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Gradient Learning for a Buyer Seller Game

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New technology allows the sellers

to collect, process and
deliver sales and customer data

to plan intelligent pricing strategies

to change prices frequently at low cost

Pricing strategies

Posted price mechanisms:

"*take-it-or-leave-it*" prices

Price-discovery mechanisms:

prices determined by bidding processes, e.g., *auctions*

Mechanism design: Optimization and game theory tools for pricing goods

- Price discrimination; e.g., nonlinear pricing
- Combinatorial auctions

Applications

Nonlinear pricing in the design of electricity tariffs (Wilson, Räsänen et. al, in the 1990's)

Dynamic pricing policies in *brick-and-mortar stores* (in the 2000's)

Combinatorial Auctions (FCC's Narrow Band Auction 1994, CAs in procurement of logistic services 1993)

Research questions

How to exploit the vast resources of online data to improve the shopping process

Theoretical work to model descriptively, predictively and normatively some aspects of the process to enable *computational advantages*

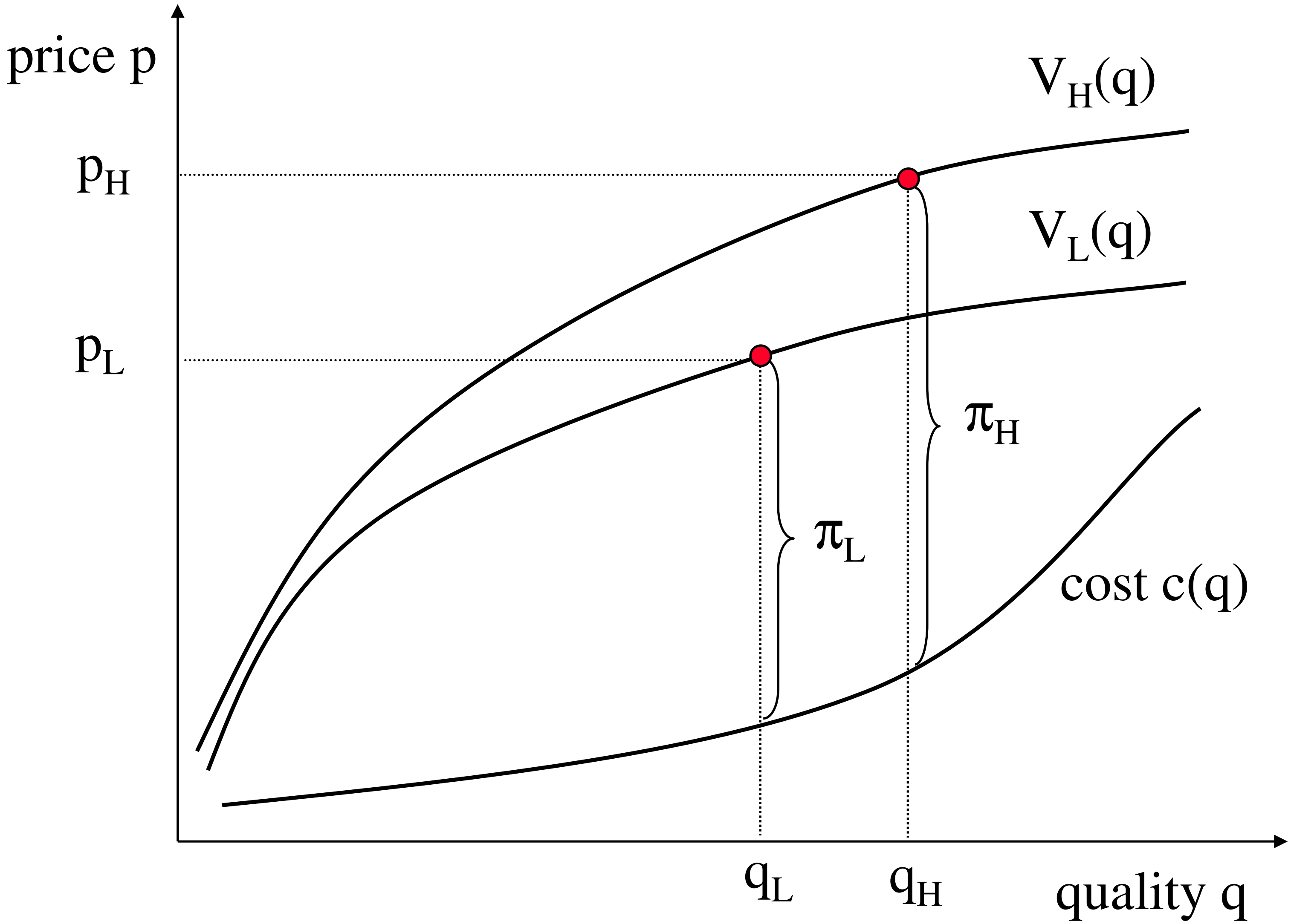
The nonlinear pricing model (Spence, 1980)

Buyers: low type (L) and high type (H)

$$U_i(q, p) = V_i(q) - p \, , \; i = L, H$$

Single-crossing condition: $V'_H(q) > V'_L(q)$

Seller: $\pi(q, p) = f_L\underbrace{[p_L - c(q_L)]}_{\pi_L} + f_H\underbrace{[p_H - c(q_H)]}_{\pi_H}$


price p
$V_H(q)$
p_H
$V_L(q)$
p_L
π_H
π_L
cost c(q)
q_L
q_H
quality q

Optimization problem

Maximize $\pi(q,p)$ under constraints:

Incentive Compatibility (IC)

$$V_i(q_i) - p_i \geq V_i(q_k) - p_k,\ k \neq i$$

Individual Rationality (IR)

$$V_i(q_i) - p_i \geq 0,\ i = L, H$$

Optimality conditions

In a typical case, it holds that $q^*_L > 0$ and

$$f_L[V'_L(q^*_L) - c'(q^*_L)] = f_H[V'_H(q^*_L) - V'_L(q^*_L)]$$

$$V'_H(q^*_H) = c'(q^*_H)$$

$$p^*_L = V_L(q^*_L)$$

$$p^*_H = p^*_L + V_H(q^*_H) - V_L(q^*_L)$$

Continuous learning dynamics

Learning path $x(t) = [q_L(t), q_H(t), p_L(t), p_H(t)]$, $t \geq 0$, $x(0) = x_0$, defines the problem solution at time t. Update defined by differential equations.

Interpretation: Continuously repeated game

Dynamics of gradient method

Direction of best profit increase locally

$$\dot{x} = \nabla \pi(x) = \left(\frac{\partial \pi}{\partial q_L}, \frac{\partial \pi}{\partial q_H}, \frac{\partial \pi}{\partial p_L}, \frac{\partial \pi}{\partial p_H} \right)(x)$$

depends on x, i.e., which constraints are active.

Four regions: (a) no constraints active, (b) IR_L active, (c) IR_L and IC_H active, (d) IC_H active

Update equations

When (a) no constraints are active, we have

$$\nabla \pi(x) = \begin{bmatrix} -f_L c'(q_L) \\ -f_H c'(q_H) \\ f_L \\ f_H \end{bmatrix}$$

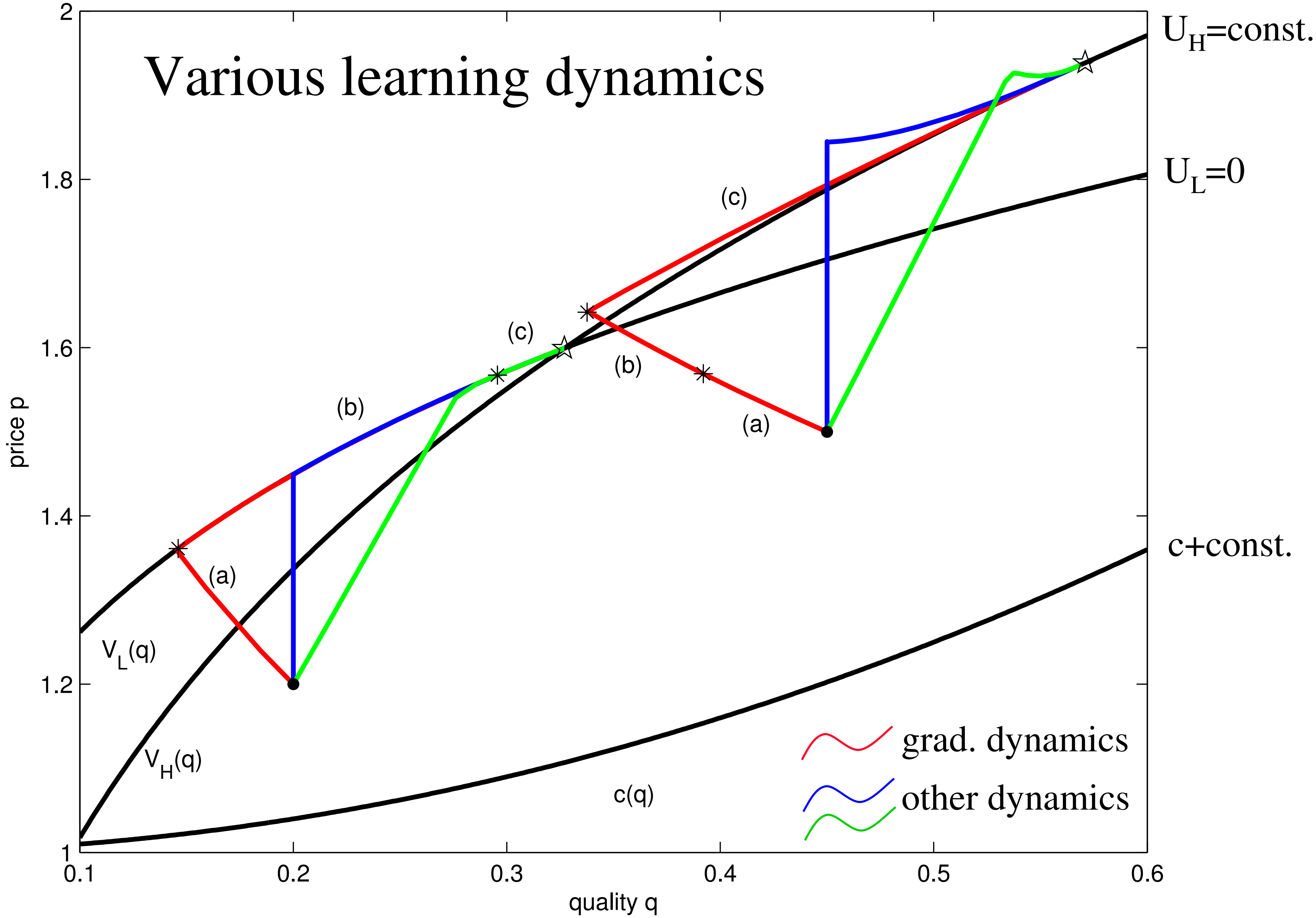
$$\pi(q, p) = f_L[p_L - c(q_L)] + f_H[p_H - c(q_H)]$$

Update equations (2)

When (b) IR_L is active, $p_L=V_L(q_L)$, we have

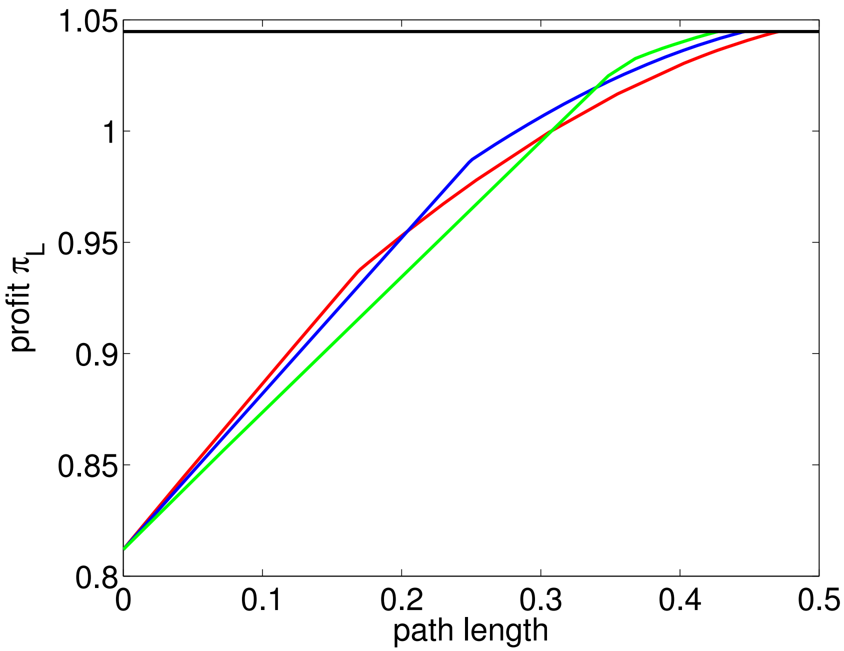
$$\nabla_R \pi(x) = \begin{bmatrix} \dot{q}_L \\ -f_H c'(q_H) \\ V_L'(q_L)\dot{q}_L \\ f_H \end{bmatrix} \qquad \frac{\partial \pi}{\partial p_L} = \dot{p}_L =$$

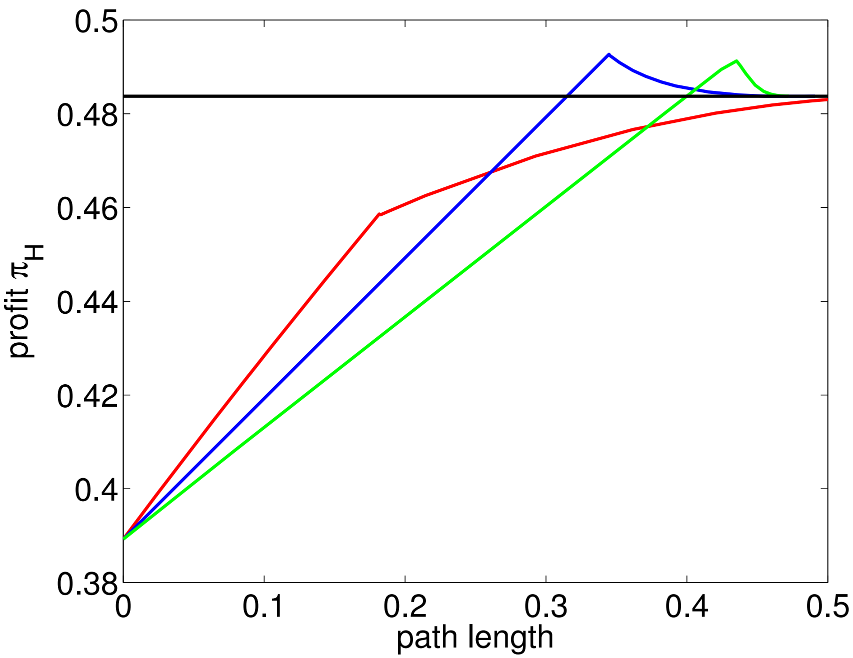
where $\dot{q}_L = \dfrac{\partial \pi}{\partial q_L} = f_L\left(V_L'(q_L) - c'(q_L)\right)$



Various learning dynamics
U_H=const.
U_L=0
c+const.
(a)
(b)
(c)
(c)
(b)
(c)
(a)
$V_L(q)$
$V_H(q)$
c(q)
grad. dynamics
other dynamics
price p
quality q
2
1.8
1.6
1.4
1.2
1
0.1
0.2
0.3
0.4
0.5
0.6

Profits from bundles L and H





Computation under limited information

In offline solution: extensive data collection is used to forecast the consumer types' utility functions in the beginning, (Wilson 1993, Räsänen et al. 1997)

In online solution: In our method (Ehtamo et al. 2008), only local demand data is used incrementally

Local information can be revealed by

- Performing price tests (take-it-or-leave-it prices) and/or marginal cost pricing in the vicinity of current bundles
- Using intelligent pricing strategies
- Performing market inquiries with pairwise comparisons, etc.

Discussion

Local methods perform well

In general many qualities and consumer classes without simplifying assumptions

In future, study of payoff landscapes, complexity issues, and computation

Literature

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