electricity can be produced in a multitude of ways, with varying cost structure and physical characteristics the cheapest possible electricity production method varies greatly in response to varying demand, temperature, hydropower supply and possible other factors, such as transmission grid bottlenecks power plant outages etc.

In practice the market price of electricity is determined by the marginal cost of producing one extra megawatt of power into the grid. This, on the other hand, depends on the facility used. Hydropower, nuclear and wind power belong to the category of low marginal costs, and once these facilities are running at full capacity, the marginal cost of electricity production is determined by condensing coal and gas powered plants. Finally the high consumption periods are covered by extremely high production cost gas turbines. Each of these have their own physical properties that further complicate the pricing mechanism. For example the time needed to get a condensing power plant up and running at full power is so large that short time scale price peaks have to be covered by hydropower plants and gas turbines.

Because of all the specialities related to electricity production the spot price of electricity is subject to very high volatility in comparison to other commodities or stock markets. These peculiar price patterns also have their effect on the modeling of the daily spot prices and option pricing. The two features not captured by the standard Black-Scholes-Merton paradigm that we consider the most relevant are mean reversion and rapid price fluctuations. Mean reversion is the phenomenon that on short time scales (hourly and daily) the spot prices appear very volatile but on longer time scales (quarterly, yearly) the spot prices are still predictable. The prices have a tendency to flow toward a base level set by the physical fundamentals such
as availability of water and fuels. Price fluctuations are a manifestation of the fact that it is somewhat easy to predict how the prices evolve on average, unless something dramatic happens. Unfortunately, dramatic, unexpected events are observed to happen. These events, such as an outage in a large nuclear facility during a cold period can triple the spot price overnight, as we have witnessed in the recent winters.
4 The Geman-Roncoroni model

4.1 Outline of the GR model

The evolution of the logarithmic price in the Geman-Roncoroni [7] model is a somewhat straightforward generalisation of the BSM-paradigm described above. It can be described by the stochastic differential equation for the log price $S$ as follows:

$$dS(t) = \frac{\partial \mu}{\partial t} dt + \theta_1 \left( \mu(t) - S(t) \right) dt + \sigma dW + h(S(t)) \, dJ$$

where the first three terms describe a mean reverting (Ornstein-Uhlenbeck) process [6]. $\mu(t)$ is the deterministic part of the log price behavior. In the limit $\theta_1 \to 0$ Orstein-Uhlenbeck process reduces to the standard Black-Scholes model. For positive $\theta_1 = 1$ the expected value of the change of the log price in a small time interval $dt$ is always in the direction of the logarithmic price in the sense that for $\theta_1$ the expected mean of $S_t$ is always dictated by the mean. In principle, the model could be extended to cover values of $\theta_1$ below zero and above 1, but such values are of little practical interest. Negative values would lead to unstable behavior where the difference between the realized and expected price grows exponentially in time. Values exceeding unity would, on the other hand, result in non-realistic oscillations around the expected price.

In addition to the continuous part the logarithmic price also has discontinuous jumps described by $dJ$. The probability of that such a jump occurs at a given time is determined by the probability function $\iota(t)$:

$$\iota(t) = \theta_2 s(t),$$

where $s(t)$ is the normalized seasonally varying function that describes the probability of a discontinuity in a given time of year and $\theta_2$ is a parameter that describes the maximal frequency of these jumps, $\theta_2 \to 0$ meaning that the system is purely described by a Ornstein-Uhlenbeck process. The prefactor $h(S(t))$ is a function that determines the direction of the jump:

$$h(x) \in \{1, -1\} \ \forall x.$$  

We will assume $h$ has the following form:

$$h(S(t)) = 1 - 2\Theta(S(t) - (\mu(t) + \Delta)),\quad (11)$$

for some constant jump threshold $\Delta$, where $\Theta$ denotes the Heaviside step function.

In order to fully describe the model one needs to specify the following factors:

- The mean reversion parameter $\theta_1$
• The probability distribution of the size of price jumps $\theta_2$

• The shape of the jump distribution. Assuming exponential distribution this can be described by one free parameter $\theta_3$

• The seasonal structure $s(t)$

A discussion on the the seasonal structure of the jumps and the time dependence of the volatility, see [8].

In addition to the parameters that describe the model, one needs to determine the following, so called structural parameters that are used in the estimation of the above mentioned properties. In practice it turns out that the daily variation is somewhat predictable, unless a jump occurs. This means that one can in a simplistic manner take the historical data and interpret all changes bigger than a given threshold to be jumps. In case of nordpool these jumps can easily be three to five sigmas large and as a result highly unlikely unless there is a jump. Thus the filtration brings about a structural parameter $\Gamma$.

In the sequel we will follow the original approach of Geman and Roncoroni and truncate the price jump price distribution and assume that no jumps bigger than a given cutoff ever occur. We will call this cutoff value $\psi$.

Now given parameters $\Gamma$ and $\psi$ one can first filter the historical discrete price series $S_n$, $n \in \{1...N\}$ into continous and discontinuous transitions:

$$C \equiv \{ n \in \mathbb{Z} : |S_{n+1} - S_n| > \Gamma, \; n \leq N \} ,$$

$$D \equiv \{ n \in \mathbb{Z} : |S_{n+1} - S_n| \leq \Gamma, \; n \leq N \} .$$

(12)

4.2 Parameter estimation

Since the continous and discontinuous part of the price are assumed to be distinguishable, one can estimate the standard volatility $\sigma$ separately from the other parameters of the model. This is done as:

$$\hat{\sigma} = \sqrt{\frac{1}{|C| - 1} \sum_{n \in C} (S_{n+1} - S_n)^2}. \quad (13)$$

As one has the volatility and a guesstimated shape along with the maximal size of the price jumps $\psi$ one can compute the logarithmic likelihood function (simply put, the logarithm of the product of individual transition
probabilities) $L$ which is, up to a unimportant constant, given by:

$$
\ln \left( \prod_i P(S_i \to S_{i+1}) \right)
= \sum_i \ln (P(S_i \to S_{i+1}))
= L(\theta) = L_C + L_D
= \sum_{n \in C} \left( \ln (1 - \theta_2 s_n) - \ln (\sigma) - \left( \frac{S_{n+1} - S_n - (\hat{\mu}_n + 1 - \hat{\mu}_n) - \theta_1 (\hat{\mu}_n - S_n)^2}{2\sigma^2} \right) \right)
+ \sum_{n \in D} \ln (\theta_2 s_n)
+ \sum_{n \in D} \left( \ln \left( \int_{-\infty}^{\infty} d\tilde{S} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(S - S_{n+1} - (\hat{\mu}_{n+1} - \hat{\mu}_n) - \theta_1 (\hat{\mu}_n - S_n))^2}{2\sigma^2}} p(S_{n+1}, \tilde{S}) \right) \right),
$$

\begin{equation}
p(\tilde{S}, S) = \frac{\theta_3 e^{-\theta_3 |S - \hat{S}|}}{1 - e^{-\theta_3 \psi}}.
\end{equation}

In practice not all the jumps are in the desired direction so an approximation is made in the definition of $p(\tilde{S}, S)$ in order clear the product in the likelihood function from a zero term that would produce $L \to -\infty$.

The parameters $\theta$ are determined by maximizing the logarithmic likelihood function. In practice this is done with different sets of structural parameters $\Gamma$, $\Delta$ and $s(t)$. Then a simulated set price paths are generated with the parameters obtained from maximizing the likelihood function and the first four moments of the price returns based on the newly simulated price paths are compared to the historical data and the structural parameters are then adjusted, if necessary.

Once the parameters of that characterize the model have been estimated, one can generate paths by standard Monte-Carlo methods, such as the Metropolis algorithm. Since there is no simple solution to the model at hand, we initially set $\mu_i = 0 \ \forall i$ when generating the log-prices that are stored into the memory and once the ensemble has been gathered, we make a correction such that the expected prices match those of our forecast:

\begin{equation}
P_i = \frac{\exp (S_i) E(P_i)}{\sum \rho \exp (S_i^\rho)},
\end{equation}

where $\sum \rho$ denotes the sum over the ensemble of the paths created.
Table 1: The historical and simulated moments of the Nord Pool system price based on 1000 trajectories generated.

<table>
<thead>
<tr>
<th>moments</th>
<th>historical</th>
<th>simulated</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.005</td>
<td>-0.001</td>
</tr>
<tr>
<td>variance</td>
<td>0.0170</td>
<td>0.0144</td>
</tr>
<tr>
<td>skewness</td>
<td>0.1834</td>
<td>0.1575</td>
</tr>
<tr>
<td>kurtosis</td>
<td>41.9895</td>
<td>39.1959</td>
</tr>
</tbody>
</table>

4.3 Structural parameters and observed moments

In practice the adjustment of the structural parameters can be done by an automated matching of the mean, variance, skewness and kurtosis of the price returns of the generated paths to the historically observed quantities.

This can be done in various search algorithms, such as minimizing the squared sum of differences of the generated and observed moments. However, for a simplified approach the following rule of thumb is observed to hold quite well: The larger the jump threshold $\Delta$, the larger the skewness. Similarly the kurtosis of the distribution can be increased by lowering the decay factor $\theta_3$ of the exponential distribution describing the size distribution of jumps.

For the remaining two structural parameters, we set as follows: $s(t)$ is assumed a phenomenological periodical function higher in winters.

$$ s(t) \propto \left(3 + \sin \left(\frac{2\pi t}{365}\right)\right)^3 $$

when time $t$ is measured in days from the mid-January when the largest jump rate is observed. The maximal jump parameter is simply set to the biggest jump observed in the historical time series.

The structural parameters are then adjusted to match in order to make the realized first four moments of a simulated data sample to match those of the historical data in the last five years. As a result, we set the following: $\psi = 2, \Delta = 0.02$ which gives us, after a maximizing the log-likelihood the moments given in table 1.

4.4 Option valuation

Once the paths have been generated, valuing options is rather simple, the profit of an option whose value depends directly on the spot price has a statistical distribution that is sampled by the ensemble of trajectories created through the monte carlo simulation.

For example, for an asian call option for a period ranging from $Q_i$ to $Q_f$, ...
whose profit to its holder is given as:

\[ R = d_f \max \left( \left( \frac{1}{Q_f - Q_i} \sum_{j=Q_i}^{Q_f} P_j \right), T \right), \quad (17) \]

where \( T \) is the strike price of the option and \( d_f \) the discount factor associated with time \( Q_f \) when the option is exercised.

The value of such an option contract can be evaluated as the expected value of the profit over the simulated ensemble of paths.

In similar fashion, the model can be used to estimate values of real options such as power generation facilities, spark and dark spreads and various other products whose values depend on the spot price that may be simulated in a similar fashion.

The true value of such approaches lies in simple evaluation of structured products, portfolios and other complicated financial instruments. Once the trajectory ensemble has been created, it is rather straightforward to compute from the sample an estimate for a confidence interval for which selling price it is worthwhile to write an asian option.

Also, in the case one has already written such option contracts, one can evaluate the the value of writing another asian option in the light of having a set of options already in one’s portfolios. Eg. assuming one has a power generation asset such as a coal condensing plant whose output one wants to hedge by writing asian options, whose liquidity is somewhat poor on the nordic market. If one has already written asian options for a part of the total output of the generation facility, one can evaluate the feasibility of writing more options (perhaps of different sort) and the risk that the two sets of options are exercised at the same time.
5 Delta hedging and the like

When the stochastic modeling is used for hedging purposes, the question whether the model at hand could be used for determining the appropriate weights for delta-neutral portfolio. The answer to this is, at least in principle, yes.

In standard delta hedging practiced in the stock market one forms a portfolio of an option and its underlying security. The weights of the option and the underlying security are determined by demanding that the derivative of the value of the portfolio with respect to the changes in the price of underlying security is zero. That is, small changes in the price of the underlying cause only second order changes in the value of the portfolio.

In the case of asian options whose underlying security is the spot price after at a certain time in the future the case is more subtle. In the case one is considering an asian option on a quarter or a month in the future, there might be liquidity for the forward contract on the same underlying security. Even in this case it is not quite clear how one should interpret the derivative of the value of the option contract with respect to the price of the underlying.

In case OTC option products the case is even more fuzzy. The only price for the underlying security is the forecast that one has. Then one can delta hedge against the changes in the forecast, for example assuming parallel changes in the price trajectories. Assuming one has a set of trajectories $P_\rho(t)$, this can be done by solving, in the case of a portfolio of a written asian call option and a short futures contract for the same period of time $t_1 \ldots t_2$

$$\alpha \alpha - 1 = \sum_\rho \partial_\rho \max \left( \frac{1}{t_2 - t_1 + 1} \sum_{t = t_1}^{t_2} P_\rho(t) - K + \Delta, 0 \right),$$

where $\alpha$ is the proportional weight to be invested in the futures product. Similar results can be derived for more complicated portfolios as well.

If the purpose is to minimize risk, the most obvious way to do this is to directly minimize a given risk measure of a portfolio, for example VaR or variance of profits. The stochastic approach gives excellent tools for this calculation. For an ensemble of trajectories, one can minimize variance of
the portfolio of the previous example simply by solving

\[
\min \sum_{\rho} \left( \alpha \frac{1}{t_2 - t_1 + 1} \sum_{t=t_1}^{t_2} P_\rho (t) - (1 - \alpha) \max \left( \frac{1}{t_2 - t_1 + 1} \sum_{t=t_1}^{t_2} P_\rho (t) - K, 0 \right) \right)
\]

(19)

\[
\alpha \frac{1}{t_2 - t_1 + 1} \sum_{t=t_1}^{t_2} P_{\rho'} (t) - (1 - \alpha) \max \left( \frac{1}{t_2 - t_1 + 1} \sum_{t=t_1}^{t_2} P_{\rho'} (t) - K, 0 \right)
\]

\[
- \frac{1}{N_{\rho'}} \left( \frac{1}{t_2 - t_1 + 1} \sum_{t=t_1}^{t_2} P_{\rho'} (t) - K, 0 \right)^2
\]

(20)

with respect to the proportional weight \( \alpha \).
6 Backtesting

In order to measure the performance of the stochastic approach we have reviewed how a fictitious trader might have performed in case he would have made his trading decisions based on the recommendations on the stochastic model described in this document.

6.1 First case - proprietary trading

In the first, most basic case is one with very loose criteria for trades to be made. In the scenario the trader is able to take a long or short position in any product he or she considers feasible. However, the investment horizon has been limited so that the trader is only allowed to invest in future products whose delivery is at least fifty days from the investment decision. Similarly, the end of the delivery period must not be more than 750 days ahead.

For the purpose of simulation, a sample of forecasts is generated for the time span of the simulated interval. These simulated forecasts are then used as the deterministic part of the stochastic differential equation.

The simulation is run for the available time span starting from August 2006 and ending in June 2010. Starting from the start of the considered period, new simulation is done with the most recent forecast available at the time and the parameters of the GR model fine tuned according to the data available at the time. Then, in the following 30 days the allowed products are bought or sold from the market at the market closing price if based on the simulation it seems that the deal would be in the mone at a probability of 60 per cent. The trades are taken at the pace of one megawatt per each calendar day assuming that the market price seems attractive for that day.

Then, after 30 days the cycle is started again and new target prices for selling and buying are determined to allow trading for the next 30 days.

The trades made are then benchmarked against the spot in order to determine if the imaginary trader would have made a profit in betting against the spot. All the trades made are presented in the cumbersome, but fancy looking diagram in fig. 1.

The trades made on a given day are presented in fig. 2 which shows the clear behavior that at the bull run before the collapse of US mortgage market the market lied well above our expectations resulting in profitable selling. However, the after the burst of the bubble, our expectation for the future power price was overly optimistic which resulted in losses from long positions on various forward contracts. After the subprime the predictability of the market, in part because of the short historical track, would have been underestimated resulting in an effective increase in the investors risk-averseness. Running the simulation with an alternative fixed long-term simulation suggests that this is at least in part an effect of the short history of historical forecasts available.
Figure 1: All trades made over the course of the backtesting exercise. The figure is created as follows: each of the lines represents a one megawatt trade that the trader would have done at the time point of the left end of the line. The right end of the line points to the time period of the delivery and the delivery price. Green lines would have resulted in positive profit, whereas red ones indicate potential losses. I.e. green rising lines indicate profitable long positions, red rising lines indicate losses in long positions and so on.

Furthermore, not all the positions that would have been opened in the late simulation have run into delivery making it impossible to effectively measure the performance of the trading algorithm at recent times.

6.2 Second case - asset backed trading

The second case is perhaps the most realistic of all the cases. The scenario simulates the behavior of a trader whose sole objective is to maximize the profits on selling the output of a constant output 200 MW power plant, such as a share in a nuclear power plant.

The trader is allowed to trade on all the same products as the trader in the previous scenario. However the asset backed trader has a tighter mandate: he is only allowed to sell short the capacity of the power plant, not more. Specifically, the trader is allowed to take a short position of 100 MW on all the quarter products as well as all the years, resulting in a total short position of 200 MW. Once the position has been opened, the trader may
Figure 2: The cumulative value creation of the proprietary trader in comparison to spot. The created value is dated to the day of the trading decision even though the physical delivery might be far in future. Thus trades in the recent years can not be evaluated at the time of writing.

close it again if the expected spot behavior or the forward price seems to support this choice.

The thresholds for selling and buying match those of the previous scenario and thus this scenario reduces to picking a subset of the trades made in the previous scenario in order to satisfy the mandate. The value creation of this case is presented in fig. 3.

6.3 Conclusions

All the scenarios described above share a great deal of similarities in terms of value creation since they have all three basic ingredients in common: the historical forward curve development, the realized spot history and the same forecasts for the time span under inspection.

It is clear that no rigorous analysis can be made based on the short time span of forecast history available, however the qualitative behavior that an imaginary trader would have followed, can be extracted.

In addition to the brevity of the time scale for which recorded forecasts are available, the data is plagued by the fact that despite its brevity, the sample contains the two coldest winters in the decade as well as the bull run
and the aftermath of the US subprime mortgage bubble.

The brevity of the history is especially tricky in the aftermath of subprime bubble. This is because in determining the volatility, most of the forecasts are from the period at which the market behavior has been historically hard to model. An analysis using a constant hard-coded volatility shows that with a longer data set the recovery after the subprime crisis would have been more dramatic, though this is somewhat hard to judge against the spot since not all the products available after the subprime have run to their delivery at the time of writing.

As a separate benchmark for the reliability of the GR model, one can, besides the backtesting scenarios, focus on the realisation of the probabilities created. If the path generation algorithm was reliable, there should be roughly the same amount of events in all probability deciles, and this can be seen to be the case as is presented in fig. 4. Some overrepresentation can be seen in the highest deciles but it is unclear whether this is an effect of the extremely cold winters we have experienced in the last years.

It should be stressed that all the simulated investment decisions were made according to data available at the time only and no parameters were cherry-picked in order to improve investment results.
7 Caveats and improvement suggestions, summary

The first and perhaps most obvious suggestion for an improvement lies in the analysis that forms the basis of the deterministic part of price evolution. In order to have reliable forecasts, one has to have a well formed estimate for the deterministic part of the spot behavior. After all, it is of little use to generate realistic volatility and fluctuations on top of the wrong mean.

Another caveat closely related to the one mentioned above, is the fact that there are no estimates for the accuracy of the forecast. One can estimate this parameter from historical data, but in the future it might be beneficial to consider alternative scenarios in terms of hydropower situation and commodity prices and use these as a basis to set the volatility based on fundamentals such as simulated supply curves.

There are factors that are left unmodeled, one of the most crucial assumptions we are making, is the assumption of constant interest rates. This could of course be attached to the study at some computational cost by studying the historical correlations between electricity prices and interest rates. However, it is our first assumption that this is not a dominant factor in the pricing of an option.

Another source for the estimates of the volatility of the price forecasts could, at least in principle, obtained from implicit at-the-money volatilities of options traded openly at the Nordpool market. This complementary view-
point, has its drawbacks, however. One of them is the fact that these implied volatilities refer to the price of a forward contract, not the spot price directly. Another problem inherent in this line of thinking is the poor liquidity of the options traded at the Nordpool making it uncertain, how sensible these volatility estimates in fact are.

In assuming the lognormal distribution, the volatility is constant when measured as a proportion of the total price. Thus, in winter times the volatility, as measured in euros per megawatt hour is larger because of two factors: the higher spike occurrence rate and the fact that the mean tends to be higher in winter times. Since the average price is matched to the expectation manually, this can result in exaggerated downside exposure. There are three possible salvations (and combinations thereof) that come immediately into mind.

The first of these is to plainly model the random walk as arithmetic instead of geometric and fit the parameters accordingly. As a result one would have, neglecting the effect of spikes, a constant volatility as measured in euros. Should this line of thinking prove inadequate, it could be extended by price-dependent or time of year dependent volatility.

In conclusion, we have presented a stochastic model aiming to model the volatile movements in the spot price in the nordic market. We have presented methodology for parametrizing the model based on historical data and added a notion for the inaccuracy of the price forecasts.

The model has shown potential as a tool for planning investment decisions in both the forward market and options trading. Despite the shortcomings listed above, we assume that the model is a welcome addition in the portfolio manager’s toolbox.

We assume that an extension of the model to evaluate various kinds of spread options and instruments with multiple underlying securities will provide a useful quantitative view on the exposures to various kinds of risks one might have in a cross-commodity trading environment.
References


