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A Simulated Annealing Algorithm for Noisy Multi-Objective Optimization

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Overview

- Noisy multi-objective optimization problem
  - Values of objective functions are uncertain
- Techniques for finding non-dominated solutions
  - Few take into account noise
  - Mostly evolutionary algorithms (EAs), e.g, [Goh&Tan, 2007]
- New simulated annealing (SA) algorithm
  - Features for handling noise and generating candidate solutions
  - **Performance superior to existing EAs in numerical tests**

Noisy multi-objective optimization problem

$$\min_{\mathbf{l} \leq \mathbf{x} \leq \mathbf{u}} [f_1(\mathbf{x}), \dots, f_H(\mathbf{x})]$$

- Values of objective functions $f_i(\mathbf{x})$ include uncertainty

$\rightarrow$ Optimization based on $f_i(\mathbf{x}) + \omega_i$ where $\omega_i \sim \mathcal{N}(0, \sigma_i^2)$

- Decision variable $\mathbf{x} \in \Re^n$
- Constraints $\mathbf{l}$ and $\mathbf{u}$

General steps of SA in multi-objective optimization

0. Define performance measure E for candidate solutions

   Select temperature T

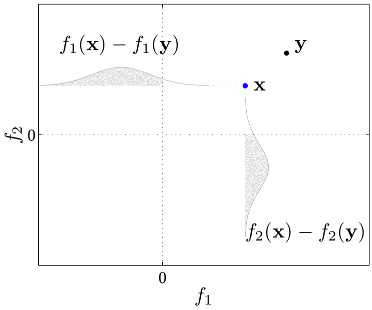
   Generate a current solution $\mathbf{x}$
1. Generate a candidate solution $\mathbf{y}$ from the neighborhood of $\mathbf{x}$
2. Set $\mathbf{x} = \mathbf{y}$ with probability $\min\left[1, \exp\left(\frac{E(\mathbf{x})-E(\mathbf{y})}{T}\right)\right]$
3. Solutions with best values of E into the non-dominated set
4. Return to step 1 or stop iteration

Features of the new SA algorithm

- Performance of a solution, *E*
  - Based on *probabilistic dominance*
  - → Takes into account multiple objectives and noise
  - Used previously in evolutionary algorithms [Hughes, 2001], not in SA
- Generation of candidate solutions
  - Using information on current non-dominated solutions
  - → Increase likelihood of obtaining new non-dominated solutions
  - Previously in deterministic single-objective SA [Sun et al., 2008]

Probabilistic dominance

- Probability that **x** dominates **y**
- Product of probabilities that objective function values are smaller for **x**
  - *M* samples
  - $\bar{f}_i(\mathbf{x})$ and $s_i^2(\mathbf{x})$ sample average and variance



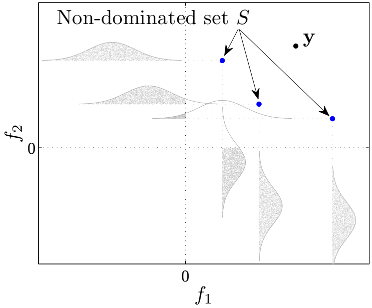
$$P(\mathbf{x} \succ \mathbf{y}) = \prod_{i=1}^{H} \Phi\left(\frac{\bar{f}_i(\mathbf{x}) - \bar{f}_i(\mathbf{y})}{\sqrt{s_i^2(\mathbf{x})/M + s_i^2(\mathbf{y})/M}}\right)$$

Φ cumulative distribution function of standard normal distribution

Performance of candidate solution $E(\mathbf{y})$

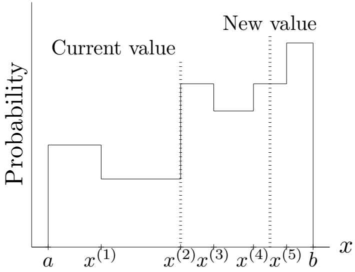
- Sum of probabilities that current non-dominated solutions dominate candidate solution **y**

$$E(\mathbf{y}) = \sum_{\mathbf{z} \in S} P(\mathbf{z} \succ \mathbf{y})$$



Generation of candidate solutions

- Use empirical data to sample new value of decision variable x
  - Recent values in non-dominated set $x^{(1)}, \ldots, x^{(m)} \in [a, b]$
  - Feasible neighborhood of current value $[a, b]$
- Concentrates to regions where likelihood of obtaining a new non-dominated solution is high

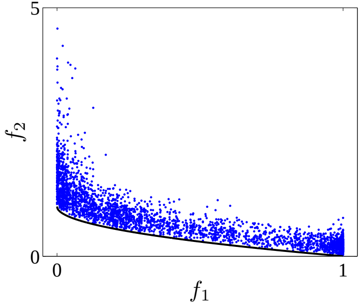


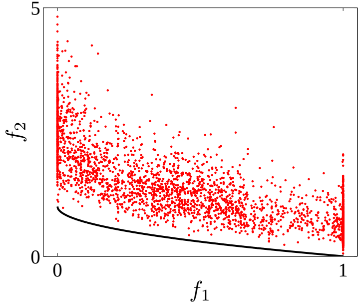
Numerical testing

- Reference algorithm
  - MOEA-RF, *Multi-Objective EA with Robust Features*
  - Outperformed several existing EAs in [Goh&Tan, 2007]
- Test problems
  - ZDT1, ZDT4, ZDT6 [Zitzler et al., 2007], FON [Fonseca&Fleming, 1998], KUR [Kursawe, 1991]
  - Four noise levels
- Performance measures used
  - *Generational distance*
  - *Spacing*
  - *Maximum spread*
  - *Hypervolume ratio*

Non-dominated solutions for ZDT1

- Highest noise level
- **SA algorithm produces solutions closer to actual non-dominated set**

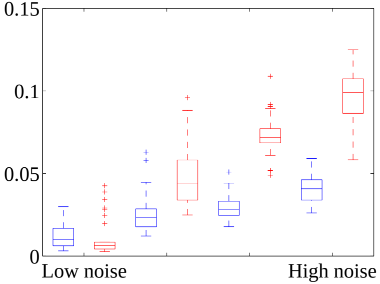




■ SA algorithm ■ MOEA-RF, 30 runs

Generational distance for ZDT1

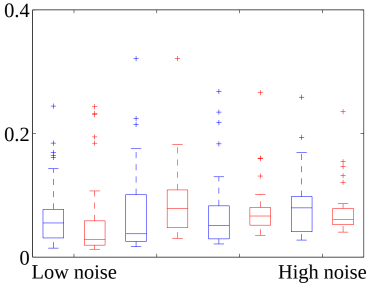
- Distance of solutions to actual non-dominated set
- **Values lower (better) for the SA algorithm**



■ SA algorithm ■ MOEA-RF, 30 runs with each noise level

Spacing for ZDT1

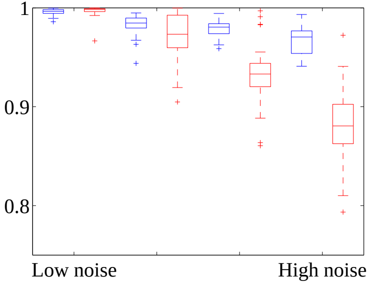
- Distance between obtained solutions
- **Values similar for the algorithms (lower is better)**



■ SA algorithm ■ MOEA-RF, 30 runs with each noise level

Maximum spread for ZDT1

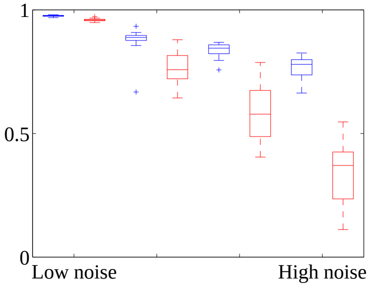
- Range of objective function values for obtained solutions
- **Values higher (better) for the SA algorithm**



SA algorithm MOEA-RF, 30 runs with each noise level

Hypervolume ratio for ZDT1

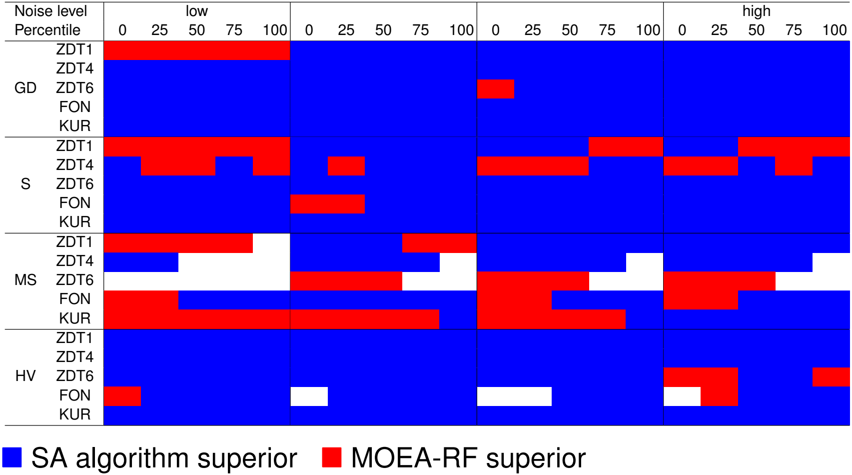
- Volume dominated by obtained solutions
- **Values higher (better) for the SA algorithm**



■ SA algorithm ■ MOEA-RF, 30 runs with each noise level

Summary of the numerical tests

Our SA algorithm superior accross test problems and noise levels



Conclusions

- New SA algorithm for noisy multi-objective optimization
  - Performance of solutions based on probabilistic dominance
  - Generation of candidate solutions using non-dominated set
- Outperformed reference EA in numerical tests
- Computational requirements comparable to the EA
- Successful application: Maintenance scheduling of aircraft [Mattila et al., 2011]
- Further development
  - Dynamic sample size for evaluating candidate solutions

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