Ratio-based Efficiency Analysis and Benchmarking of Healthcare Services

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Ranking Intervals and Dominance Relations for Ratio-Based Efficiency Analysis

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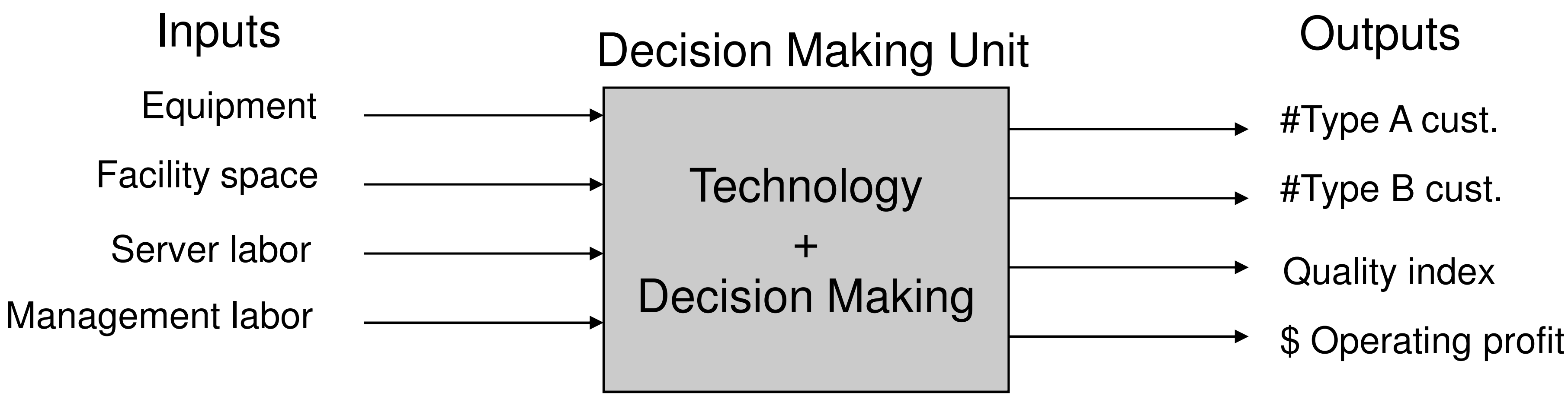
We develop comparative results for ratio-based efficiency analysis (REA) based on the decision-making units' (DMUs') relative efficiencies over sets of feasible weights that characterize preferences for input and output variables. Specifically, we determine (i) *ranking intervals*, which indicate the best and worst efficiency rankings that a DMU can attain relative to other DMUs; (ii) *dominance relations*, which show what other DMUs a given DMU dominates in pairwise efficiency comparisons; and (iii) *efficiency bounds*, which show how much more efficient a given DMU can be relative to some other DMU or a subset of other DMUs. Unlike conventional efficiency scores, these results are insensitive to outlier DMUs. They also show how the DMUs' efficiency ratios relate to each other for *all* feasible weights, rather than for those weights only for which the data envelopment analysis (DEA) efficiency score of *some* DMU is maximized. We illustrate the usefulness of these results by revisiting reported DEA studies and by describing a recent case study on the efficiency comparison of university departments.

Key words: efficiency analysis; data envelopment analysis; preference modeling
History: Received April 2, 2009; accepted September 6, 2010, by Teck-Hua Ho, decision analysis.

Efficiency

- Efficiency defined as the ratio between outputs and inputs

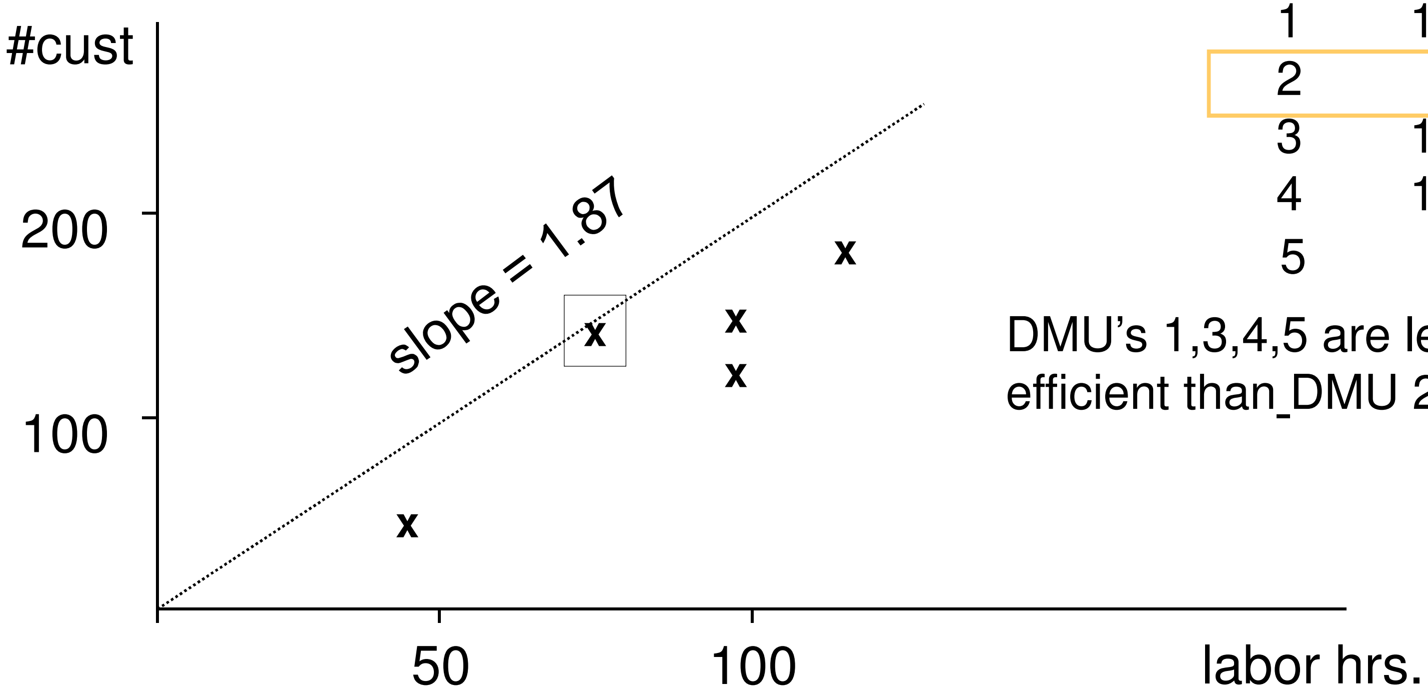
$$\text{Productivity} = \frac{\text{Outputs}}{\text{Inputs}}$$



Data Envelopment Analysis (Charnes, Coopers & Rhodes '78)

- DMU = Decision Making Unit
- A method for measuring the efficiency of DMUs which consume inputs in order to produce outputs

DMU	labor hrs.	#cust.	#cust/hr.
1	100	150	1.50
2	75	140	1.87
3	120	160	1.33
4	100	140	1.40
5	40	50	1.25



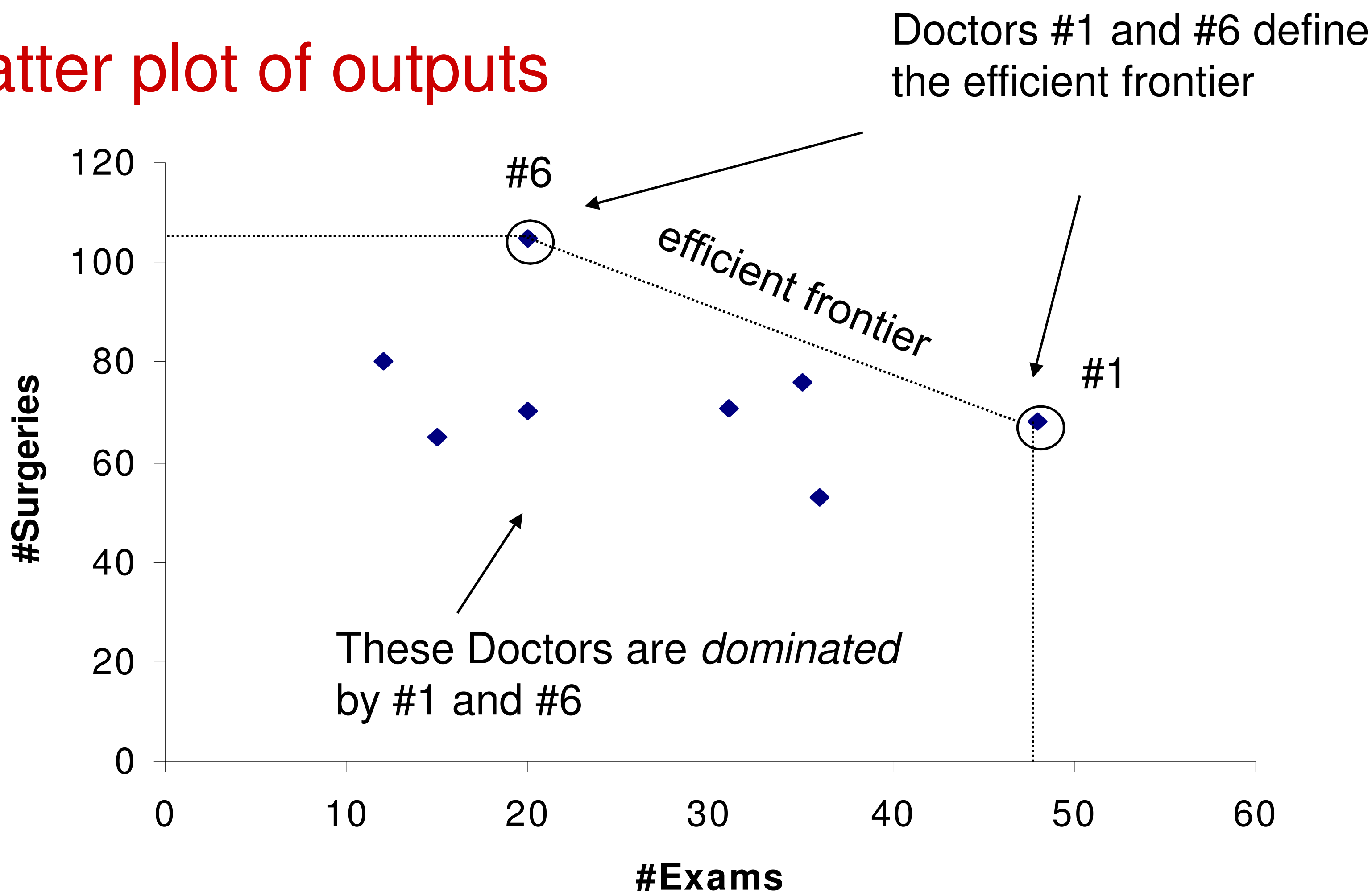
DMU's 1,3,4,5 are less efficient than DMU 2.

Extending to multiple outputs ..

- 8 Doctors work for 160 hrs in a month.
  - Each performs exams and surgeries
  - Which doctors are most "productive"?

Doctor	#Exams	#Surgeries
1	48	68
2	12	80
3	35	76
4	31	71
5	20	70
6	20	105
7	36	53
8	15	65

Scatter plot of outputs



On the efficient frontier, it is impossible to increase the production of one output without a compensating decrease in some other output

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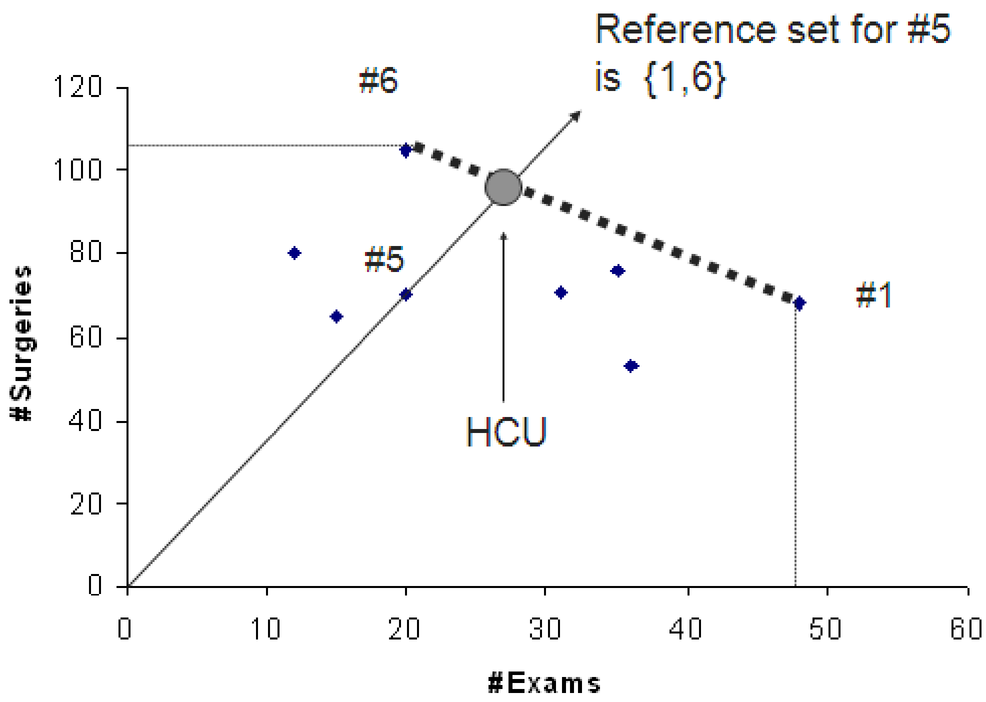
Reference set

- "Nearest" efficient DMUs define
  - ❶ a reference set and
  - ❷ linear combination of the reference set inputs and outputs of a hypothetical composite unit (HCU)



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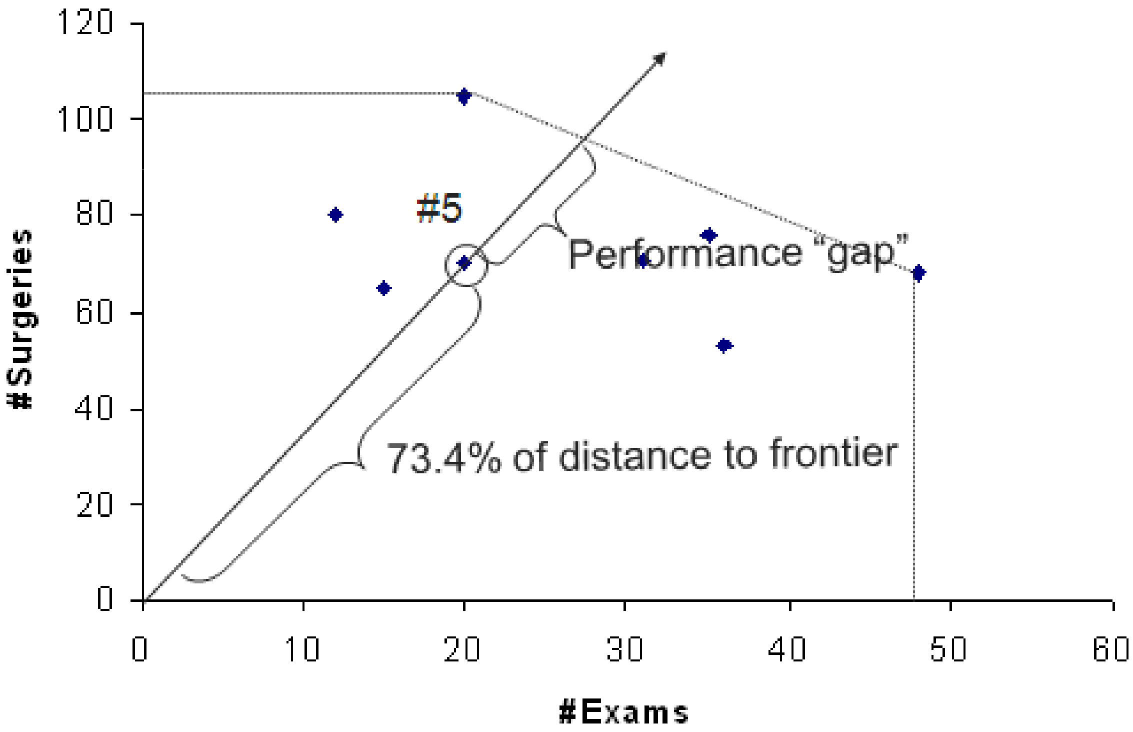
Performance gaps

- How “bad” are inefficient Doctors relative to efficient ones?
- How large are the output gaps?



Notation

- Data

K #operating units (DMUs) $k = 1,\ldots,K$

M # inputs $m = 1,\ldots,M$

N #outputs $n = 1,\ldots,N$

y_{nk} observed level of output n from DMU k

x_{mk} observed level of input m from DMU k

- Model variables

v_m weight of input m

u_n weight of output n

E_k efficiency of DMU k (0-100%)

$$E_k = \frac{\sum_{n=1}^{N} u_n y_{nk}}{\sum_{m=1}^{M} v_m x_{mk}}$$

Evaluating the CCR efficiency of DMU k

- Choose input/ouput weights to $\max E_k$ subject to

$$E_l \leq 1, \qquad l = 1,\ldots,K$$

- Optimization problem

$$\max \quad \frac{\sum_n u_n y_{nk}}{\sum_m v_m x_{mk}}$$

subject to

$$\frac{\sum_n u_n y_{nl}}{\sum_m v_m x_{ml}} \leq 1, \qquad l = 1,\ldots,K$$

$$u_n, v_m \geq 0$$

$\Rightarrow$

$$\max \quad \sum_n u_n y_{nk}$$

subject to

$$\sum_m v_m x_{mk} = 1$$

Weighted input of DMU k is normalized to one

$$\sum_n u_n y_{nl} \leq \sum_m v_m x_{ml}, \qquad l = 1,\ldots,K$$

$$u_n, v_m \geq 0$$

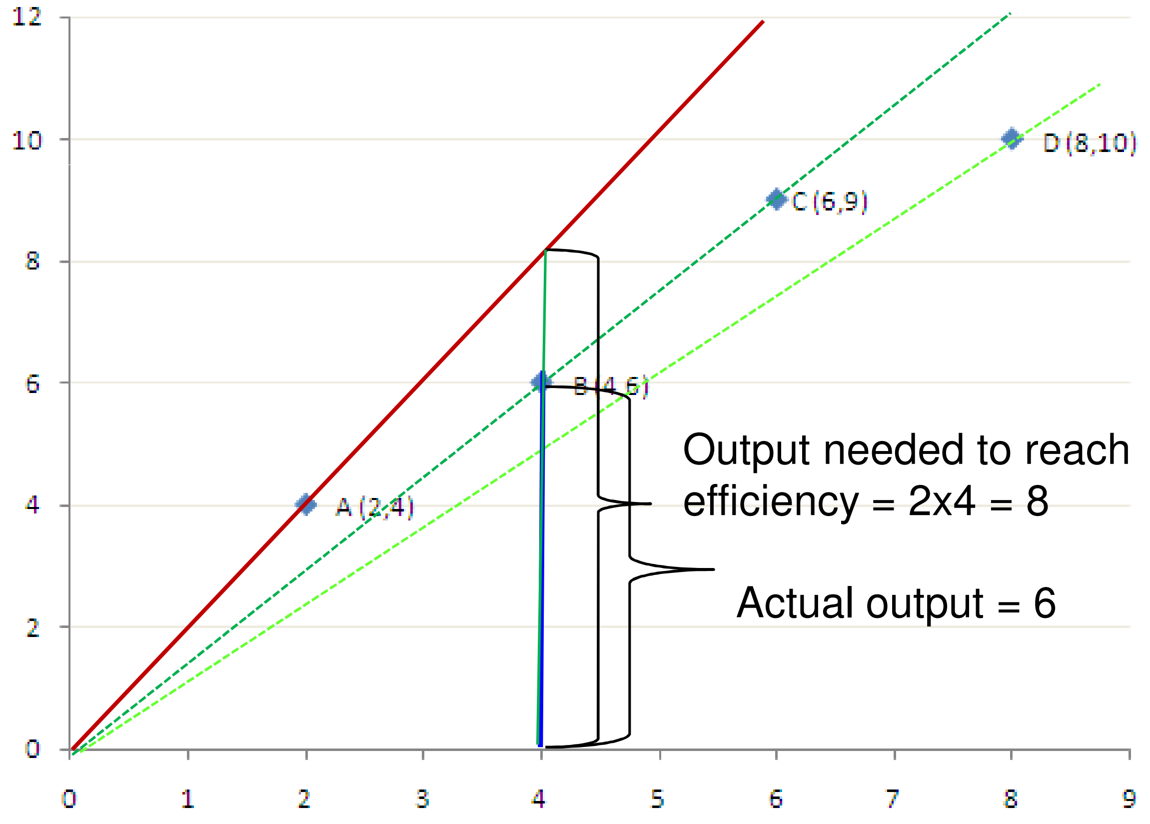
An example with 4 DMUs

- Four DMUs, one input, one output
- The efficiency ratio is highest for DMU A
  - – Max.efficiency = 1
  - ⇒ Input weight twice as high as output weight
  - ⇒ Efficiencies of other DMUs

$E_B = 6/8 = 0.75$

$E_C = 9/12 = 0.75$

$E_D = 10/16 = 0.625$



Variants of DEA Models

- CCR Model
  - Charnes, Cooper, and Rhodes (1978)
  - Assumes constant returns to scale in production possibilities: an increase in the amount of inputs leads to a proportional increase in outputs

- BCC Model
  - Banker, Charnes and Cooper (1984)
  - Constant returns to scale are not assumed, efficiency depends on the scale of operations

- Superefficiency model

- DEA models with weight information

Superefficiency model

- Helps determine how much more efficient an efficient DMU is relative to other DMUs

$$\max \sum_n u_n y_{nk} \quad \text{subject to}$$

$$\sum_m v_m x_{mk} = 1$$

$$\sum_n u_n y_{nl} \le \sum_m v_m x_{ml}, \quad l = 1,\ldots,K, \; l \neq k$$

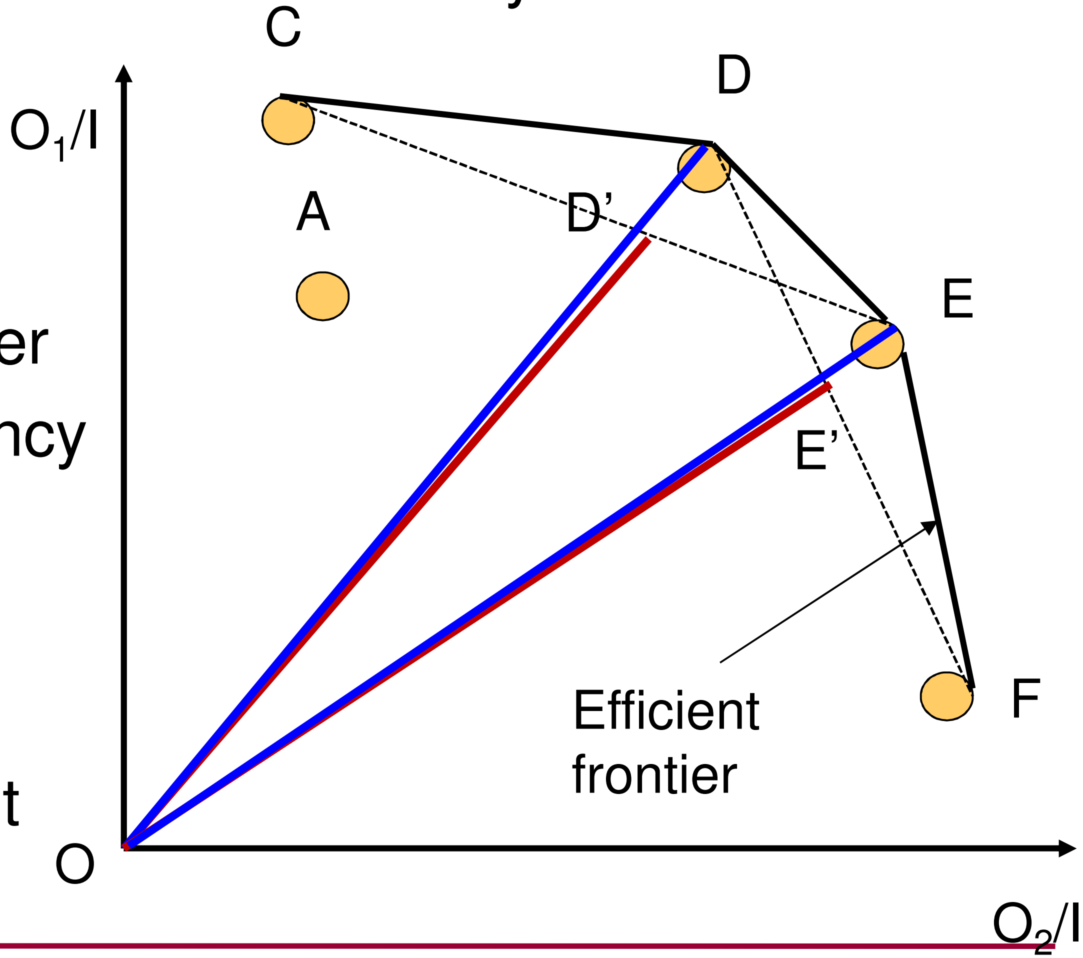
$$u_m, v_n \ge 0$$

DMU k under evaluation is removed from the constraint set thereby allowing its efficiency score to exceed a value of 1.00

- The model does not help rank inefficient DMUs

Super efficiency illustrated

- D evaluated relative to the frontier defined by C-E-F
- Superefficiency defined by the distance OD/OD’
- Similarly E evaluated in comparison with the frontier C-D-F and its superefficiency defined by the distance OE-OE’
- By visual inspection, D is slightly more superefficient than D



DEA models with weight information

- DMUs may attain their efficiency scores for ‘extreme’ weights in conventional DEA models
- Preference information can be captured through statements about the relative values of ❶ inputs and ❷ outputs
- Examples
  - “A Dissertation is as at least as valuable as 2 Master’s Theses, but not more valuable than 7 master’s theses”

    ➔ $u_{doctoral} \geq 2u_{master's}$, $u_{doctoral} \leq 7u_{master's}$
  - “An article in a refereed journal is more valuable than a Master’s Thesis”

    ➔ $u_{article} \geq u_{master's}$

Example of a DEA model with weight restrictions

$$\max \sum_n u_n y_{nk} \qquad \text{subject to}$$

$$\sum_m v_m x_{mk} = 1$$

$$\sum_n u_n y_{nl} \leq \sum_m v_m x_{ml}, \qquad l = 1,\ldots,K$$

$$\alpha_m v_m \leq v_1 \leq \beta_m v_m, \qquad m = 1,\ldots,M$$

$$a_n u_n \leq u_1 \leq b_n u_n, \qquad n = 1,\ldots,N$$

$$u_m, v_n \geq 0$$

Summary of DEA

- Productivity is defined relative to the efficient frontier
- Efficiency score defined as the relative distance to the frontier
- This score based on weights that are most favorable to the DMU
- “Nearest point” on the frontier is the efficient comparison unit
- There are no comparisons among inefficient DMUs

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Ratio-based Efficiency Analysis (REA)[1]

- Observations on DEA efficiency scores
  - Production possibilities may not be easy to characterize
  - Do not show how low the DMUs efficiency scores become for unfavorable weights
  - Introduction of an outlier DMUs may disrupt efficiency scores

- REA
  - Does not necessitate assumptions what the set of production possibilities is beyond the set of DMUs under comparison
  - Evaluates the relative efficiencies of DMUs for all feasible weights
  - Provides several efficiency results

[1] Ahti Salo and Antti Punkka (2011). *Ranking Intervals and Dominance Relations for Ratio-Based Efficiency Analysis*, Management Science 57(1), 200-214.

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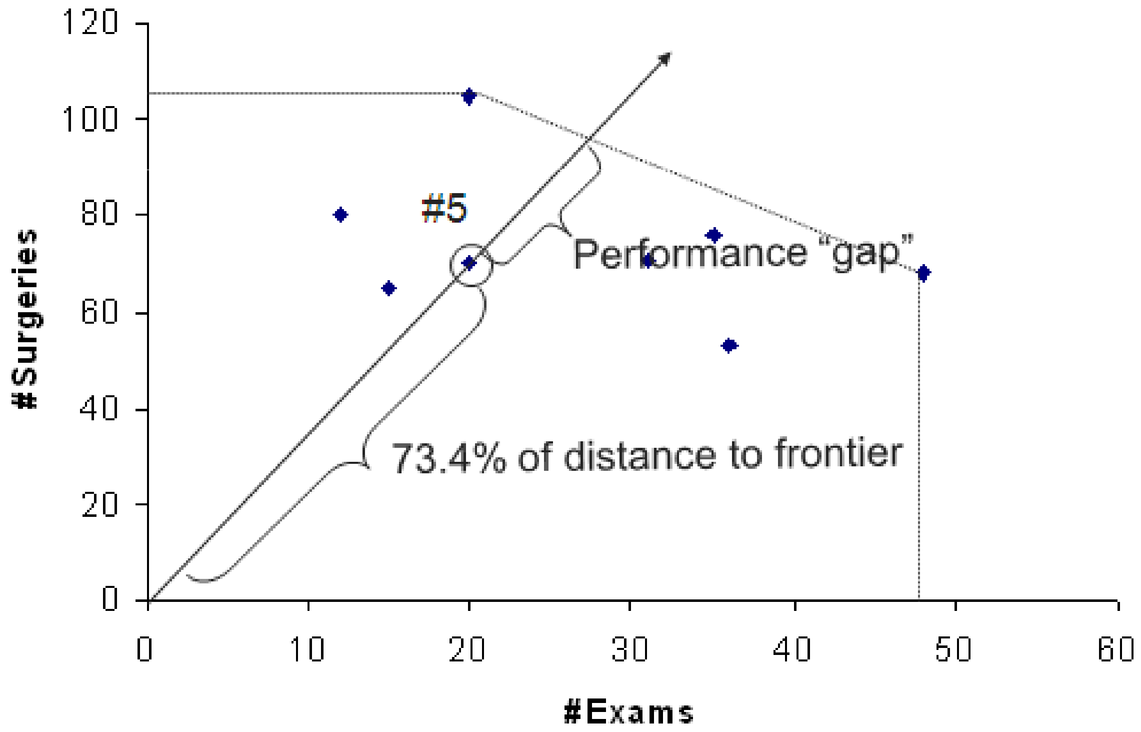
Concerns in benchmarking w.r.t. the efficient frontier

- Outliers may disrupt the frontier and hence efficiency scores, too



Ratio-Based Efficiency Analysis (REA)

- Given a pair of DMUs, is the first DMU more efficient than the second for all weights?

  → **Dominance relations**

- What are the best/worst possible rankings a DMU can attain?

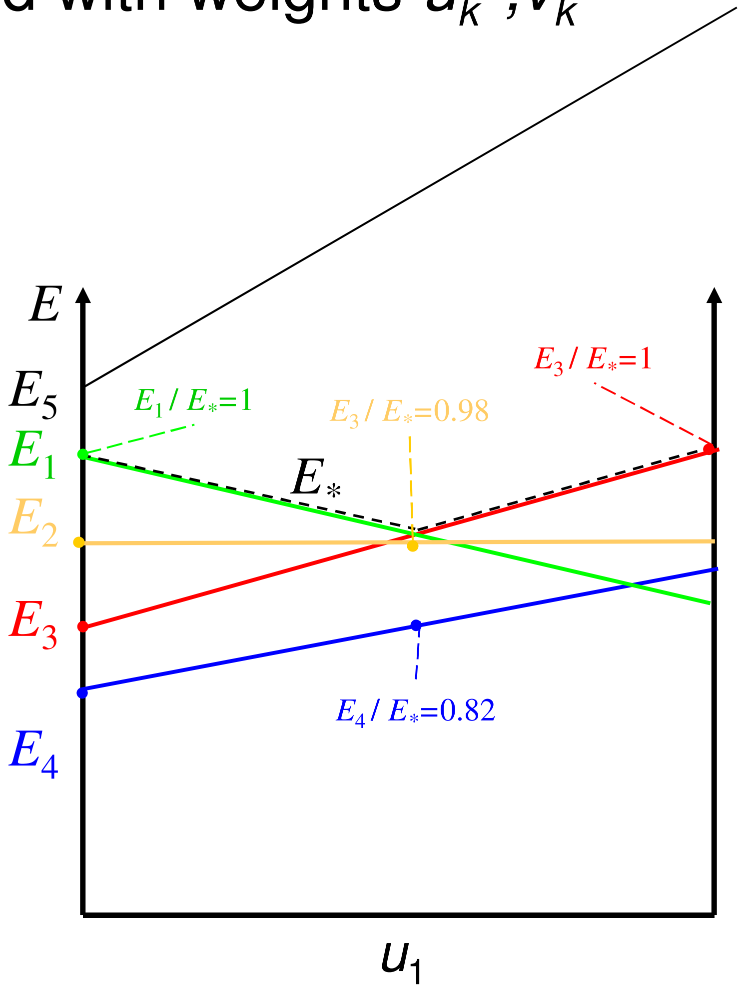
  → **Ranking intervals**

- How much more/less efficient a DMU is in comparison with other DMUs across the set of feasible weights?

  → **Efficiency bounds**

Efficiency ratios in CCR-DEA

- Efficiency score of DMU_k is computed with weights u_k^*, v_k^* to maximize $E_k/[\max_{l=1,\ldots,K} E_l]$
  - Does not provide information about the efficiencies for other weights
  - These weights depend on the set of DMUs
  - ➔ Changing the set of DMUs can influence the relative efficiency ranking of DMUs



- DMU_1 and DMU_3 are efficient
  - If DMU_5 is included, then DMU_2 becomes more efficient than DMU_3 in terms of its DEA score

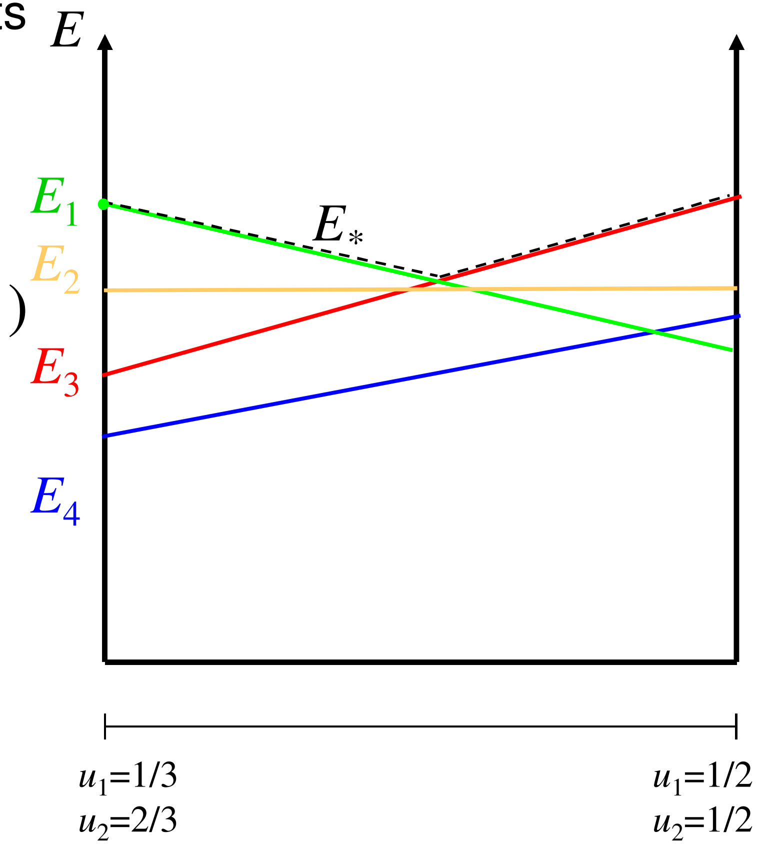
Efficiency dominance (1/2)

- DMU$_k$ dominates DMU$_l$ iff
  - (i) its efficiency ratio is at least as high as that of DMU$_l$ for all feasible weights
  - (ii) higher for some feasible weights

$$E_k(u,v) \geq E_l(u,v) \quad \text{for all } (u,v) \in (S_u, S_v)$$

$$E_k(u,v) > E_l(u,v) \quad \text{for some } (u,v) \in (S_u, S_v)$$

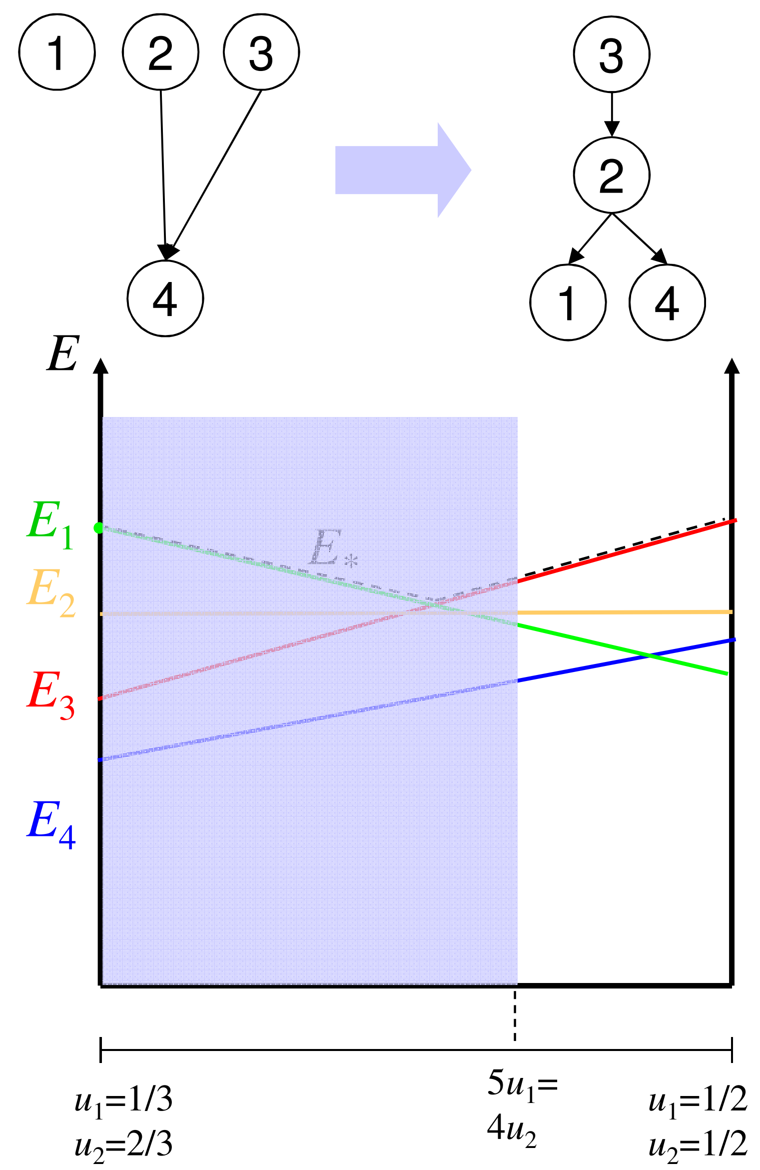
- Example, 2 outputs, 1 input
  - Feasible weights such that $2u_1 \geq u_2 \geq u_1$
  - DMU$_3$ and DMU$_2$ dominate DMU$_4$
  - DMU$_2$ is inefficient but **non-dominated**



Efficiency dominance (2/2)

- A graph to show dominance relations among DMUs
  - Transitive: If A dominates B and B dominates C, then A dominates C
  - Asymmetric: (i) If A dom. B, then B does not dom. A and (ii) no DMU dominates itself

- Additional preference information often leads to more conclusive dominance relations
  - Statement $5u_1 \geq 4u_2$ leads to new dominance relations



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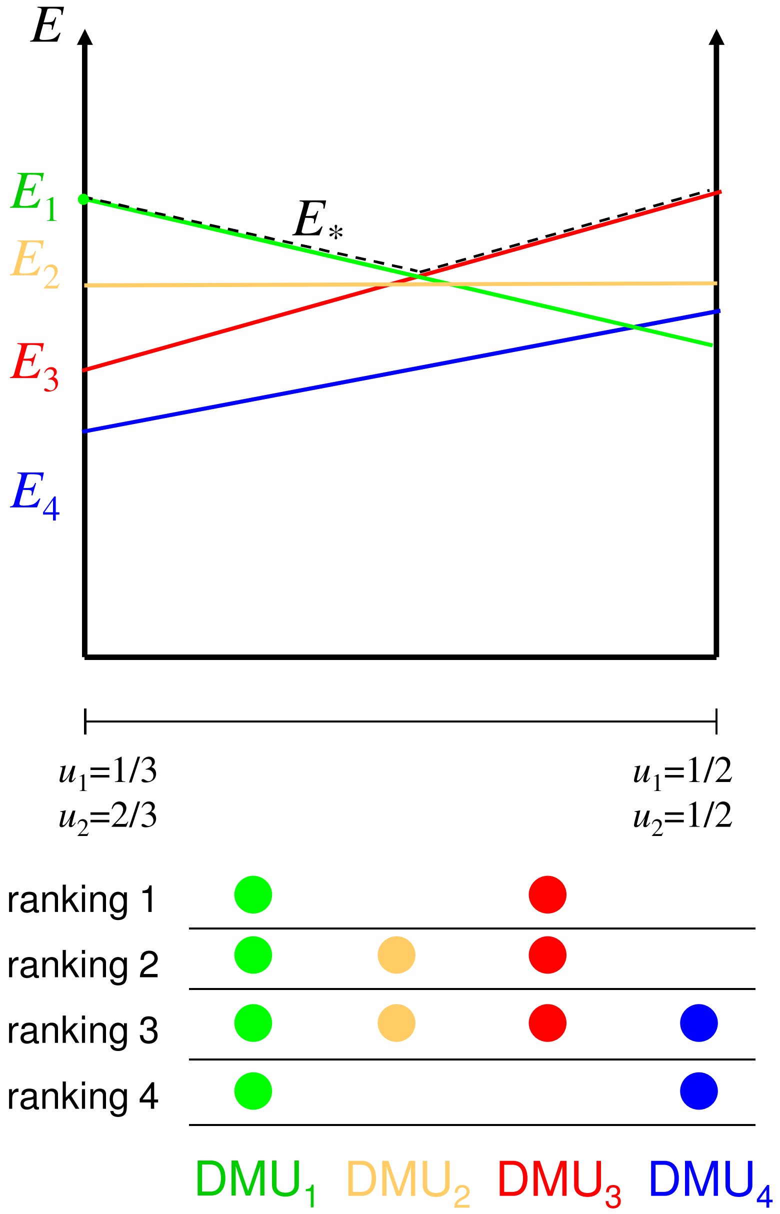
Ranking intervals

- For any feasible weights (u,v), DMUs can be ranked based on their efficiency ratios
  - **The minimum ranking** of DMU$_k$, r_k^{min}, is obtained for weights such that the number of DMUs with strictly higher efficiency ratios is minimized
  - **The maximum ranking** of DMU$_k$, r_k^{max}, is obtained for weights such that the number of DMUs with higher or equal efficiency ratios is maximized



Ranking intervals (2/2)

- Properties
  - Provide a holistic comparative view of relative efficiencies at a glance
  - Show how inefficient DMUs can be, a worst
  - Are insentitive to the introduction of outlier DMUs

- Additional weight information can narrow (but not widen) ranking intervals

- CCR-DEA-efficient DMUs have the highest efficiency ratio for some weights $\Rightarrow$ their minimum ranking is 1

Computation of dominance relations (1/2)

- How to determine whether DMU$_k$ dominates DMU$_l$

$$E_k(u,v) \geq E_l(u,v) \quad \forall (u,v) \in (S_u, S_v) \text{ and}$$

$$E_k(u,v) > E_l(u,v) \quad \text{for some } (u,v) \in (S_u, S_v)\,?$$

$$E_k(u,v) \geq E_l(u,v) \quad \forall (u,v) \in (S_u, S_v) \text{ if}$$

$$\min_{(u,v)\in(S_u,S_v)} [E_k(u,v) - E_l(u,v)] \geq 0 \Leftrightarrow$$

$$\min_{(u,v)\in(S_u,S_v)} \frac{E_k(u,v)}{E_l(u,v)} \geq 1 \Leftrightarrow \ldots$$

(S_u,S_v) is open, not bounded, and the objective function non-linear...

How to solve the optimization problem?

Computation of dominance relations (2/2)

- Normalize weights so that
  - The value of inputs of DMU_k=1
  - The value of outputs of DMU_l is equal to its value of inputs
- Feasible weights now bounded, closed by linear constraints, plus a linear objective function
- If the minimum is exactly 1, maximize the same objective function to see whether there exists weights such that $E_k > E_l$

$$\min_{\substack{A_u u \le 0 \\ A_v v \le 0}} \frac{\sum_{n=1}^{N} u_n y_{nk}}{\sum_{m=1}^{M} v_m x_{mk}} / \frac{\sum_{n=1}^{N} u_n y_{nl}}{\sum_{m=1}^{M} v_m x_{ml}} \ge 1 \Leftrightarrow$$

$$\min_{\substack{A_u u \le 0 \\ A_v v \le 0 \\ \sum v_m x_{mk} = 1 \\ \sum v_m x_{ml} = \sum u_n y_{nl}}} \sum_{n=1}^{N} u_n y_{nk} \ge 1$$

Computation of ranking intervals and efficiency bounds

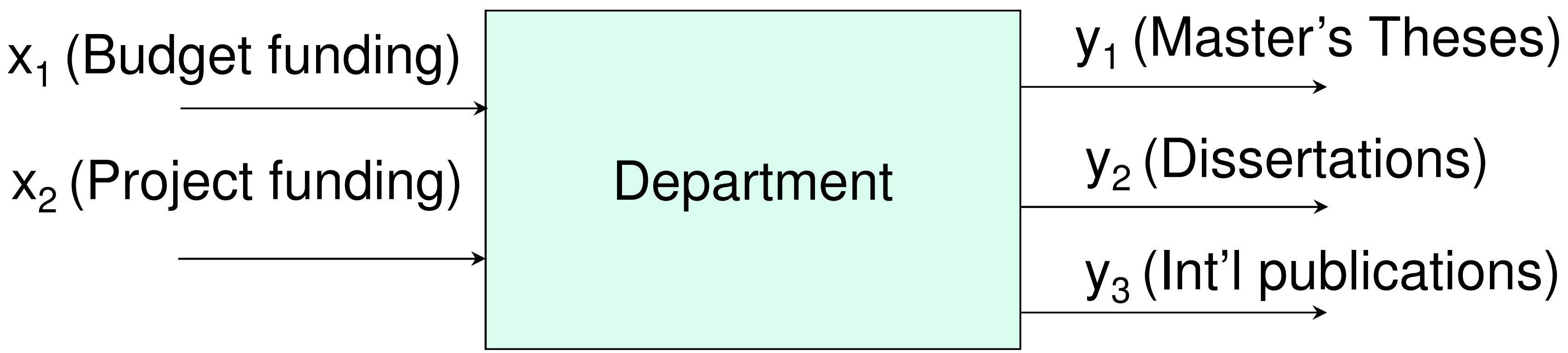
- Minimum (best) rankings for DMU$_k$
  1. For all other DMUs, define binary variables z_l so that $z_l = 1$ if $E_l > E_k$

$$E_l(u,v) \leq E_k(u,v) + Cz_l, \quad C >> 0$$

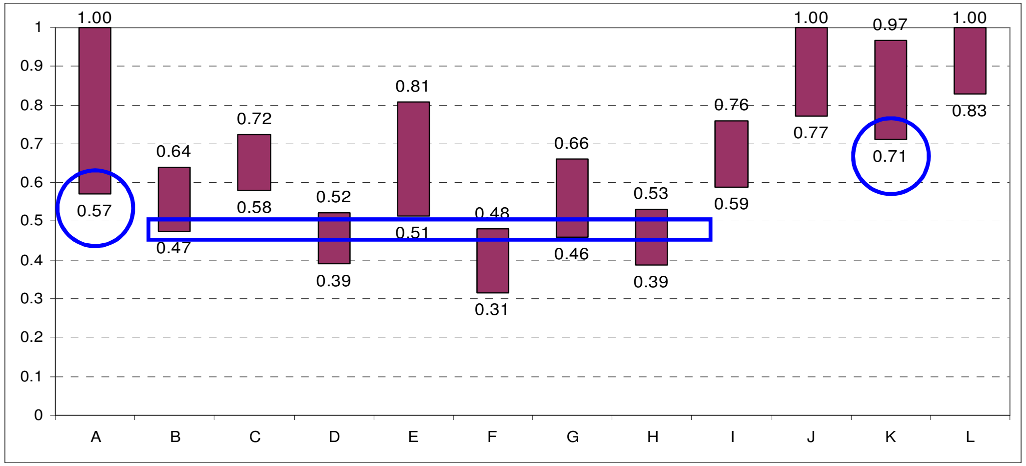
  2. Choose a suitable normalization to come up with a MILP model
  3. The minimum is 1 + the minimum of z_l over (S_u, S_v)
  – Maximum rankings with a corresponding model
- Efficiency bounds compared to the most efficient DMU
  – Maximum with LP similar to the computation of DEA scores
  – Minimum
    1. Minimize the linear model used for the computation of dominance relations against all DMUs in the benchmark group
    2. The smallest of these is the minimum
  – Comparisons to the least efficient DMU with corresponding models

Example: Efficiency analysis of TKK’s departments

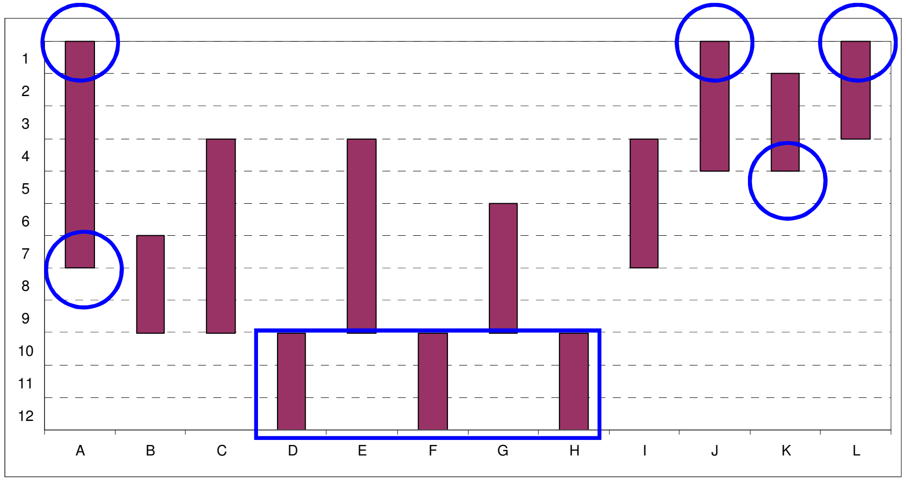
- Departments consume inputs in order to produce outputs
  - Data from TKK’s reporting system
  - 2 inputs, 44 outputs



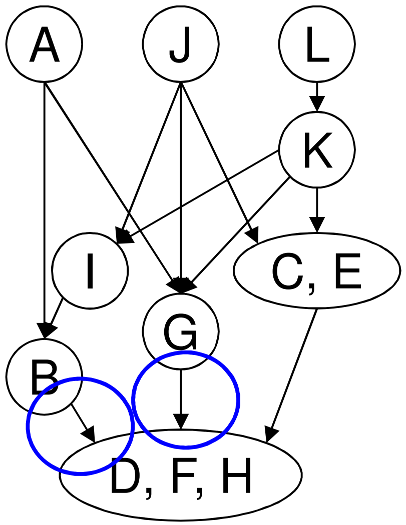
- Preferences from 7 members of the Resources Committee
  - Ex: What is the value of a Master’s thesis relative to a dissertation.?
  - Each member responded to elicitation questions, which yielded crisp weights
  - The feasible weights were then modeled as all possible convex combinations of these weightings



Efficiency bounds compared to the most efficient department



Ranking intervals



Dominance relations

- Departments A, J and L are efficient
  - But A can attain ranking 7 > 4, the worst ranking of the inefficient department K
  - There are feasible weights so that the Efficiency Ratio of A is only 57 % of that of the most efficient department
    - » For K, the corresponding ratio is 71%
- The efficiency intervals of D, F and H overlap with those of B and G
  - Yet, for all feasible weights the Efficiency Ratios of D, F and H are smaller than those of B and G

EXAMPLE - ANALYZING THE EFFICIENCY OF DENTAL CARE

Case: Efficiency Analysis in Dental Healthcare

- Finnish municipalities are responsible for public health care services, including dental care
- The situation of public health care is not particularly good
  - Long queues, budgets are getting tighter
- Pressures to improve processes towards higher quality and cost-efficiency
- Benchmarking yields insights into which – and what kinds of – processes are efficient

Yrjänä's Thesis on uses of REA in benchmarking

- Objectives
  - Analyze the overall efficiency of dental care in municipalities
  - Identify top-achievers that seem promising for closer benchmarking
  - Explore which factors contribute to efficiency/inefficiency

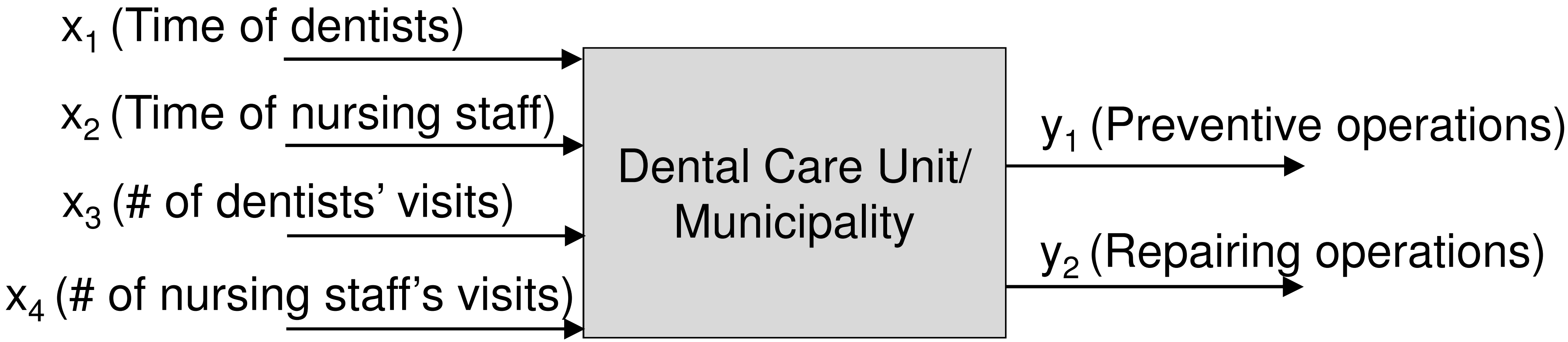
- Results targeted at medical directors
  - Responsible for the provision of dental care in a municipality
  - Empowered to implement changes and to reorganize processes

Data

- 11 municipalities
  - 16 000 – 200 000 inhabitants in each
  - Total 100 public dental care units
- Extensive data system
  - Information of every single operation done to every patient
  - Number and duration of visits
  - Etc.
- Scope limited to basic dental care and root treatments
  - Orthodontics and surgery excluded, among others
  - The scope still covers about 80% of all operations

Model structure

$$\text{Efficiency score, } E_k = \frac{\sum_{n=1}^{N} u_n y_{nk}}{\sum_{m=1}^{M} v_m x_{mk}}$$



- "Nursing staff" = Dental hygienists & assistants
- "Time" = The time reserved for patient visits
- "Preventive/repairing operations" = The weighed sum of operations. The more resources (time, materials) an operation demands, the larger its weight; this takes the case-mix into account.

The structure of model (2)

- Reasons for variable choices:
  - Usually the efficiency is calculated as a ratio between operations and either time or visits. Our model combines the both ways.
  - The salary of a dentist can be substantially higher than the one of dental hygienist or assistant. Thus the use of resources has to be considered separately for dentists and nursing staff.
  - The purpose of health care is to improve public health. The preventive care is an essential factor in executing this purpose.
    - The shares of preventive and repairing operations has to be in balance. The unit which does only repairing care at the expense of preventive care cannot be exemplary.
- NÄMÄ ASIAT VOIVAT TARVITTAESSA TULLA ILMI PUHEESSA EDELLISEN KALVON KOHDALLA.

Weight restrictions

- Weights represent the “value” of variables
- Constraints help eliminate irrelevant weight combinations

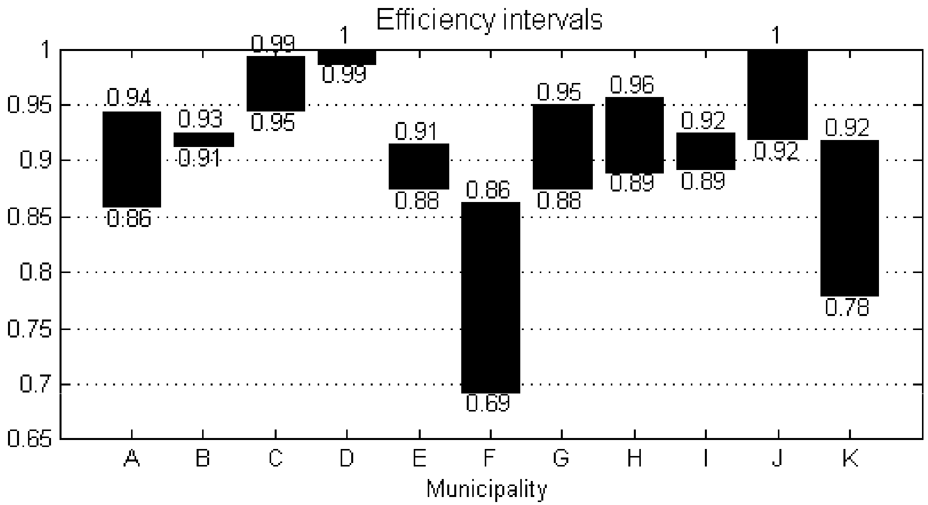
$$\tfrac{1}{2} \leq \frac{u_1}{u_2} \leq 2 \qquad \tfrac{1}{2} \leq \frac{\text{preventive operations}}{\text{repairing operations}} \leq 2$$

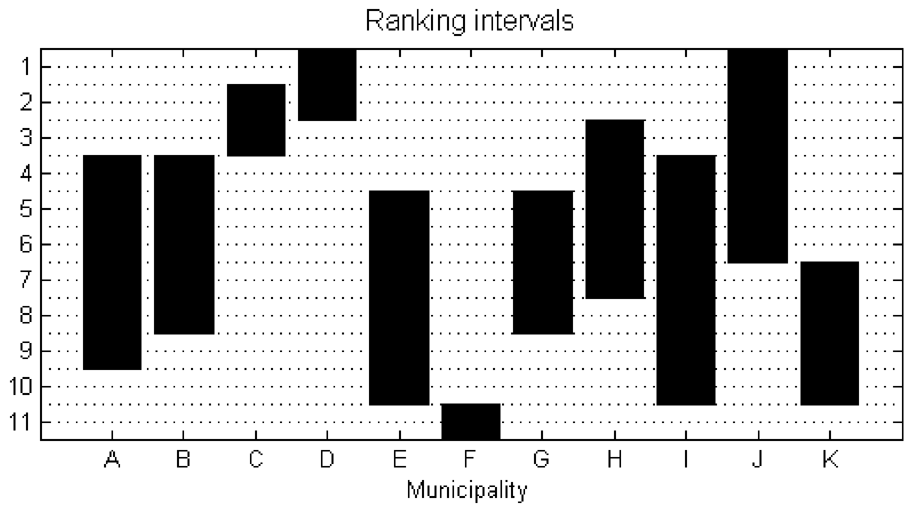
$$\tfrac{1}{3} \leq \frac{v_1 + v_2}{v_3 + v_4} \leq 3 \qquad \tfrac{1}{3} \leq \frac{\text{time}}{\text{visits}} \leq 3$$

$$1{,}5 \leq \frac{v_1}{v_2} = \frac{v_3}{v_4} \leq 3 \qquad 1{,}5 \leq \frac{\text{dentists}}{\text{nursing staff}} \leq 3$$

Efficiency score and ranking intervals

- Considerable variation in CCR-REA scores and the width of efficiency intervals
- The narrower the interval, the more “balanced” the DMU
- Traditional DEA scores contained in REA results

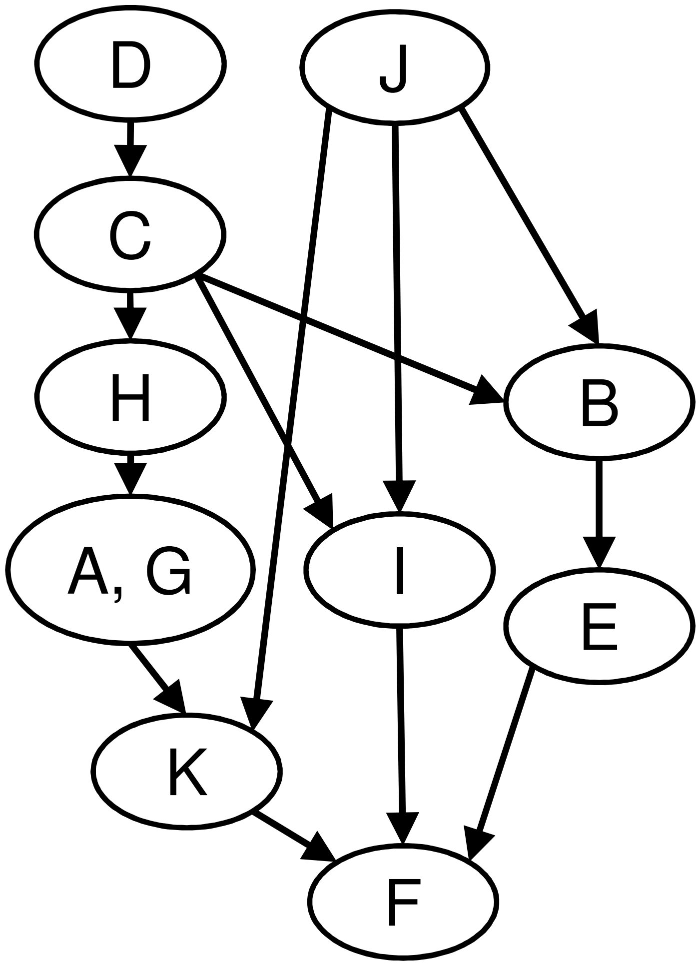




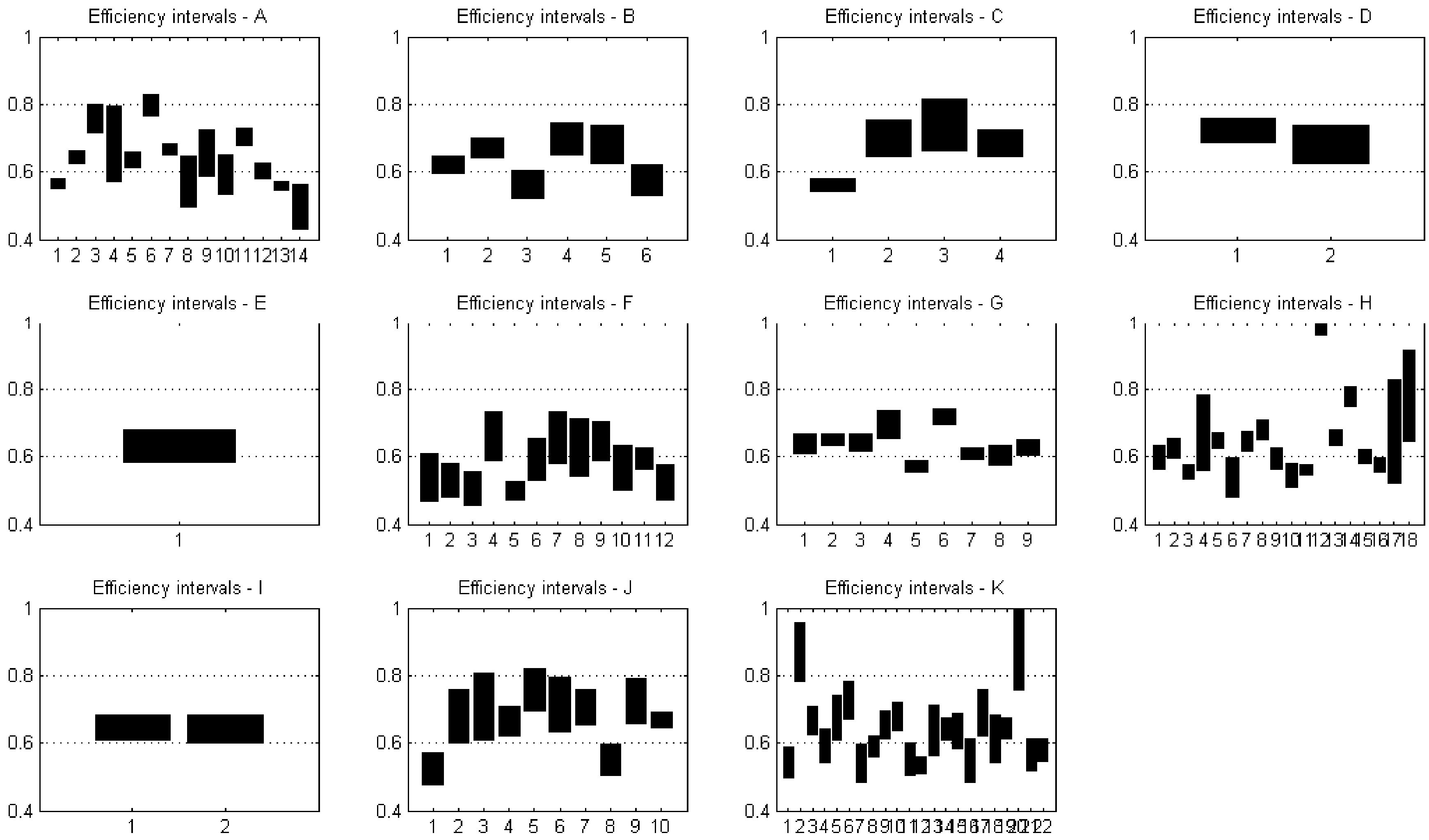
Dominance relations

- For example, D dominates C
  - D is more efficient than C for all feasible input /output weights

- Dominance relations may exist even if efficiency intervals or ranking intervals overlap



Efficiencies of care units in different municipalities



Uses of results

- Benchmarking best units
  - Discussions among medical directors about processes and management practices
  - Cost-efficient processes easier to identify at the level of dental care units rather than municipalities (there can be very diverse care units in a large municipality)

- Identifying causes for efficiency/unefficiency
  - Regression analyses between efficiency scores and explanatory variables

- Developing and implementing efficiency-improving action plans

Conclusion

- REA results use all feasible weights to evaluate DMUs
  - **Dominance relations** supports pairwise comparisons of DMUs
  - **Ranking intervals** show which rankings can be attained by DMUs
  - **Efficiency bounds** show how efficient a DMU can be compared to the DMUs in a benchmark group
  - Computed with LP and MILP models
- Uses of preference information
  - Implausible extreme weights can be excluded
  - Additional information leads to more conclusive results
- Results not sensitive to the introduction of outliers