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Comparing topological and probabilistic node importance measures in Mandl's Swiss network

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Abstract

Disruptions in critical transportation infrastructures are a problem that can be approached by modelling transportation networksas graphs. This thesis aims to identify important nodes in transportation networks and comparing the different node importance measures calculated.

A small benchmark network, Mandl's Swiss transit network with 15 nodes and 21 edges, where the nodes represent bus stops and the edges represent road segments, is used as a case study. The network and the operationality of nodes, paths and terminal pairs are defined mathematically. To assess the importance of nodes in a network, topological node importance measures - degree centrality, closeness centrality and betweenness centrality - are calculated from the structure of the network, whereas probabilistic importance measures - risk achievement worth (RAW) and risk reduction worth (RRW) - incorporate the additional data of node disruption probabilities and origin-destination traffic demand. Additionally, a sensitivity analysis on RAW is performed.

The results indicate that different node importance measures identify many of the same nodes as important, even though notable differences can be observed their rankings. RAW and RRW exhibited the strongest positive correlation, supported well by the literature. Topological node importance measures correlated also positively, but not as strongly. When comparing topological and probabilistic measures to each other, much weaker correlation was observed, indicating that RAW and RRW are highly dependent on traffic flow amounts and node disruption probabilities, where as centrality measures are not. The sensitivity analysis indicated rather high robustness of RAW, as well as the sensitivity of RAW highly correlating with the value of RAW itself.

In conclusion, topological and probabilistic node importance measures provide complementary perspectives on transportation network vulnerability. In this case study, topological measures efficiently detect the most central and connective nodes in the network, whereas RAW and RRW, measured by the total change in traffic volume, give additional insight into the operational consequences of node disruptions and reveal the potential transportation hubs and traffic bottlenecks.

Keywords transportation network, node importance measures, topological, probabilistic, centrality, disruption probability

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Tiivistelmä

Häiriöt kriittisessä liikenneinfrastruktuurissa ovat ongelma, jota voidaan lähestyä mallintamalla liikenneverkkoja verkkoteorian avulla. Tämän diplomityön tavoitteena on identifioida tärkeät solmut liikenneverkossa ja vertailla eri tärkeysmittoja keskenään.

Tapaustutkimuksena käytetään pientä vertailuverkkoa, Mandlin sveitsiläistä liikenneverkkoa, jossa on 15 solmua ja 21 kaarta. Solmut edustavat linja-autopysäkkejä ja kaaret teitä tai katuja niiden välillä. Verkko, sekä solmujen, polkujen ja terminaaliparien toimivuus määritellään matemaattisesti. Solmujen tärkeyden arviointiin, lasketaan solmuille topologiset tärkeysmitat verkossa: astekeskeisyys, läheisyyskeskeisyys ja välillisyyskeskeisyys, jotka ovat laskettavissa verkon rakenteesta, sekä todennäköisyyspohajiset tärkeysmitat: riskinnousukerroin ja riskinalennuskerroin, jotka hyödyntävät solmujen vikaantumistodennäköisyyksiä sekä pysäkkien välisiä liikennemääriä. Lisäksi suoritetaan riskinnousukertoimen herkkyysanalyysi.

Tulokset osoittavat, että eri tärkeysmittojen perusteella samat solmut ovat tyypillisesti tärkeitä, mutta merkittäviä eroja on kuitenkin havaittavissa. Riskinnousukertoimella ja riskinalennuskertoimella oli voimakkain positiivinen korrelaatio. Topologiset tärkeysmitat korreloivat keskenään myös positiivisesti, mutta eivät niin voimakkaasti. Topologisten ja todennäköisyyspohjaisten tärkeysmittojen korrelaatio oli huomattavasti vähäisempää, mikä osoittaa että riskinnousukerroin ja riskinalennuskerroin ovat riippuvaisia liikennemääristä ja solmujen häiriintymistodennäköisyyksistä, kun taas keskeisyysmitat eivät ole. Herkkyysanalyysin valossa riskinnousukerroin on robusti muuttuja, ja sen herkkyys korreloi riskinnousukertoimen arvon itsensä kanssa.

Päätelmänä topologiset ja todennäköisyyspohjaiset solmujen tärkeysmitat tarjoavat täydentävät näkökulmat liikenneverkkojen haavoittuvuuteen. Tässä tapaustutkimuksessa topologiset tärkeysmitat löytävät tehokkaasti keskeiset ja yhdistävät solmut verkosta, kun taas riskinnousukerroin ja riskinalennuskerroin, joita mitataan kokonaisliikennemäärän muutoksen avulla, antavat lisätietoa solmujen vikaantumisten seuraamuksista verkon toimivuudessa ja paljastavat liikenteen solmukohtia ja pullonkauloja.

Avainsanat liikenneverkko, solmujen tärkeysmitat, topologinen, todennäköisyyspohjainen, keskeisyys, häiriötodennäköisyys

Preface

This thesis was completed as part of the Master's Degree Programme in Systems and Operations Research at Aalto University. The thesis compares topological and probabilistic node importance measures in Mandl's Swiss network.

I would like to express my deepest gratitude to my supervisor, Professor Ahti Salo, and to my advisor, Leevi Olander, for valuable guidance, feedback and support throughout the project of writing this thesis.

I would also like to thank faculty and staff of Aalto University for providing great learning environment, new friends and unforgettable student events.

Finally, I wish to thank my family and friends for supporting me and being part of my life.

Contents

1 Introduction

Transportation networks are important in enabling the connectivity of people and essential services, facilitating the mobility of energy, goods, and information, and allowing society to achieve goals of economic growth, energy sustainability, urban development, telecommunication and both local and global integration. Transportation networks consist of infrastructures that enable movement of transportation vehicles or units. Examples of transportation networks include road networks, urban transit networks consisting of railways, trams, and buses, air or maritime networks, utility networks such as power grids and pipelines, and information networks.

Due to society's high demand of transportation networks, they require regular maintenance and immediate attention in the case of disruptions. Disruptions in networks can force the use of alternative routes and connections, increase congestion, and decrease the predictability of flow, thus causing additional expenses and further increasing the probability of disruptions. These disruptions can be unplanned and unwanted events, such as natural disasters, for example, floods, earthquakes or landslides, infrastructure failures and intentional attacks [36]. As an example, the collapse of I-35W bridge over the Mississippi river in 2007 caused an interruption in the route of 140000 daily vehicles for 66 days [8], causing the estimated daily costs to users to be between 71000 and 220000 dollars. Planned disruptions can happen due to the need for maintenance or construction work, or managing arrangements of a large-scale event [36]. Disruptions can occur at different magnitudes, and their impacts on infrastructure performance may vary largely. Depending on factors such as delays, safety, financial costs of repairing infrastructure and temporary rerouting, and the number of passengers they affect, the protocol of managing disruptions should support prioritizing the maintenance the most critical and influential elements in the network.

In the case of public transport networks, disruptions tend to result in a decrease in the amount of enabled traffic, causing severe costs to the service provider. Thus, it is important from the viewpoint of both passengers and service providers to assess and examine the location, frequency, and passenger delay of potential disruptions in the network [34]. This has led to the development of different methods of determining the most important elements in the network. An example of this is vulnerability analysis of public transport networks by Cats et al. in 2014 [5].

Transportation networks comprise several components, services and links. In order to mitigate the effects of events hindering the level of service enabled by the network, it is often useful to model the network and to demonstrate the risks related to it. Graph theory is helpful at describing transportation networks in a compact form. Depending on the type of infrastructure, nodes can represent, for instance, train stations or power stations, whereas edges connecting the nodes can correspond to railway tracks, pipelines or power lines. The main benefits of graphs is that quantitative methods can be used to assess the network. If the network is spatial, such as public transit networks, vectors can be used to show the direction and scale [25]. The importance of transportation infrastructure that is represented by a node or an edge, can not always be uniquely determined. There are multiple topological metrics to assess the importance of nodes, such as closeness centrality and betweenness centrality. However, some nodes being more important than others in a topological sense, does not necessarily reflect their importance on the level of service enabled by a transportation infrastructure. Research has been done to develop different probabilistic importance measures to assess the impact on the level of service enabled by a transportation infrastructure. Measures such as risk achievement worth (RAW), risk reduction worth (RRW) have been used as importance measures in transport industry by Mahboob et al. [22], for instance.

This thesis helps to understand and examine the impacts of disruptions on the level of service enabled by a transportation network. Different topological and probabilistic metrics are used to measure the importance of nodes in Mandl's Swiss bus network. A key research hypothesis is that there are different outcomes resulting from different topological and probabilistic node importance measures.

2 Literature review

2.1 Transportation networks

2.1.1 Background

Transportation networks are spatial structures of transport systems. They consist of interconnected transportation infrastructure and connections that enable transportation between them. They enable the movement of passengers and freight, and the distribution of services, goods, utilities and information [26]. Many transportation networks are critical infrastructures that are necessary in order to achieve goals in areas such as economic and social development, energy sustainability, and societal security [35]. Transportation is considered one of the most important human activities worldwide, and its essential function in our society is to overcome spatial and geographical constraints.

There are four key components of transportation: networks, infrastructures, modes and flows [26]. Networks are systems of connected locations, representing the spatial and functional organization of transportation. Infrastructures are the physical components of the networks, such as railway tracks, highways, power lines and airports, which enable the mobility of transport modes. Modes represent the units that support the mobility of passengers and goods, such as vehicles. Flows are the movement of people, freight, or utilities in network, having origins, destinations and intermediary locations. [26]

The transportation infrastructures are prone to various types of disruptions. They can be affected by planned or unexpected disruptions ranging from catastrophic natural hazards and extreme weather conditions to terrorist attacks or a temporary infrastructure closure due to construction work or organization of a big event. Disruptions may vary in duration, scale, and impact, and increasing the resilience of transportation infrastructures against these disruptions has become an important topic of research that respond to the needs of public authorities [1].

Previous literature proposes that the impact of disruption can be assessed in terms of its impact on the level of service enabled by the network, which can be estimated based on, for example, expected increase travel costs compared to the baseline scenario without disruptions.[6] [5].

Concept of resilience has received a large amount of attention in the literature comprising transportation networks, because it is one of the primary objectives of transportation network operators. Freckleton et al. have defined transportation resilience system's ability preserve an acceptable level of service or to return to that level in a specified window of time. In other words, it is an ability to accommodate unexpected conditions, and to absorb the outcomes of disruptions. Resilience implies that transportation network operators should maintain the enabled level of service of a transportation system even when the system is disrupted [14].

2.1.2 Modelling networks as graphs

A systematic and modelled representation of transportation networks is helpful for predicting and reinforcing transportation infrastructures against potential disruptions. Representing networks as graphs offers the presentation of important information, such as connectivity, directionality, travel times, travel distances, and traffic flow. Formally, a graph is a set of vertices and edges. Vertices, that can also be called nodes, are conventionally represented by labeled geometrical points in the plane. Edges, or sometimes links, are connections, that are represented by unordered pairs distinct nodes in the set of vertices V [12]. For graphs, one can calculate different metrics for nodes and edges and apply algorithms to minimize, for example, travel time or travel costs. [13]

In existing studies, graph theory has been widely applied in the context of transportation networks. Casoetto et al. (2011) have researched the French electricity and gas network operator GRTSUEZ, and demonstrated the difficulties of commercializing reliable transmission capacities [4]. The study by Johansson et al. (2010) used graph theory to model interdependent transportation infrastructures in the context of vulnerability analysis by modelling fictional electrified railway network and the functional and geographical interdependencies between different networks [18].

The figures below illustrate simple graphs and how nodes and edges are connected to each other. Figure 1 shows the general structure of a graph and figure 2 illustrates the application of graphs in transportation networks.

2.2 Topological measures of node importance

Graph theory offers a framework for analyzing transportation networks in the context of topology. A graph that has many nodes with various degree of connectivity, and transportation modes passing through them, requires methods for measuring importance of each node based on how critical or central it is in the network. Centrality measures have widely been applied in the literature on transportation networks. Centrality measures are properties of a node, which measure how connected the node is to other nodes in the network. In general, a higher value of centrality indicates greater significance for transportation mode flow and the absence of a node with high

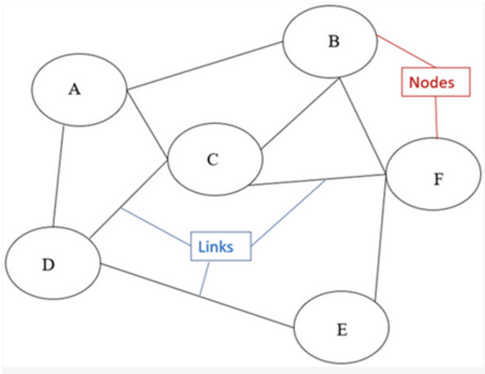

Figure 1: *An example of a simple undirected graph with nodes and edges (links) connecting the nodes.*

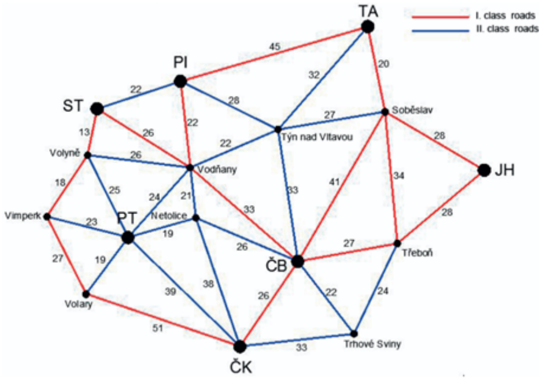

Figure 2: *The bus network in South Bohemian region as a graph with nodes as towns, edges as routes between towns, and a travel cost (distance) associated to each edge. [17]*

centrality is more important to the total performance of the network.

2.2.1 Degree centrality

The degree of a node is the number of adjacent nodes connected by edges. Degree centrality is the degree of a node normalized by the size of the network. High value of degree centrality indicates that the node is connected to a large number of other nodes in the network, assessing the importance of a node by the number of its direct connections. [10]

Land based physical transportation networks, such as bus and railway networks, have dimensional constraints, hence limiting the value of degree centrality [10]. This occurs especially in the case of planar graphs, where edges do not intersect each other. Such graph could represent a street network without bridges or underpasses, and nodes would represent intersections.

Degree centrality in urban transport networks has been researched by Wang and Fu in 2017, based on real data of Guangzhou bus network, where bus stops are considered as nodes [33]. The study observed that the majority of the nodes have degree of 2, which indicates that such nodes are intermediate nodes on some route and there is only one possible way to pass through them. Degree larger than 2 indicates that the node is more important in the network, having connections in multiple directions, thus working as a transportation hub [33]. A node with high degree centrality can act as an important transfer point of different lines and modes of transportation. However, the usefulness of degree centrality is considered very limited due to its consideration of local properties only and the independence of traffic loads served by the nodes [28].

2.2.2 Closeness centrality

Closeness centrality measures the importance of nodes by calculating how close a node is to other nodes in the network. Technically, closeness centrality of the node is the inverse sum of the shortest routes to all the other nodes [28]. In transportation contexts, closeness centrality can be interpreted as the cost of accessing locations, services, and activities [30].

Closeness is considered to be one of the most important centrality measures and has been well documented in the literature. According to the study of Guangzhou bus network, nodes with large closeness centrality have a tendency to exert more influence on other nodes [33]. Another study by Cheng et al. in 2015 compared different centrality measures in Singapore's subway network and found that nodes with closeness centrality are more likely to be located near the center of the network, while it is possible for nodes with high degree and betweenness centralities to be further away [7]. A bivariate correlation analysis of different centrality measures for a study of Greek transportation network by Tsiotas et al. (2015) showed that closeness centrality is a quite weak predictor of other centrality measures, confirming their very different behaviors [30].

Closeness centrality helps determine which nodes have the quickest access to other nodes, making them candidates for important transportation hubs. One limitation of closeness centrality is that its application is limited in disconnected networks. [28]

2.2.3 Betweenness centrality

Betweenness centrality is an node importance measure that equals the ratio of the number of shortest paths that pass through the node to the number of all the shortest paths in the network between all other nodes [21]. A high value of betweenness centrality means that transportation modes will choose to pass through a node more frequently, reflecting the requirement of increased load capacity of that node [33]. Betweenness centrality can be used to identify bottlenecks or chokepoints in a transportation network [28].

Derrible (2012) studied centralities in different metro systems and found out that betweenness centrality becomes more evenly distributed when the size of the network increases. The same study also showed that the share of shortest paths crossing a central node in larger networks is smaller, hence reducing betweenness centrality and making large transportation networks more robust to disruptions. The decay of betweenness was also discovered to follow a power law with respect to the size of the network, where the exponent for the highest value of betweenness centrality was low and vice versa, meaning that increasing the network size preserves the centrality of important nodes much better than peripheral nodes[9]. In a study of centrality in the Kazakhstani urban transport network by Goremyko et al. (2018), stated that betweenness centrality can be used to determine geopolitical importance. Interestingly, the study also discovered that there is barely any correlation between betweenness and degree centralities [16].

The main limitation of betweenness centrality is its complete dependency on shortest paths as the definition of importance. In addition, many nodes in the network, especially terminal nodes, are not located on a single shortest path, which can lead to a low betweenness centrality [28]. A research on traffic flow in a Chinese taxi network by Gao et al. (2013) has also shown that betweenness centrality is a weak predictor for urban traffic flow. [15]

2.3 Probabilistic risk assessment

2.3.1 Overview of probabilistic risk assessment

Probabilistic risk analysis or probabilistic risk assessment (PRA) is a systematic, comprehensive and quantitative methodology for evaluating risks associated with every aspect in the life cycle of a complex system. PRA attempts to find solutions for three basic questions: What are such potential risks associated with the studied system, how severe are the consequences, and how likely are these consequences. [27] PRA offers a quantitative basis for decision making and prioritizing actions of risk reduction.

PRA is applied in various areas with risks of detrimental or fatal consequences, such as nuclear power plants, aerospace engineering, construction of transportation infrastructures, chemical processing, military, and healthcare [3]. The concept of PRA originated from aerospace sector in the late 1960s, when three astronauts were killed in the fire of Apollo test AS-204. In 1969, the Space Shuttle Task Group developed criteria for evaluating quantitative safety goals. At the time, quantitative risk assessment quickly got drawback, since numbers and probabilities being treated as absolute judgements could not guarantee safety. [3] The requirement for safety assurance started to become more urgent due to the growth or emergence of nuclear power plants in the late 1960s and early 1970s. The first full-scale applications of PRA started to rise on popularity later in the 1970's, when the US Nuclear Regulatory Commission (NRC) published Reactor Safety Study WASH-1400 (1975), consisting of extensive analysis of accident consequences. It is considered to be the beginning of modern PRA. [3][19]

During the last decades, PRA has been widely applied in the field of transportation. Transportation infrastructures are constantly at risk of malfunctioning and damage due to external aspects, especially unpredictable environmental hazards. Amini and Padgett (2023) applies PRA in debris impact caused by hurricanes on coastal transportation infrastructure [2]. The study classified the model into three different groups according to the aforementioned three basic questions of PRA methodology, which is shown in Figure 3. The hazard models consist of event selection and intensity estimation, the exposure models contain debris volume and dispersion models, which evaluate the damage to transportation infrastructure caused by debris, and the impact models assess the impact on infrastructure performance, such as road closure rate and emergency response. [2]

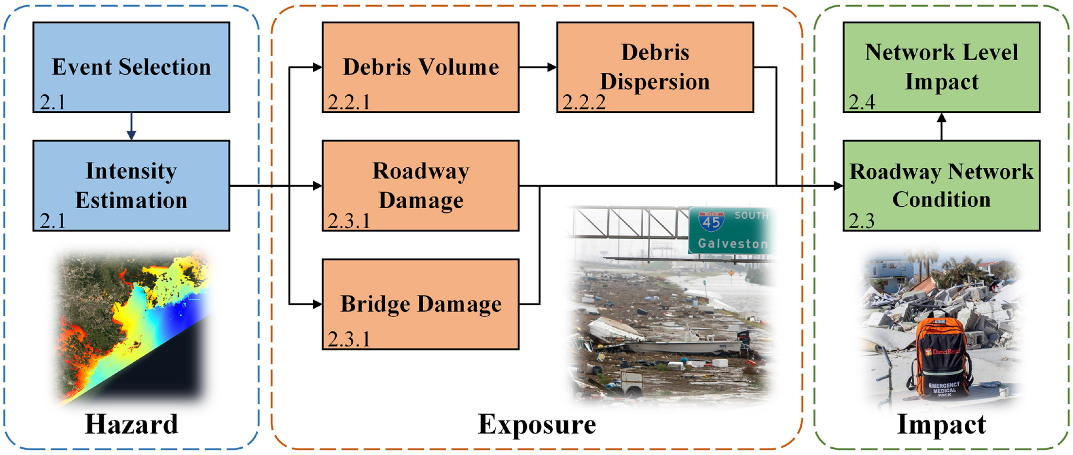

Figure 3: Example of an overview chart of probabilistic risk assessment methodology that shows the progression in finding answers to the three basic questions: What are the risks, what are the consequences, and how severe and likely these are. [2]

2.3.2 Probablistic importance measures in critical infrastructures

PRA utilizes different numerical metrics to assess the potential increases and decreases of risks. Most widely utilized risk importance measures are risk achievement worth (RAW) and risk reduction worth (RRW). RAW quantifies how much the overall risk of failure of the system would increase if a given component were assumed to fail completely, thus indicating the importance of keeping the entire system safe. RRW denotes the potential risk decrement if a given component was assumed to always function reliably. It quantifies the improvement potential of the system if a disrupted component was fixed [32].

Table 1 shows the utility of RAW and RRW, displays their differences, and meaning of high and low values. Threshold for values of RAW and RRW can be determined

	RAW	RRW
Measures	How much does the risk of the system increase if component failed?	How much does the risk decrease if component was perfectly reliable?
Reflects	System dependence of a component	Improvement potential of a system
Used for	Identifying critical single components and bottlenecks	Prioritizing maintenance of critical components
High value means	High risk concentration, requires immediate attention	High potential for improvement, should be prioritized
Low value means	Nearly negligible for the risk of entire system	No significant benefits from improvement

Table 1: *Properties and uses of RAW and RRW.*

such that above certain value, maintenance and repairing actions are required. RAW and RRW can offer a prioritization hierarchy for component management attention.

RAW and RRW have been widely used to quantify the contribution of individual components to overall system risk. Although originally developed in the context of nuclear safety, these measures have been increasingly applied to various different critical infrastructure systems. Espiritu et al. (2007), used RAW and RRW to evaluate the importance of components such as transmission lines, transformers, and circuit breakers. They models the power system using reliability block diagrams and fault trees, allowing the computation of system failure probabilities. [11] In more recent studies, RAW and RRW have been extended to the reliability assessment of complex systems, that are modeled using non-homogeneous Markov processes. [23] These studies emphasize that RAW and RRW enable quantitative ranking, which is essential in large-scale infrastructure where resources are constrained. Pandit et al. (2025) evaluated supply chain shortages using PRA framework along with probabilistic importance measures [24]. Vrbanic et al. (2019) showed how RAW and RRW are derived from conditional probabilities of system and component disruptions. The two importance measures have a relationship, and if one of them is known, the other can be calculated. [32] RAW and RRW along with other probabilistic importance measures, Fussell-Vesely importance (FV), and Birnbaum importance (BI) have been evaluated based on sensitivity coefficient for PRA. [29]

3 Methods

In this thesis, the importance of the nodes of the Mandl's Swiss transit network, which enables transportation volumes between bus stop origin-destination pairs, are evaluated with commonly used topological node importance measures and with probablistic node importance measures presented in literature. The node importance measures are compared to each other, and differing results are explored and analyzed, and a sensitivity analysis is performed.

3.1 Mandl's Swiss network

Mandl's Swiss network was originally developed as a test network based on a real transit system in Switzerland. It is a bus network including $n_n = 15$ bus stops between which transportation is enabled, and $n_e = 21$ road segments on which buses travel between stops. The traversal durations between bus stops are known.

The network has been used, for example, to research demand based route generation algorithms for public transit network design by Kılıç and Gök (2014) [20], and bus line planning problem with integrated passenger routing by Vermeir et al. (2021) [31]. Mandl's Swiss Network is a widely applied benchmark network used in transportation research and transit network design. Due to its modest size and complexity, it is highly applicable for experimentation with transit network design and route optimization related algorithms. It is used as a standard test case to compare different methods of designing routes, allocating and directing traffic, optimizing cost efficiency to maintain network performance, and prioritizing critical and vulnerable nodes. The structure, edges and nodes with their respective ID's of Mandl's Swiss network are visualized in Figure 4.

3.1.1 Graph representation

The Mandl's Swiss network can be modeled as a graph $M = (V, E)$, $E \subseteq \{ (v_1, v_2) \mid v_1, v_2 \in V \}$, where V is the set of nodes and E is the set of undirected edges. Let M represent the network.

The node $v_i \in V$ is a vertex with index i, and an edge is represented with a pair of indexed nodes v_i and v_j. The set of edges E is a subset of all possible pairs of nodes v_i and v_j in set of nodes V. Terminal nodes are nodes that can function as a starting or ending node of a path. Transportation is enabled between all nodes, this the set of terminal pairs T is:

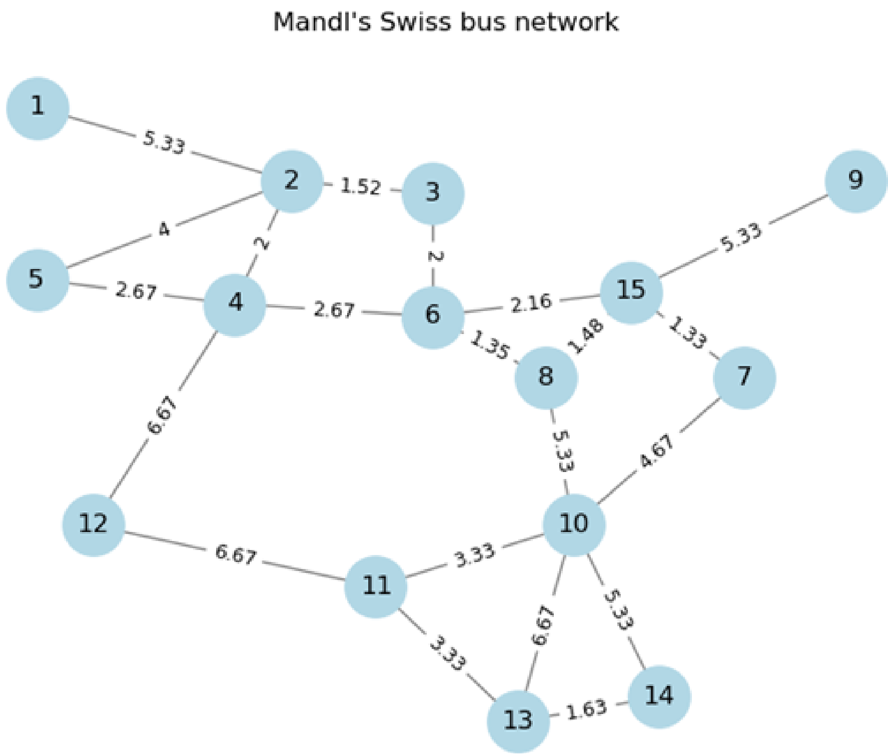

Figure 4: *Mandl's Swiss network is visualized with node ID's and edge weights. The weights represent traversal durations between nodes in minutes.*

$$T = \{\, (v_1, v_2) \mid v_1 \in V,\ v_2 \in V,\ v_1 \neq v_2 \,\}$$

The total number of terminal pairs is thus $n_n \cdot (n_n - 1)/2 = 15 \cdot (15 - 1)/2 = 105$, when (v_1, v_2) and (v_2, v_1) represent the same terminal pair.

A path in the network is defined as a sequence of nodes connected by edges such that path does not contain same node more than once. Such paths are called simple paths. Let a path $p_i \in P$ have length l_i, then $p_i = (v_{i,1}, v_{i,2}, ..., v_{i,l_i-1}, v_{i,l_i})$ such that $v_{i,j} \in V$, $(v_{i,j}, v_{i,j+1}) \in E, j = 1, ..., l_i - 1$, and $v_{i,j} \neq v_{i,k}, j \neq k$. P denotes the set of feasible paths. The set of paths for a terminal pair $t = (v_1, v_2)$ is $P_t \subset P$, and accordingly $t = (v_{i,1}, v_{i,l_i}), v_{i,j} \in p_t$.

3.1.2 Network operationality

The main purpose of a transportation network is to enable transportations between bus stops . The performance is reflected by the ability of the network to transfer large volumes of passengers. However, the network and its nodes can become disrupted. Disrupted nodes disturb the flow of traffic, force and redirect passenger routes or prevent them entirely.

Each node in the network has an operational status that can be represented by a binary variable.

$$O(v_i) = \begin{cases} 1, & \text{if } v_i \text{ is operational,} \\ 0, & \text{if } v_i \text{ is disrupted.} \end{cases} \tag{1}$$

For a passenger to be able to traverse to, from, or through a certain node, requires the status of the node to be operational. A status of a path p_i is operational if all nodes $v_{i,j} \in p_i$ are also operational.

$$O(p_i) = \begin{cases} 1, & O(v_{i,j}) = 1, j = 1, 2, ..., l_i, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

A terminal pair can have multiple paths connecting the terminal nodes. At least one operational path must exist between these nodes in order to the terminal pair to be operational. Thus, the operational status of a terminal pair is

$$O(t) = \begin{cases} 1, & \sum_{p_i \in P_t} O(p_i) > 0, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

The status of an entire transportation network can be operational only if it is possible to travel from any node to any other node in the network. Thus, there must exist at least one operational path for all terminal pairs $t \in T$.

$$O(M) = \begin{cases} 1, & O(t) > 0, \forall t \in T \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

3.1.3 Disruption probabilities

As stated previously, intentional or unintentional disruptions can change the operational status of a node to zero. A large degree of uncertainty is related to disruptive events, which requires assigning probabilities for operational statuses of nodes. These probabilities can be elicited by careful expert judgement using statistical methods and data analysis. For the sake of simplicity, we assume that the disruption probabilities do not change in time and that the probabilities are independent from each other. The disruption probability of a node v is denoted by $\mathbb{P}[O(v) = 0]$. Hence, the disruption probability of a path $p_i \in P$ can be expressed in following way.

$$\mathbb{P}[O(p_i) = 0] = 1 - \prod_{v_{i,j} \in p_i} \mathbb{P}[O(v_{i,j}) = 0].$$

Terminal pair $t \in T$ is disrupted if all paths $p_i \in P_t$ are disrupted. The probability of this event is

$$\mathbb{P}[O(t) = 0] = \mathbb{P}[\bigwedge_{p_i \in P_t} O(p_i) = 0].$$

3.1.4 Network demand

The Mandl's Swiss network carries a flow of passengers, and the volume of flow in different locations varies throughout the network. Let d_t denote the demand for a terminal pair $t = (v_i, v_j)$. We can form a $n_n \cdot n_n$ demand matrix D for Mandl's Swiss network based on its respective demand data. In this network, the terminal pairs are undirectonal based on the data, which means that the demands between the nodes are equal from both ends, in other words the demand of (v_i, v_j) the same as of (v_j, v_i). This makes the demand matrix symmetric along the diagonal. The origin–destination traffic demand matrix for the Mandl's network is shown in table 2.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	400	200	60	80	150	75	75	30	160	30	25	35	0	0
2	400	0	50	120	20	180	90	90	15	130	20	10	10	5	0
3	200	50	0	40	60	180	90	90	15	45	20	10	10	5	0
4	60	120	40	0	50	100	50	50	15	240	40	25	10	5	0
5	80	20	60	50	0	50	25	25	10	120	20	15	5	0	0
6	150	180	180	100	50	0	100	100	30	880	60	15	15	10	0
7	75	90	90	50	25	100	0	50	15	440	35	10	10	5	0
8	75	90	90	50	25	100	50	0	15	440	35	10	10	5	0
9	30	15	15	15	10	30	15	15	0	140	20	5	0	0	0
10	160	130	45	240	120	880	440	440	140	0	600	250	500	200	0
11	30	20	20	40	20	60	35	35	20	600	0	75	95	15	0
12	25	10	10	25	15	15	10	10	5	250	75	0	70	0	0
13	35	10	10	10	5	15	10	10	0	500	95	70	0	45	0
14	0	5	5	5	0	10	5	5	0	200	15	0	45	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 2: *Origin-destination traffic demand matrix for the Mandl network is shown in the table. Each element in the matrix represents a number of passengers traveling from origin node to destination node in a unit of time.*

3.2 Node importance measures

3.2.1 Topological node importance measures

Centrality measures provide a formal way to quantify the importance of nodes in a network. Let the Mandl's network be represented as a graph $M = (V,E)$, where V is the set of nodes and E is the set of edges, with $|V| = n$ being the total number of nodes in the network. Next, we present mathematical definitions of degree centrality, closeness centrality, and betweenness centrality.

Degree centrality, determined by the degree of the node or its direct connections, can defined as

$$C_{deg}(v_i) = \frac{1}{n-1}\sum_{j=1}^{n}\alpha_{i,j}$$

where $\alpha_{i,j}$ gets a value of 1 when node v_j is connected to node v_i and a value of 0 when node v_j is not connected to node v_i. A normalization factor $\frac{1}{n-1}$ is included to factor the size of the network.

Closeness centrality is considering all shortest paths from other nodes to node v_i. Let $d(i, j)$ be the length of the shortest path from node v_i to node v_j. It is defined as

$$C_{cls}(v_i) = \frac{n-1}{\sum_{j\in V, j\neq i} d(i,j)}.$$

Betweenness centrality of node v_i is a measure of how often v_i appears on the shortest path between nodes v_j and v_k, where j, k are all the possible pairs of nodes in the network excluding v_i itself. The formula for calculating betweenness centrality is

$$C_{btw}(v_i) = \sum_{v_j\in V}\sum_{v_k\in V, k\neq j\neq i}\frac{g_{jk}(v_i)}{g_{jk}}.$$

Here g_{jk} denotes the number of paths between nodes v_j and v_k, and $g_{jk}(v_i)$ denotes the number of paths between nodes v_j and v_k that pass through node v_i. [7]

3.2.2 Probabilistic node importance measures

We establish RAW and RRW measures for M for measuring the increase or decrease in the expected traffic volume in the network. Let d_t be the traffic volume between the nodes of terminal pair t. If there exists a path p that connects t, the traffic volume will equal the demand. If such a path does not exist, the traffic volume is zero.

Each terminal pair t is weighed according to its traffic volume d_t. RAW of node i is thus

$$RAW(v_i) = \frac{\sum_{t\in T} d_t \times \mathbb{P}[O(t) = 0 \mid O(v_i) = 0]}{\sum_{t\in T} d_t \times \mathbb{P}[O(t) = 0]}$$

where $\mathbb{P}[O(t) = 0 \mid O(v_i) = 0]$ is a conditional probability that the terminal pair is disrupted given that the node v_i is disrupted. $RAW(v_i)$ denotes the relative increase in the loss of traffic volume in M given that node v_i is always disrupted. Accordingly, RRW of node v_i is

$$RRW(v_i) = \frac{\sum_{t \in T} d_t \times \mathbb{P}[O(t) = 0]}{\sum_{t \in T} d_t \times \mathbb{P}[O(t) = 0 \mid O(v_i) = 1]}$$

where $\mathbb{P}[O(t) = 0 \mid O(v_i) = 1]$ is the conditional probability that the terminal pair t is disrupted given that node v_i is operational. $RRW(v_i)$ denotes the relative increase in the traffic volume in M given that node v_i is operational.

3.2.3 Comparison of node importance measures

The relationships between the node importance measures are investigated using pairwise scatterplots and correlation matrix. Scatterplots were generated for all combinations of degree centrality, closeness centrality, betweenness centrality, RAW and RRW. Each point in a scatter plot represents a node in the Mandl's network. Positive linear relationships appear as points clustered around an increasing trend, where as weak relationships are characterized by a more spread out distribution of points. The plots also identify individual nodes that differ substantially from the overall trend.

For quantifying the strength of the relationship between node importance measures, we use Pearson correlation coefficients and form a correlation matrix based on them. Pearson correlation coefficient between two variables X and Y is defined as

$$r = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^{n} (y_i - \bar{y})^2}},$$

where x_i and y_i are the values of the two measures for node i, and $\bar{x}$ and $\bar{y}$ are their respective means. The coefficient r ranges from -1 to 1. A value of 1 indicates a perfect positive linear relationship, a value of -1 indicates a perfect negative linear relationship, and a value of 0 implies there is no correlation at all. The calculated Pearson correlation coefficients were arranged into a correlation matrix, which provides a compact representation of pairwise relationships of all node importance measures.

3.2.4 Implementation and assumptions

The aformentioned topological and probabilistic importance measures are calculated using python. Calculating degree centrality is straightforward and requires only the information of the structure of the network: how nodes are connected to each other with edges. Closeness and betweenness centralities utilize the information about edge lengths to calculate the shortest paths. This is an alternative for calculating shortest paths based on giving each edge the same weight, which would result in multiple paths having the same lengths. For betweenness centrality, we examine exactly $(15 - 1) * (15 - 2) = 182$ pairs of nodes, since the path must strictly pass through node v_i without the node being a terminal node itself. The topological node importance measures are solely based on the network structure, and does not take the traffic volume into account.

For RAW and RRW, a default node disruption probability of 0.01 will be used. Thus $\mathbb{P}[O(t) = 0] = 0.01$ is true for all nodes independently in the default case. RAW of a node examines the case where $\mathbb{P}[O(t) = 0] = 1$ and RRW examines the case where $\mathbb{P}[O(t) = 0] = 0$ for a certain node. Precisely calculating the state of the entire Mandl's network and the existence of a path between a terminal pair, each combination of operational and disrupted nodes are considered. The important assumption for the purpose of this study is that if a terminal pair can be connected by operational nodes, the traffic volume between them is always d_t regardless of the length of that path. Only if enough nodes become disrupted to disable all possible operational paths between a terminal pair, the traffic volume for that terminal pair is 0. The key measure for evaluating RAW and RRW is the enabled traffic volume, thus calculating them requires the input of initial traffic volume data between terminal pairs. However, the lengths of the edges are not required for calculating RAW and RRW, since we assumed a passenger will take any operational path instead of skipping the trip due to the lack of the shortest path.

Brute force approach to loop through all 2^{15} different states of the network is used to calculate RAW and RRW. For each state, we check if there exists an operational path between every possible terminal pair in the network. If it exists, we multiply the probability of the state of the network by the traffic volume, and add it to the total. If the path does not exists, nothing is added. The total number yielded is the total traffic volume in the network. This can be used to determine RAW and RRW values.

3.2.5 Sensitivity analysis

A sensitivity analysis is conducted to evaluate how changes in the assumed disruption probabilities affect RAW. The objective of the analysis is to assess the robustness of calculated RAW values with respect to uncertainty in the disruption probability assumptions.

In the baseline scenario, the disruption probability of each node was assumed to be 0.01. A local one-at-a-time sensitivity analysis is then performed by increasing the disruption probability of one node at a time from 0.01 to 0.05 while keeping the disruption probabilities of all other nodes unchanged. After each probability modification, the corresponding RAW and RRW value of the modified node is recalculated. This process is repeated for all 15 nodes in the network.

The sensitivity of the probabilistic importance measures is quantified by comparing recalculated values with the corresponding baseline values. The relative change is calculated as

$$\frac{RAW_{\text{new}} - RAW_{\text{baseline}}}{RAW_{\text{baseline}}},$$

where RAW_{baseline} denotes the importance measure obtained using the baseline disruption probability (0.01) and RAW_{new} denotes the recalculated value after increasing the disruption probability of the corresponding node to 0.05.

The results of the sensitivity analysis are presented using a tornado diagram, in which the nodes are ordered according to the amount of the relative change. This visualization emphasizes the nodes with the greatest influence on RAW values under disruption probability changes.

4 Results

4.1 Values and node rankings

The topological node importance values were obtained by calculating degree centrality, closeness centrality and betweenness centrality values and the probabilistic node importances were obtained by calculating RAW and RRW for all nodes in the Mandl's Swiss network. The results are presented in Table 3.

Degree centrality identified node 10 as the most connected node in the network with a value of 0.35714. Nodes 2, 4, 6 and 15 formed the second highest group with degree centrality values of 0.28571. Nodes 1 and 9 received the lowest degree centrality values.

Closeness centrality produced slightly different ranking of values. Node 6 achieved the highest closeness centrality value of 0.15864, followed by node 8 (0.15746) and node 15 (0.14846). These nodes are therefore located in positions that provide relatively short average distances to the rest of the network.

The highest betweenness centrality was obtained again by node 6, getting a value of 0.48352. Node 10 achieved the second highest value (0.32967), followed by nodes 8 (0.26373) and 15 (0.23077). These results indicate that node 6 isparticularly important in connecting different parts of the network through shortest paths. Nodes 1, 5, 9, 12 and 14 received a betweenness centrality value of 0, indicating that these nodes only function as terminal nodes, not intermediate nodes.

Although the three topological node importance measures produced different rankings, several nodes consistently appeared among the highest-ranked positions. This suggests that certain nodes occupy structurally important positions regardless of the specific centrality measure applied.

Risk Achievement Worth (RAW) and Risk Reduction Worth (RRW) were calculated for all nodes in Mandl's network using the model described in section 3.2.2. They were based on an independent default node disruption probability of 0.01 for each node. RAW for a certain node assumes the disruption probability is 1 and RRW assumes the disruption probability is 0. In this study, RAW and RRW are a measure of potential change in traffic volume, thus they are heavily influenced by the initial traffic volume data observed for terminal pair of nodes.

The RAW results indicate the risk increase factor, which translates into the expected losses of total enabled traffic volume in the network. Node 10 received by far the highest RAW value. This can be justified by the fact that node 10 was a terminal node that was used by a substantially larger number of passengers than any of the other nodes. The failure of node 10 would have the largest effect on total traffic volume.

Node	Degree centrality	Closeness centrality	Betweenness centrality	RAW	RRW
1	0.071	0.078	0	1.203	1.00170
2	0.286	0.128	0.176	1.360	1.00267
3	0.143	0.137	0.198	1.119	1.00107
4	0.286	0.136	0.198	1.122	1.00110
5	0.143	0.103	0	1.066	1.00062
6	0.286	0.159	0.484	1.330	1.00251
7	0.143	0.139	0.088	1.147	1.00129
8	0.214	0.157	0.264	1.147	1.00129
9	0.071	0.086	0	1.041	1.00040
10	0.357	0.122	0.330	2.156	1.00542
11	0.214	0.099	0.154	1.163	1.00142
12	0.143	0.094	0	1.075	1.00070
13	0.214	0.079	0.022	1.117	1.00106
14	0.143	0.084	0	1.039	1.00038
15	0.286	0.148	0.231	1.042	1.00041

Table 3: *The values of topological importance measures; degree centrality, closeness centrality and betweenness centrality, as well as probabilistic importance measures; RAW and RRW, are displayed for each node in Mandl's network.*

Nodes 2 and 6 also had a significantly large RAW values, indicating a disruption of these nodes having a major effect. Nodes such as 14, 9 and 15 had small RAW values, thus disrupting them would have only minimal effect on the whole network.

The RRW results describe the potential improvement in enabled traffic volume if individual nodes were assumed to be perfectly operational. Similar to RAW, node 10 ranked highest in RRW value, suggesting that improvements to this node would provide the greatest reduction in overall network risk, translating into the greatest expected increase in total traffic volume. Nodes 2 and 6 were ranked relatively high, and nodes 14, 9 and 15 were ranked on the bottom receiving small RRW values.

A comparison of the RAW and RRW rankings reveals substantial similarity between the two measures. The nodes that received the largest RAW and RRW values are largely identical, although minor differences in ranking order are observed.

4.2 Comparison and correlation

Pairwise scatter plots of node importance measures are visualized in Image 5 to show the relative similarities and differences between node importance measures compared to each other. To quantify the similarities between the node importance measures, a correlation analysis was performed. Pearson correlation coefficients were calculated and the resulting correlation matrix is presented in Image 6.

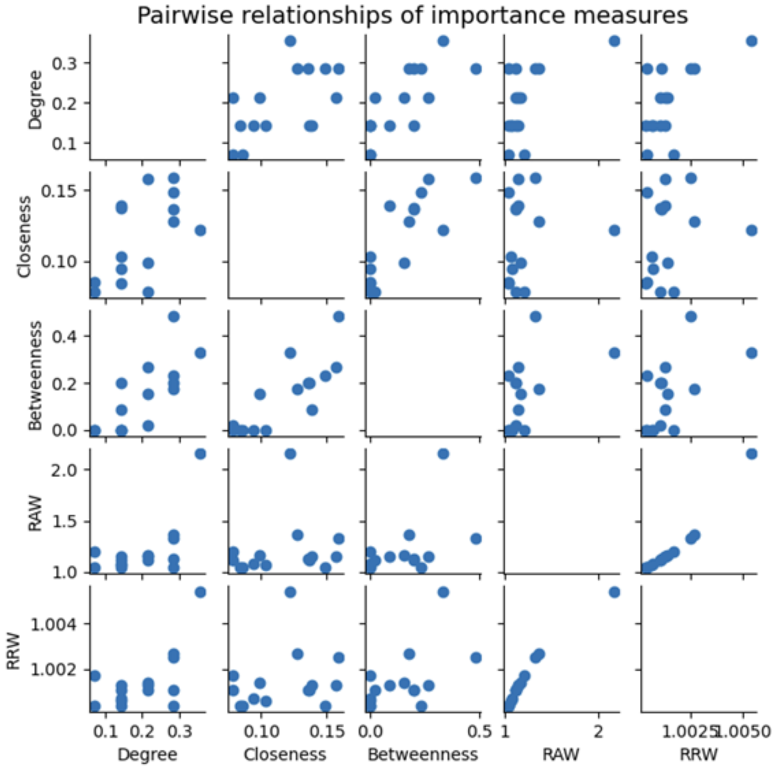

Figure 5: *Pairwise scatterplots of importance measures show relationships between them.*

Among the topological measures, the strongest relationship was observed between closeness centrality and betweenness centrality, with r value of 0.81. Degree centrality also exhibited significantly strong correlation with betweenness centrality (0.76) and closeness centrality (0.57). These results indicate that the topological measures generally identify similar nodes to be central in the network, although the strength of the relationships varies. Since betweenness and closeness centralities use the lengths of edges to determine the shortest paths, and degree centrality does not, the correlation coefficients will be dependent on the individual lengths of edges.

By far the strongest correlation were observed between RAW and RRW, which appeared to be nearly perfect with a value of 0.98. This result is backed up in the study by Vrbanic et al. that studies the basic relation of RAW and RRW and states that one can be derived from another [32]. The weakest correlation was between close centrality to RAW and RRW, with values 0.17 and 0.23 respectively, indicating that closeness centrality is nearly unrelated to the probabilistic node importance measures.

Overall the results indicate that while some topological measures are partially related to operational importance, substantial differences remain between topological and probabilistic assessments of node importance.

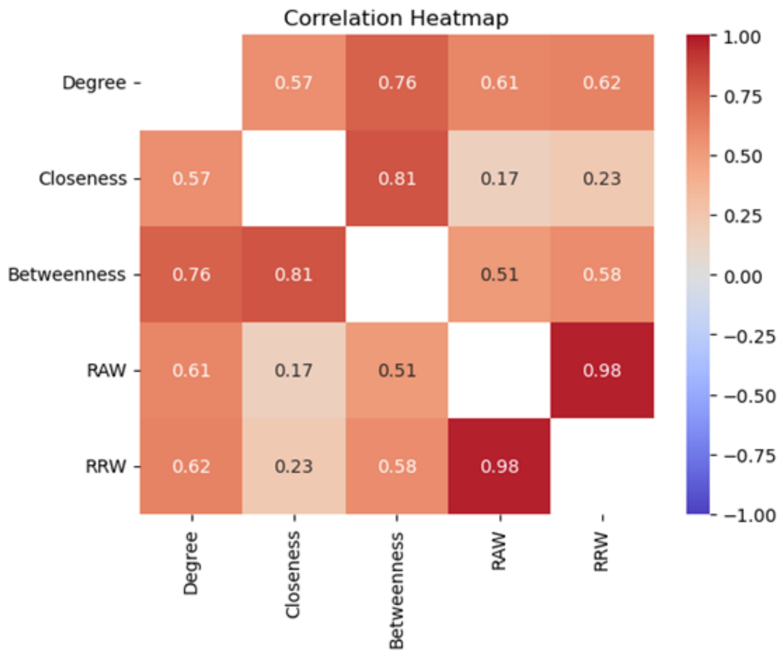

Figure 6: *Correlation matrix visualized as a heatmap shows correlations between importance measures. Shades of red indicate positive correlation. Darker shades mean stronger correlation and lighter shades mean weaker correlation.*

4.3 Sensitivity analysis

The sensitivity of RAW was evaluated by individually increasing the disruption probability of each node from 0.01 to 0.05 while maintaining the baseline probability of 0.01 for all remaining nodes. The resulting changes in the RAW values are illustrated in Figure 7 using a tornado diagram.

For all nodes, increasing the disruption probability resulted in a decrease in the corresponding RAW value. This happens because the failure rate of the network has already grown larger due to an increase in the disruption probability of one node, and thus turning the node entirely unoperational is a smaller change from a node with an already higher likelyhood of disruption. However, the magnitude of the decrease varied between the nodes. The largest relative decrease was observed for node 10, whose RAW value decreased by approximately 2.17%. Nodes 2 and 6 exhibited the next largest decreases of approximately 1.07% and 1.00%, respectively. The remaining nodes experienced smaller changes, with relative decreases ranging from approximately 0.15% to 0.68%.

Despite these differences, the overall ranking of the nodes remained the same. Node 10 retained the highest RAW value, followed by nodes 2 and 6, while the least important nodes also preserved their relative positions. The sensitivity analysis also indicates that moderate changes in the assumed disruption probabilities have only a limited influence on the calculated RAW values and do not alter the identification of the most critical nodes within the Mandl network. Another interesting observation is that the ranking of nodes based on how much RAW changed under disruption probability changes, reflects surprisingly accurately the original ranking of RAW values of the nodes.

The relatively small changes observed in the RAW values suggest that the RAW is rather robust with respect to the disruption probability assumptions considered in this study. Therefore, the conclusions drawn from the baseline analysis are not substantially affected by small uncertainty in the node disruption probabilities.

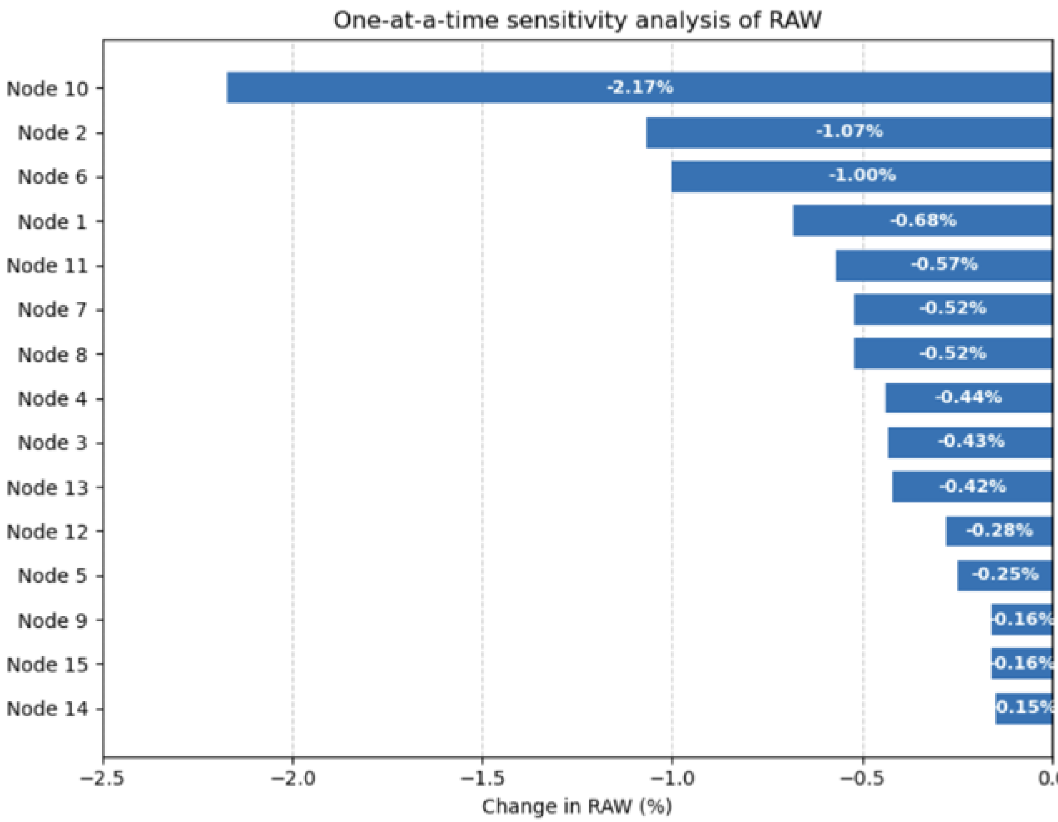

Figure 7: *One-at-a-time sensitivity analysis was performed and nodes were arranged based on the largest change in RAW, when their initial disruption probability were changed from 0.01 to 0.05. Results were displayed in a tornado diagram.*

5 Discussion

5.1 Limitations

Several limitations should be taken into account. First, the analysis was conducted using the Mandl's network, which is a relatively small benchmark transportation network consisting of only 15 nodes and 21 edges. Although it is widely used in transportation research, the network is considerably simpler than many real-world public transport systems.

A plenty of simplifications and assumptions were made. Shortest paths were calculated based on edge lengths, that represented the travel time and the waiting time at stops were ignored. The model assumes each terminal pair is connected by a direct connection, but in often a real world transit networks, multiple distinct paths form the total path, representing changes in vehicles.

Modelling node disruption with equal and independent probability values is also a strong simplification, in real world the probabilities vary, fluctuate in time, and most importantly correlate with each other, since the disruptions might have been caused by the same event such as a major flood or large construction project.

The assumption that traffic volume remains constant as long as there is an operational path would not be entirely applicable to the real world. If the length of a shortest path grows significantly larger due to disrupted nodes, we could assume the traffic volume to decrease gradually according to that, since the threshold of passengers to take that path in the first place grows larger.

An important consideration in the interpretation of probabilistic node importance measures is that the calculated RAW and RRW values include traffic demand for which the disrupted node serves as either an intermediate node or a terminal node. A portion of the increase in risk associated with the disruption of a node is associated to trips that directly originate from or terminate to the disrupted node, rather than to its role in facilitating traffic through the network.

5.2 Future research

Future work could extend the analysis to larger transportation networks and investigate the performance of node importance measures under more realistic conditions. Additional work could also consider dynamic passenger demand, time-dependent disruptions and cascading or correlating node disruption effects.

The model could also be improved by considering capacity of edges, a maximum load of traffic that can pass through an edge in a unit of time. One could also impose time threshold such that when the time to travel a designated path under node disruptions exceeds the time of the shortest path by a greater amount than the threshold, the customer selects not to travel the path. Both of these issues could lead to the development of interesting path selection decision making procedures.

Furthermore, other centrality measures and reliability-based importance measures not considered in this study, could be included to provide a broader comparison of approaches for identifying critical transportation infrastructure.

No single measure alone can fully depict the quality of the network performance, but getting perspective from different approaches and combining their advantages, could assist the creation of new importance metrics to better capture and solve the objective of a specific performance related problem related to transportation networks and infrastructures.

6 Conclusions

The objective of this thesis was to compare different node importance measures using the Mandl's Swiss bus transit network. Three of the measures were based on network topology: degree centrality, closeness centrality and betweenness centrality. The remaining two measures, risk achievement worth (RAW) and risk reduction worth (RRW), also considered disruption probabilities and origin-destination traffic demand. Comparison between these made it possible to evaluate the similarities and differences between topological and probabilistic approaches.

The calculated node importance values showed certain nodes being consistently identified as important by all measures. In particular, node 10 was ranked as one of the most critical nodes throughout the analysis, whereas nodes 2 and 6 received also relatively high rankings. From a topological perspective, this means that the nodes are have a potential to be central transportation hubs in the network. Considering probabilistic importance measures, highest ranked nodes are the nodes that most affect the traffic volume in the network.

Comparison of the node values and rankings was supported by pairwise scatter plots and Pearson correlation coefficient analysis. The strongest correlation was found between RAW and RRW. The topological measures also showed relatively strong correlations with each other. The correlations between the topological and probabilistic measures however, were weaker. This suggests that the choice of importance measures used is highly dependable on operational factors, such as traffic demand and disruption probabilities, which are changing variables even when the network structure stays the same. Thus RAW and RRW utilize the additional information from traffic demand data and node disruption assumptions, which the network topology alone cannot fully represent.

A local one-at-a-time sensitivity analysis for RAW was performed by increasing the baseline disruption probability of one node at a time from 0.01 to 0.05. the resulting changes in the RAW values were relatively small. A significant increase in disruption probabilities leading only to a small decrease in RAW values indicates that RAW is a rather robust measure in this context. The important observation was to detect the relative differences in changes in RAW values. These relative changes reflected the original node rankings based on RAW, which indicates that the larger the RAW value of a node is, the more sensitive the RAW of that node is to changes in node disruption probability.

Overall, the results show that different node importance measures complement each other instead of providing identical or completely opposite results. The choice of importance measure depends on the purpose of the analysis. Whereas network connectivity is captured well by topological importance measures, the assessment of operational vulnerability and disruption impacts is better demonstrated by examining probabilistic importance measures.

In conclusion, this thesis demonstrates that comparing multiple node importance measures helps better understand transportation network vulnerability than relying on a single measure. The presented methodology can also be applied to other transportation networks and may support future studies on network reliability, resilience and critical infrastructure analysis.

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