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Modelling Incomplete Information about Baselines in Portfolio Decision Analysis

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Multi-criteria project portfolio selection

- Choose a subset (=a portfolio) of projects from a set of proposals
  - Projects evaluated on multiple criteria
  - Maximize portfolio value function subject to resource constraints
- Additive-linear portfolio value (Golabi et al., 1981)
  - Widely used in applications; e.g., Healthcare (Kleinmuntz, 2007), R&D (Golabi et al., 1981), infrastructure asset management (Liesiö et al., 2007), military (Ewing et al., 2006)
  - Decision recommendations depend on the specification of the **baseline value**, i.e., the value of not doing a project (Clemen & Smith, 2009)
- We develop methods for
  - Specifying the baseline value
  - Analyzing how sensitive decision recommendations are to changes in the baseline value

Linear-additive portfolio value

- Projects $j = 1, \dots, m$ evaluated w.r.t. criteria $i = 1, \dots, n$
  - Measurement scales $X_1, \dots, X_n$ with least (most) preferred level $x_i^{\,0} (x_i^{\,*})$
  - $x_i^{\,j} \in X_i$: performance of project j w.r.t. criterion i

- Value of project j: $v\left(x^j\right) = \sum_{i=1}^{n} w_i v_i(x_i^{\,j})$
  - $v_i: X_i \rightarrow [0,1]$: criterion-specific value functions ($v_i(x_i^{\,0}) = 0$, $v_i(x_i^{\,*}) = 1$)
  - w_i: weight of criterion *i* ($\sum_{i=1}^{n} w_i = 1$)

→ $v(x^0) = 0$ and $v(x^*) = 1$

- Value of portfolio z: $V(z, v^B) = \sum_{j=1}^{m} [z_j v\left(x^j\right) + \left(1 - z_j\right) v^B]$
  - Binary decision variable $z_j = 1$ iff project j is included in the portfolio
  - → Optimization problem: $\max_z V(z, v^B)$ subject to resource constraints
  - ***v^B*: baseline value** defining the value of not doing a project

The baseline value v^B matters

Project	Financial contribution x_1^j	Risk x_2^j	Fit to strategy x_3^j	Days required c_j	$v_1(x_1^j)$	$v_2(x_2^j)$	$v_3(x_3^j)$	$v(x^j)$
A $(j = 1)$	\$200000	uncertain	5	800	0.47	0	1	0.6175
B $(j = 2)$	−\$13750	probable	5	250	0	0.5	1	0.625
C $(j = 3)$	\$12500	safe	4	700	0.3	1	0.75	0.7
D $(j = 4)$	\$307500	safe	3	650	0.7	1	0.5	0.675
E $(j = 5)$	−\$1250	safe	2	350	0.03	1	0.25	0.3825
F $(j = 6)$	\$393000	uncertain	2	800	0.89	0	0.25	0.3475
G $(j = 7)$	\$442500	uncertain	2	600	1	0	0.25	0.375
H $(j = 8)$	\$26500	probable	1	400	0.61	0.5	0	0.2775

Example from Kleinmuntz (2000)

- Solving $\max_z \{V(z, v^B) \mid \sum_{j=1}^{m} z_j c_j \leq 2500\}$ yields
  - {B,C,D,E,H}, if $v^B = v(x^0) = v(-\$13750, \text{uncertain}, 1) = 0$
  - {A,B,C,D}, if $v^B = v(\$0, \text{safe}, 1) \approx 0.258$

Specifying the baseline value

- Golabi et al. (1981): Ask the DM to define a project $x \in X_1 \times \cdots \times X_n$ such that she is indifferent between doing and not doing the project
  - →E.g. "I am indifferent between doing and not doing project with performance ($0,safe,1)"
  - →v^B = ($0,safe,1) ≈ 0.258
- Such a project can be difficult to define
- More general approach: establish constraints on the baseline value
  - E.g. "I would not do project ($0,safe,1) but I would do ($0,safe,2)"
  - →0.258 ≈ ($0,safe,1) < v^B < ($0,safe,2) ≈ 0.383

What if the baseline value v^B is below $v(x^0)$?

- E.g. selecting which bridges to repair
  - 2 damage indexes X_1 = {I,II,III,IV}, X_2 = {A,B,C}
  - If the DM would repair a bridge with performance (I,A):

    $\rightarrow v^B < v(\text{I,A}) = v(x^0) = 0 \leq v(x)\ \forall\, x \in X_1 \times X_2$

    $\rightarrow$Not possible to specify a bridge x, s.t., the DM would be indifferent between repairing and not repairing it

- Baseline value can be constrained by comparing **portfolios**
  - Any preference between two portfolios with unequal number of projects yields a constraint $V(z, v^B) \geq V(z', v^B)$ for the baseline value
  - E.g., “A portfolio of five (I,A) bridges is preferred to a portfolio of three (IV,C) bridges”
    - $5v(x^0) + (m-5)v^B \geq 3v(x^*) + (m-3)v^B \;\Rightarrow v^B \leq -3/2$

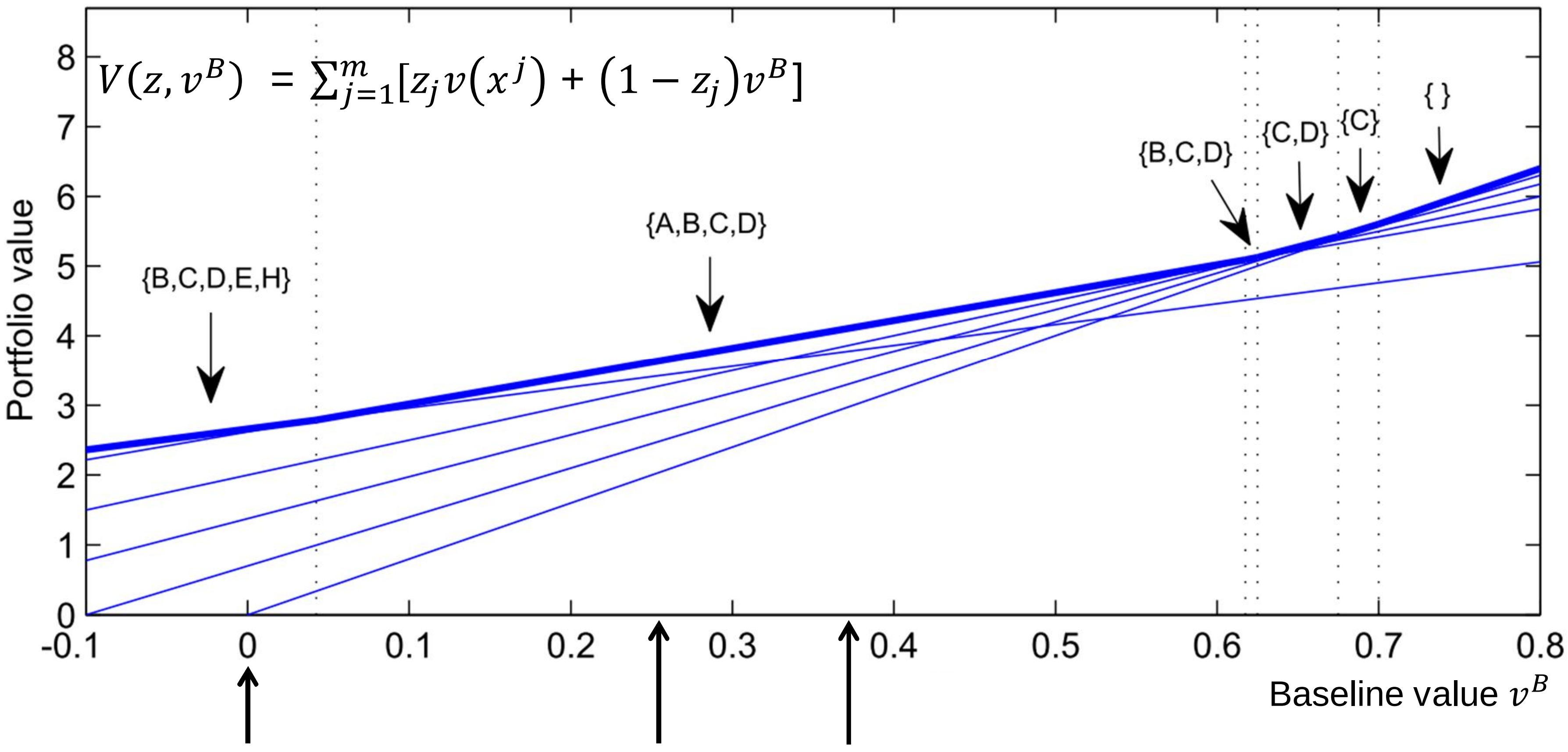
Potentially optimal (PO) portfolios

- Which portfolios can be optimal if the baseline value is incompletely defined?

- How sensitive the decision recommendation are to small changes in the baseline value?

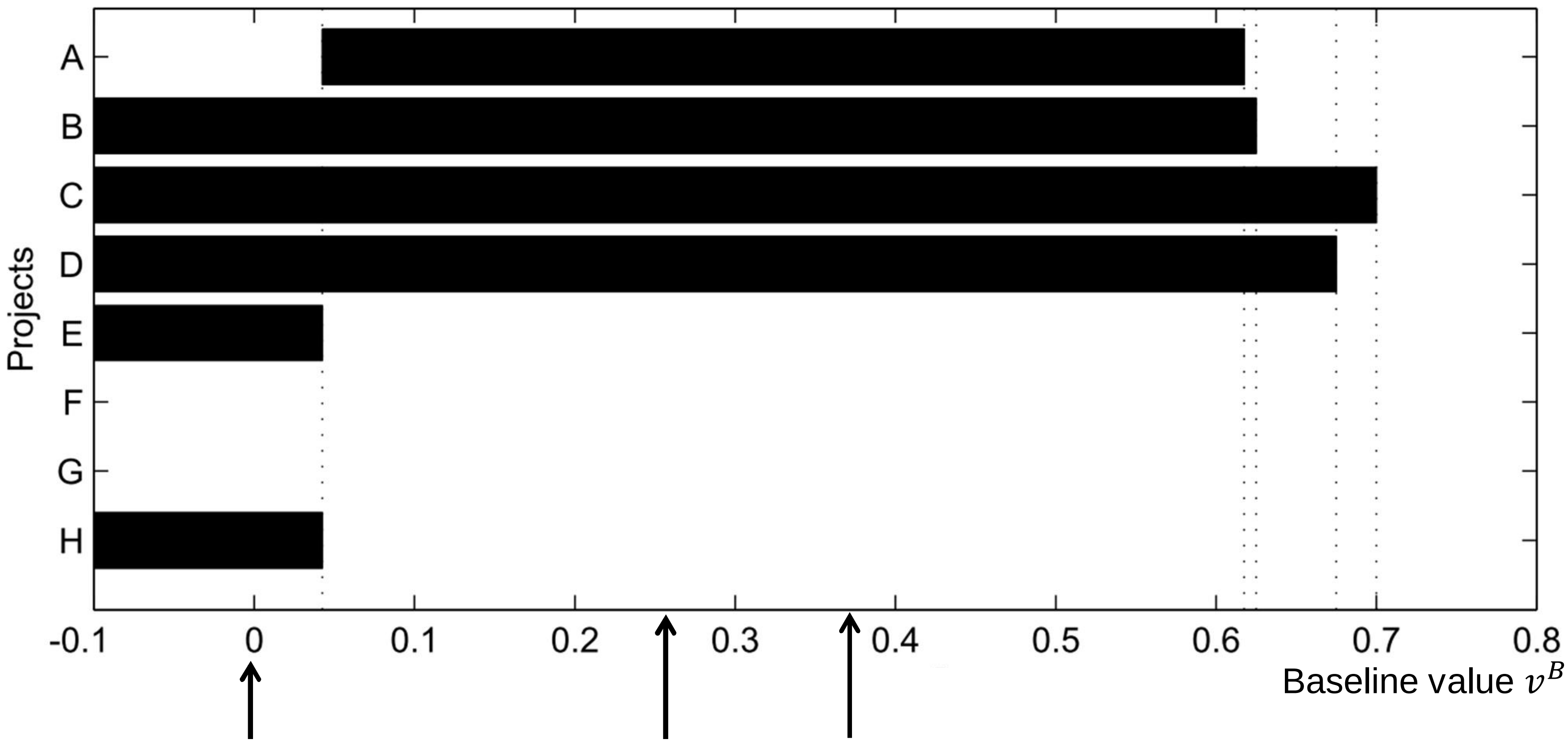
→ **Definition.** A feasible portfolio z is *potentially optimal* if it maximizes $V(z, v^B)$ for some baseline values v^B
  - A feasible portfolio satisfies the resource constraints

Example: Potentially optimal portfolios



v(-$13750,uncertain,1) *v*($0,safe,1) *v*($0,safe,2)

Example: Potentially optimal portfolios



v(-$13750,uncertain,1) *v*($0,safe,1) *v*($0,safe,2)

Algorithm for identifying PO portfolios

- **Lemma**: The value difference of two portfolios containing the same number of projects is constant for all $v^B \in \mathbb{R}$:
  $V(z, v^B) - V(z', v^B) = \sum_{j=1}^{m} z_j v(x^j) - \sum_{j=1}^{m} z'_j v(x^j)$

→Algorithm sketch:

1. Solve the optimal portfolio of each size $k = 0, \ldots, m$ with ILP:
   $\max_z \{V(z,\cdot) \mid \sum_{j=1}^{m} z_j c_j \leq b, \sum_{j=1}^{m} z_j = k\}$
2. Use (simple) pairwise checks to identify the PO portfolios

- Solving some 130 PO portfolios for a problem with $m = 200$ projects takes about 13 seconds

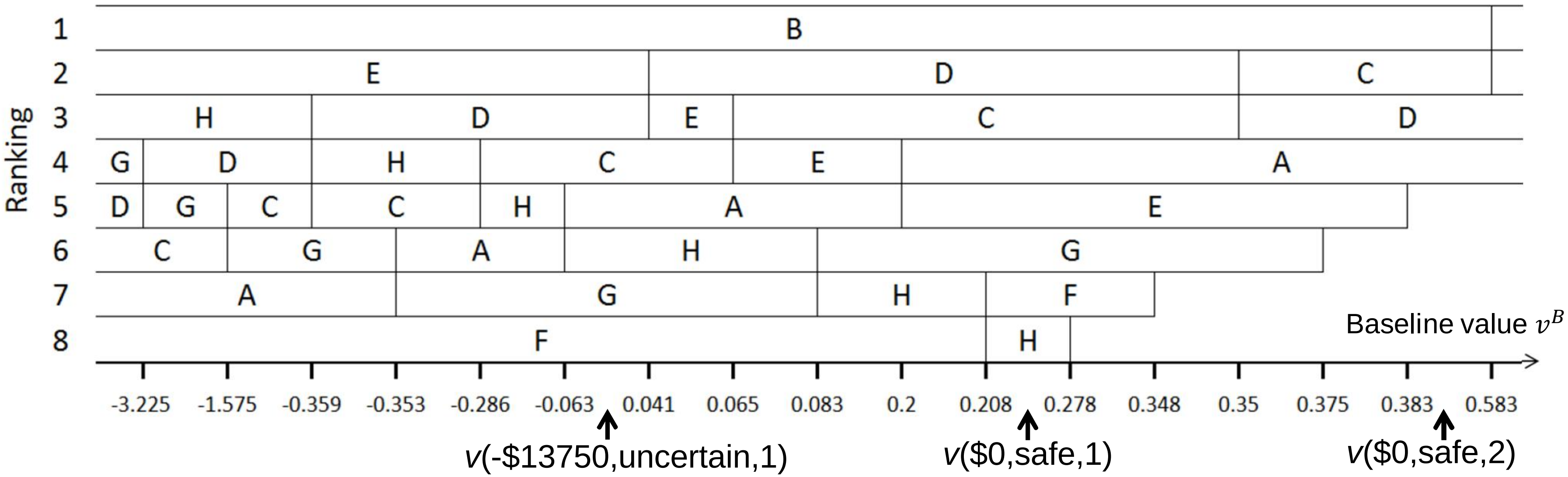
Value-to-Cost ratios

- In applications with a single budget constraint, ratios $v(x^j)/c_j$ are often used to prioritize projects
  - Clemen & Smith (2009): Use of $v(x^j)/c_j$ assumes $v^B = 0$
- Value-to-cost ratio should be defined as $\frac{v(x^j)-v^B}{c_j}$:
  - Take any baseline value v^B and let portfolio z^* include the projects with the highest (positive) value-to-cost ratios

→ z^* is an optimal solution to $\max_z\{V(z,v^B) \mid \sum_{j=1}^m z_j c_j \le b\}$, where b = $\sum_{j=1}^m z_j{}^* c_j$

Computing all possible Value-to-Cost orderings

- The ordering can change only at points v^B in which
  1. Two projects have equal (positive) ratio: $\frac{v(x^j)-v^B}{c_j} = \frac{v(x^k)-v^B}{c_k}$
  2. Ratio of some project is zero: $v(x^j) - v^B = 0$



Conclusions

- The baseline value $v^B \in \mathbb{R}$ defines the value of not doing a project
- General techniques for specifying the baseline value
  - Applicable also if the baseline value is below $v(x^0)$
  - Ordinal preference statements can be modeled as constraints on the baseline value
- Computational tools for analyzing how project and portfolio decision recommendations depend on the baseline value
  - Allows sensitivity analysis / incompletely specified baseline value
  - Applicable for problem instances with hundreds of projects

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