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AN ADJUSTMENT PROCESS IN A BUYER-SELLER GAME

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Abstract. *We give a new approach in modeling the incomplete information in a buyer-seller game. We assume that the seller does not know the buyer's utility function at all. Usually the problem is solved by determining the Bayesian Nash equilibrium of the game, where it is assumed that the buyer's utility function has only some parameters unknown to the seller, and the seller knows the distribution of these parameters. Instead, we assume that the seller faces different types of buyers repeatedly, and the seller learns the buyers' preferences. We present an adjustment process that leads to Bayesian Nash equilibrium by using linear tariffs to extract enough information. This approach motivates the Bayesian Nash equilibrium of the buyer-seller game.*

1 INTRODUCTION

Adjustment processes and learning models are used to analyze how and what equilibria are reached in economics. Cournot [6] studied an equilibrium resulting from an adjustment process in a strategic environment in 1838, Walras [21] studied the stability of a market with a tâtonnement process in 1874, and both Brown [4] and Robinson [19] studied how Nash equilibrium could be reached by fictitious play in 1951. Our study gives a motivation to Bayesian Nash equilibrium of a buyer-seller game, and studies the computational side of the problem. The adjustment process can be seen as a form of reinforcement learning, where the principal adjusts her tariff so that i) her information about the buyers' utilities increases, we call this experimenting, see Fudenberg and Levine [8]; and thus, ii) the potential profit increases. By potential profit, we mean that the principal could construct a tariff that gives better profit. But the principal keeps on experimenting until she has enough information to construct the optimal tariff. The process can be interpreted as a preplay of the Bayesian game, or as a gradual movement towards the equilibrium, like Walras' tâtonnement process.

Kalai and Lehrer [14] studied rational learning in infinitely repeated strategic form games. They assumed that the players have subjective prior beliefs, update their beliefs by Bayes rule, and maximize the expected payoff. The result was that the players learn to predict the future play of the game, and the players learn to play the Nash equilibrium. However, they assumed perfect monitoring, and that the prior beliefs contain grain of truth, that is, do not assign probability zero to events that might happen in the game. Additionally, they found that in games of incomplete information, the learning leads to Nash equilibrium instead of Bayes Nash equilibrium. Jordan [13] offered the same results with slightly different assumptions, for he assumed that the initial expectations are given by a common prior over player types.

Gilli's [10] model of imperfect monitoring is more related to the situation that we study. He relaxes the assumption that the players can observe all the moves of other players and nature. In these cases, he argues that conjectural equilibrium, see Hahn [12] and Battigalli [3], describes the situation better than Nash equilibrium. He also studies learning towards the conjectural equilibrium.

In the buyer-seller game, a principal, acting as a monopolist, offers a tariff, which defines the price of the good as a function of quantity, to an agent, who chooses the amount he wishes to consume, see Mussa and Rosen [17], Baron and Myerson [2], Maskin and Riley [15], Fudenberg and Tirole [9]. The principal's problem is to construct the optimal tariff that maximizes her expected profit without knowing the buyer's preferences. The problem can be seen as i) Akerlof's [1] adverse selection, where a buyer lacks the information of the quality of the car, see also Grossman and Hart [11] for principal-agent models; or ii) a problem of mechanism design, where a principal designs an optimal mechanism that makes the agents act as she wishes, see Conitzer and Sandholm [5] for computational side of mechanism design.

Suppose that there are n different types of agents, where each type corresponds a different utility function, and the principal knows the discrete probability distribution over these types. The principal's problem is to construct a tariff without knowing which of these types is chosen, or equally, construct a tariff that serves all the different types of agents. With n different types, it suffices to offer n price-amount bundles instead of the whole continuous tariff. We will use this bundle approach to solve the Bayesian Nash equilibrium of the game.

We simplify the problem and derive its necessary conditions of the problem by making standard assumptions on the form of agents' utility functions. These assumptions were presented by Mussa and Rosen [17], and the theoretical background comes from Mirrlees's [16] theory of optimal income taxation and Spence's [20] nonlinear prices. This simplification of the problem is presented in Fudenberg and Tirole [9] for analogical model of continuous types. With the necessary conditions, the principal can adjust her tariff in the ascent direction of the profit with the limited information she has.

We relax the assumption that the principal knows the possible types of agents. We assume that the principal does not know the utility functions at all. Instead of the previously described stage game, we study a repeated game, where the distribution of types is static. We will also relax the observability of the principal; after each stage, the principal will only observe the amount the agent chose. This is why Gilli's [10] imperfect monitoring game is more related to our model than Kalai and Lehrer's [14]. We assume that there is a population of different agents and the agents behave myopically; this enables the principal to extract enough information. We will show that the principal can reach the Bayesian Nash equilibrium of the stage game by offering linear tariffs that i) differentiate the different types; thus, the principal can eventually learn which responses correspond to which types of the agent; and ii) extract the slope of the agent's utility function. The use of linear tariffs was introduced in Ehtamo et al. [7].

With this information, the principal can evaluate the necessary conditions, which consists of slopes of utility functions, at a specific point. This local information is sufficient in determining the ascent direction of the profit, the direction of adjustment in reinforcement learning. By remembering the previous iterations, the principal learns the optimal amounts one by one.

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