An Adjustment Process in a Buyer-Seller Game

Kimmo Berg

Helsinki University of Technology

Systems Analysis Laboratory

Co-authors: Harri Ehtamo and Mitri Kitti

The Model

- One-dimensional monopoly pricing
- Monopoly price discriminates different buyers by offering a tariff specifying price $t(x)$ for amount x
- Different buyers are indexed by $i \in J = \{1, \ldots, n\}$ and their utilities are $U_i(x; t(x)) = V_i(x) - t(x)$, $i \in J$, where $V_i(x)$ is buyer i's surplus
- Buyers SELF-SELECT x from the tariff $t(x)$ by maximizing their utility $U_i(x)$

Optimization Problem

- Instead of designing a tariff, the monopoly designs price-amount bundles $(x;t) \triangleq (x_1,\ldots,x_n;t_1,\ldots,t_n)$
- The monopoly faces two kinds of constraints:

$$V_i(x_i) - t_i \geq U_i(0,0) = 0,\ \forall i \in I \qquad (IR)$$

$$V_i(x_i) - t_i \geq V_i(x_j) - t,\ \forall i,j \in I, j \neq i \qquad (IC)$$

- Individual Rationality (IR); buyers should prefer buying the product
- Incentive Compatibility (IC); buyers should prefer their own bundle the most

Optimization Problem (2)

- Monopoly's profit

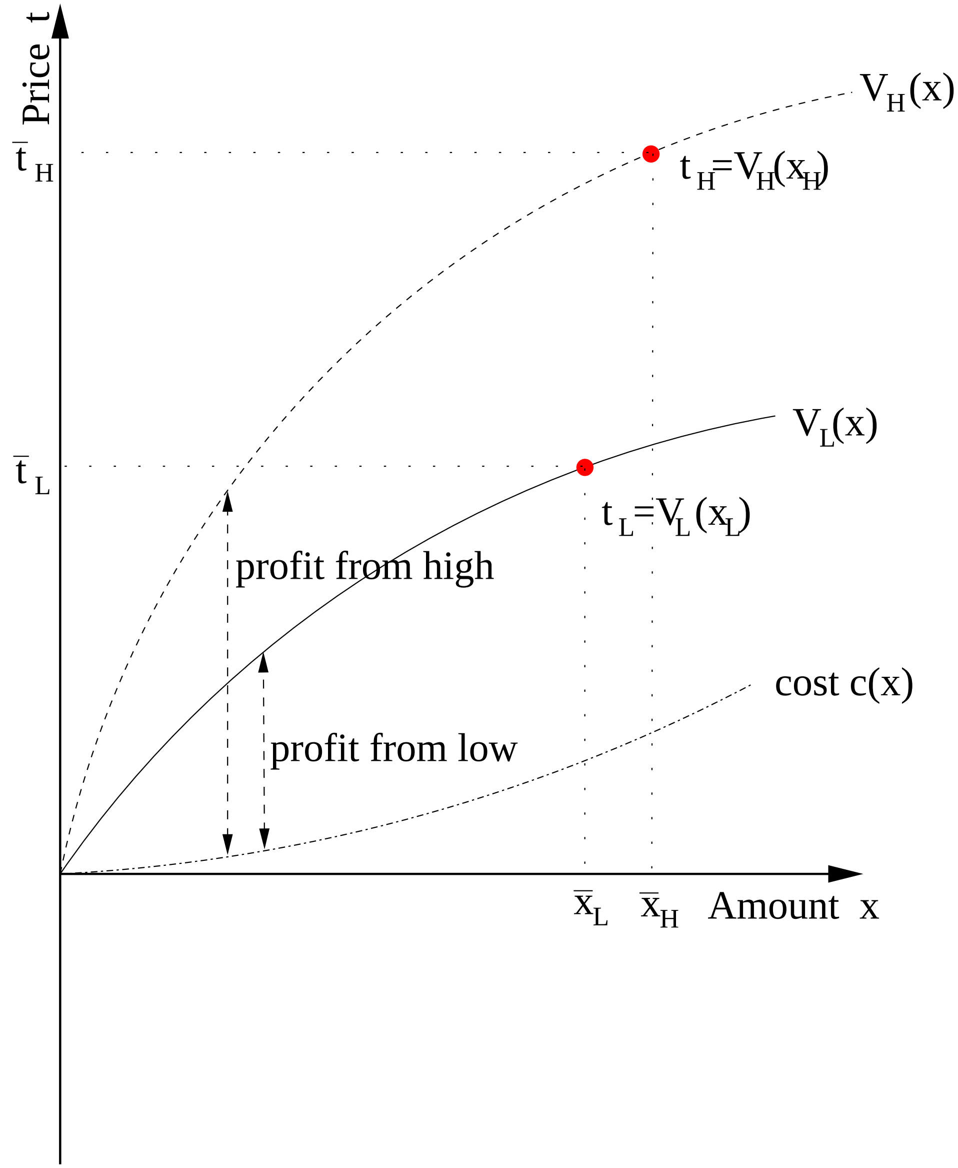
$$\pi(x,t) = \sum_{i=1}^{n} p_i[t_i - c(x_i)],$$

where p_i is the fraction (probability) of buyer i in the population and $c(x)$ is the cost of producing amount x

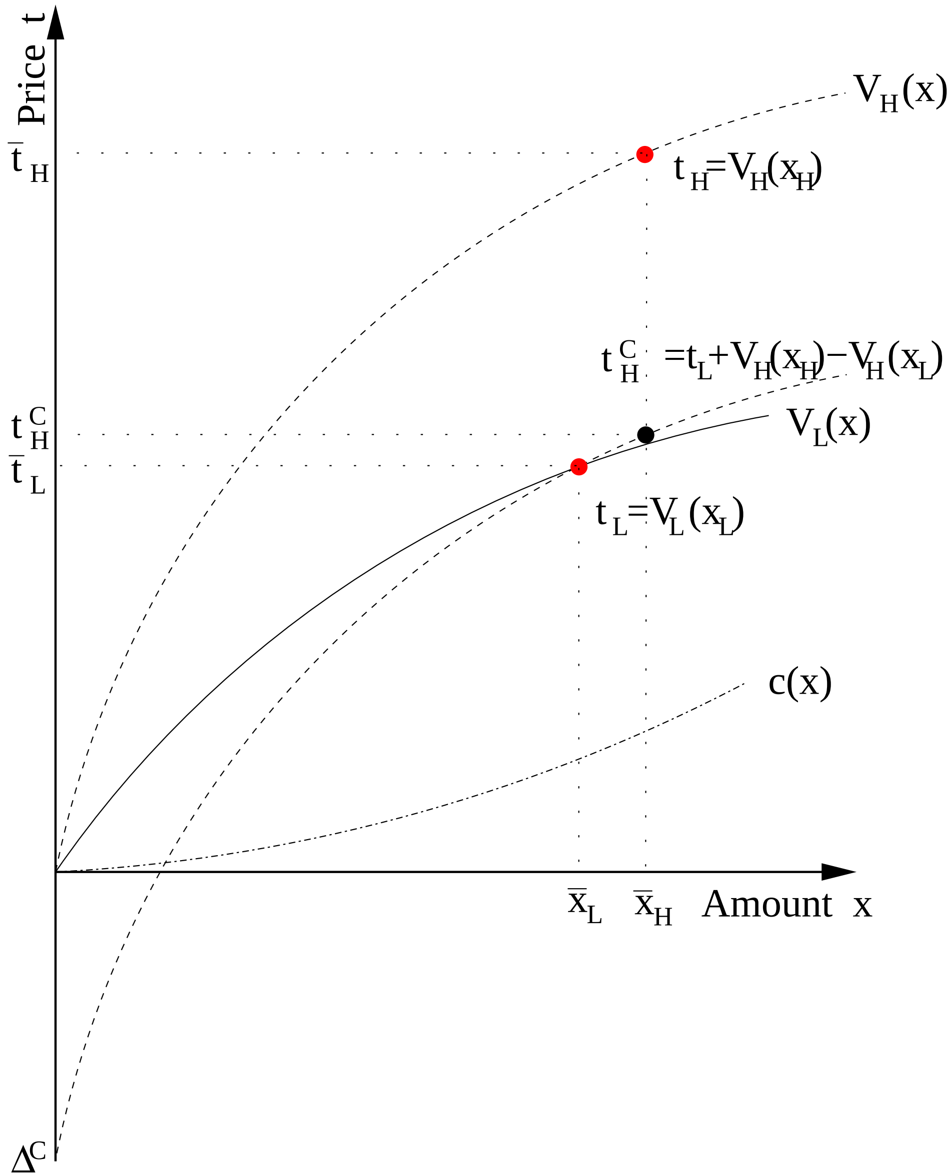
- Now, the maximization problem is

$$\begin{aligned} \max_{x,t} \quad & \pi(x,t) \\ s.t. \quad & V_i(x_i) - t_i \geq 0,\ \forall i \in I && (IR) \\ & V_i(x_i) - t_i \geq V_i(x_j) - t,\ \forall i,j \in I, j \neq i && (IC) \end{aligned}$$

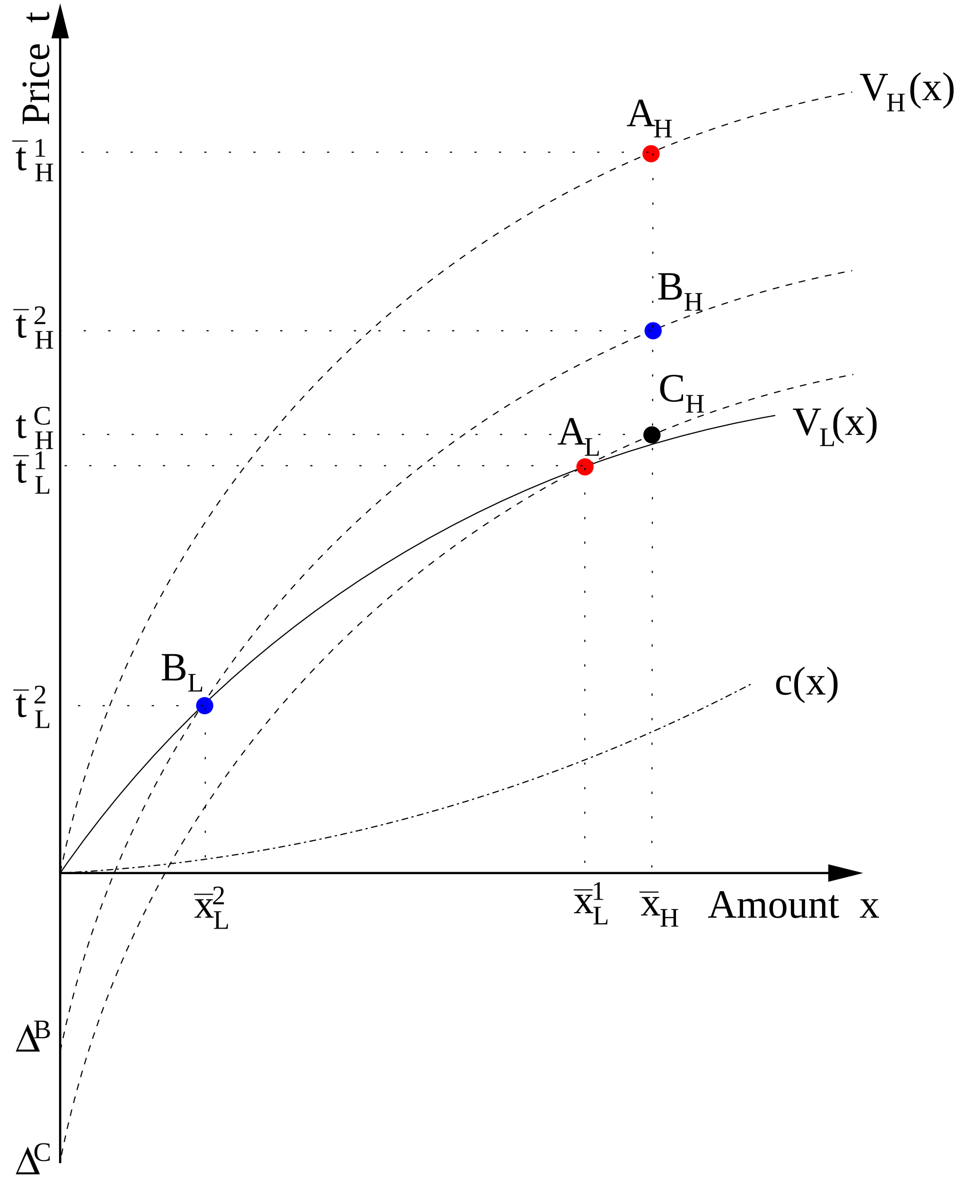
Example: First-degree pricing



Example: Second-degree pricing



Example: Second-degree pricing (2)



Example: Second-degree pricing (3)

- When designing the low bundle, the monopoly maximizes profit $V_L(x) - c(x)$ and minimizes Informational Rent $V_H(x) - V_L(x)$

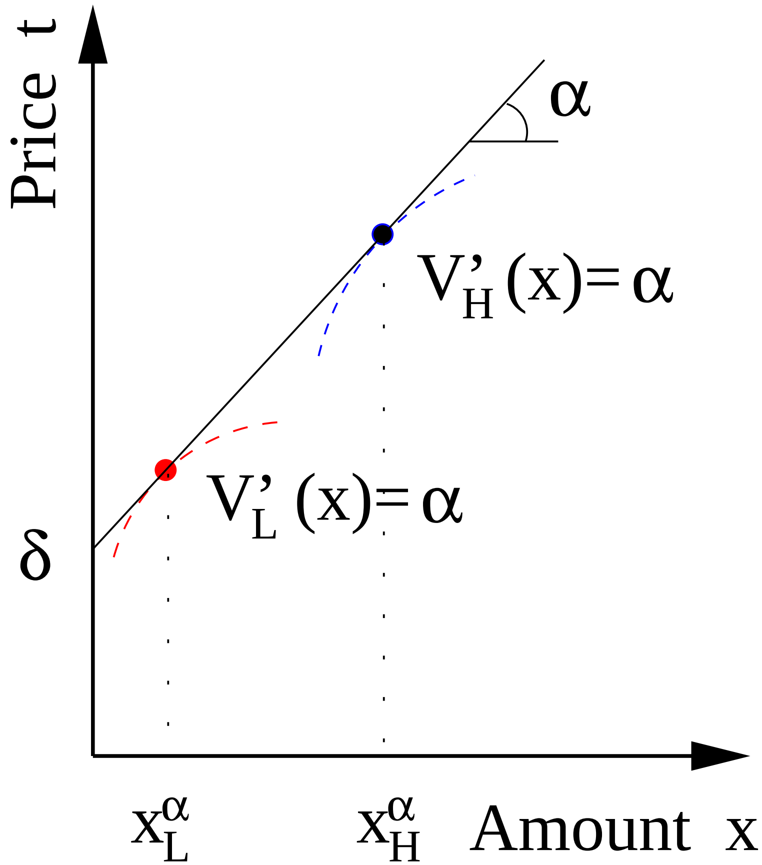
- Thus, optimality condition:
$p_L[V_L'(\overline{x}_L) - c'(\overline{x}_L)] = p_H[V_H'(\overline{x}_L) - V_L'(\overline{x}_L)]$

- And generally (with some assumptions):

$$p_i[V_i'(\overline{x}_i) - c'(\overline{x}_i)] = \sum_{k=i+1}^{n} p_k[V_{i+1}'(\overline{x}_i) - V_i'(\overline{x}_i)]$$

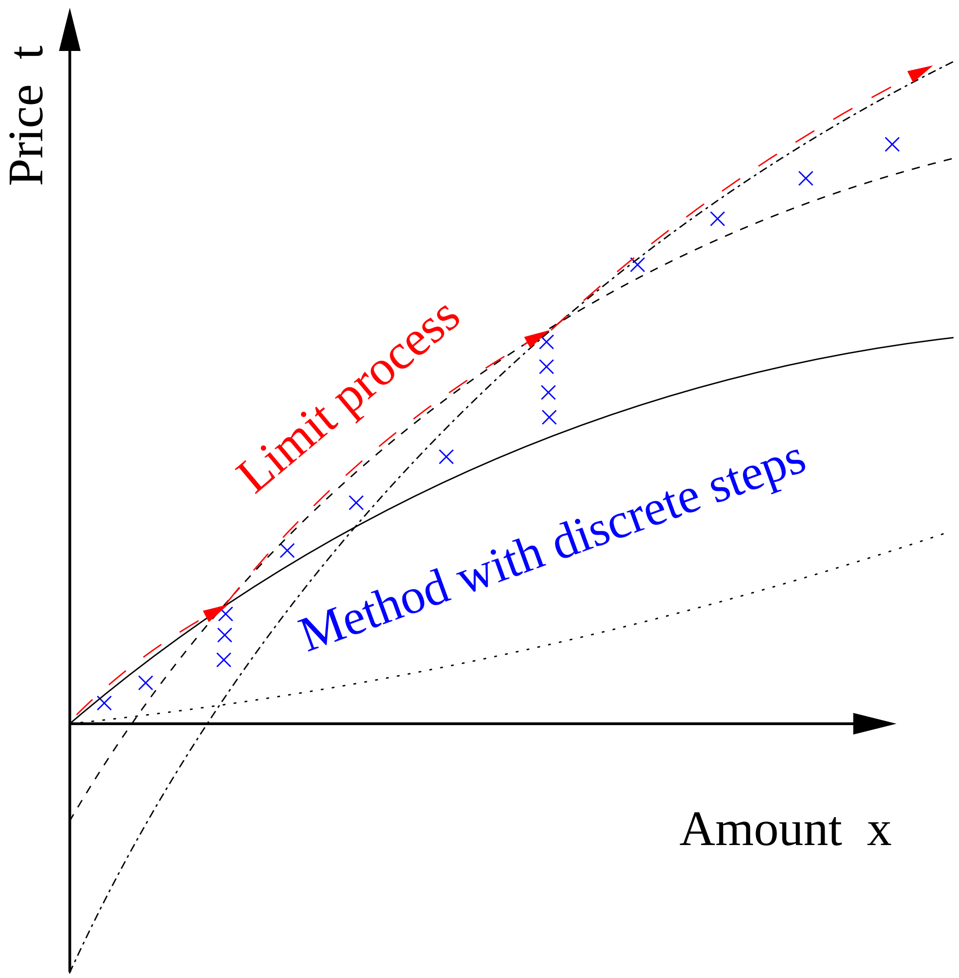
Extracting information

- Offer a linear tariff: $t(x) = \alpha x + \delta$



- Thus, optimality condition can be evaluated

Illustration of the method



Conclusions

- We suggest an iterative solution method
- Buyers' valuations $V_i(x)$ need not to be known
- Explains how equilibrium can be reached under incomplete information
- We motivate Bayesian Nash equilibrium