# Simulation Metamodeling with Bayesian Networks

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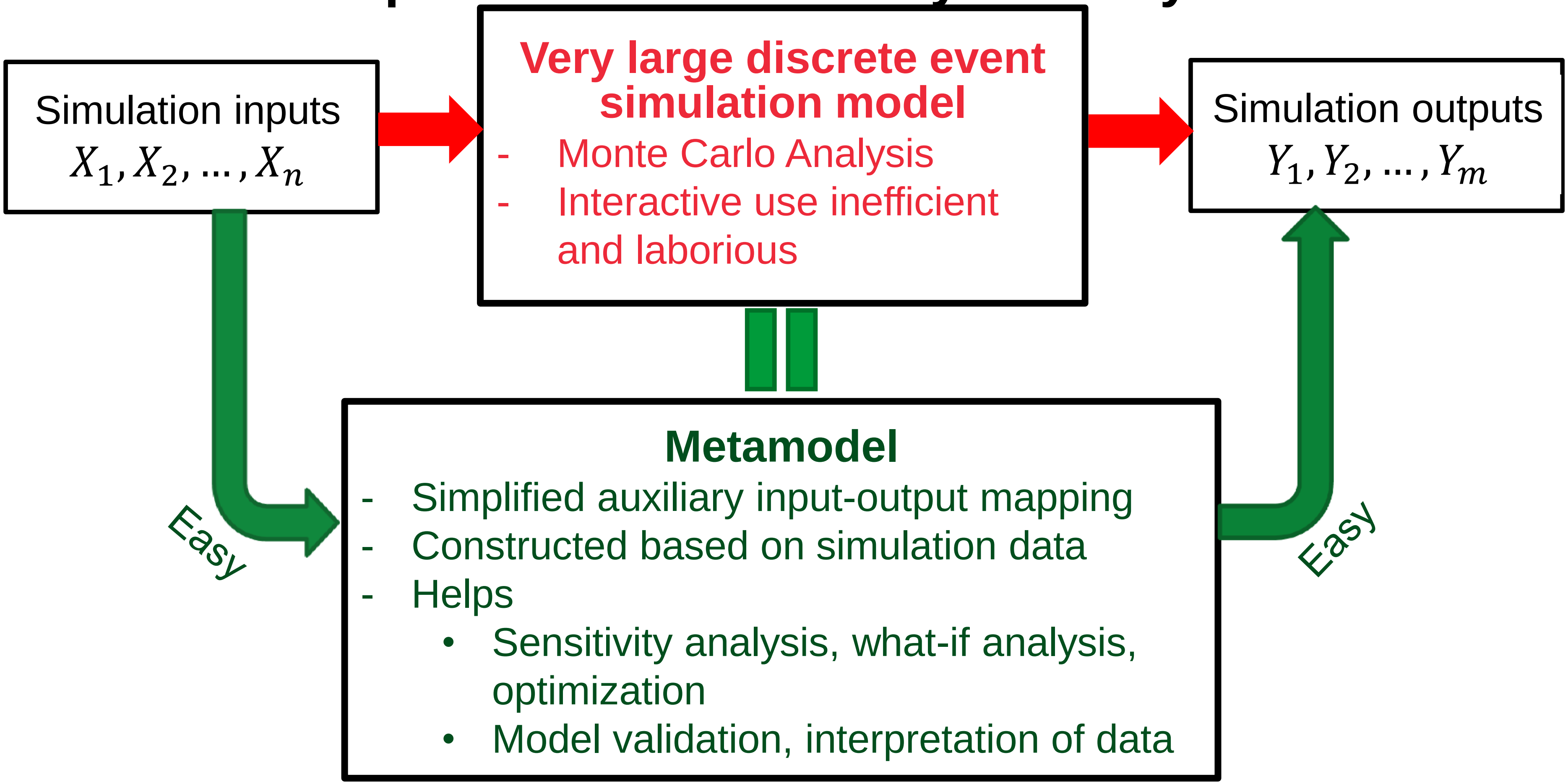
# Simulation metamodeling

## Complex and stochastic dynamic system



# Increasing the complexity of models increases the need for metamodeling

- Several existing approaches
  - Regression models, kriging models, splines, games, neural networks…

    → Mappings from inputs to **the expected values** of outputs

    → LOSS OF INFORMATION!

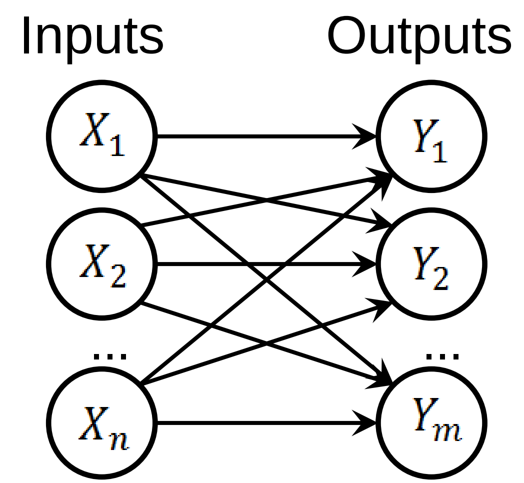
## New features allowed by Bayesian networks

Joint probability distribution of all inputs and outputs

→

"Mappings between input and output distributions"

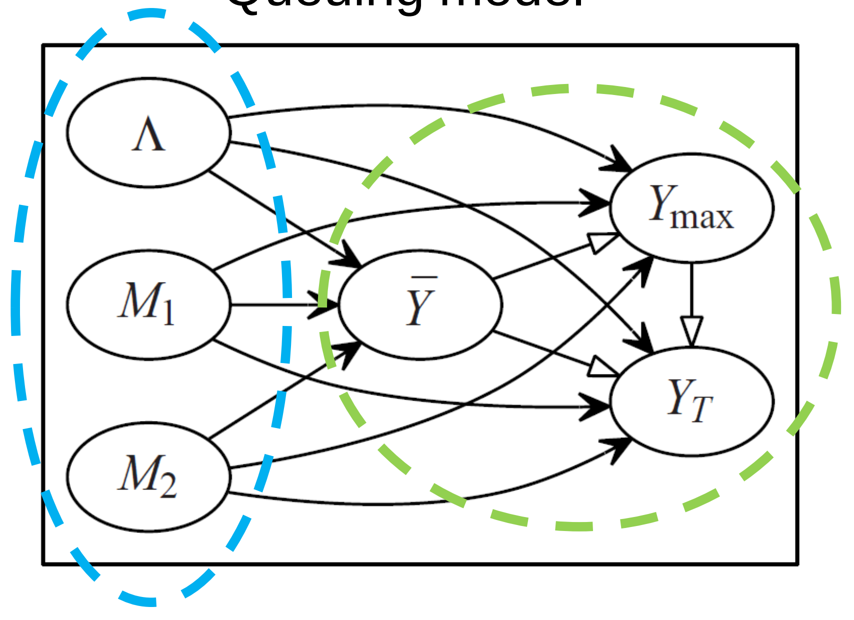
- Probabilistic dependencies between variables, in particular between outputs
- Efficient calculation of conditional probability distributions → Versatile what-if analyses



# Bayesian networks (BNs) as simulation metamodels

- Joint probability distribution of random variables
- Simulation inputs and outputs
  - Nodes
  - Discrete random variables
- Dependencies
  - Arcs
  - Conditional probabilities
- Available algorithms
  - Calculation of conditional probability distributions
  - Construction of BNs (sparse networks more efficient)

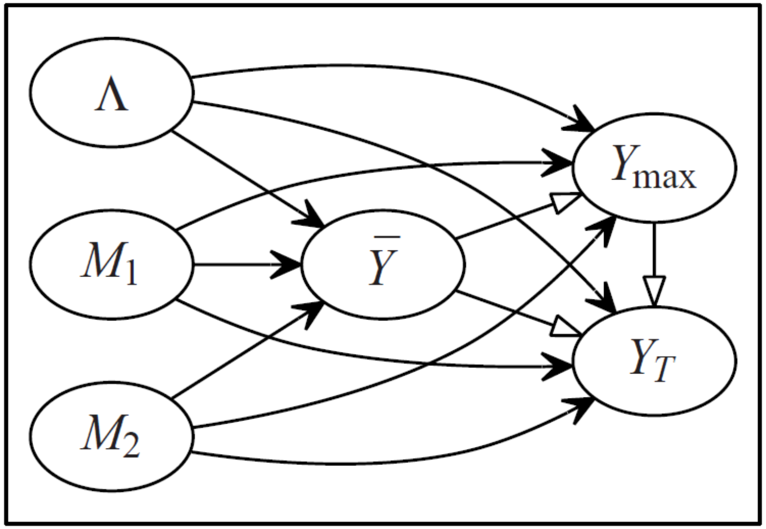
Queuing model



# Construction of BN Metamodels

1) Selection of variables
   - Simulation inputs and outputs
2) Collection of simulation data
   - All feasible input combinations simulated
3) Determination of network structure
   - Initial structure:
   - Dependencies found in data:
4) Estimation of probabilities
   - Conditional probability distributions for outputs
   - Input uncertainty

+) Validation



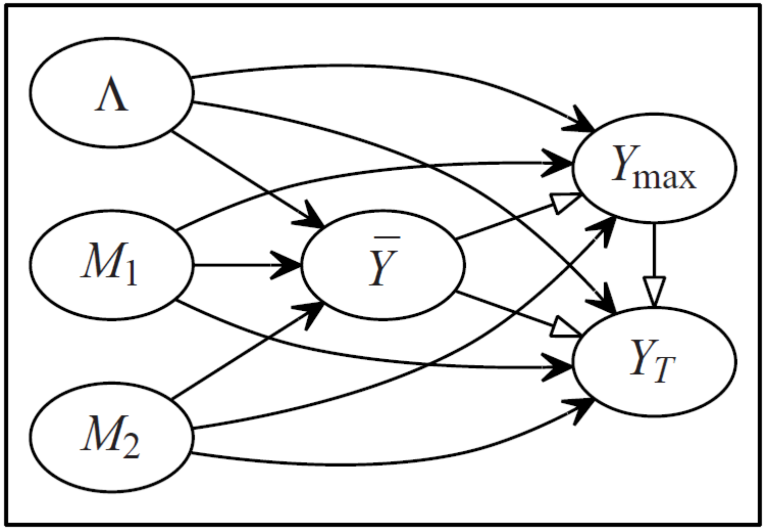
# Utilization of BN Metamodels

- What-if analysis

  Values of some variables fixed →

  Conditional probability distributions for other variables updated

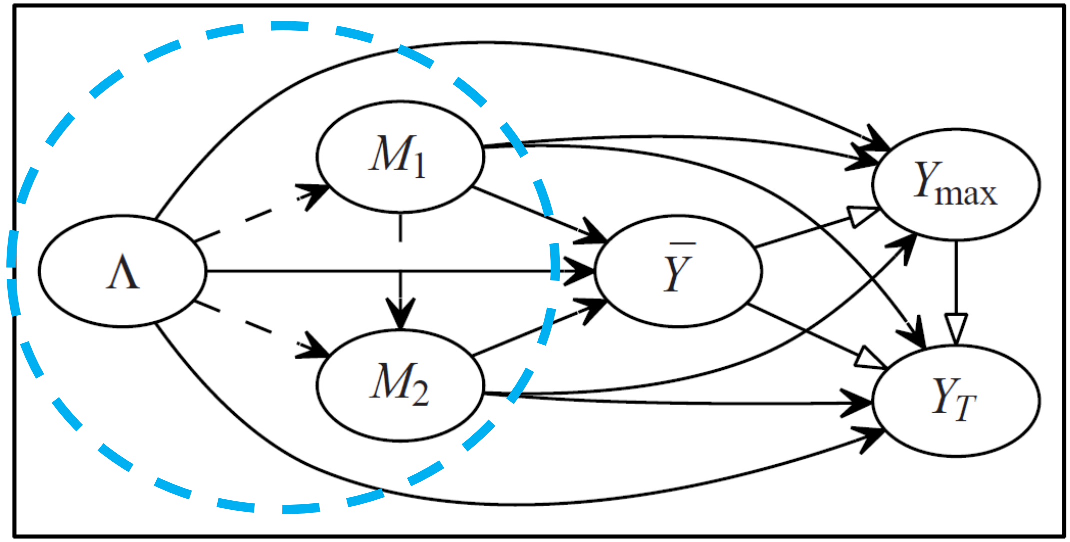
- Applications of what-if analyses
  - Dependence between inputs and outputs
  - Effect of input uncertainty on outputs
  - Dependence between outputs
  - Inverse reasoning



# Example: Queuing model

Single queue with Poisson arrivals and
two servers with exponential service times

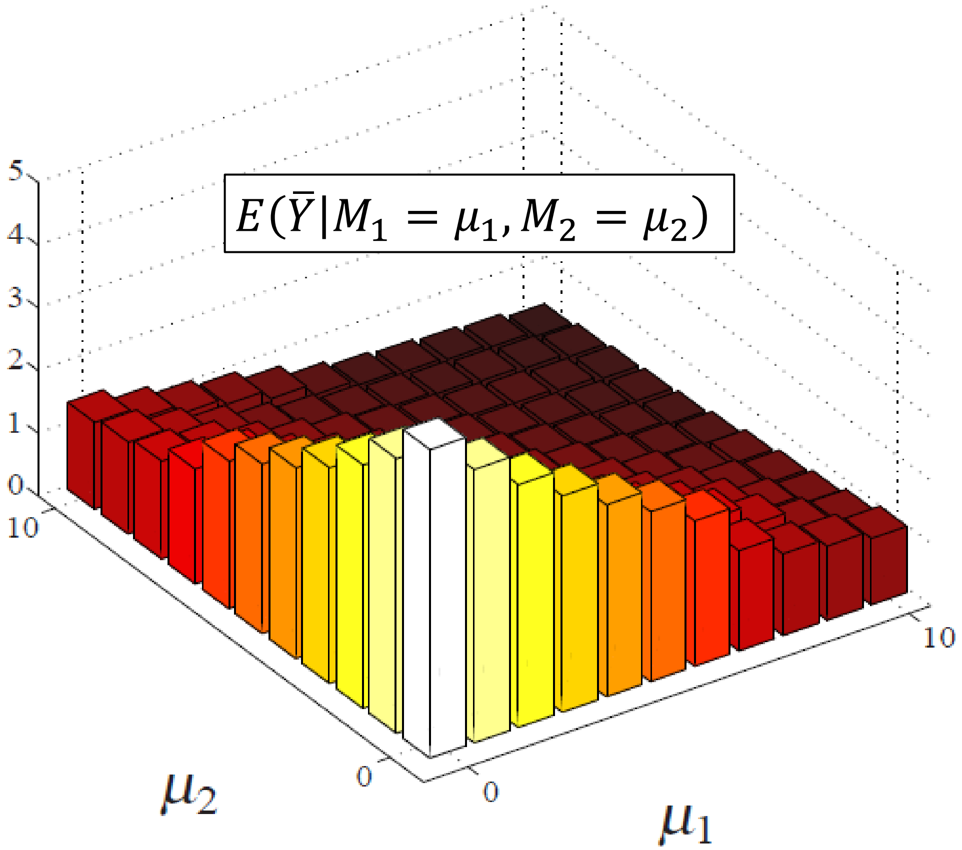
DEPENDENCE BETWEEN INPUTS




- Simulation inputs: $\Lambda, M_1, M_2$ (Arrival intensity; Service intensities)
- Simulation outputs: $\bar{Y}, Y_{max}, Y_T$ (Average and maximum number of customers; Number of customers at the end of simulation)

# Dependence between Inputs and Outputs

- Studied by calculating conditional probability distributions of outputs for fixed values of inputs

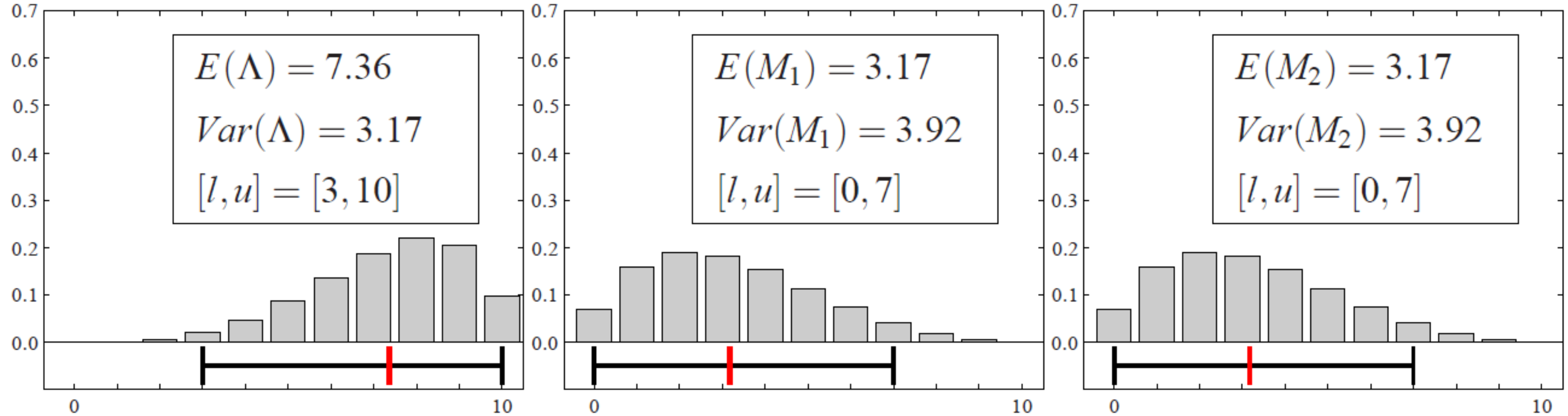



**Only the conditional expected value is presented**

# Effect of Input Uncertainty...

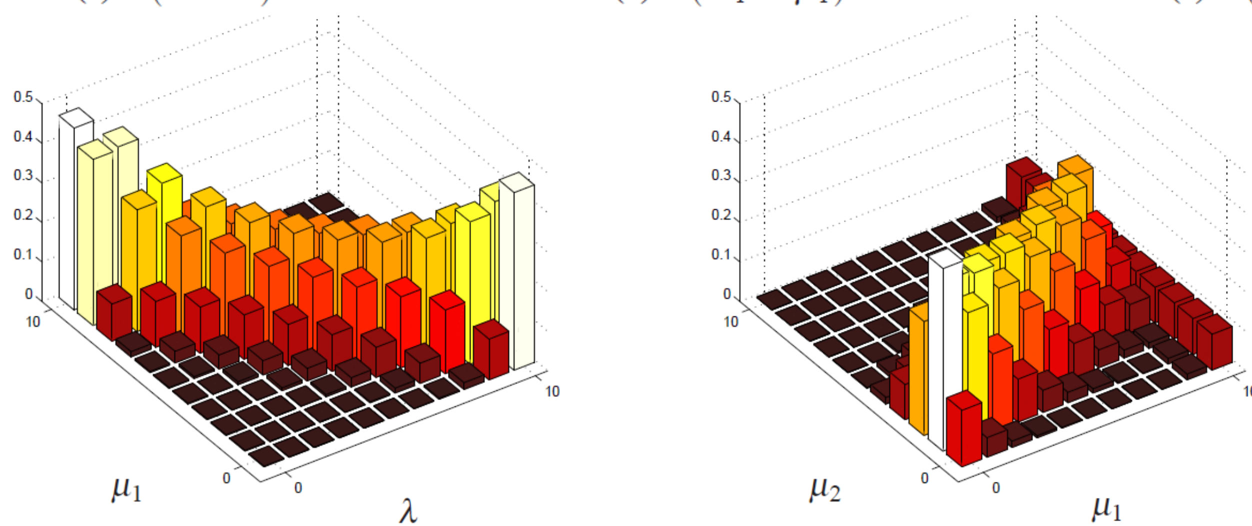
- Inputs considered as dependent random variables
  - ”Prior uncertainty” assessed by subject matter expert

MARGINAL DISTRIBUTIONS



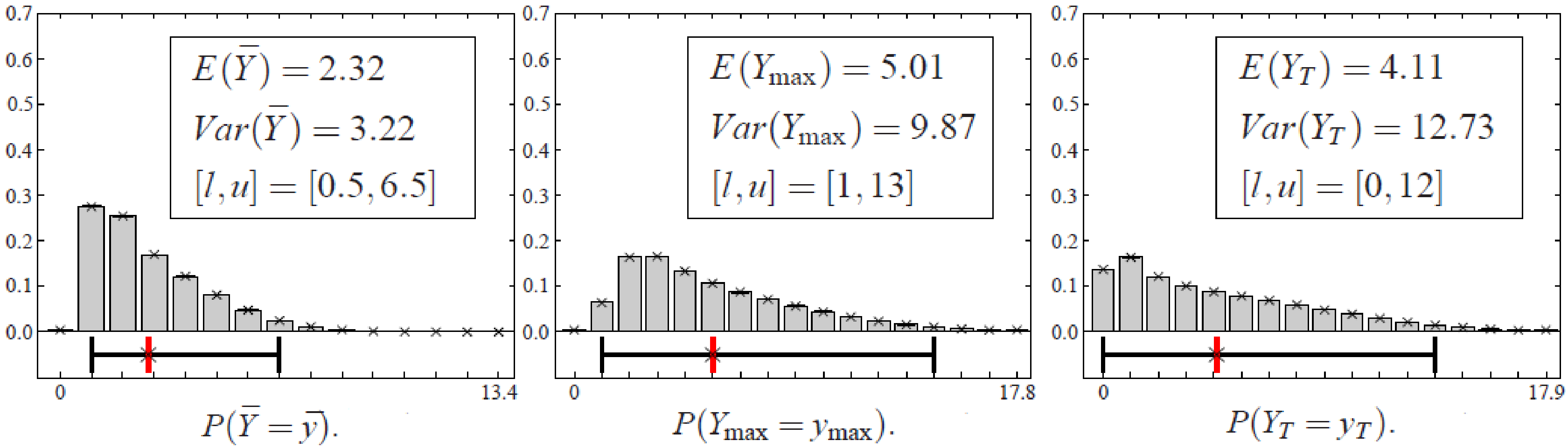

(a) $P(\Lambda = \lambda)$. (b) $P(M_1 = \mu_1)$. (c) $P(M_2 = \mu_2)$.

CONDITIONAL DISTRIBUTIONS




(a) $P(M_1 = \mu_1 | \Lambda = \lambda)$. (b) $P(M_2 = \mu_2 | \Lambda = 8, M_1 = \mu_1)$.

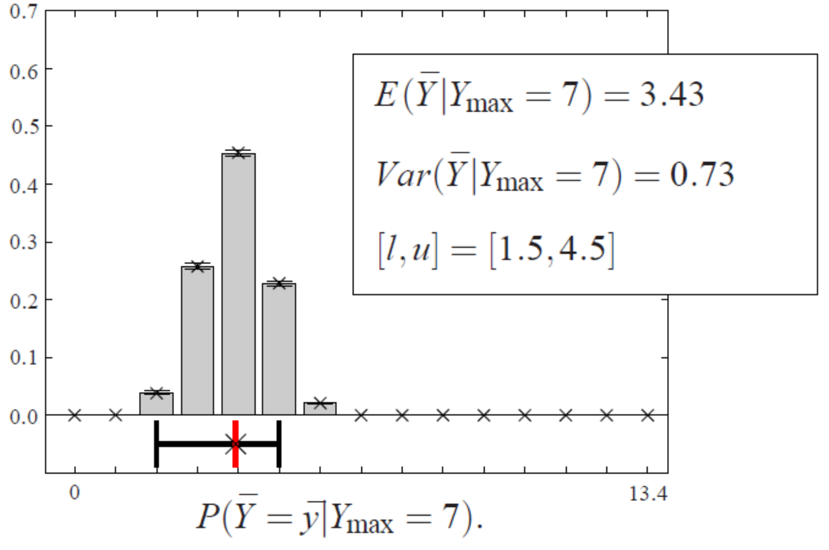
# ... on Distributions of Outputs

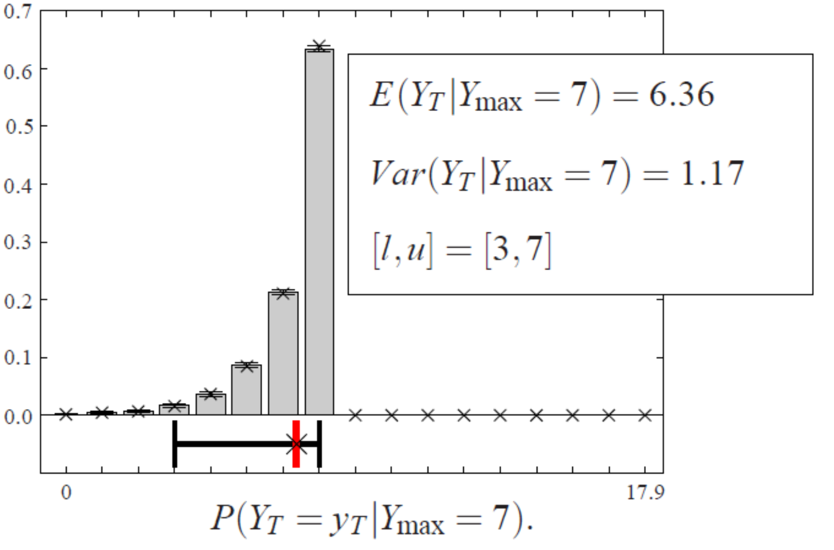



- Probability distributions include the effect of input uncertainty
- Enables calculation of descriptive statistics such as expected values, variances, and quantiles

# Dependence between Outputs

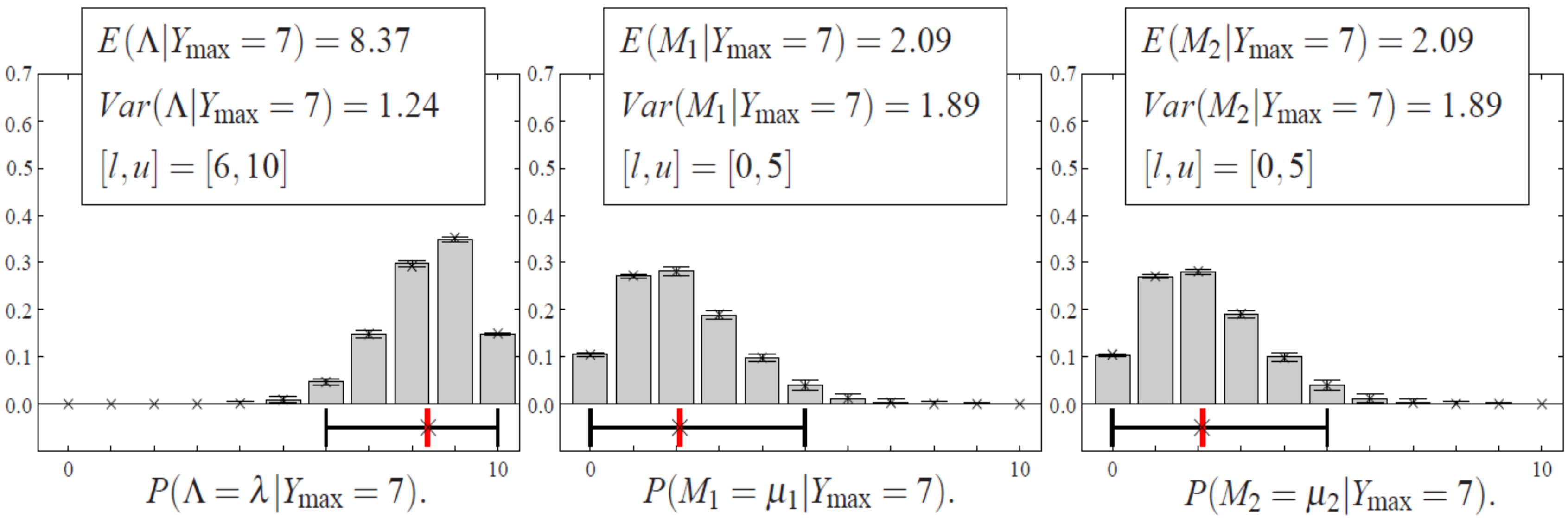
- Studied by calculating conditional probability distributions for fixed values of outputs (for example: $Y_{max} = 7$)



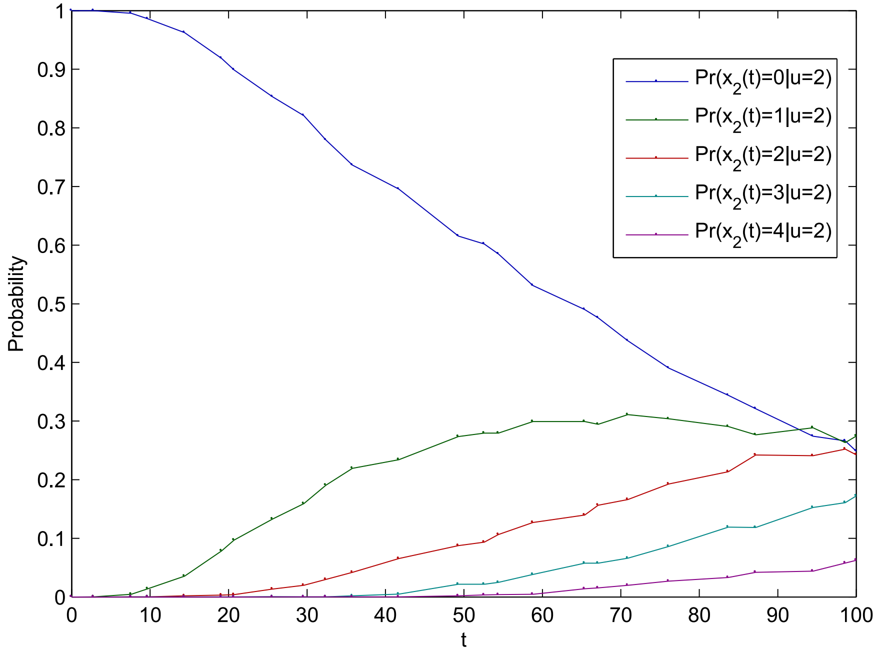
# Inverse reasoning

- Probability distributions of inputs updated conditional on fixed values of outputs (for example: $Y_{max} = 7$)
  - ”Posterior uncertainty” (cf. ”prior uncertainty” related to inputs)



# Dynamic Bayesian networks (DBNs) as simulation metamodels

- Time-dependent state variables of the simulation model
- Key time instants treated precisely - approximations used for other time instants
- Allows for a broader range of analyses
  - Time evolution of probability distributions
- Construction and utilization more demanding
  - DBN metamodeling tool



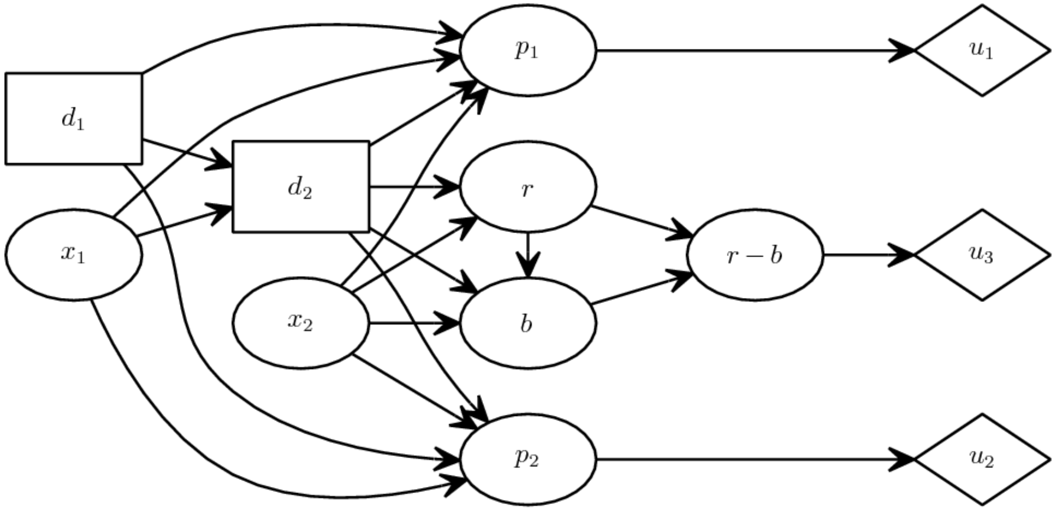
# Conclusion

- Simulation metamodeling benefits from BNs
  - Complete representation of probability distributions
    - No loss of information
      - cf. existing metamodels representing only expected values
  - New analysis capabilities
    - For example, input uncertainty and inverse reasoning
  - Available software with readily implemented algorithms
- Limitations
  - Construction necessitates large simulation data sets
  - Continuous variables have to be discretized

# Future research

- BN metamodeling
  - Continuous variables
    - Discretization
    - Interpolation
  - Sequential sampling
- Multi-criteria influence diagrams
  - Tool for decision support

Multi-Criteria Influence Diagram



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Pousi, J., Poropudas, J., Harju, M., and K. Virtanen. 2013. Simulation metamodeling with Bayesian networks, *Journal of Simulation*, to appear.