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Prioritizing Failure Events in Fault Tree Analysis Using Interval-valued Probability Estimates

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Uncertainty in probability estimates

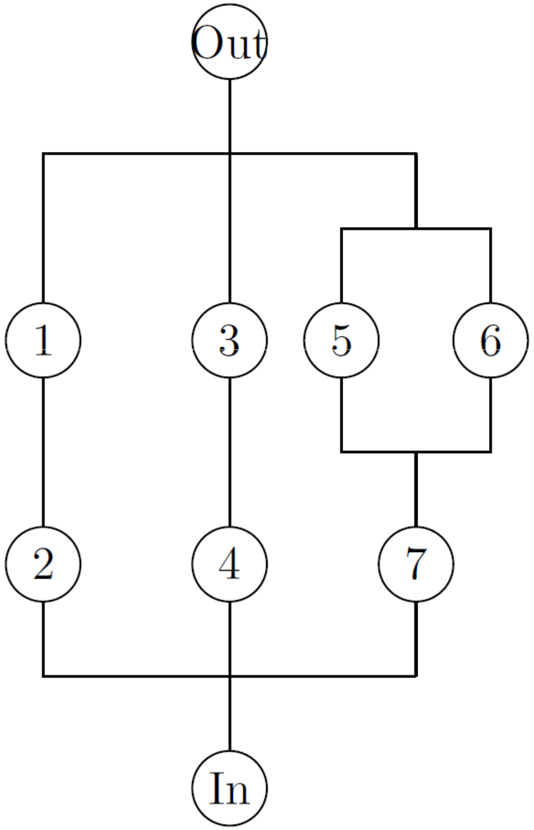
- Risk importance measures help prioritize failure events
  - E.g. Fussell-Vesely, Birnbaum
- These are typically computed using crisp probabilities
- Probability estimates can be uncertain
  - Statistical data, simulation models, expert opinions
  - If component fails iid 20 times out of 1000 → 95 % confidence interval of probability is [0.012, 0.031] (Pearson-Klopper method)
- What is the impact of this uncertainty?

Prioritization of failure events with interval probabilities

- Interval-probabilities define a set of values that the ”true” probability can have
  - We use confidence intervals
- A dominates B iff
  - Risk importance of A is at least as great as the risk importance of B for all probabilities within the intervals, and
  - Risk importance of A is strictly greater than the risk importance of B for some probabilities within the interval
- Dominance relation determine an incomplete ordering of the failure events
  - Extends prioritization based in crisp values

Illustrative example (1/2)

System



Traditional

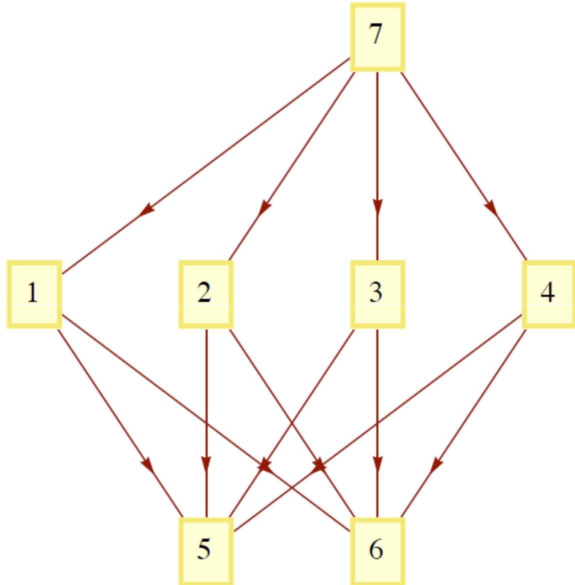
$$p_j = 0.02 \forall j$$

Comp	FV
1	5,00E-01
2	5,00E-01
3	5,00E-01
4	5,00E-01
5	1,96E-02
6	1,96E-02
7	9,80E-01

Interval-probability

Fussell-Vesely dominance

$$0.01 \leq p_j \leq 0.03 \forall j$$



With a wider interval

$$0.01 \leq p_j \leq 0.04 \forall j$$

no component dominates the other

Illustrative example (2/2)

Traditional

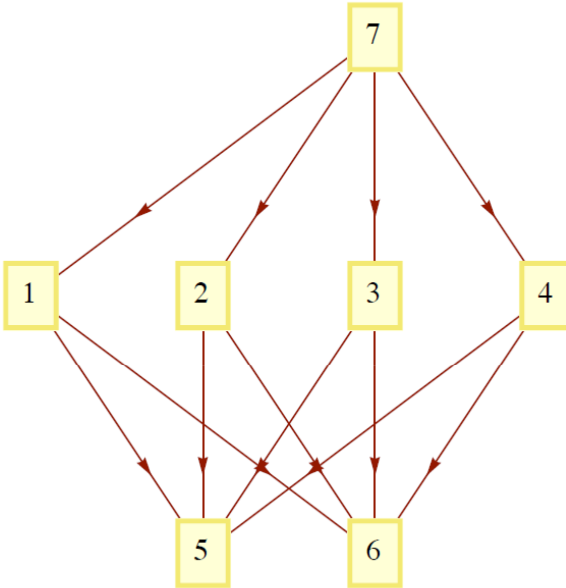
$$p_j = 0.02 \forall j$$

Comp	FV	Birnbaum
1	5,00E-01	8,16E-04
2	5,00E-01	8,16E-04
3	5,00E-01	8,16E-04
4	5,00E-01	8,16E-04
5	1,96E-02	6,34E-05
6	1,96E-02	6,34E-05
7	9,80E-01	1,57E-03

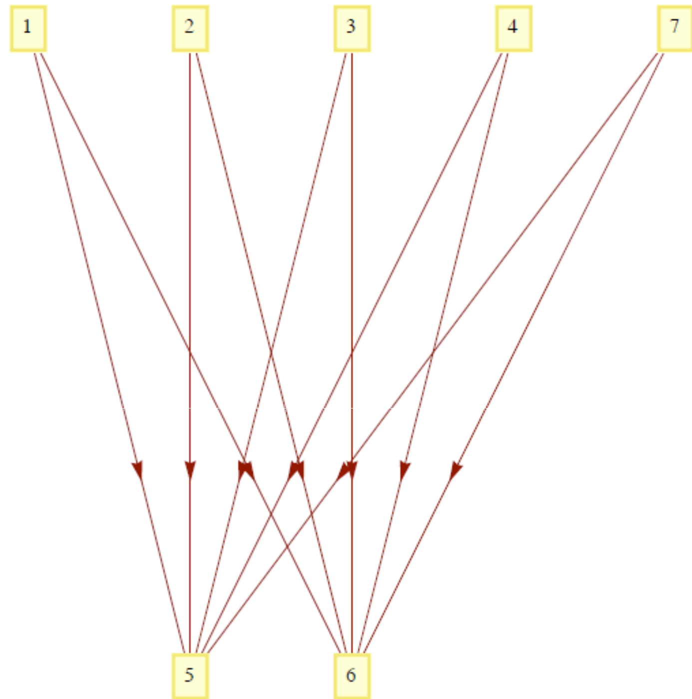
Interval-probability

$$0.01 \le p_j \le 0.03 \forall j$$

Fussell-Vesely



Birnbaum



With a wider interval $0.01 \le p_j \le 0.04 \forall j$ no component dominates the other

Computation of dominances

- How to check if $FV_1 > FV_5$ when $0.01 \le p_j \le 0.03 \forall j$?

$$FV_A = \frac{\sum_{MCS_i | A \in MCS_i} \prod_{j \in MCS_i} p_j}{P(T)}$$

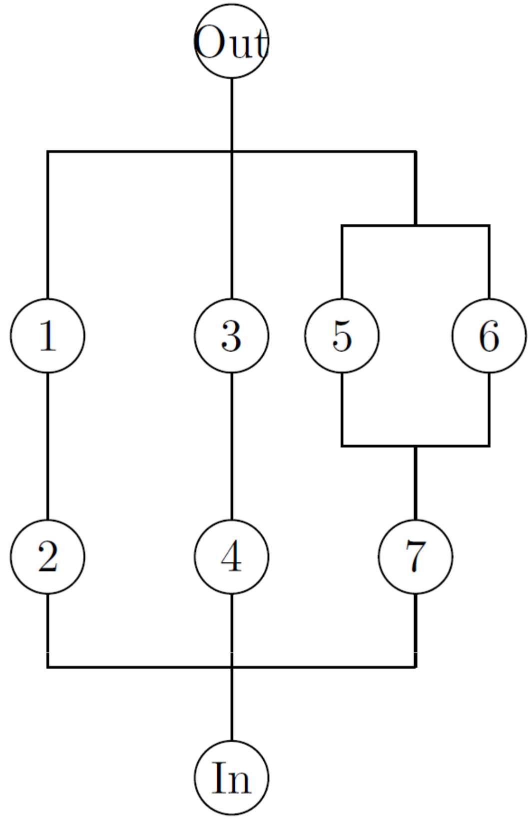
$$FV_1 - FV_5 < 0? \Rightarrow$$

$$p_1 p_3 p_7 + p_1 p_4 p_7$$

$$- p_2 p_3 p_5 p_6 - p_2 p_4 p_5 p_6 < 0$$

$$\underline{p}_1 \underline{p}_3 \underline{p}_7 + \underline{p}_1 \underline{p}_4 \underline{p}_7$$

$$- \overline{p}_2 \underline{p}_3 \overline{p}_5 \overline{p}_6 + \overline{p}_2 \underline{p}_4 \overline{p}_5 \overline{p}_6 < 0 \Rightarrow FALSE \Rightarrow$$ 1 dominates 5

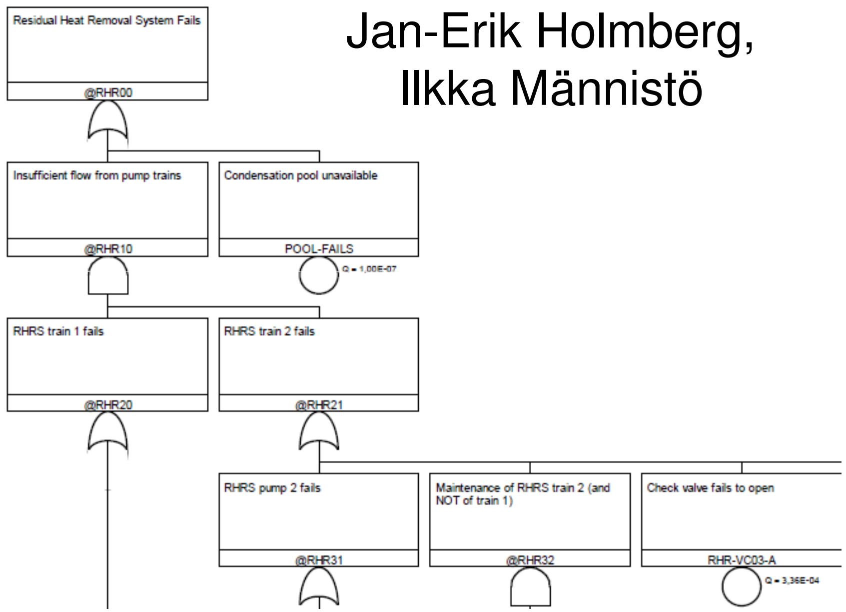


Minimal cut sets 1356, 137, 1456, 147, 2356, 237, 2456, 247

Application on the Residual Heat Removal System (RHRS)

- Medium sized fault tree
  - 31 basic events (BEs)
  - 147 minimal cut sets of 1-3 BEs
  - Each component typically belongs to 1-13 of the MCS
- Probability interval equals the 90 % confidence interval
- Dominances computed using our algorithm
  - Implemented in Mathematica

Case data by Technical Research Centre of Finland (VTT)
Jan-Erik Holmberg, Ilkka Männistö

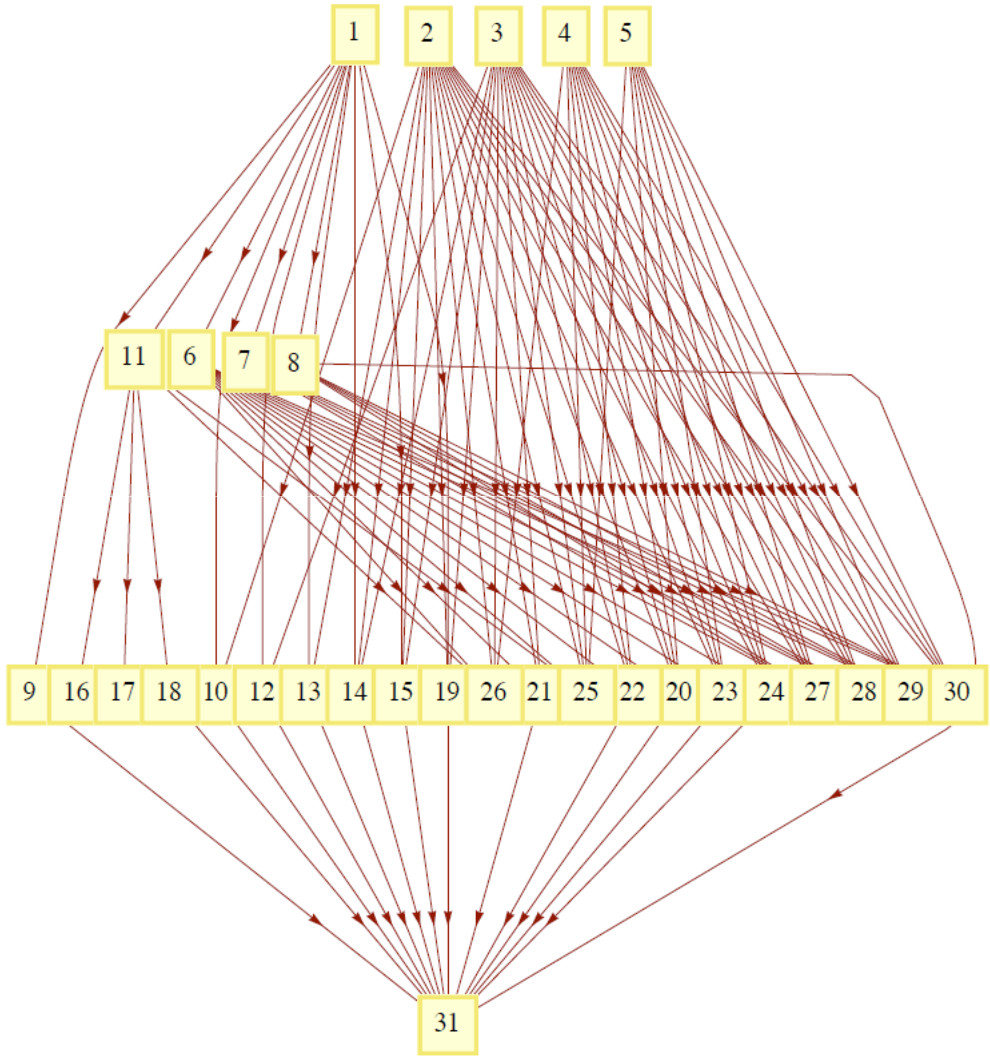


Part of the RHRS fault tree

RHRS Fussell-Vesely dominances

- Basic events labeled by their conventional FV risk importance ranks
- Dominances define an incomplete order
  - Eg. 5 and 12 are equal in the sense that neither dominates the other, even if with crisp probabilities

    FV_5 = 5.86E-02 and FV_{12}=5.21E-03



Lessons learned from the RHRS case

- Our method computatinally viable
  - RHRS case model (medium size) solved under a minute
- Model data readily available
  - MCS, probability confidence intervals from standard fault tree analysis
- The dominance graph gives an overview of the sensitivity of the priorities
  - In RHRS case relatively few dominances → uncertainty has large impact in priorities
  - Transparent, conservative and justifiable

Further research

- Derive lower and upper bounds for the relative risk ranking of components
  - Which rankings are attainable?(cf. Salo and Punkka, 2011)
- Apply current method for prioritizing and ranking minimal cut sets
- Apply methods to large fault trees

References

- Salo, A., J. Keisler and A. Morton (eds) (2011). Portfolio Decision Analysis: Improved Methods for Resource Allocation, Springer, New York
- Salo, A. and A. Punkka (2011). Ranking Intervals and Dominance Relations for Ratio-based Efficiency Analysis Management Science, Vol. 57, No. 1, pp. 200-214.
- Walley, Peter (1990). *Statistical Reasoning with Imprecise Probabilities*. London: Chapman and Hall.