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Spatial multi-attribute decision analysis with attribute specific spatial weights and incomplete preference information

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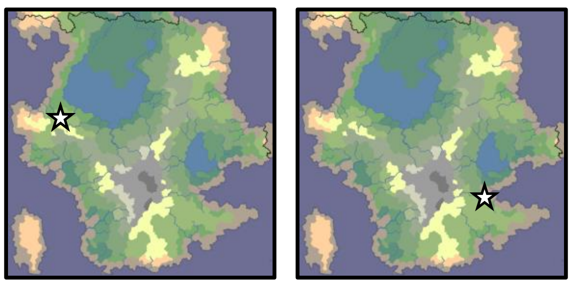
Spatial decision analysis

Typically used in site selection problems

- E.g., what is the best site for a rescue helicopter base?



Alternative 1 Alternative 2

Conventional MCDA in spatial problems

- Decision alternatives are judged directly based on properties of site candidates
- E.g., additive value representation

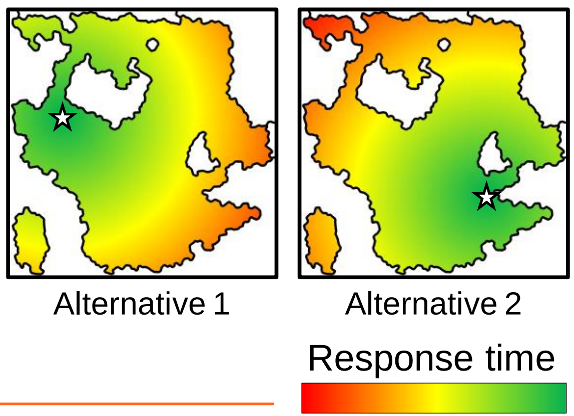
$$v(z) = \sum_{j=1}^{m} b_j v_j(z_j)$$

Spatial value function

Simon, Kirkwood, Keller 2014
Harju, Liesiö, Virtanen 2019

Decision alternatives judged based on their impact across the region

- Treated as functions that assign a consequence to each location



Spatial value representation

- Locations $s \in S$, subregions $S' \subseteq S$
- Consequences $\mathrm{c} \in C$
- Decision alternatives $\mathrm{z} \in Z = C^S$

$$V(z) = \int v(z(s))d\alpha(s)$$

- *Spatial value function* $V: Z \to \mathbb{R}$
- *Consequence value function* $v: C \to \mathbb{R}$
- *Spatial weighting measure* $\alpha: 2^S \to [0,1]$

Multiple attributes

E.g., additive multi-attribute consequence value function

$$v(c) = \sum_{i=1}^{m} b_j v_j(c_j)$$

$$V(z) = \int \sum_{i=1}^{m} b_j v_j\left(z_j(s)\right) d\alpha(s)$$

Are the underlying preference assumptions still valid?

- “The same consequence value function can be applied everywhere”
- What if the relative importance of attributes varies across the region?

Attribute-specific spatial weighting

$$V(z) = \sum_{j=1}^{m} b_j V_j(z_j) = \sum_{j=1}^{m} b_j \int v_j\left(z_j(s)\right) d\alpha_j(s)$$

Attribute-specific spatial value functions V_j

- Aggregated based on attribute weights b_j

Attribute-specific consequence value functions v_j

- Gives the value of consequences with respect to the attribute

Attribute-specific weighting measures α_j

- Gives the importance of subregions with respect to the attribute

Incomplete preference information

Preferences represented by $V(z) = \sum_{j=1}^{m} b_j \int v_j\left(z_j(s)\right) d\alpha_j(s)$

- Consequence value functions v_j assumed to be known
- Spatial weightings α_j (and attribute weights b_j) not known

Incomplete weighting information

- Preference statements provided by the DM
- Feasible weightings compatible with all preference statements

Non-dominated alternatives

- Based on the set of all feasible weightings
- More preference information → fewer non-dominated alternatives

Spatial preference statements

Technically comparisons of pairs of alternatives

- The alternatives are identical in all but one attribute

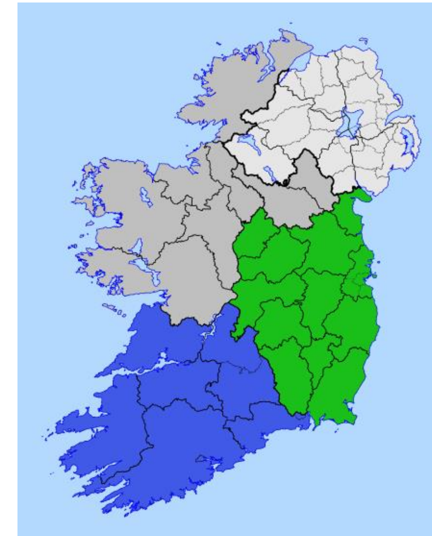
"S^1 is more important than S^2 with respect to j":

$$z^1 \succcurlyeq z^2 \Leftrightarrow V(z^1) \geq V(z^2) \Leftrightarrow \cdots \Leftrightarrow \alpha_j(S^1) \geq \alpha_j(S^2)$$

Magnitudes of importance

"S^1 at least twice as important as S^2"

- Direct statement of importance problematic
  - *Is the statement unambiguous?*
  - *Are there biases?*
- Indirect statement also possible
  - *"S^1 can be split into two parts, each of them at least as important as S^2"*
  - *The exact partition is irrelevant*



Can be generalized to "S^1 at least $\frac{k_1}{k_2}$ times as important as S^2"

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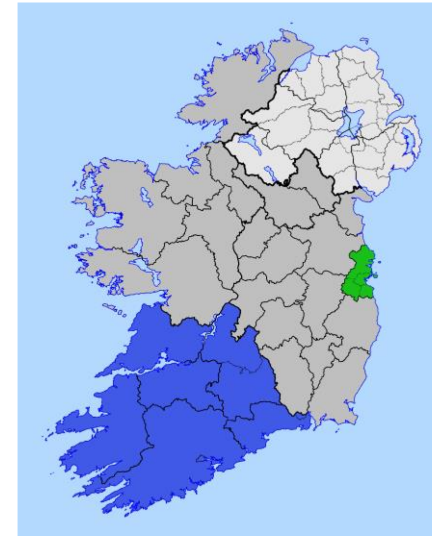

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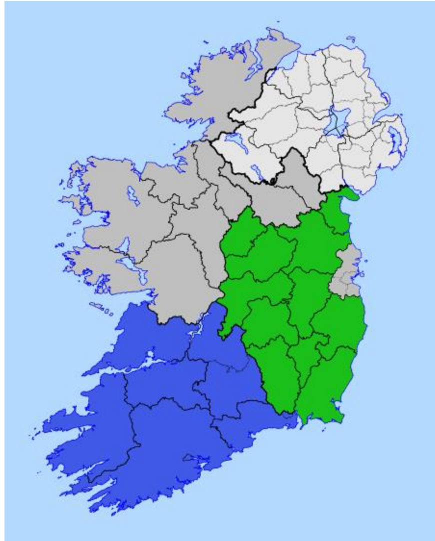

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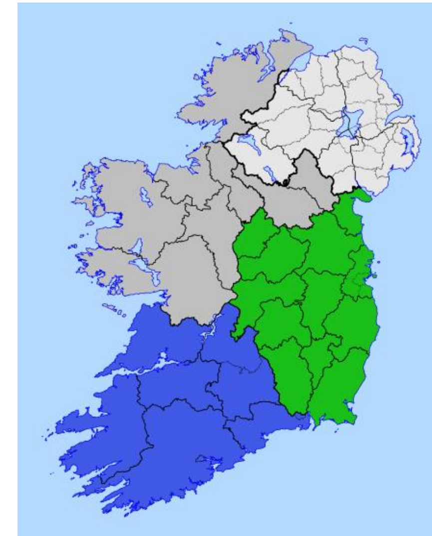


$$\alpha_j(S^{1a}) \geq \alpha_j(S^2),$$
$$\alpha_j(S^{1b}) \geq \alpha_j(S^2)$$
$$\Rightarrow \alpha_j(S^1) \geq 2\alpha_j(S^2)$$

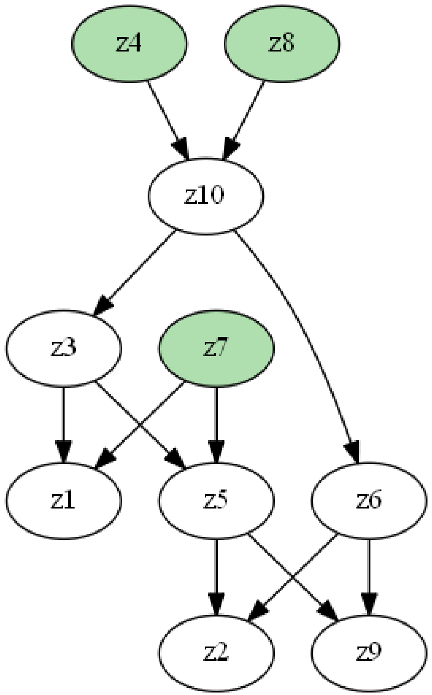
Non-dominated alternatives

Decision alternative z dominates z' if

- $V(z) \geq V(z')$ for every feasible weighting
- $V(z) > V(z')$ for some feasible weighting
- Identified by determining the bounds of the difference in value of z and z'

Decision alternative z is non-dominated if no other alternative dominates it

- Identified by performing all necessary pairwise comparisons between alternatives



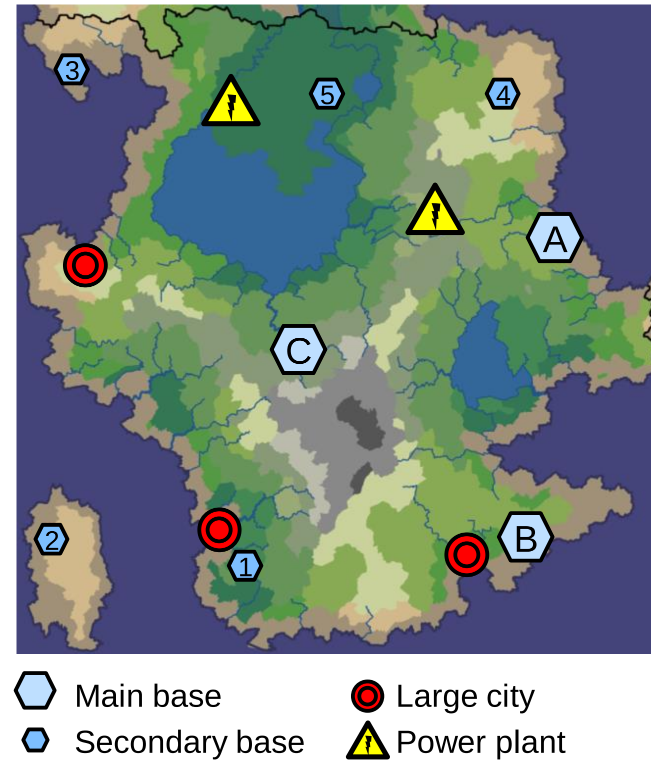
Illustrative example

Application related to air defense

- Select positions for air bases (2 main, 3 secondary) from a list of candidates
- 30 possible combinations, or decision alternatives

Two attributes

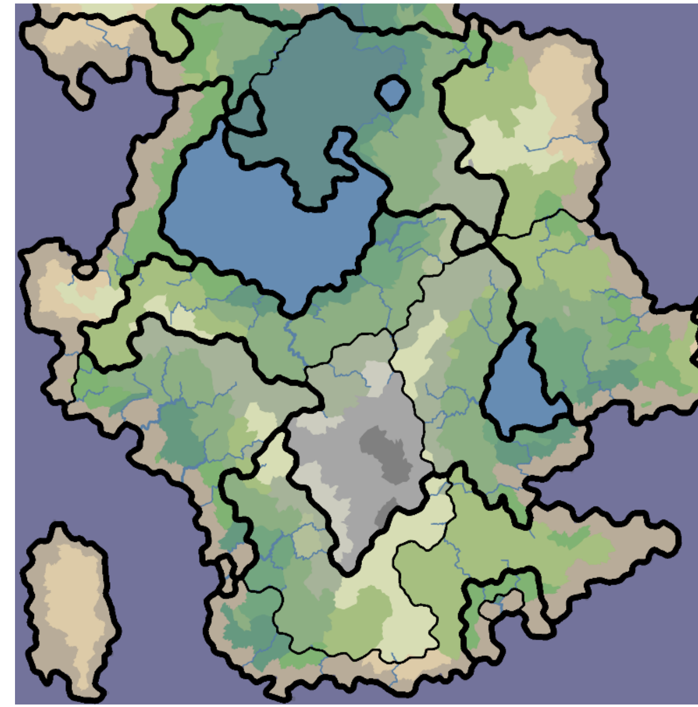
- Capability to withstand prolonged attacks
- Capability to intercept the initial attack



Capability to withstand prolonged attacks

Hierarchical preference information

- Upper level: Region split into 9 areas
- Lower level: Each area split into 1-3 subareas



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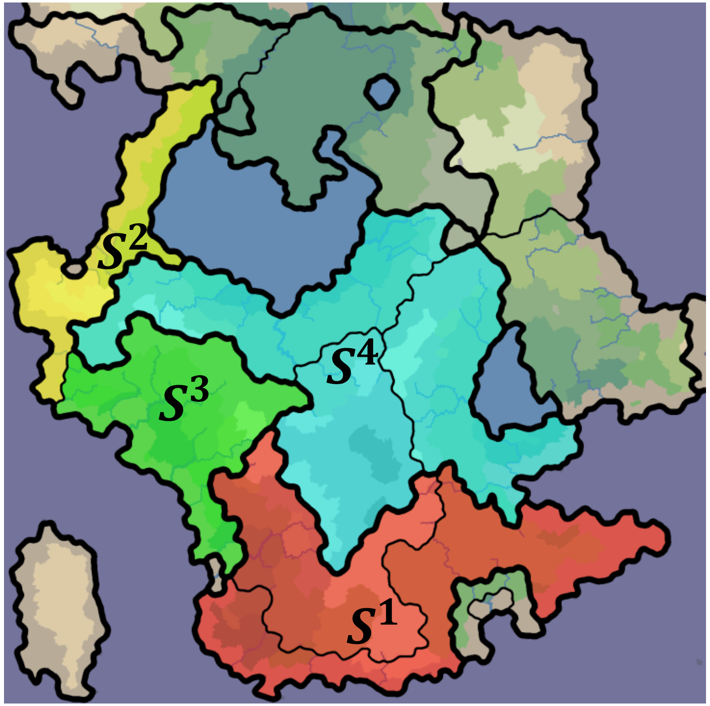
$$\frac{2}{3}\alpha_j(S^1) \leq \alpha_j(S^2) + \alpha_j(S^3) \leq \alpha_j(S^1)$$



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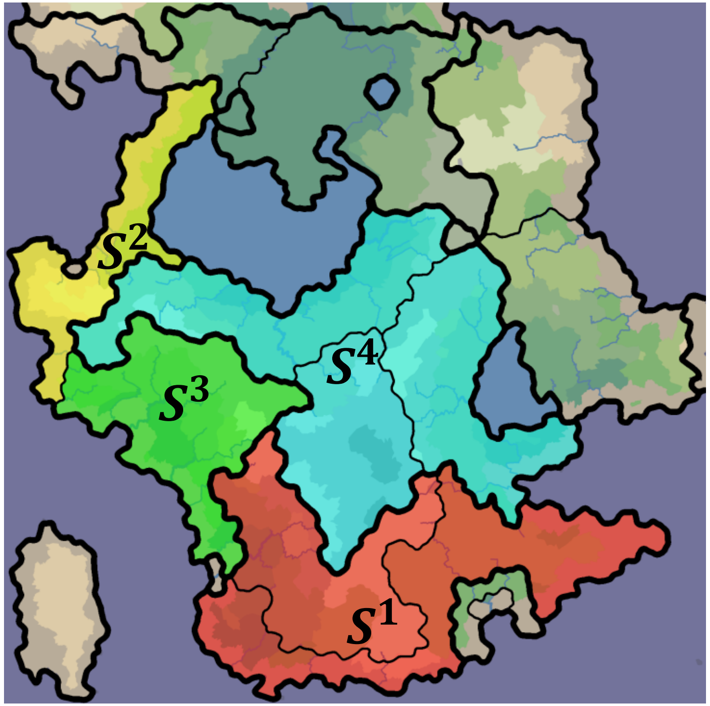
$$\alpha_j(S^4) \le \alpha_j(S^2) + \alpha_j(S^3) \le \frac{3}{2}\alpha_j(S^4)$$



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$$\alpha_j(S^3) \leq \alpha_j(S^2) \leq \frac{3}{2}\alpha_j(S^3)$$

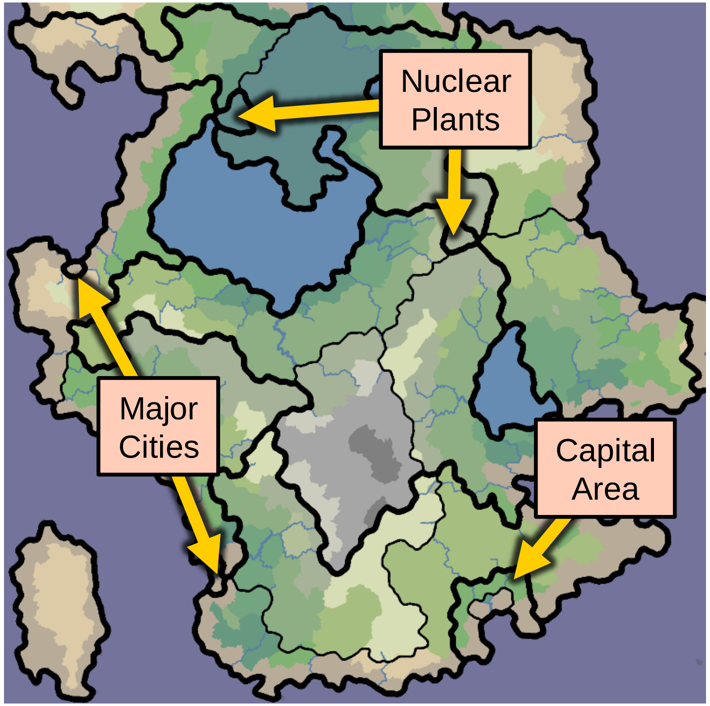
Capability to intercept the initial attack

The attack is assumed to originate from the west or south

- Bases close to these coasts offer the best protection

High value targets particularly important

- Capital area, major cities, nuclear power plants
- Likely to be focus of the initial attack



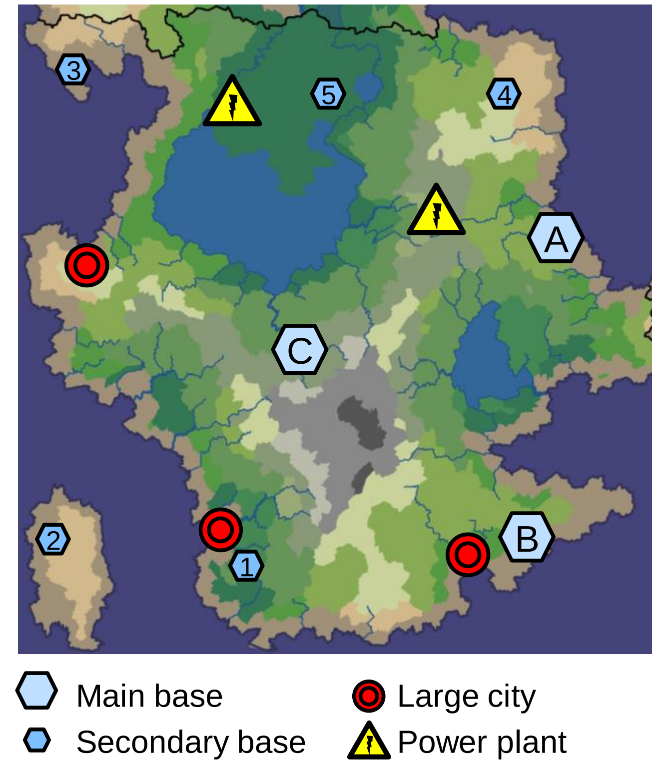
Results

5 non-dominated configurations

- BC123, BC134, BC135, BC234, BC235
- Main bases at positions B and C
- Secondary bases at 3, and 1 or 2
- Final secondary base at any candidate

Selecting one alternative

- Elicit additional preference information
- Examine the 5 non-dominated alternatives directly



Conclusion

Spatial multi-attribute additive value function

- Attribute-specific spatial weighting
- Incomplete preference information

Behavioral considerations

- Elicitation of spatial weightings
- Target for potential future research

References

- *Simon, J., Kirkwood, C.W., and Keller, L.R., 2014. Decision analysis with geographically varying outcomes: Preference models and illustrative applications. Operations Research, 62(1)*
- *Harju, M., Liesiö, J., and Virtanen, K., 2019. Spatial multi-attribute decision analysis: Axiomatic foundations and incomplete preference information. European Journal of Operational Research, 275(1)*