Bayesian networks for scenario analysis of nuclear waste repositories

Edoardo Tosoni[a,b], Ahti Salo[a], Enrico Zio[b,c]

a. Systems Analysis Laboratory, Dept of Mathematics and Systems Analysis - Aalto University

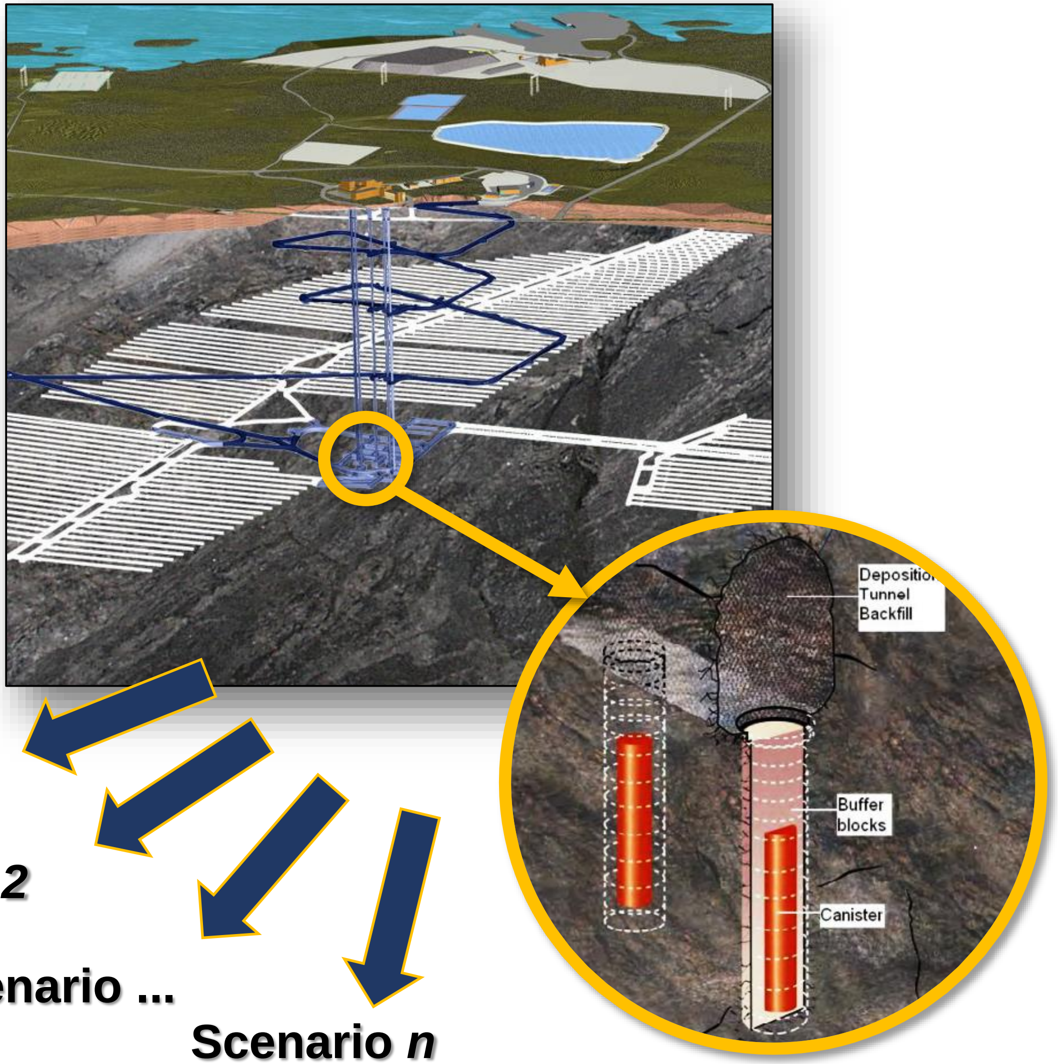
b. Laboratory of signal and risk analysis, Dipartimento di Energia - Politecnico di Milano

c. Chair on Systems Science and the Energetic Challenge - École Centrale Paris and Supelec

November 2, 2017

Background

- Deep geological disposal of nuclear waste
- For licensing a repository, safety assessment
- Large aleatory uncertainty about the evolution of the disposal system
- Typically addressed by scenario analysis



Safety ? ← **Scenario Analysis** ←
- **Scenario *1***
- **Scenario *2***
- **Scenario ...**
- **Scenario *n***

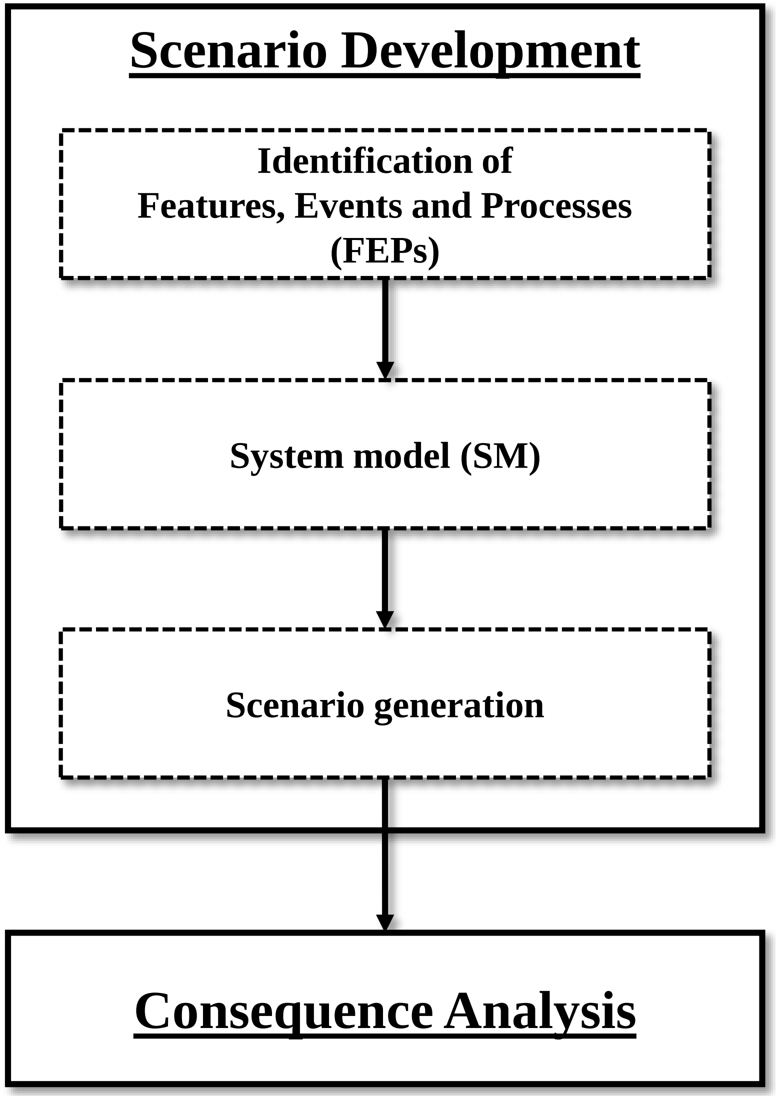
Motivation

- Spent-nuclear-fuel repository at Olkiluoto, Finland
- Emphasis on comprehensiveness
- TURMET project objectives:
  - Systematize scenario analysis for nuclear waste repositories
  - Bring methodological advancements to help achieve comprehensiveness in scenario analysis

Aalto University
School of Science
POLITECNICO DI MILANO

Scenario analysis process

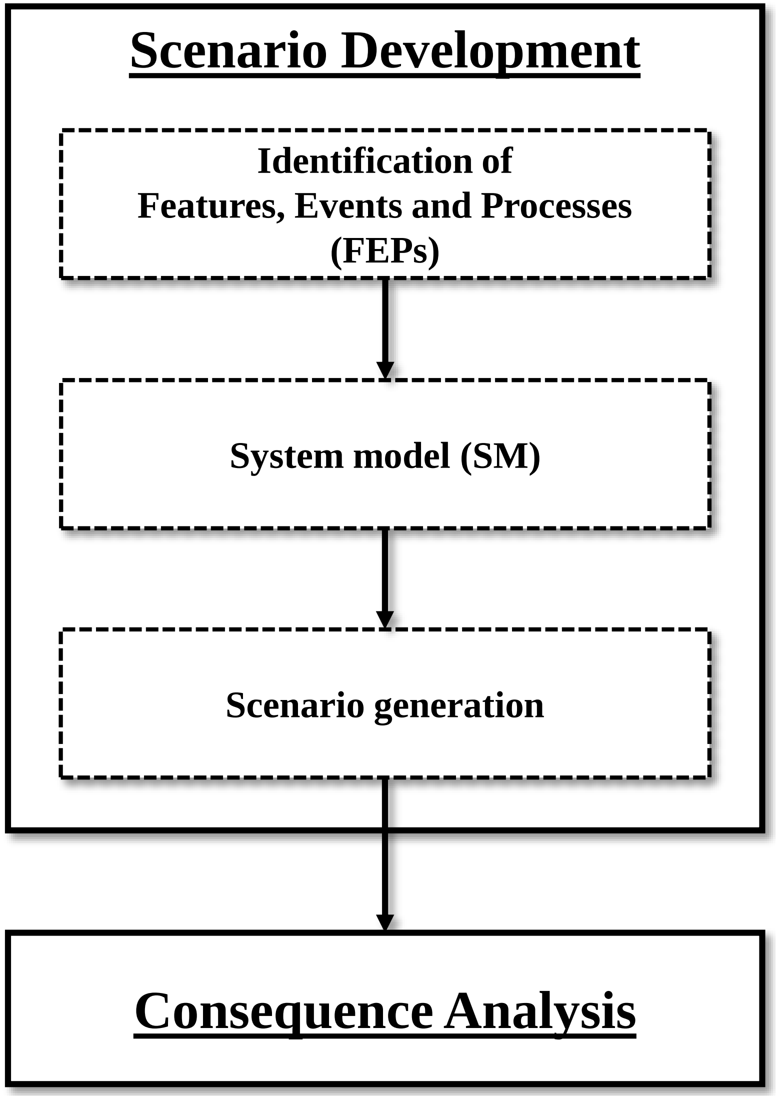
- Structure of the process:
  - Scenario development
    - Identification of the Features, Events & Processes (FEPs)
    - System model of the disposal system
    - Scenario generation
  - Consequence analysis
- Approaches to scenario generation:
  - Pluralistic
  - Probabilistic



Challenges

- Methodological challenges in scenario analysis:

1. Building a **system model** as a framework for scenario generation
2. Achieving **comprehensiveness**
3. Treating the **epistemic uncertainties**



FEPs and safety target

- Set of nodes – FEPs and safety target – and arcs
- Random variables with discrete states $z^i \in \mathbb{S}^i$

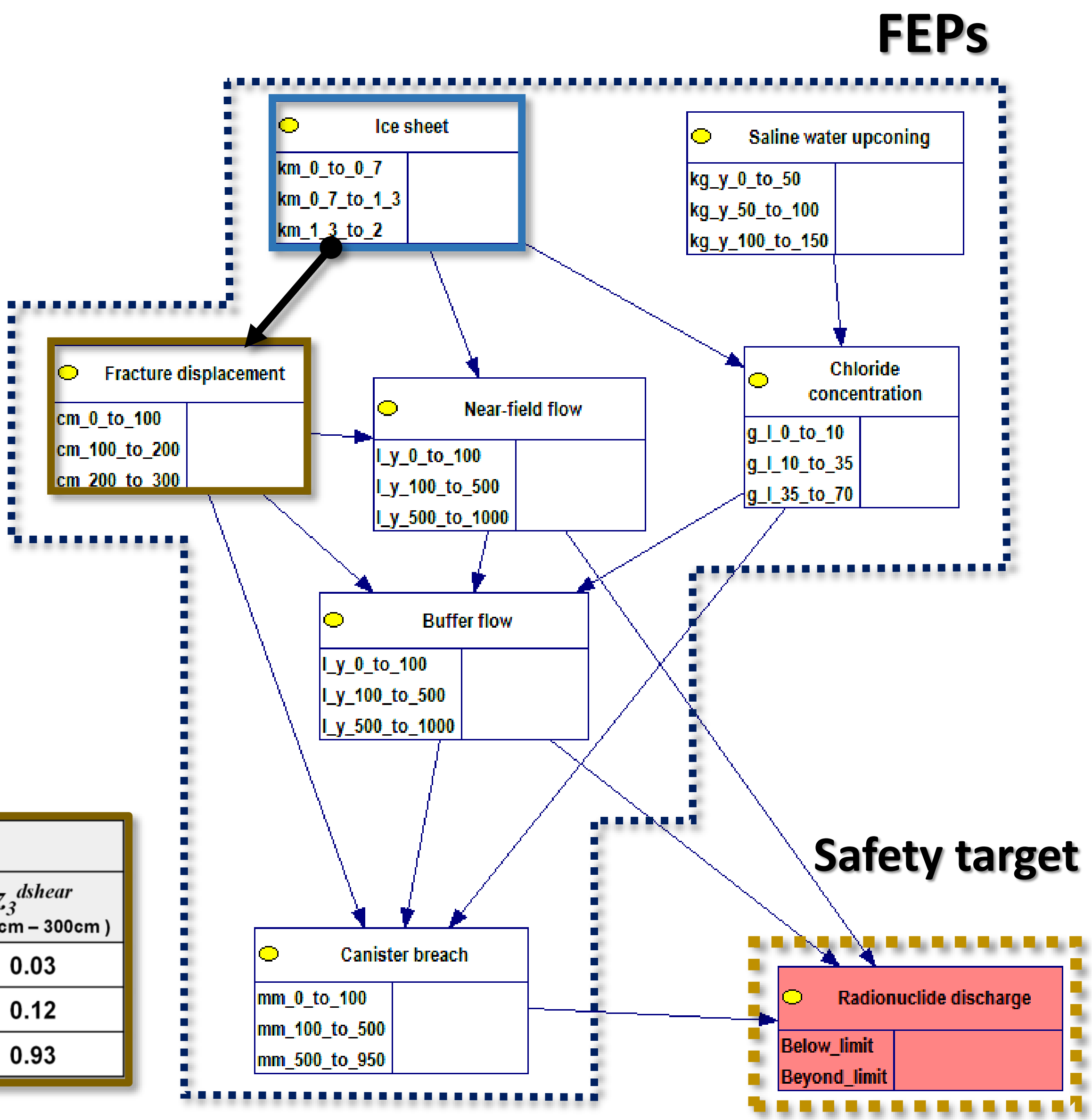
$$x^{d_{shear}} \in \mathbb{X}^{d_{shear}} = [\ 0\ ,\ 300\ cm\ ]$$

$$Z^{d_{shear}} = z_1^{d_{shear}} \Leftrightarrow X^{d_{shear}} = x^{d_{shear}} \in [\ 0\ ,\ 100\ cm\ )$$

- State probabilities:
  - Independent nodes → Unconditional $p^i_{z^i}$
  - Dependent nodes → Conditional $p^{i|\mathbb{V}^i_-}_{z^i|\mathbf{z}^i_-}$

Ice sheet		
z^e_1 [0 – 0.7 km)	z^e_2 [0.7 km – 1.3 km)	z^e_3 [1.3 km – 2 km)
0.56	0.28	0.16

Ice sheet	Fracture displacement		
	z_1^{dshear} [0 – 100 cm)	z_2^{dshear} [100cm – 200cm)	z_3^{dshear} [200cm – 300cm)
z^e_1	0.91	0.06	0.03
z^e_2	0.49	0.39	0.12
z^e_3	0.02	0.05	0.93



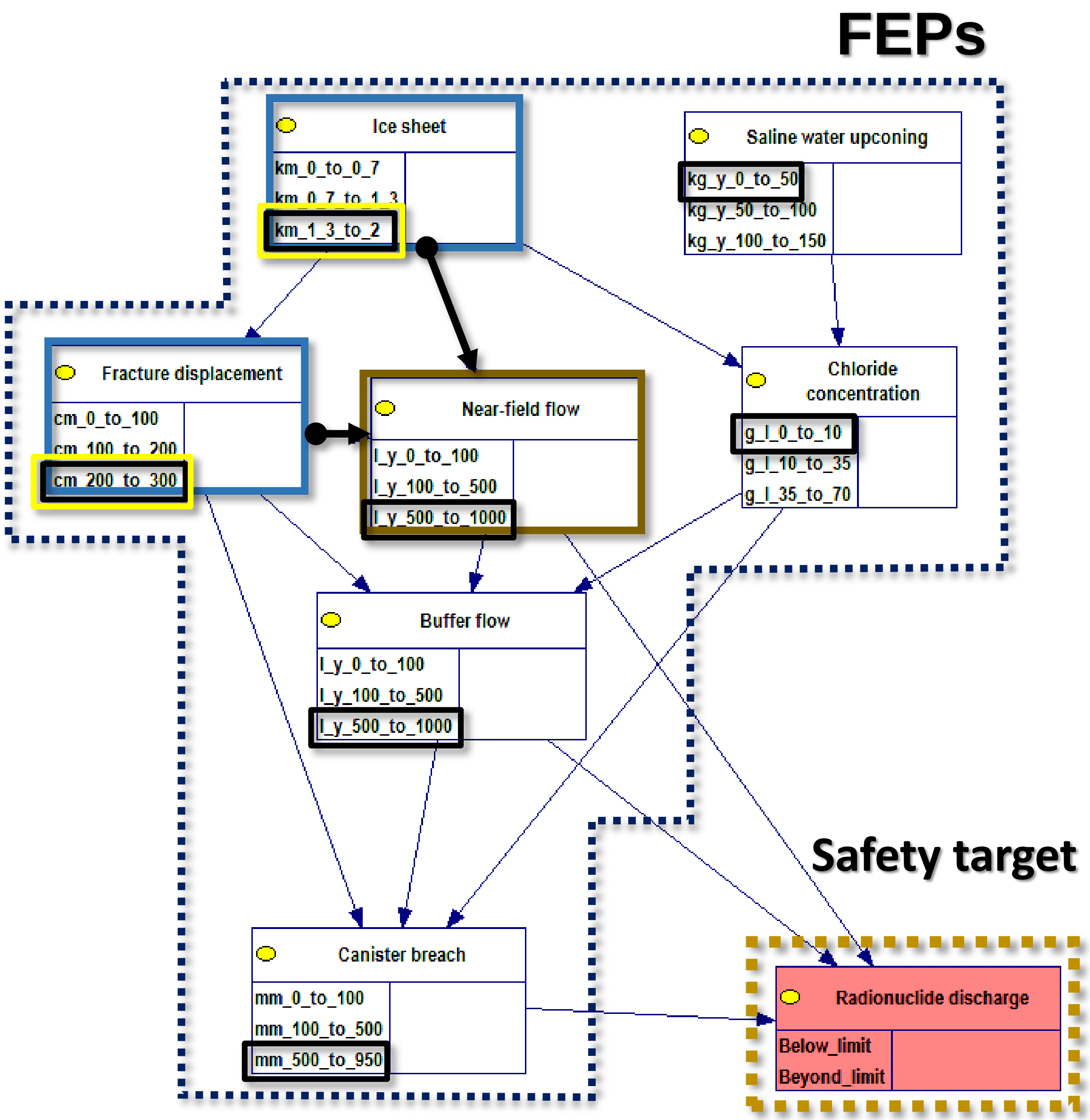
Scenarios and subscenarios

- A scenario is a combination of FEP states

$$\mathbf{z} = \left(z^1, \ldots, z^{n_{FEP}} \right)$$

- For a dependent node, a subscenario is a combination of states of its parents

$$\mathbf{z}_{-}^{i} = \left\{ z^j \right\}, \ \forall \ j \in \mathbb{V}_{-}^{i}, \ i \in \mathbb{V}^{D}$$



Safety

- State of the safety target indicating *failure*
- Total *failure probability* of the disposal system

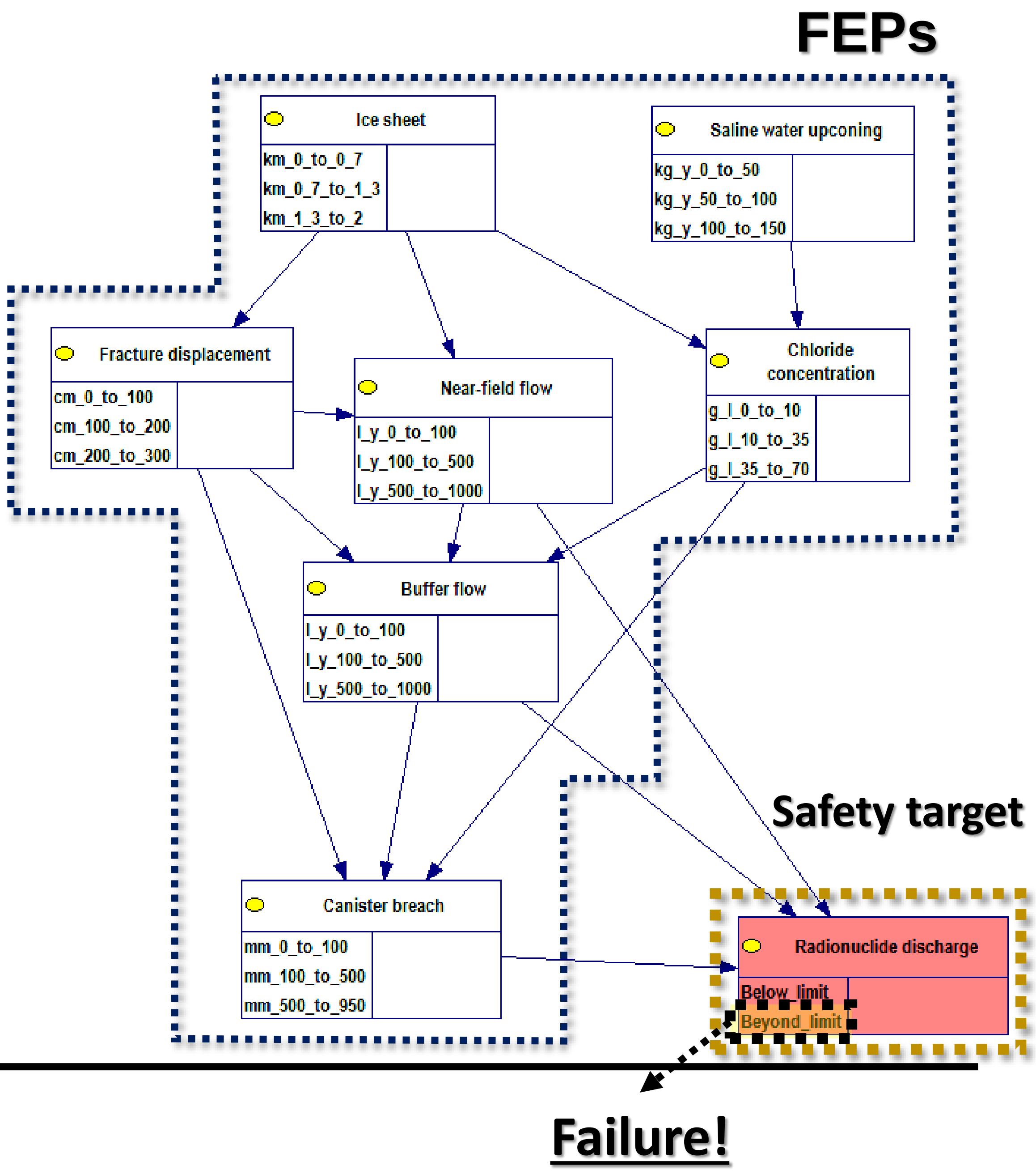
$$p_{fail}(\mathbf{p}) = \underbrace{\sum_{\mathbf{z}\in\mathbb{S}^{\mathbf{z}}}}_{\text{aggregate over all scenarios}} \underbrace{\prod_{i\in\mathbb{V}^{\mathrm{I}}} p^{i}_{\mathbf{z}_{\perp i}} \cdot \prod_{j\in\mathbb{V}^{\mathrm{D}}\setminus C} p^{i|\mathbb{V}^{i}_{-}}_{\mathbf{z}_{\perp i}|\mathbf{z}_{\perp \mathbb{V}^{i}_{-}}} \cdot p^{C|\mathbb{V}^{C}_{-}}_{z^{C}_{fail}|\mathbf{z}_{\perp \mathbb{V}^{C}_{-}}}}_{\text{joint probability of scenario } z \text{ and failure state}}$$

aggregate over all scenarios

joint probability of scenario z and failure state

Propagation

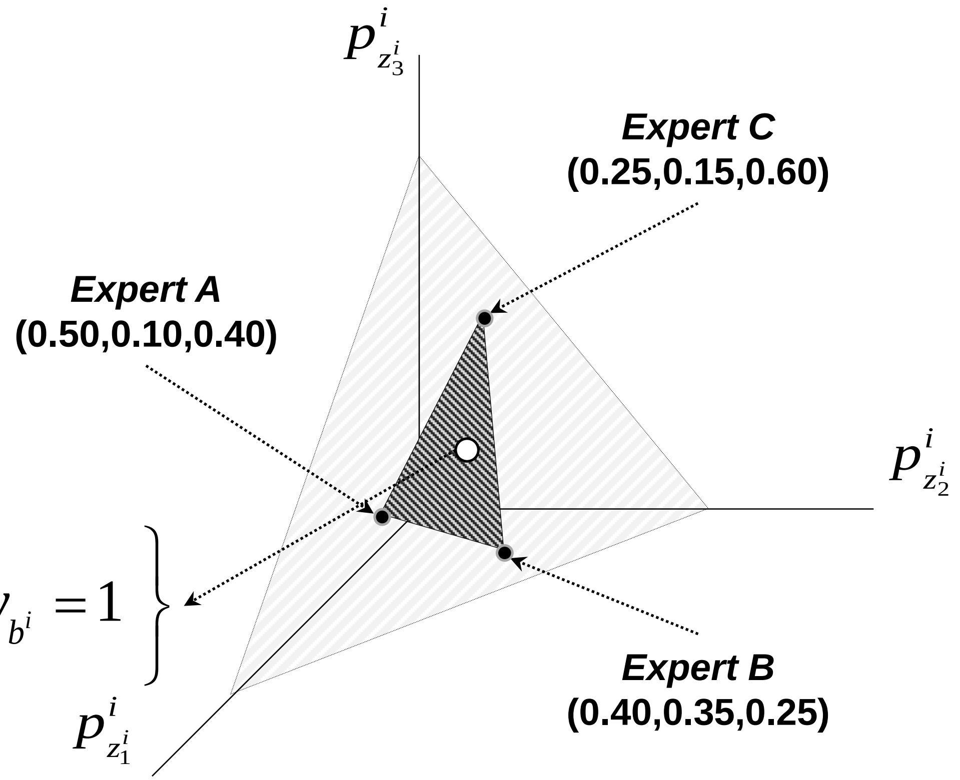
- Safety: $p_{fail}(\mathbf{p}) < \varepsilon_{fail}$



Expert judgment

- At a given node, set of experts
- Suppose one is estimating $p^i_{z^i}$
- For the state-probability vector, multiple experts' beliefs
- Feasible region for the state probabilities:
  - Convex combination of experts' beliefs

$$\mathbb{P}^i = \left\{ \boldsymbol{p}^i = \left(p^i_{z^i_1}, \dots, p^i_{z^i_{N_i}}\right) \; : \; \boldsymbol{p}^i = \sum_{b^i \in \mathbb{B}^i} w_{b^i} \boldsymbol{p}^i\left(b^i\right), w_{b^i} \geq 0 \, , \sum_{b^i \in \mathbb{B}^i} w_{b^i} = 1 \right\}$$



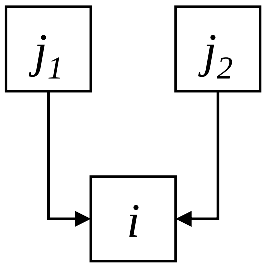
Simulations

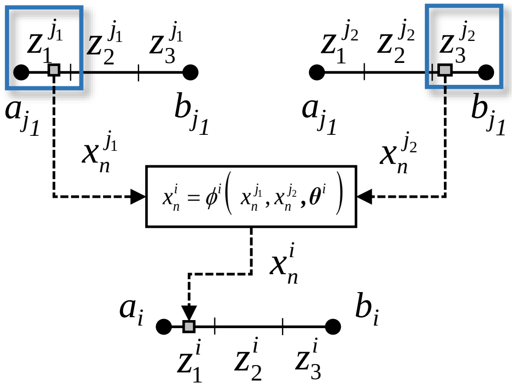
- Relationship between the continuous values of a node and of its parents

$$x^i = \phi^i\left(\boldsymbol{x}_-^i, \boldsymbol{\theta}^i \right)$$

$$x^i \in \mathbb{X}^i = [a_i, b_i]$$
$$\boldsymbol{x}_-^i \in \mathbb{X}_-^i = \underset{j \in \mathbb{V}_-^i}{\times} [a_j, b_j], \quad \boldsymbol{\theta}^i \in \Theta^i$$
$$\phi^i : \left(\mathbb{X}_-^i \times \Theta^i \right) \to \mathbb{X}^i$$
$$\left| \Theta^i \right| = 1$$

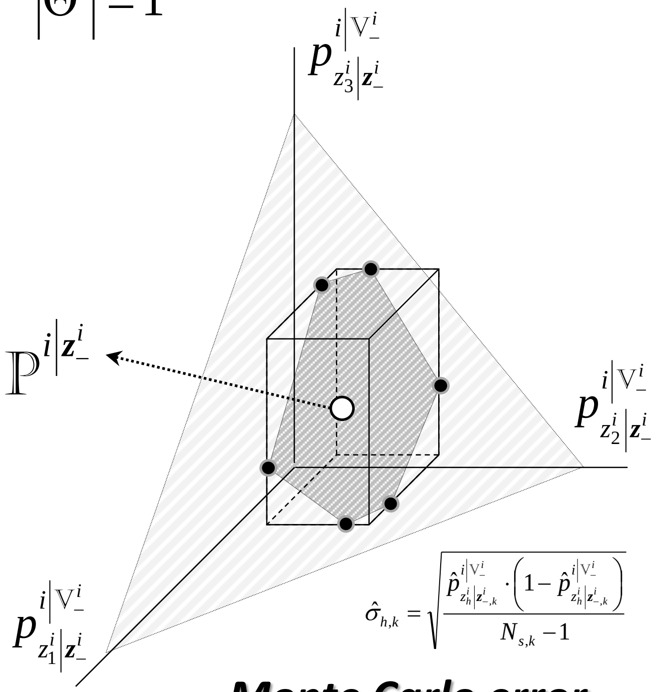
- Suppose one is estimating $p_{z^i|\boldsymbol{z}_-^i}^{i|\mathbb{V}_-^i}$
- Repeated Monte Carlo sampling
- Feasible region for the state probabilities:
  - Belong to their intervals
  - Sum up to one





subscenarios		z_1^i	z_2^i	z_3^i
$z_1^{j_1}$	$z_3^{j_2}$	2	3	5

$$\hat{p}_{z_1^i|z_1^{j_1}, z_3^{j_2}}^{i|j_1, j_2} = 0.2$$
$$\hat{p}_{z_2^i|z_1^{j_1}, z_3^{j_2}}^{i|j_1, j_2} = 0.3$$
$$\hat{p}_{z_3^i|z_1^{j_1}, z_3^{j_2}}^{i|j_1, j_2} = 0.5$$

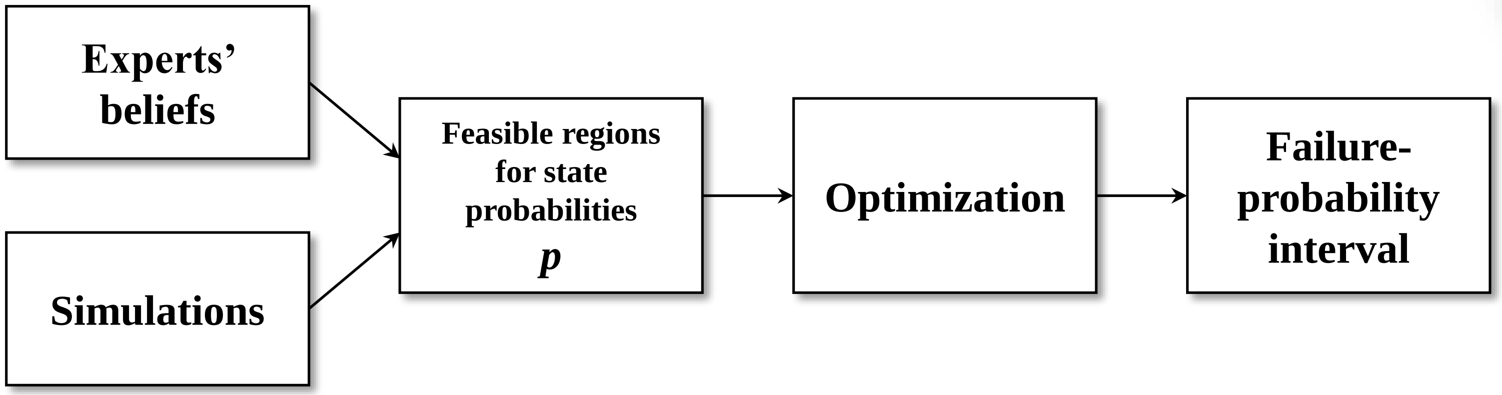


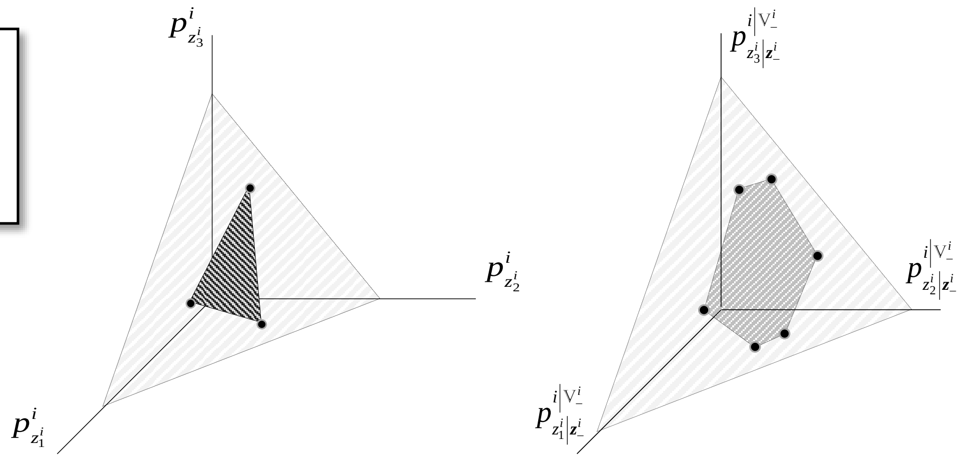
Monte Carlo error

Failure-probability interval

- Optimization to estimate bounds to the failure probability
- Customized algorithm: simplex + reduced gradients
- Thread expert judgment & simulations – failure-probability interval

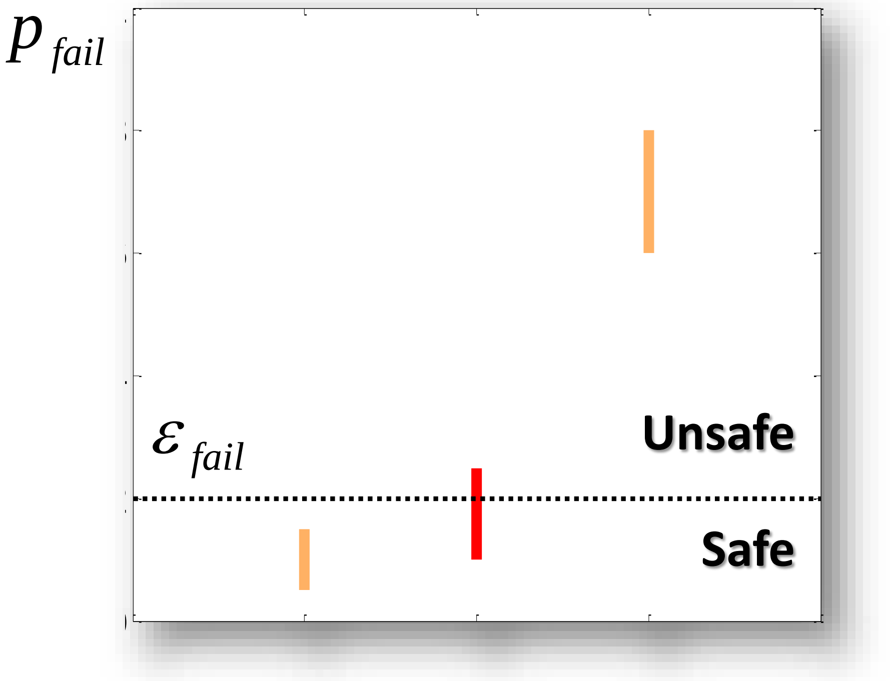
Estimation of the failure-probability interval		
Bound	Lower	Upper
Objective function	$\min_{\boldsymbol{p}} p_{fail}(\boldsymbol{p})$	$\max_{\boldsymbol{p}} p_{fail}(\boldsymbol{p})$
Constraints	$\boldsymbol{p}^i \in \mathbb{P}^i$	$\forall\ i \in \mathbb{V}^{\mathrm{I}}$
	$\boldsymbol{p}^{i\|\boldsymbol{z}_-^i} \in \mathbb{P}^{i\|\boldsymbol{z}_-^i}$	$\forall\ \boldsymbol{z}_-^i \in \mathbb{S}_-^i,\ \forall\ i \in \mathbb{V}^{\mathrm{D}}$





Conclusiveness & comprehensiveness

- It can be challenging to assess safety
- The failure-probability interval is conclusive if it lies either:
  - entirely below the maximum acceptable threshold - Safe
  - entirely above the maximum acceptable threshold - Unsafe
- Comprehensiveness:

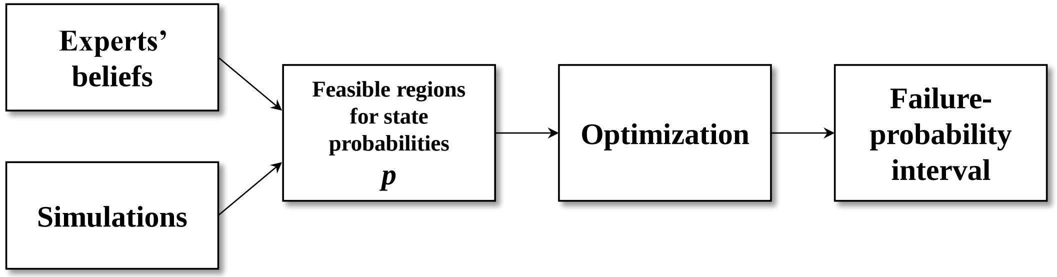


$$\mathbb{P} : \left(\forall \boldsymbol{p} \in \mathbb{P} \Rightarrow p_{fail}(\boldsymbol{p}) < \varepsilon_{fail} \right) \vee \left(\forall \boldsymbol{p} \in \mathbb{P} \Rightarrow p_{fail}(\boldsymbol{p}) \geq \varepsilon_{fail} \right)$$

$$\mathbb{P}^i, \ \forall i \in \mathbb{V}^I$$

$$\mathbb{P}^{j|\boldsymbol{z}_-^j}, \ \forall \boldsymbol{z}_-^j \in S_-^j, \ \forall j \in \mathbb{V}^D$$

$$\mathbb{P} = \left(\underset{i \in \mathbb{V}^I}{\times} \mathbb{P}^i \right) \times \left(\underset{j \in \mathbb{V}^D, \ \boldsymbol{z}_-^j \in S_-^j}{\times} \mathbb{P}^{j|\boldsymbol{z}_-^j} \right)$$



Comprehensiveness & simulations

- Achieving comprehensiveness can be challenging if there are limits to the number of simulations

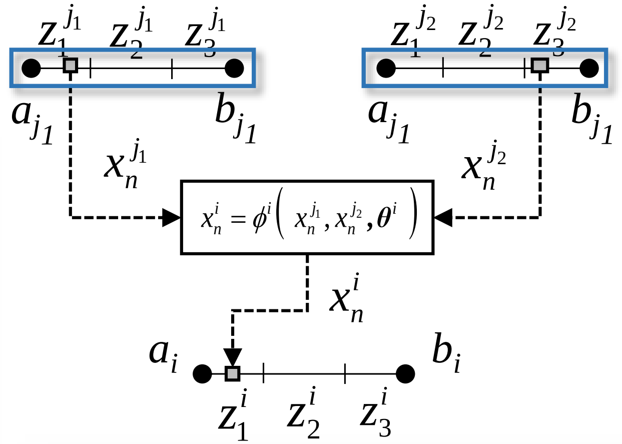
- For instance, if the subscenarios to be simulated are sampled randomly:
  - few simulations for all subscenarios
  - large Monte Carlo error
  - wide state-probability intervals
  - wide, possibly nonconclusive, failure-probability interval

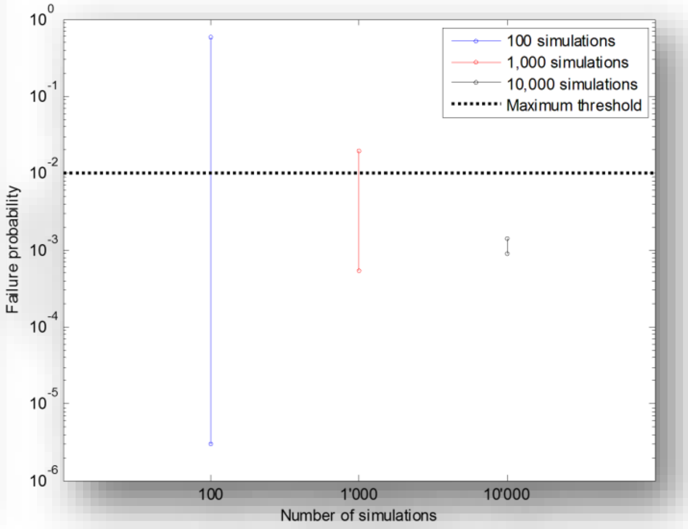
$$\hat{\sigma}_{h,k} = \sqrt{\frac{\hat{p}^{i|\mathrm{V}^i_-}_{z^i_h|\mathbf{z}^i_{-,k}} \cdot \left(1-\hat{p}^{i|\mathrm{V}^i_-}_{z^i_h|\mathbf{z}^i_{-,k}}\right)}{N_{s,k}-1}}$$

- Can simulations be performed for a restricted set of responsibly selected subscenarios?

- For instance, identified by risk-importance measures

Subscenarios: Ice sheet	Subscenarios: Fracture displacement	Simulations
z_1^1	z_1^2	11
z_2^1	z_1^2	12
z_3^1	z_1^2	15
z_1^1	z_2^2	7
z_2^1	z_2^2	11
z_3^1	z_2^2	8
z_1^1	z_3^2	13
z_2^1	z_3^2	13
z_3^1	z_3^2	10
Total		100





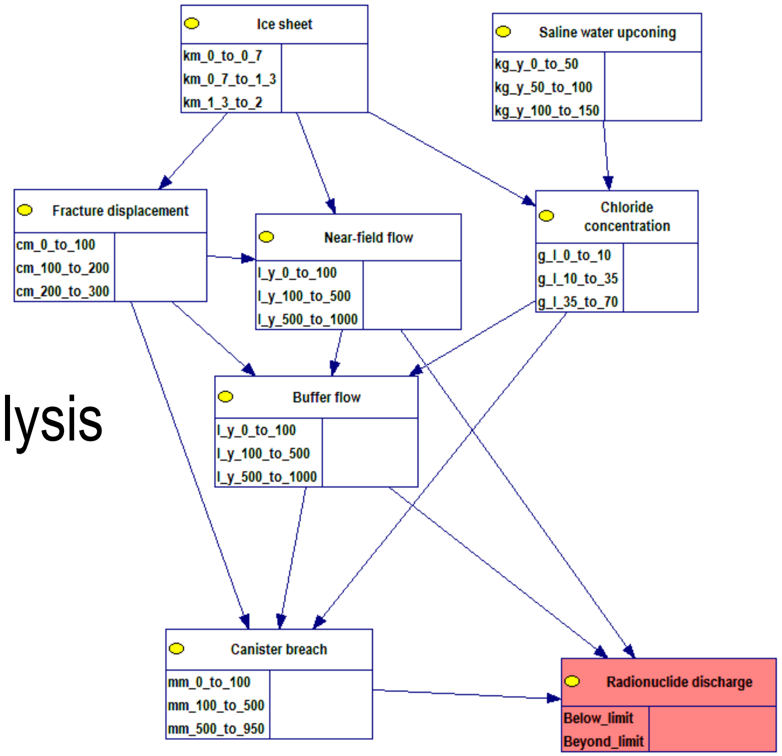
Addressing challenges in scenario analysis

- Recall the methodological challenges in scenario analysis:

1. Building a **system model** as a framework for scenario generation → Bayesian network of FEPs, in which scenarios and subscenarios are defined
2. Achieving **comprehensiveness** → The subscenarios to be analyzed with more simulations to obtain a conclusive failure-probability interval are identified
3. Treating the **epistemic uncertainties** → The epistemic uncertainty about the values of the state probabilities is characterized by feasible regions

Comparison to former approaches

- Pluralistic
  - Scenarios selected by judgment
  - Representative/illustrative of the future
- Here, probabilistic scenario analysis



- Probabilistic (e.g., Yucca Mountain)
  - Rigorous mathematical framework
  - Great computational availability
  - Large sample from initial nodes, then simulations in cascade
- Here, less computational availability:
  - integrate expert judgments and simulations
  - identify the regions of the probability space (subscenarios) to be analyzed


Thank you for attending!