MS-E2108 Independent Research Projects in Systems Analysis

Comparison of Safety Stock Policies

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Turkka Anttonen

Instructor: Professor Ahti Salo
Supervisor: Professor Ahti Salo

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1 Introduction

Supply chain networks are collections of organizations that are interconnected via flows of information, money, and physical goods. There are many studies where the performance of a supply chain is assessed as a whole and not only from a single organization's point-of-view. In many cases, it has been observed that centralizing the decision making in a supply chain (or setting rules that enforce or incentivise to follow a centralized policy) improves performance (e.g. Cachon and Zipkin 1999). Lee et al. (1997) present reasons for the amplification of the demand variance in a multi-tier supply chain. This amplification is called the bullwhip effect, which has drawn the attention of many.

Production-inventory systems are a common in supply chain research. The studies vary in the objectives, supply chain modelling, and methods. The objective is often formulated as an optimization problem, either as cost minimization or profit maximization. Supply chain parameters and assumptions or simplifications also vary. Decisions are made about which constraints or system dynamics are included in the model and how. One significant decision is that of defining which parts of the model are stochastic and which are deterministic. In reality, practically all parts of a supply chain exhibit uncertainty, but being able to accurately model such uncertainty can make the model unnecessarily complicated. It can also mean that analytical methods become impractical and numerical analysis and simulation are necessary.

Inventory-production models can be broken into subcategories. Some studies focus on interactions between two parties, a supplier and a customer, while some studies extend the supply chain to cover multiple tiers of supply chain. These are called multi-echelon inventory models. Another difference that distinguishes approaches is whether the supply chain is a distribution network for one (commodity) product or an assembly system, where each assembly node requires a unique input from each upstream node in order to start producing the assemblies. The third fundamental difference between models is whether continuous time is assumed or whether decisions are made at discrete points in time. The latter are referred to as periodic review models.

Combining stochastic demand and constraints on production lead into the consideration of safety stocks. Stock required for a functioning inventory operation is divided into cycle stock and safety stock. Cycle stock is the stock required to fulfill demand during the lead time of replenishing inventory, e.g. expected one-month demand if the order lead time is one month. Safety stock reduces the likelihood of a stockout when demand turns out to be greater than expected. Required safety stock is typically expressed in terms of service level that describes the probability of a stockout, the expected backlog, or the fraction of expected demand fulfilled. These are called α, γ, and β service measures, respectively (Lagodimos 1992).

Two different stocking strategies are compared in this study. Both describe the same multi-echelon assembly network facing uncertain customer demand but in one of the strategies, there are stockpoints only on the first supplier tier. The upstream supply chain is modelled as a single production process that has the same lead time as the longest path in the original model and the same cost accumulation profile. Because the lead time is long enough so that the upstream supply chain can react to any demand in lead time, no safety stocks are needed upstream. The other one models the whole supply chain with safety stocks at each assembly node. The objective is to study the qualitative implications of approaching safety stocks differently in the supply chain network. The main interest is in understanding how the different policies drive capital allocation in the supply chain and how delivery performance towards external customers is impacted.

2 Modelling the problem

The supply chain network used in this study consists of the manufacturer of a single product and the suppliers of components needed to assemble the product. The components suppliers' suppliers are modelled to varying degree. The physical flow of goods is assumed to be deterministic, i.e. when production is started, the corresponding delivery is made after a fixed lead time. However, production starts are constrained by the availability of components from upstream. Any production starts that cannot be made in accordance to the centralized policy are backlogged. There are no capacity constraints and ordering cost is assumed to be zero. The nodes that do not require input from other nodes draw from an infinite pool of resources so there are no constraints on their production decisions. There is a cost for holding stock and having materials in the production pipeline, but there is no separate cost for backlog.

The demand that the manufacturer faces is stochastic. The external demand process is stationary and follows the normal distribution. The manufacturer as well as the rest of the supply chain know the exact parameters of the process generating demand. Information flow is immediate and each node has identical accurate information on the external demand when it is observed at the beginning of a period. Each node will order and/or start producing according to a centralized policy, which is to match production starts with the external demand at each period, i.e., following an order-up-to policy where the inventory (position) target is the cycle stock plus safety stock. Each assembly node will keep safety stock of the components it needs. This matches the (S, T)-system described in Schneider (1981). The supply chain, along with the lead times of the nodes, is shown in figure 1. This model is denoted by $\mathcal{M}_1$.

The simulation is started by generating initial orders at each node in such a way that at the beginning of the first period with external demand, all the nodes have the set safety stock level available and enough production to cover the expected demand during the following periods. After that, the system is updated using the following algorithm:

(i) Stock is increased by the amount of products whose lead time is up.

(ii) External demand for the period is observed and each node in the supply chain network will start production to cover that demand and any

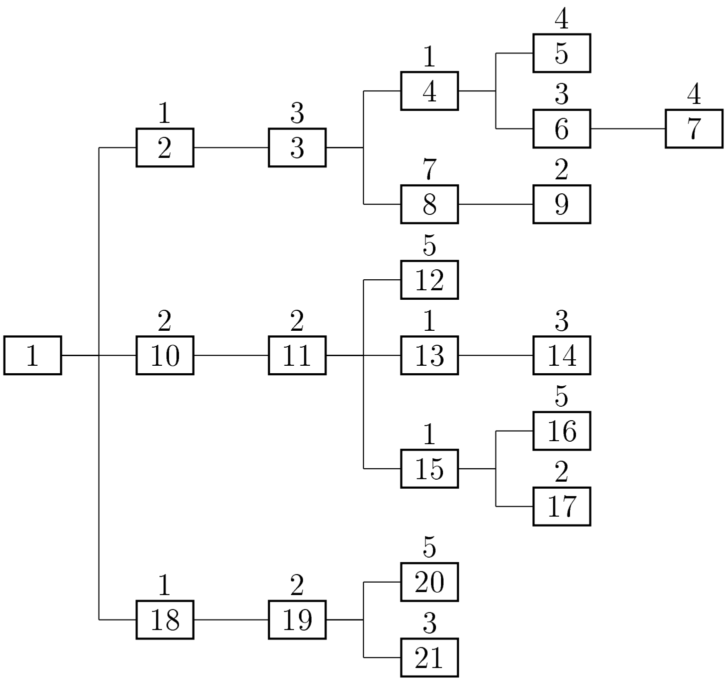

Figure 1: Supply chain network used in this study. Node 1 faces external customer demand and receives goods from nodes 2, 10, and 18. Lead times for each production process are above the respective nodes.

backlog from previous period. Any amount that cannot be satisfied because of resource constraints is backlogged.

(iii) Stock is reduced by the amount used for production.

The following notation is used in this study:

$s(i,t)$ = stock of component i at the beginning of period t
$p(i,t)$ = production starts for component i during period t
$o(i,t)$ = orders placed for component i during period t
$b(i,t)$ = backlog of production starts for component i at the beginning of period t
$d(t)$ = external demand for period t (observed at the beginning of period t)
μ = expected value of external demand
σ = standard deviation of external demand
$l(i)$ = lead time for component i
$f(i)$ = safety stock for component i
$C(i)$ = set of child nodes for node i
t_0 = first period with external demand

For demand, we can write the definition of t_0 in relation to $d(t)$ and the distribution of $d(t)$ as

$$d(t) = 0 \forall t < t_0 \tag{1}$$
$$d(t) \sim \mathcal{N}(\mu, \sigma^2) \forall t \geq t_0. \tag{2}$$

Safety stock is

$$f(i) = z\sqrt{l(i)\sigma^2}, \tag{3}$$

where z is a service factor that quarantees an α type service level. In this study, safety stock is kept at each assembly node.

Setting up the supply chain is started by defining the composite lead times. We assume that the nodes are numbered so that the upstream nodes have always bigger numbers, i.e. $i > j \forall i \in C(j)$. The composite lead times $L(i)$ are set recursively from the smallest to the largest i

$$L(i) = l(i) + L(j), i \in C(j). \tag{4}$$

The composite lead times are used to schedule the initial orders so that there is no backlog prior to t_0. We also define the cumulative lead time $\hat{L}(i)$ and the value $v(i)$ of each component recursively from the largest to the smallest

$$\hat{L}(j) = l(j) + \max \hat{L}(i), i \in C(j) \tag{5}$$

$$v(j) = l(j) + \sum_{i \in C(j)} v(i). \tag{6}$$

In this study, value is proportional to the lead time of the component and it is increased by one unit in each round after a production start. For an assembly, the value after the first round of production is the sum of child parts' values plus one.

In the same way, we define the initial orders to create safety stock

$$O(i) = f(i) + O(j), i \in C(j). \tag{7}$$

These orders are timed so that each node has safety stock at the beginning of period t_0. For simplicity, safety stock is created in one period at each node, not minimizing the inventory held prior to t_0 which will not impact the results because the performance measurement is started from period t_0. Orders prior to t_0 are thus

$$o(i,t) = \begin{cases} 0, & \text{when } t < t_0 - L(i) \\ O(i) + \mu, & \text{when } t = t_0 - L(i) \\ \mu, & \text{when } t_0 > t > t_0 - L(i). \end{cases} \tag{8}$$

The update algorithm for $t \geq t_0$ is

$$s(i,t) = s(i,t-1) + p(i,t-l(i)) - \sum_{i \in C(j)} p(j,t-1) \tag{9}$$

$$o(i,t) = d(t) \forall t \geq t_0 \tag{10}$$

$$p(i,t) = \min\big(o(i,t) + b(i,t), s(k_1,t), \ldots, s(k_n,t)\big), \text{where } k_1, \ldots, k_n \in C(i) \tag{11}$$

$$b(i,t) = b(i,t-1) + o(i,t-1) - p(i,t-1). \tag{12}$$

Equations (9), (11), and (12) are used to update the states also when $t < t_0$.

The model $\mathcal{M}_1$ is compared against a variation, where the supply chain network is simplified by aggregating the nodes 2 to 9, 10 to 17, and 18 to 21 into single nodes with the lead times equal to the cumulative lead times in $\mathcal{M}_1$. As the lead time is long enough for the chains to react to the orders coming from node 1, there is no need to keep safety stocks of the components in the chains beyond nodes 2, 10, and 18. The tradeoff is that the supply chain is slower to respond to changes in demand. The varied model is called $\mathcal{M}_2$ and is shown in figure 2.

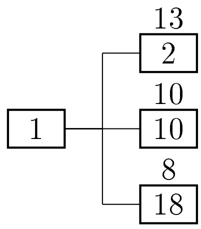

Figure 2: The alternative supply chain where upstream chains beyond nodes 2, 10, and 18 are replaced by increasing the respective lead times to the sum of lead times on the critical (longest) paths.

The performance of the supply chain can be measured using multiple metrics. Hwarng et al. (2005) studied a multi-echelon distribution system but the metrics are applicable to an assembly system as well. They measured average stock level, average backlog level, and average cost. We do not associate cost with backlog in this study, but we do calculate metrics related to both average stock and backlog levels. In an assembly system, the value of the components and assemblies vary. The value of an assembly is greater than or equal to the sum of values of its components. We have made such an assumption in (6) by modelling the cost accumulation as a linear function of the lead times, i.e. the value being incremented by one unit during each period that the component or assembly is in the production process. Due to the unequal value of components, average stock does not describe the performance of an inventory system from financial point-of-view. Hence, we follow the capital tied in the system. Capital tied in a node includes the stock at the beginning of the period and the production starts that have not turned into stock yet. It is

$$C(i,t) = s(i,t)v(i) + \sum_{k=1}^{l(i)-1} \big(v(i) - k\big)p(i,t-k). \tag{13}$$

Measuring capital tied is started at $t = t_0$ and ends after n periods.

Backlog is only measured for node 1. While backlog is possible in the other assembly nodes as well, only backlog at node 1 is relevant for measuring the performance of the supply chain network as it is perceived by the customers generating the external demand. We calculate the mean and maximum backlogs over the n periods. We also count the number of periods where there was backlog at node 1 in order to calculate the α type service level.

In order to describe the delivery performance, we also analyze the backlog by measuring incidents. An incident is defined as a range of consecutive periods where backlog is non-zero. We count the incidents and calculate the mean and max backlogs per period during the incidents and the mean duration of the backlog incidents in periods.

3 Numerical results

Supply chain is initialized as described earlier. Service level is set at 95% and the corresponding $z = 1.64$. The cumulative lead time for node 1 $\hat{L}(1) = 13$ so t_0 can be set as $t_0 = 14$ so that there is enough time for the supply chain to be initialized. We set $n = 100000$ and draw n random numbers from normal distribution with mean $\mu = 500$ and standard deviation $\sigma = 100$. These form $d(14), \dots, d(100013)$. Then the same simulation is run for both supply chains. The results are presented in table 1.

From the results, we see that there is on average 21.8% more capital tied in $\mathcal{M}_1$. On the other hand, with the exception of the service level measure, the delivery performance of $\mathcal{M}_1$ is greater than $\mathcal{M}_2$. There were 131% more

Table 1: The results from the numerical analysis. Backlog and incident measures refer to backlog at node 1.

Measure	$\mathcal{M}_1$	$\mathcal{M}_2$
Total capital tied	2.30×10^{10}	1.88×10^{10}
Backlog		
- Mean	5.2	11.9
- Max	466	890
- Periods with backlog	8875	8409
- α service level	91.1%	91.6%
Incidents		
- Count	5673	2578
- Mean duration	1.56	3.26
- Mean	28.6	64.8
- Mean Max	59.1	135.1

backlog units in $\mathcal{M}_2$ and the maximum backlog was 91% higher. While there were more periods with backlog in $\mathcal{M}_1$, the backlog issues were clearly more severe in $\mathcal{M}_2$, which is described by the measures for backlog incidents.

From the results, we can formulate a relationship between the holding cost and the backlog penalty. Let us assume that the penalty is only imposed on the backlog at node 1. The difference of backlog units is 67506 units. Dividing the difference in capital tied with the difference in backlog, we conclude that the ratio with which the models would yield the same total cost is 6094. A simple numerical example: is we assume that the value of the finished product is \$100, annual holding cost is 25%, and we assume the period to equal a week. The relationship between dollars and unit cost in the model is obtained by dividing by $v(1) = 57$. For the cost-equalizing backlog penalty, we get

$$\frac{\$100}{57} \times \frac{25\%}{52} \times 6094 = \$51.4.$$

With smaller values of backlog penalty, $\mathcal{M}_2$ is more cost efficient.

Service levels do not follow the set service level parameters. For assembly nodes with one predecessor that has no predecessors, the service level equals the parameter. However, for nodes with multiple inputs and nodes further downstream, the service levels were lower. The service levels calculated using (3) do not take into account the possibility of backlog in the input node. By running the simulation on different service levels, we found out that both models gave 95% service level when service level parameter was set to 97.1%.

4 Summary

Supply chain is a rich topic for operations research. Various models have been developed and they cover different aspects of supply chains using different assumptions and methods. Inventory-production models are used to study optimal ordering and production policies. In this study, we have created a supply chain model where there are production and assembly nodes subject to resource constraints. The supply chain faces uncertain external demand and maintains a service level by installing safety stocks at each assembly node. The decision making in the supply chain has been centralized and every node follows the same order-up-to policy. The model is compared to a variation where the same supply chain is modelled as a two-level system. The second level is the aggregation of the respective level and the whole upstream supply chain of the first model. The question is: how does reducing the stockpoints and moving safety stock upstream affect the performance of the supply chain?

The model is created and random numbers from the known demand distribution generated for the supply chain simulation. Results show that reducing safety stock leads into less capital being tied in the supply chain.

However, the delivery performance towards the external customer is considerably worse. Estimating the correct penalty for backlog is difficult as backlog incurs different kinds of costs that are both direct and indirect. Thus, we do not make an assumption in this study but we calculate the penalty for which the two models would give cost-wise the same result. The backlog is also presented in terms of incidents which makes it possible to calculate the duration of a backlog situation that practitioners may find relevant for comparing the operational implications of the tradeoff.

Creating and using this approach for studying different safety stock strategies is easy. It is possible to model this type of a supply chain in a spreadsheet. However, in a practical setting, there are more sources of uncertainty. Assuming stationary demand is also a simplification in many cases and modelling a nonstationary demand forecast can make the model too complicated to be handled in a spreadsheet. The methods used in this study are applicable to the same problem with extensions. Further research could focus on these four: making the lead times and production yields stochastic, constraining the capacities of the nodes, and modelling the external demand as non-stationary.

References

Cachon, G. P. and P. H. Zipkin (1999). Competitive and cooperative inventory policies in a two-stage supply chain. *Management Science 45*(7), 936–953.

Hwarng, H. B., C. S. P. Chong, N. Xie, and T. F. Burgess (2005). Modelling a complex supply chain: Understanding the effect of simplified assumptions. *International Journal of Production Research 43*(13), 2829–2872.

Lagodimos, A. G. (1992). Multi-echelon service models for inventory systems under different rationing policies. *International Journal of Production Research 30*(4), 939–956.

Lee, H. L., V. Padmanabhan, and S. Whang (1997). Information distortion in a supply chain: The bullwhip effect. *Management Science 43*(4), 546–558.

Schneider, H. (1981). Effect of service-levels on order-points or order-levels in inventory models. *The International Journal of Production Research 19*(6), 615–631.