Modelling Incomplete Information about Baselines in Portfolio Decision Analysis

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Multi-criteria project portfolio selection

- Choose a subset (=a portfolio) of projects from a set of proposals
  - Projects evaluated on multiple criteria
  - Maximize portfolio value function subject to resource constraints

- Additive-linear portfolio value (Golabi et al., 1981)
  - Widely used in applications; e.g., Healthcare (Kleinmuntz, 2007), R&D (Golabi et al., 1981), infrastructure asset management (Liesiö et al., 2007), military (Ewing et al., 2006)
  - Decision recommendations depend on the specification of the baseline value, i.e., the value of not doing a project (Clemen & Smith, 2009)

- We develop methods for
  - Specifying the baseline value
  - Analyzing how sensitive decision recommendations are to changes in the baseline value
Linear-additive portfolio value

- Projects $j = 1, \ldots, m$ evaluated w.r.t. criteria $i = 1, \ldots, n$
  - Measurement scales $X_1, \ldots, X_n$ with least (most) preferred level $x_i^0(x_i^*)$
  - $x_i^j \in X_i$: performance of project $j$ w.r.t. criterion $i$

- Value of project $j$: $v(x^j) = \sum_{i=1}^{n} w_i v_i(x^j)$
  - $v_i: X_i \rightarrow [0,1]$: criterion-specific value functions ($v_i(x_i^0) = 0$, $v_i(x_i^*) = 1$)
  - $w_i$: weight of criterion $i$ ($\sum_{i=1}^{n} w_i = 1$)
  $\rightarrow v(x^0) = 0$ and $v(x^*) = 1$

- Value of portfolio $z$: $V(z, v^B) = \sum_{j=1}^{m} [z_j v(x^j) + (1 - z_j) v^B]$
  - Binary decision variable $z_j = 1$ iff project $j$ is included in the portfolio
  $\rightarrow$ Optimization problem: $\max_z V(z, v^B)$ subject to resource constraints
  - $v^B$: baseline value defining the value of not doing a project
The baseline value $v^B$ matters

<table>
<thead>
<tr>
<th>Project</th>
<th>Financial contribution</th>
<th>Risk</th>
<th>Fit to strategy</th>
<th>Days required</th>
<th>$v_1(x_1)$</th>
<th>$v_2(x_2)$</th>
<th>$v_3(x_3)$</th>
<th>$v(x^j)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A ($j = 1$)</td>
<td>$200000$</td>
<td>uncertain</td>
<td>5</td>
<td>800</td>
<td>0.47</td>
<td>0</td>
<td>1</td>
<td>0.6175</td>
</tr>
<tr>
<td>B ($j = 2$)</td>
<td>$-13750$</td>
<td>probable</td>
<td>5</td>
<td>250</td>
<td>0</td>
<td>0.5</td>
<td>1</td>
<td>0.625</td>
</tr>
<tr>
<td>C ($j = 3$)</td>
<td>$12500$</td>
<td>safe</td>
<td>4</td>
<td>700</td>
<td>0.3</td>
<td>1</td>
<td>0.75</td>
<td>0.7</td>
</tr>
<tr>
<td>D ($j = 4$)</td>
<td>$307500$</td>
<td>safe</td>
<td>3</td>
<td>650</td>
<td>0.7</td>
<td>1</td>
<td>0.5</td>
<td>0.675</td>
</tr>
<tr>
<td>E ($j = 5$)</td>
<td>$-1250$</td>
<td>safe</td>
<td>2</td>
<td>350</td>
<td>0.03</td>
<td>1</td>
<td>0.25</td>
<td>0.3825</td>
</tr>
<tr>
<td>F ($j = 6$)</td>
<td>$393000$</td>
<td>uncertain</td>
<td>2</td>
<td>800</td>
<td>0.89</td>
<td>0</td>
<td>0.25</td>
<td>0.3475</td>
</tr>
<tr>
<td>G ($j = 7$)</td>
<td>$442500$</td>
<td>uncertain</td>
<td>2</td>
<td>600</td>
<td>1</td>
<td>0</td>
<td>0.25</td>
<td>0.375</td>
</tr>
<tr>
<td>H ($j = 8$)</td>
<td>$26500$</td>
<td>probable</td>
<td>1</td>
<td>400</td>
<td>0.61</td>
<td>0.5</td>
<td>0</td>
<td>0.2775</td>
</tr>
</tbody>
</table>

Example from Kleinmuntz (2000)

- Solving $\max_z \{ V(z, v^B) | \sum_{j=1}^{m} z_j c_j \leq 2500 \}$ yields
  - $\{B, C, D, E, H\}$, if $v^B = v(x^0) = v(-13750, \text{uncertain}, 1) = 0$
  - $\{A, B, C, D\}$, if $v^B = v(0, \text{safe}, 1) \approx 0.258$
Specifying the baseline value

- Golabi et al. (1981): Ask the DM to define a project $x \in X_1 \times \cdots \times X_n$ such that she is indifferent between doing and not doing the project
  - E.g. “I am indifferent between doing and not doing project with performance ($0,\text{safe},1)$”
  - $v^B = ($0,\text{safe},1) \approx 0.258$

- Such a project can be difficult to define
- More general approach: establish constraints on the baseline value
  - E.g. “I would not do project ($0,\text{safe},1)$ but I would do ($0,\text{safe},2)$”
  - $0.258 \approx ($0,\text{safe},1) < v^B < ($0,\text{safe},2) \approx 0.383$
What if the baseline value $v^B$ is below $v(x^0)$?

- E.g. selecting which bridges to repair
  - 2 damage indexes $X_1 = \{I,II,III,IV\}$, $X_2 = \{A,B,C\}$
  - If the DM would repair a bridge with performance (I,A):
    $v^B < v(I,A) = v(x^0) = 0 \leq v(x) \forall x \in X_1 \times X_2$
    → Not possible to specify a bridge $x$, s.t., the DM would be indifferent between repairing and not repairing it

- Baseline value can be constrained by comparing portfolios
  - Any preference between two portfolios with unequal number of projects yields a constraint $V(z, v^B) \geq V(z', v^B)$ for the baseline value
  - E.g., “A portfolio of five (I,A) bridges is preferred to a portfolio of three (IV,C) bridges”
    - $5v(x^0) + (m - 5)v^B \geq 3v(x^*) + (m - 3)v^B \Rightarrow v^B \leq -3/2$
Potentially optimal (PO) portfolios

- Which portfolios can be optimal if the baseline value is incompletely defined?

- How sensitive the decision recommendation are to small changes in the baseline value?

→ **Definition.** A feasible portfolio $z$ is *potentially optimal* if it maximizes $V(z, v^B)$ for some baseline values $v^B$
  - A feasible portfolio satisfies the resource constraints
Example: Potentially optimal portfolios

\[ V(z, \nu^B) = \sum_{j=1}^{m} [z_j \nu(x^j) + (1 - z_j) \nu^B]\]
Example: Potentially optimal portfolios

\[ v(-\$13750, \text{uncertain}, 1) \quad v(\$0, \text{safe}, 1) \quad v(\$0, \text{safe}, 2) \]
Algorithm for identifying PO portfolios

- **Lemma**: The value difference of two portfolios containing the same number of projects is constant for all $v^B \in \mathbb{R}$:
  \[
  V(z, v^B) - V(z', v^B) = \sum_{j=1}^{m} z_j v(x^j) - \sum_{j=1}^{m} z'_j v(x^j)
  \]

→ Algorithm sketch:
  1. Solve the optimal portfolio of each size $k = 0, \ldots, m$ with ILP:
     \[\max_z \{V(z, \cdot) \mid \sum_{j=1}^{m} z_j c_j \leq b, \sum_{j=1}^{m} z_j = k\}\]
  2. Use (simple) pairwise checks to identify the PO portfolios

- Solving some 130 PO portfolios for a problem with $m = 200$ projects takes about 13 seconds
Value-to-Cost ratios

- In applications with a single budget constraint, ratios $\frac{v(x^j)}{c_j}$ are often used to prioritize projects
  - Clemen & Smith (2009): Use of $\frac{v(x^j)}{c_j}$ assumes $v^B = 0$

- Value-to-cost ratio should be defined as $\frac{v(x^j) - v^B}{c_j}$:
  - Take any baseline value $v^B$ and let portfolio $z^*$ include the projects with the highest (positive) value-to-cost ratios
  - $z^*$ is an optimal solution to $\max_z \{ V(z, v^B) \mid \sum_{j=1}^m z_j c_j \leq b \}$, where $b = \sum_{j=1}^m z_j^* c_j$
Computing all possible Value-to-Cost orderings

- The ordering can change only at points $\nu^B$ in which
  1. Two projects have equal (positive) ratio: $\frac{\nu(x^j) - \nu^B}{c_j} = \frac{\nu(x^k) - \nu^B}{c_k}$
  2. Ratio of some project is zero: $\nu(x^j) - \nu^B = 0$
Conclusions

• The baseline value $v^B \in \mathbb{R}$ defines the value of not doing a project

• General techniques for specifying the baseline value
  – Applicable also if the baseline value is below $v(x^0)$
  – Ordinal preference statements can be modeled as constraints on the baseline value

• Computational tools for analyzing how project and portfolio decision recommendations depend on the baseline value
  – Allows sensitivity analysis / incompletely specified baseline value
  – Applicable for problem instances with hundreds of projects
References:
Kleinmuntz, D. N., (2000). CBA associates. Department of Business Administration, University of Illinois at Urbana-Champaign.