Improving Construction of Conditional Probability Tables for Ranked Nodes in Bayesian Networks
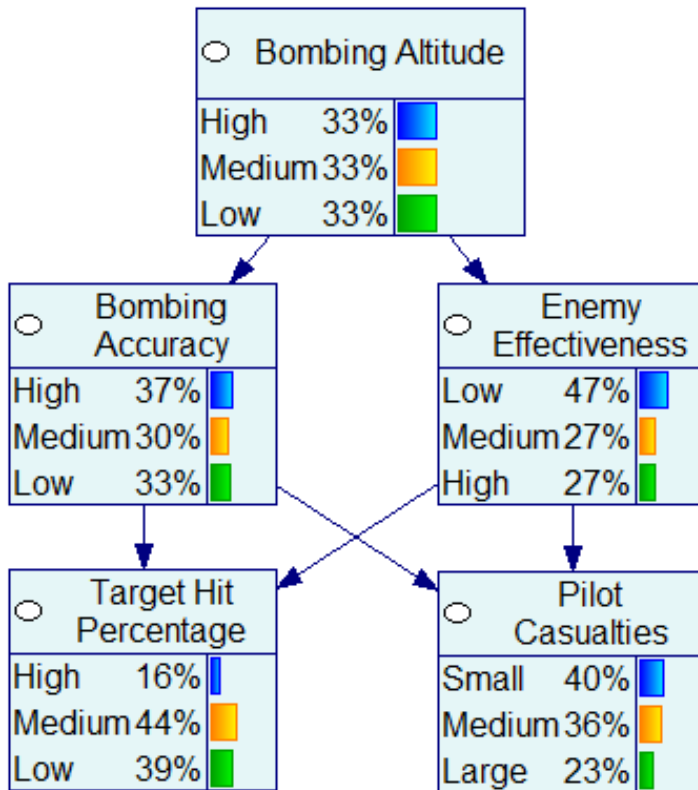
Pekka Laitila, Kai Virtanen

pekka.laitila@aalto.fi, kai.virtanen@aalto.fi

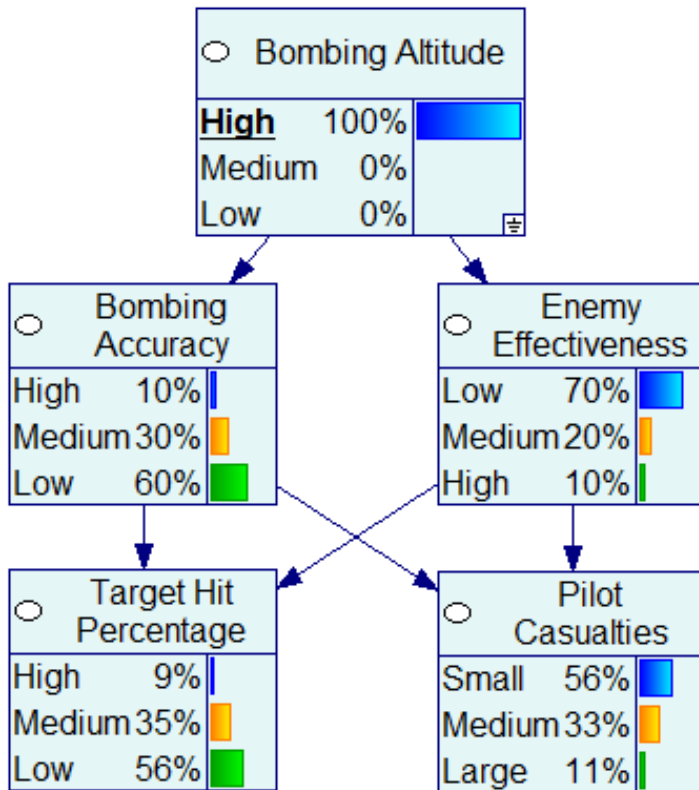Systems Analysis Laboratory, Department of Mathematics and Systems Analysis

Bayesian networks (BNs)

➤ Represent uncertain knowledge
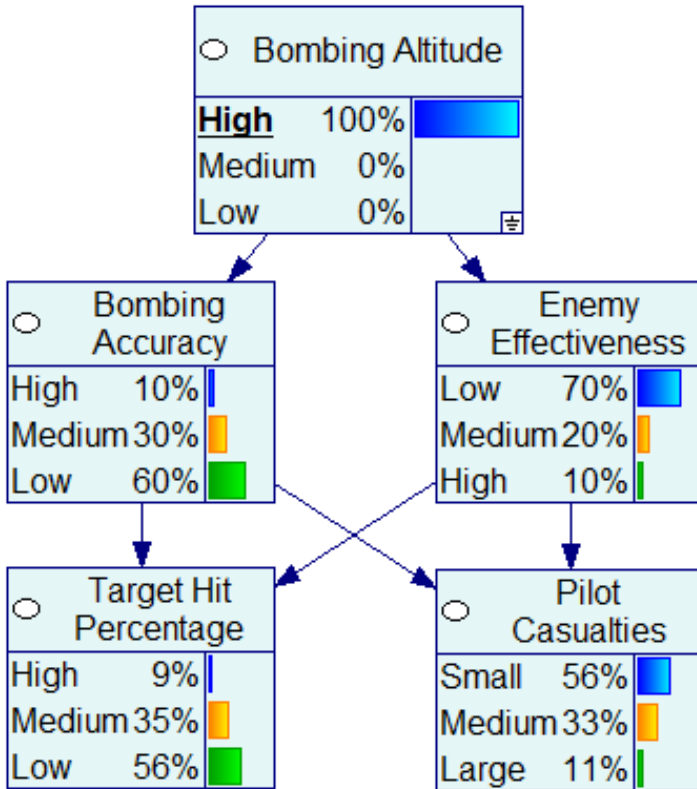
➤ Reasoning under uncertainty

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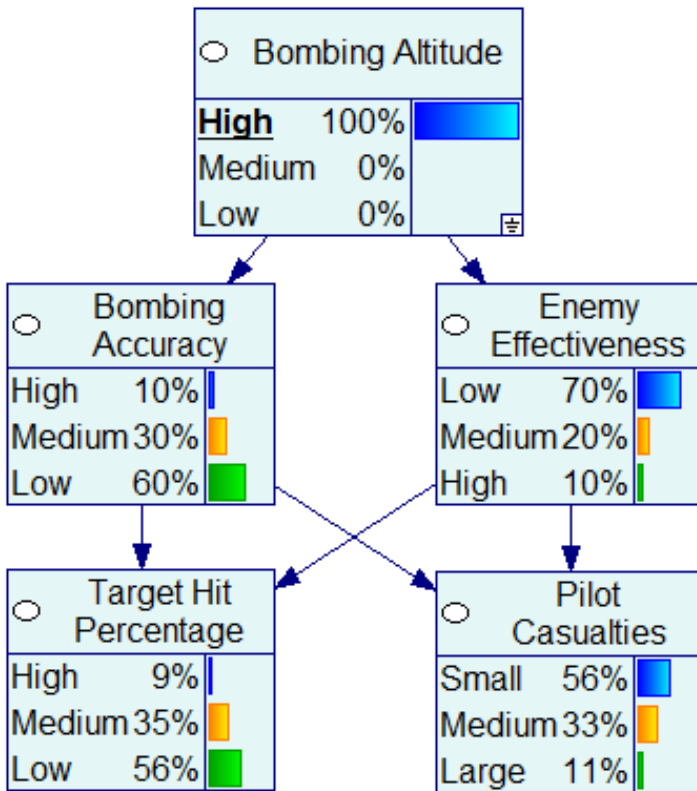
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Conditional probability tables (CPTs)

➢ Quantify dependence between linked nodes

	Bombing Altitude	High	Medium	Low
Bombing Accuracy	High	0.1	0.3	0.7
	Medium	0.3	0.4	0.2
	Low	0.6	0.3	0.1

Bombing Altitude

High	100%
Medium	0%
Low	0%

Bombing Accuracy

High	10%
Medium	30%
Low	60%

Enemy Effectiveness

Low	70%
Medium	20%
High	10%

Target Hit Percentage

High	9%
Medium	35%
Low	56%

Pilot Casualties

Small	56%
Medium	33%
Large	11%

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Challenge of expert elicitation
Construction of CPTs based on
expert elicitation is
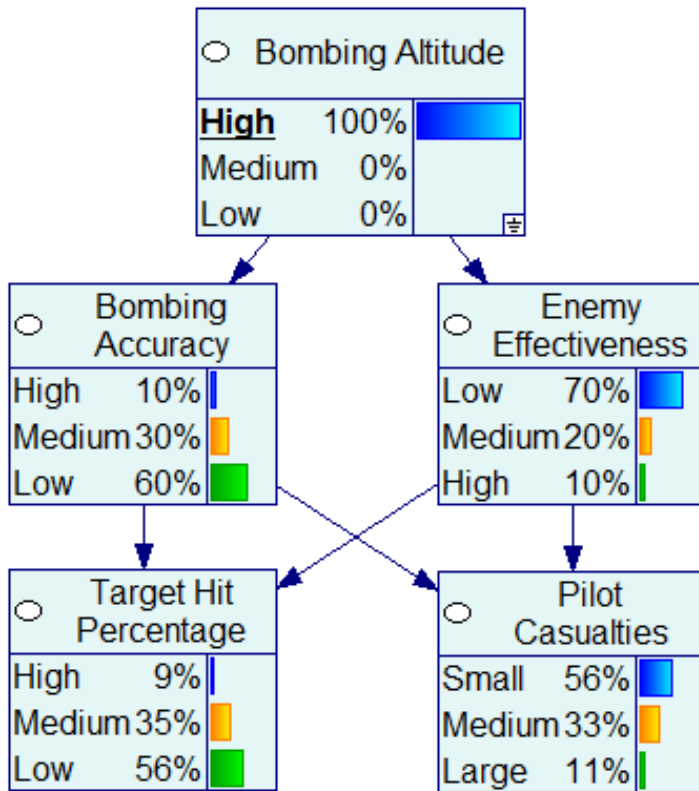time consuming and prone to biases

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Challenge of expert elicitation

• Direct estimation
• Probability scale methods
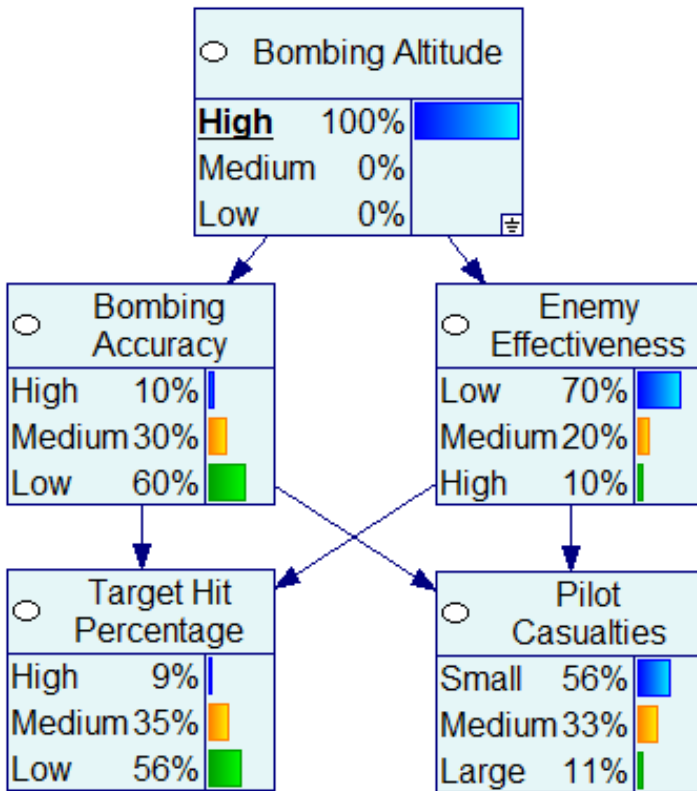• Gamble-like methods
• Probability wheel
• etc.

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Challenge of expert elicitation
- Direct estimation
- Probability scale methods
- Gamble-like methods
- Probability wheel
- etc.

Inadequate!

Construction of CPTs with parametric methods

- **Idea:**
  1. Probabilistic relationship between nodes fits a standard pattern ⎤
  2. Expert assigns parameters characterizing the pattern ⎦ **CPT**
- **Benefit:**
  - Number of parameters << Number of CPT entries ➔ Expert saves time!

Construction of CPTs with parametric methods

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    1. Probabilistic relationship between nodes fits a standard pattern
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        } **CPT**

- **Benefit:**
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- **Challenges we have recognized in "Ranked Nodes Method" (RNM)**
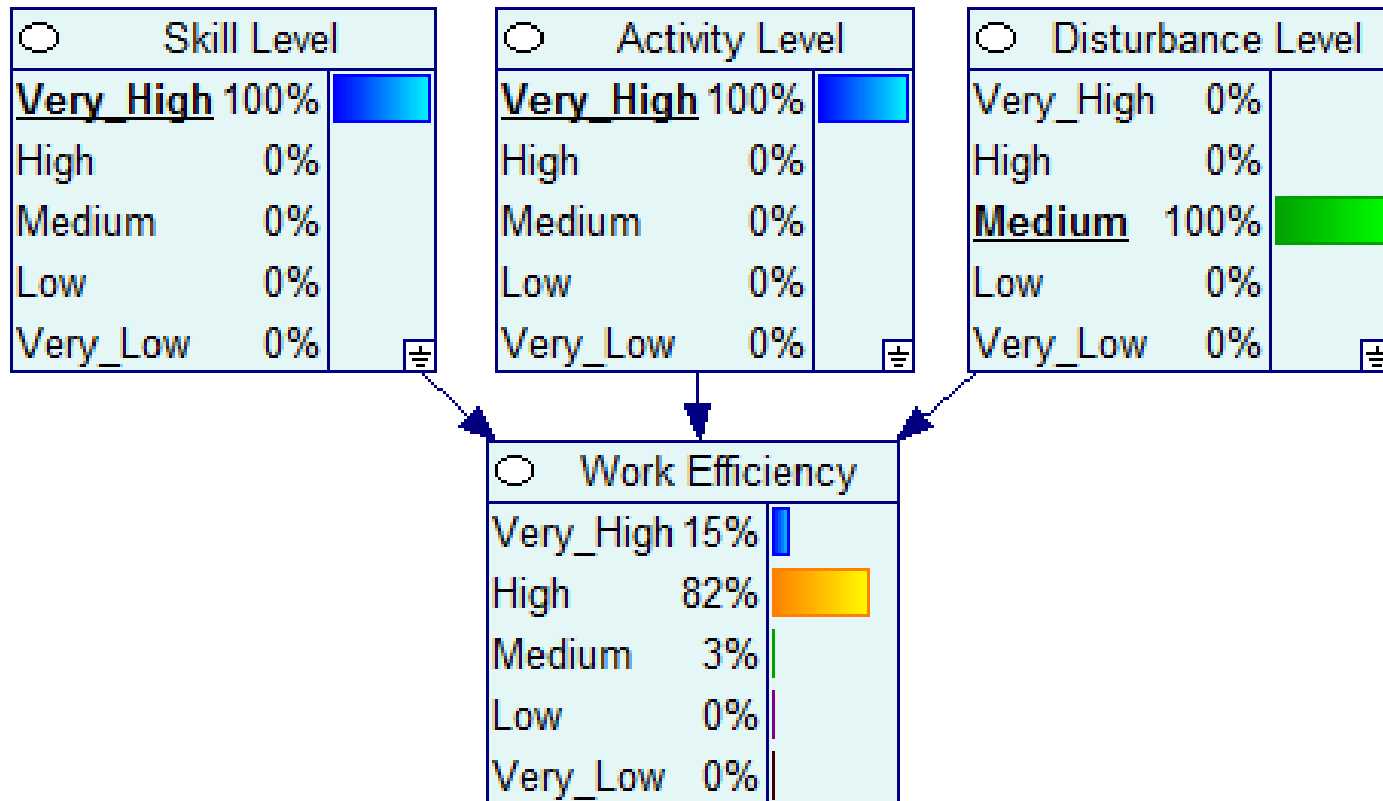   - Parameters lack clear interpretations ➔ Hampers assignment
    - Application requires technical insight ➔ Use inefficient

- **Our contribution for alleviating efforts of the expert**
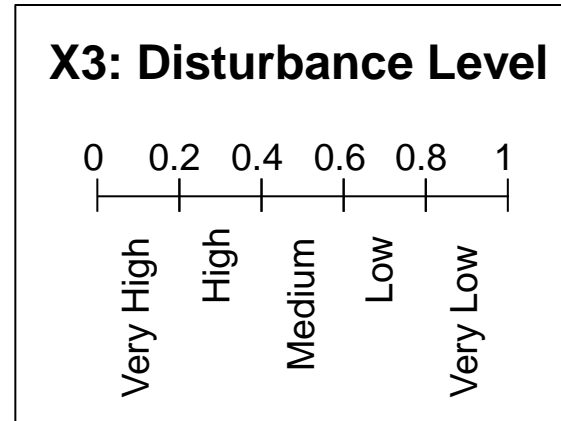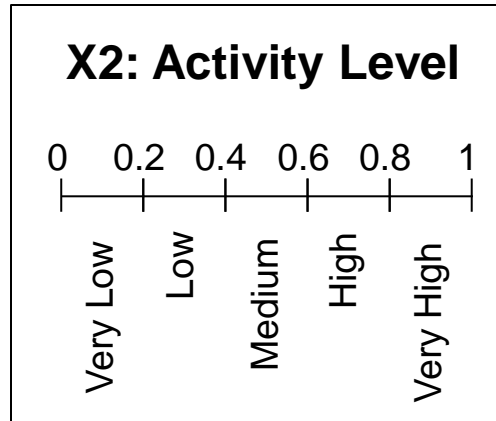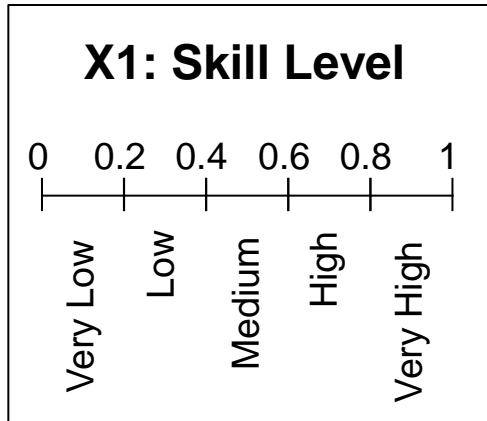   - Interpretations that facilitate determination of parameters
    - Guidelines for efficient use of RNM
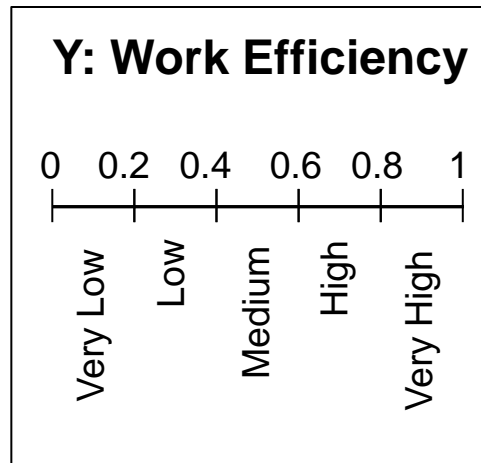
Ranked Nodes (Fenton, Neil, and Caballero, 2007)

Represent by ordinal scales continuous quantities
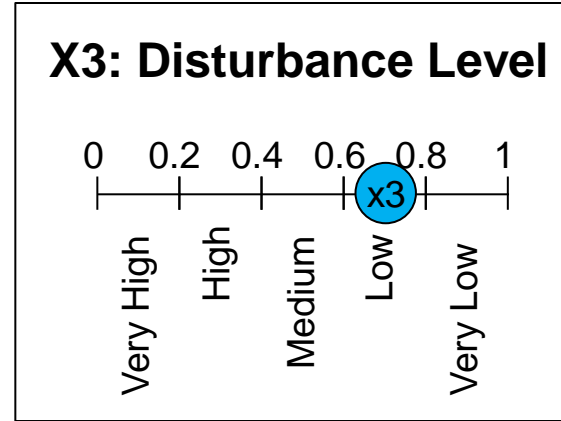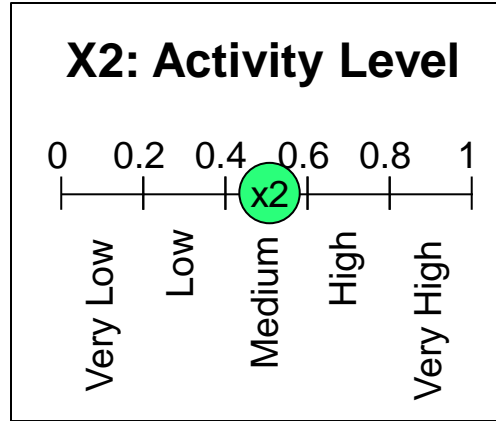that lack a well-established interval scale

Ranked Nodes Method (RNM) (Fenton, Neil, and Caballero, 2007)

Ranked Nodes Method (RNM) (Fenton, Neil, and Caballero, 2007)

X1: Skill Level

0   0.2   0.4   0.6   0.8   1

x1

Very Low | Low | Medium | High | Very High

X2: Activity Level

0   0.2   0.4   0.6   0.8   1

x2

Very Low | Low | Medium | High | Very High

X3: Disturbance Level

0   0.2   0.4   0.6   0.8   1

x3

Very High | High | Medium | Low | Very Low

Parameters to be elicited

- Aggregation function F

- Weights of parent nodes
  w1 w2 w3

- Uncertainty parameter σ

Y: Work Efficiency

μ

0   0.2   0.4   0.6   0.8   1

Very Low | Low | Medium | High | Very High

$TNormal(\mu, \sigma, 0, 1),$

$\mu = F(x1, x2, x3, w1, w2, w3)$

P(Y=Low | X1=Low, X2=Medium, X3=Low)

Challenges recognized with RNM

1. Parameters lack interpretations ➔ Expert must determine values by trial and error ➔ Slow and difficult!

2. Application to nodes with interval scales: Ignorant user may form ordinal scales that prevent construction of sensible CPTs

RNM and nodes with interval scales:
New approach (Laitila and Virtanen, 2016)
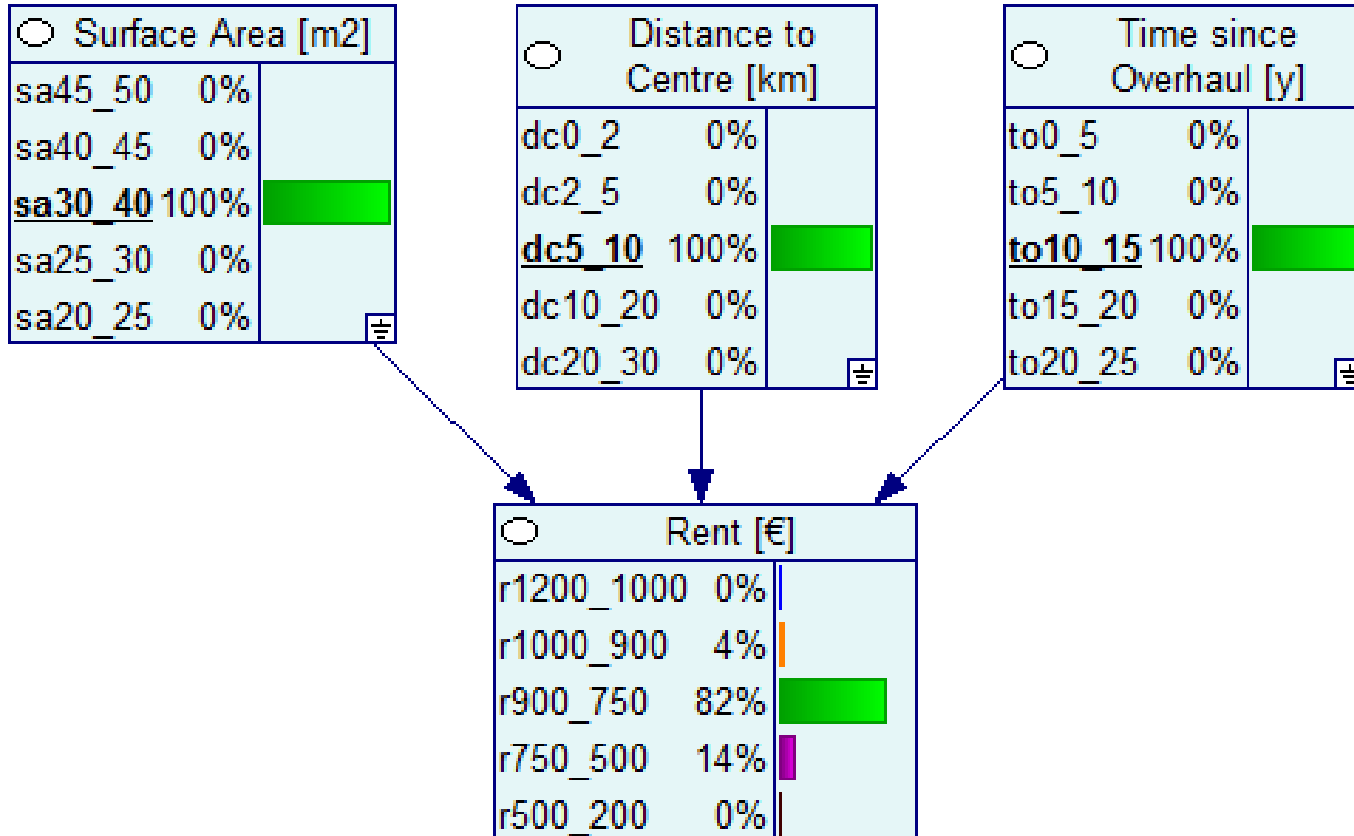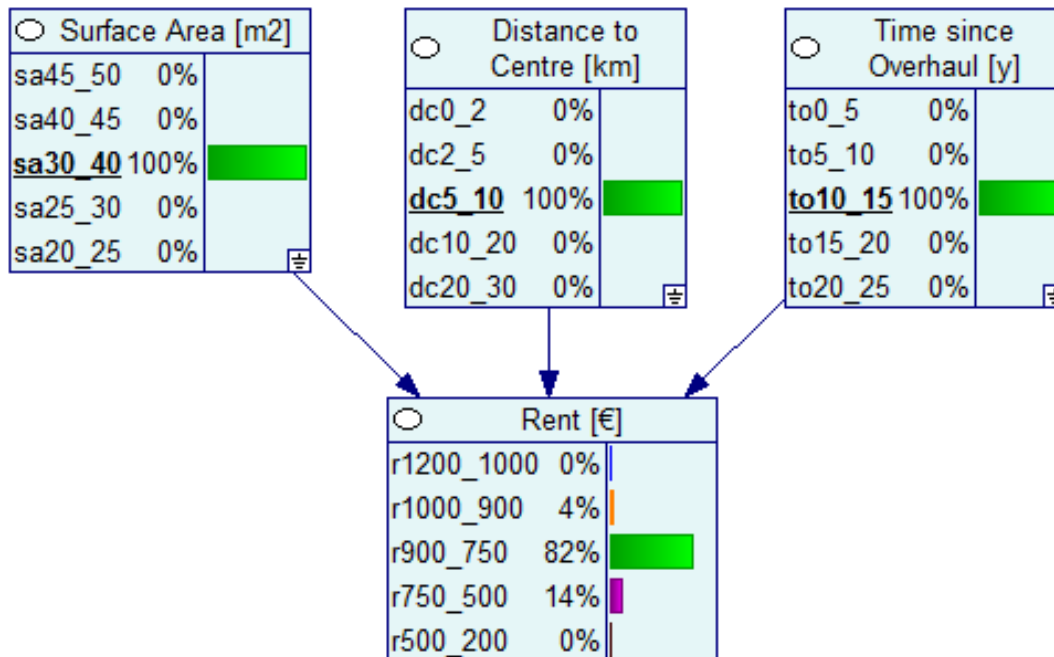
Formation of suitable ordinal scales

– Divide interval scales freely into equal amount of subintervals

– Ask the expert about the mode of child node in scenarios corresponding to equal ordinal states of parent nodes

➔ Update discretizations accordingly

○ Surface Area [m2]	
sa45_50	0%
sa40_45	0%
sa30_40	**100%**
sa25_30	0%
sa20_25	0%

○ Distance to Centre [km]	
dc0_2	0%
dc2_5	0%
dc5_10	**100%**
dc10_20	0%
dc20_30	0%

○ Time since Overhaul [y]	
to0_5	0%
to5_10	0%
to10_15	**100%**
to15_20	0%
to20_25	0%

"What is the most likely rent for a 40 m2 apartment that is 5 km from the centre and has 10 years since overhaul?"
"I'd say its 900 €."

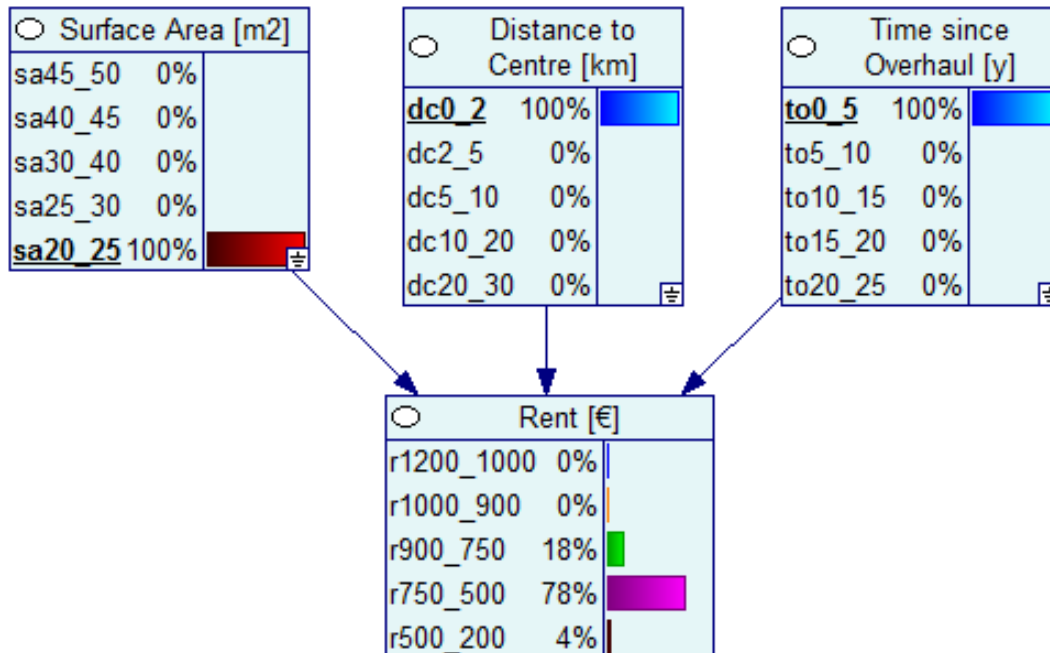| ○ Rent [€]	
r1200_1000	0%
r1000_900	4%
r900_750	82%
r750_500	14%
r500_200	0%

RNM and nodes with interval scales:
New approach (Laitila and Virtanen, 2016)

Determination of aggregation function F and weights w1,...,wn

– Ask the expert about the mode of child node in scenarios corresponding to extreme ordinal states of parent nodes

➔ F and w1,...wn determined based on interpretations derived for weights

○ Surface Area [m2]	
sa45_50	0%
sa40_45	0%
sa30_40	0%
sa25_30	0%
sa20_25	**100%**

○ Distance to Centre [km]	
dc0_2	**100%**
dc2_5	0%
dc5_10	0%
dc10_20	0%
dc20_30	0%

○ Time since Overhaul [y]	
to0_5	**100%**
to5_10	0%
to10_15	0%
to15_20	0%
to20_25	0%

○ Rent [€]	
r1200_1000	0%
r1000_900	0%
r900_750	18%
r750_500	78%
r500_200	4%

"What is the most likely rent for a 20 m2 apartment that is right in the centre and has just been renovated?"
"I'd say its 600 €."

Conclusion

- Parametric methods ease up construction of CPTs for BNs by expert elicitation

- New approach facilitates use of RNM

  ➔ Further relief to expert elicitation

  – Currently applied in a case study concerning performance of air surveillance network

  – Applicable to BNs and Influence Diagrams
    ➔ Supports decision making under uncertainty

- Future research

  – Human experiment: new approach vs. direct parameter estimation

  – Generalisation of the approach to nodes without interval scales

References

- P. Laitila and K. Virtanen, "Improving Construction of Conditional Probability Tables for Ranked Nodes in Bayesian Networks," IEEE Transactions on Knowledge and Data Engineering, vol. 28, no. 7, pp. 1691–1705, 2016

- N. Fenton, M. Neil, and J. Caballero, "Using Ranked Nodes to Model Qualitative Judgments in Bayesian Networks," IEEE Transactions on Knowledge and Data Engineering, vol. 19, no. 10, pp. 1420–1432, 2007

- S. Russell and P. Norvig, Artificial Intelligence: A Modern Approach. Prentice Hall, 2003