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Hierarchical Portfolio Optimization of Reinforcement Actions in Transportation Networks

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Abstract

Transportation networks, such as railway networks, are critical infrastructure enabling the transportation of goods and people. Their importance highlights the need to reinforce them against disruptions caused by the deterioration of their components or deliberate attacks. Reinforcing the network often comprises of reinforcing its components, for example, its nodes. Given limited resources allocated to reinforcing, identifying those collections of reinforcement actions, which have the greatest positive impact relative to the cost of implementing these actions, is crucial. Such collections of reinforcement actions are called cost-efficient portfolios.

Large transportation networks often have multiple decision makers responsible for separate parts of the network. This thesis proposes a hierarchical portfolio optimization model for computing cost-efficient portfolios of reinforcement actions in partitionable transportation networks that reflect the real-world management of responsibilities. The objective is to maximize expected enabled traffic volume while minimizing reinforcement costs. Only probabilistic node disruptions are considered, and they are assumed to occur independently of other disruptions. Portfolios consisting of reinforcement actions, which decrease the disruption probabilities of nodes of the network, are considered.

The proposed model is illustrated with a case study on a part of the Finnish railway network comprising ten stations in Northern Savonia. Most of the nodes of the network represent railway switches and the edges connecting them correspond to railway tracks. The results indicate that the reinforcement of some switches are included in a relatively high share of cost-efficient portfolios, suggesting a higher importance of them in enabling traffic. Conversely, the reinforcement of some switches appear in no cost-efficient portfolios, suggesting that they have less impact relative to their cost.

Overall, the proposed hierarchical portfolio optimization model is a powerful framework for supporting decision-making in the reinforcement of critical transportation networks, and additionally, it may better reflect real-world management of responsibilities than approaches with a single decision maker. The results help identify key reinforcement actions and exclude actions that do not appear in any cost-efficient portfolio.

Keywords Transportation network, hierarchical portfolio optimization, decision analysis

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Tiivistelmä

Liikenneverkot, kuten rautatieverkot, ovat kriittinen osa infrastruktuuria ja mahdollistavat tavaroiden ja ihmisten kuljetuksen. Niiden tärkeys korostaa niiden vahvistamisen arvoa häiriöitä vastaan, jotka voivat johtua verkon komponenttien rappeutumisesta tai tahallisista hyökkäyksistä. Verkon vahvistaminen toteutetaan usein vahvistamalla sen komponentteja, kuten verkon vaihteita. Vahvistukseen allokoidut rajatut resurssit tulisi kohdentaa sellaisiin vahvistustoimiin, joilla on suurin positiivinen vaikutus suhteessa kustannuksiin. Tällaisista vahvistustoimista koostuvia kokoelmia kutsutaan kustannustehokkaiksi portfolioiksi.

Suurissa liikenneverkoissa on usein useita päätöksentekijöitä vastuussa omasta osastaan verkkoa. Tässä työssä esitetään hierarkkinen portfolio-optimointimalli kustannustehokkaiden portfolioiden määrittämiseen liikenneverkoissa, jotka ovat jaettavissa aliverkkoihin. Mallin rakenne vastaa paremmin todellista resurssien jakoa. Tavoitteina on maksimoida liikenneverkon odotusarvollisesti mahdollistettua liikennöintimäärä ja minimoida vahvistustoimien kustannukset. Vain solmujen vikaantumisia tarkastellaan ja niiden oletetaan tapahtuvan riippumattomasti muista vikaantumisista. Vahvistustoimet pienentävät solmujen vikaantumistodennäköisyyttä.

Esitettyä mallia havainnollistetaan esimerkkitapauksella, joka on osa Suomen rataverkkoa ja koostuu kymmenestä asemasta Pohjois-Savossa. Suurin osa verkon solmuista vastaa rautatievaihteita ja niitä yhdistävät kaaret puolestaan kiskoja. Tulokset osoittavat, että joidenkin rautatievaihteiden vahvistaminen kuuluu suhteelliseen suureen osaan kustannustehokkaista portfolioista, joka viittaa siihen, että näiden rautatievaihteiden vahvistaminen lisää verkon mahdollistamaa liikennöintimäärää enemmän. Toisaalta joidenkin rautatievaihteiden vahvistaminen ei kuulu yhteenkään kustannustehokkaista portfolioista, joten näiden vahvistaminen on vähemmän tärkeää.

Tämä hierarkkinen portfolio-optimointimalli osoittautuu tehokkaaksi työkaluksi tukemaan päätöksentekoa kriittisten liikenneverkkojen vahvistamisessa. Lisäksi se saattaa paremmin vastata hallinnollista vastuunjakoa kuin yhden päätöksentekijän malleissa. Tulokset mahdollistavat sekä keskeisten vahvistustoimien identifioimisen, että sellaisten vahvistustoimien poissulkemisen, jotka eivät esiinny missään kustannustehokkaassa portfoliossa.

Avainsanat Liikenneverkko, hierarkkinen portfolio-optimointi, päätösanalyysi

Preface

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Symbols

Symbol	Description
$G = (V, E)$	Transportation network
$V = V^{\mathrm{S}} \cup V^{\mathrm{T}}$	Set of nodes
V^{S}	Set of physical nodes
V^{T}	Set of terminal nodes
N	Number of physical nodes
E	Set of undirected edges
k	Number of subnetworks
$G_j = (V_j, E_j)$	jth subnetwork
V_j^{S}	Physical nodes of subnetwork j
V_j^{T}	Terminal nodes of subnetwork j
N_j	Number of physical nodes in subnetwork j
t	Terminal pair
$\mathcal{T}$	Set of all terminal pairs in the transportation network
$\mathcal{T}_j$	Set of terminal pairs in subnetwork j
π	Path
$\mathcal{P}^t$	Set of paths for a specific terminal pair
$f = (f^t)_{t \in \mathcal{T}}$	Vector of traffic volumes for all terminal pairs
$f_j = (f_j^t)_{t \in \mathcal{T}_j}$	Vector of traffic volumes for subnetwork j
$O(v)$	Operational status of node $v \in V^{\mathrm{S}}$
$O(\pi)$	Operational status of path π
$O(t)$	Operational status of terminal pair t
$q_j = (q_{j,v})_{v \in V_j^{\mathrm{S}}}$	Portfolio for subnetwork j
$C(q_j)$	Cost of portfolio q_j
$\mathcal{Q}_j$	Set of all possible portfolios for subnetwork j
$\mathcal{Q}_j^{\mathrm{CE}}$	Set of cost-efficient portfolios for subnetwork j
$Q_j = (q_1, \ldots, q_j)$	Combined portfolio for first j subnetworks
$c_v = (c_{v,1}, \ldots, c_{v,r})$	Cost vector for reinforcing node v
r	Number of resource types
$b = (b_1, \ldots, b_r)$	Budget vector
$\succ$	Dominance relation between portfolios

1 Introduction

Modern society relies on the transportation of goods and people, which is supported by the level of service enabled by the routes of transportation networks. These networks enable sectors such as energy, public transportation, and the media to function. The importance of such sectors motivates decision makers (DMs) in, for example, energy companies or national governments to not just maintain but to improve the level of service enabled by transportation networks, as discussed in Newman [1].

Disruptions to the components of transportation networks may occur due to factors such as component deterioration, accidents, or external hazards, including deliberate attacks. Transportation networks can be reinforced against these disruptions, for example, by replacing network components to increase their reliability or by adding new components to enable new routes. Cost-efficient reinforcement may require identifying the most important components with respect to the reliability of the network as discussed in Olander et al. [2]. It is meaningful to consider the importance of combinations of components, because the disruption of a single component may not cause significant harm to the level of service enabled by the network, but the disruption of a selected combination of components may do so.

In this thesis, we consider reinforcement actions to reduce the disruption probability of the nodes of the network. We compute cost-efficient portfolios of reinforcement actions that maximize the expected enabled traffic volume of the network while minimizing the cost of implementing these actions. This problem is presented as a multi-objective optimization problem in Section 3, where the necessary concepts of reinforcement actions, reliability, and cost-efficiency are also introduced.

Many real-world transportation networks are large, which can pose significant challenges in terms of computation time due to the complexity of the optimization problems. We introduce a hierarchical approach, which is based on transportation networks that can be partitioned into subnetworks, which, for example, represent railway stations within a railway network. Additionally, this hierarchical approach may better represent the delegation of responsibilities and associated allocation of resources in transportation networks. The computation of the cost-efficient portfolios of reinforcement actions for the entire transportation network is done in two stages: first, we compute the cost-efficient portfolios of reinforcement actions for all the subnetworks separately, and then, in the second stage, evaluate the combinations of those cost-efficient portfolios to compute the cost-efficient portfolios of reinforcement actions for the entire transportation network.

This thesis has the following structure. Section 2 provides an overview of the background and earlier approaches to computing cost-efficient portfolios of reinforcement actions for transportation networks. Section 3 presents the developed hierarchical portfolio optimization model, which utilizes multicriteria decision analysis to quantify the reliability of a transportation network for identifying cost-efficient portfolios of reinforcement actions for large-scale transportation networks. Section 4 presents a case study for the developed hierarchical model on a part of the Finnish railway network and the reinforcement of its switches in a cost-efficient manner to improve the level of service enabled by the train routes of the network. Section 5 discusses the content of the thesis and presents some future research directions. Lastly, Section 6 concludes this thesis.

2 Background

2.1 Transportation Networks

Transportation networks enable the transportation of people, goods, and information. Transportation networks, such as highway systems or railway networks, can be modeled as graphs, which consist of nodes and edges, as discussed in Newman [1]. Nodes represent, for example, bus stops or interchanges of a highway system. Edges are the connections between nodes of the network. They may represent railway tracks or a road segments between two stops of a bus line. The nodes and edges of a transportation network may be prone to disruptions to their function due to, for example, mechanical failures. Such disruptions can be modeled with probabilities, which enables the analysis of the network using probabilistic measures. Often, it is beneficial to consider reinforcing the nodes or edges of the transportation network to reduce the probability of disruption. Reinforcing can have significant costs, and often, limited resources are available to reinforcing the network.

In addition to nodes and edges, transportation networks may have additional attributes associated with transportation, such as travel volumes, reliabilities of their components, or capacities that limit the amount of travel volume that can pass through the corresponding component. Transportation networks are often partitioned into separate subnetworks based on administrative responsibilities related to the operation and preventive maintenance of subnetworks. For example, the railway network of mainland Europe is partitioned into subnetworks within each country, which can be further partitioned into networks consisting of a single station.

Cappanera and Scaparra [3] introduce a game-theoretic framework for reinforcing the edges of networks in a shortest-path transportation network to maximize its robustness against disruptions, utilizing a multilevel optimization model. Jenelius et al. [4] investigate methods for measuring the reliability of a transportation network. They introduce the importance of a component (e.g., a node or an edge) of the network as a consequence of the disruption of the component and the exposure of the component to the likelihood of that disruption affecting the traffic that the transportation network enables.

2.2 Reliability Engineering

Kapur and Pecht [5] present reliability engineering as a field that focuses on assessing, managing, and preventing failures in critical systems. Reliability engineering of transportation networks can focus on the structure and the individual components of these networks. The components are analyzed and studied from the viewpoint of their reliability, specifically their ability to function as intended within a given time frame. This definition of reliability may vary based on the context and preferences of the DM regarding what constitutes the proper function of a component, as discussed in Pant et al. [6]. Topological measures like the degree of a node in a network (see, e.g., Latora and Marchiori [7], Haritha and Anjaneyulu [8]) can be used as heuristics to guide reliability analysis or one can use probabilistic measures utilizing methodologies like probabilistic risk assessment (PRA) as in Olander et al. [2]. Henry and Ramirez-Marquez [9] provide an alternative to reliability with a formal definition of resilience as a function of time. In contrast, topological metrics do not typically take time into consideration.

Ip and Wang [10] utilize the concepts of resilience and friability, where they define resilience as the average number of independent paths that enable transportation in the network, and they define friability as the average decrease in resilience when one node is disrupted at a time. In their paper, they consider two types of actions: reinforcing existing edges and adding new edges to the network. They present an optimization model for selecting actions to maximize resilience while minimizing the fragility of a transportation network. This bi-objective optimization problem is solved using a weighted sum approach and a genetic algorithm. Additionally, the model includes a constraint on the total cost of the implemented actions.

Olander et al. [2] adapt the concept of terminal pair reliability (see, e.g., Yoo and Deo [11]) to define a PRA-based importance measure for identifying the nodes of a transportation network that are more important to its ability to enable transportation. They apply this importance measure to selecting which railway switches to reinforce in an illustrative railway station in Finland.

2.3 Multicriteria Decision Analysis

Multicriteria Decision Analysis (MCDA) tackles the challenge of solving decision problems with multiple criteria (see, e.g., Zionts [12]). MCDA differs from single criterion decision analysis in that there rarely exists a single optimal solution, but rather multiple solutions that together form the set of non-dominated solutions. Roy [13] presents his philosophy in MCDA as supporting decision analysis rather than optimizing it. Roy argues that the goal of MCDA is not to find a single optimal solution for the DM, but instead to support their decision-making by presenting them with many alternatives to choose from, together with descriptive information about them, their consequences, and assumptions. For example, Roy [13] pioneered the ELECTRE family consisting of methods that rank alternatives based on outranking relations.

Multiple criteria can be combined into a single criterion. For example, criteria weight elicitation aims to determine weights for different criteria, enabling them to be aggregated into a single criterion, as discussed in Riabacke et al. [14]. Multiattribute value functions can be utilized for the same task as proposed in Dyer and Sarin [15], which is further discussed in Keeney and Raiffa [16]. This approach, which incorporates utility theory, is also explored in the context of MCDA. Morton et al. [17] present a formal framework utilizing MCDA methods for the task of portfolio selection. MCDA is widely applied within the transportation sector, as presented in the state-of-the-art literature review by Yannis et al. [18].

Morton [19] describes a decision analysis process that uses criteria weight elicitation and presents an illustrative example of its usage. Salo and Hämäläinen [20] present an approach for eliciting weights using imprecise ratio statements. Their approach enables robust decision analysis in the absence of complete preference information. Other approaches for tackling uncertainty in MCDA include the decision rules *minimax regret* and *maximin* (see, e.g., Greco et al. [21]) and uncertainty sets (see, e.g., Bertsimas and Brown [22]).

2.4 Portfolio Optimization

Salo et al. [23] outline the history of portfolio optimization in both finance and operations research. They define portfolios as collections of individual assets or projects. Portfolios are often associated with costs and benefits (e.g., cash flows or improving system reliability). Salo et al. define portfolio optimization as the use of mathematical programming methods to support the selection of a portfolio of assets, taking into account the preferences of the decision maker and any possible constraints on the chosen portfolio. When constructing a portfolio, there is often a large number of assets or projects to choose from; thus, the number of portfolios is also often large. The basis for modern portfolio theory is the Markowitz model [24], which identifies efficient portfolios of financial assets by maximizing the expected return of the portfolio while minimizing the variance of the returns. Efficient portfolios have since been adapted to various fields evidenced by, for example, Salo et al. [25], Levine [26], and Salo et al. [23].

Ghasemzadeh et al. [27] formulate project portfolio selection as an optimization problem with binary variables that represent the selection of projects or assets in the portfolio, thereby establishing a link between the Markowitz model and operations research. It provides a foundation that later work, such as Liesiö et al. [28], has utilized in their research in portfolio optimization. Liesiö et al. [28] present a MCDA technique called robust portfolio modelling to handle incomplete information in portfolio selection. They define information sets to represent incomplete information and then identify the non-dominated portfolios within the information set to support robust decision-making. This approach is extended in Liesiö et al. [29] to cases with possible project interdependencies and in the absence of complete cost information. They introduce the concept of the core index, which is defined for a project as the proportion of non-dominated portfolios that contain it. A core index of one identifies a core project, which is included in all non-dominated portfolios, while an index of zero identifies an exterior project, which is included in none. They propose that these core projects can be recommended to a rational DM regardless of their preferences, and those projects that do not belong to any non-dominated portfolios can be discarded from the selection. De la Barra et al. [30] use MCDA for portfolio optimization to select reinforcement actions in infrastructure networks. They also employ core indexes of reinforcement action to create recommendations to the DM.

De la Barra et al. [31] explore the reinforcement of transportation networks through cost-efficient portfolios of reinforcement actions, which reduce the disruption probability of nodes in the network. A general framework for modeling transportation networks, comparing portfolios, and an algorithm for identifying the set of cost-efficient portfolios is presented. In their paper, they apply the framework to a case study on identifying the cost-efficient portfolios for reinforcing the railway switches of an illustrative Finnish railway station.

2.5 Hierarchical Optimization

Hierarchical optimization can be used to address problems that have multiple levels of decision makers, as discussed in Anandalingam and Friesz [32]. Additionally, hierarchical optimization can be useful in complex problems where solving the entire problem at once is computationally intractable (see, e.g., Sobieszczanski-Sobieski [33]). These large and complex problems are often partitioned into subproblems, which are then solved separately and combined into a solution for the whole problem. A key class of hierarchical optimization is bi-level optimization (see, e.g., Bard [34], Sinha et al. [35]). There are also more complex hierarchical optimization methods such as tri-level optimization and decomposition based approaches (see, e.g., Benders [36]). Many of these approaches are not guaranteed to yield optimal solutions for all problems. If an approximate solution is acceptable, hierarchical optimization can still be suitable for problems that cannot be partitioned into separate subproblems such that the optimal solutions of the subproblems do not depend on each other.

Bi-level optimization models have been applied to problems related to transportation networks (see, e.g., Fan and Machemehl [37], Patriksson [38]). Jing et al. [39] introduce a hierarchical optimization approach to improve path finding in transportation networks by partitioning a large network into smaller separate networks. Du et al. [40] solve a dynamic pickup and delivery problem in transportation networks using their proposed hierarchical optimization framework.

3 Modelling Transportation Networks

This thesis considers transportation networks that enable the transportation of people or goods between a set of terminal pairs. Each terminal pair is associated with a traffic volume. Additionally, these transportation networks are assumed to be partitionable into $k \in \mathbb{Z}_+$ disjoint parts called subnetworks.

3.1 Transportation Networks

Let the graph $G = (V, E)$ represent a transportation network, where $V = V^{\mathrm{S}} \cup V^{\mathrm{T}}$ denotes the set of nodes consisting of the physical nodes V^{S}, which represent, for example, road intersections or railway switches, and the terminal nodes V^{T}, which are used for modelling the start and end points for the traffic of the network. The number of physical nodes is $N = |V^{\mathrm{S}}|$, and the set of undirected edges is $E \subseteq \{(v, v') \mid v, v' \in V\}$, which represent, for example, roads or railway tracks, between the nodes of the network.

A terminal pair is a pair of terminal nodes $t = (v_1, v_n) \in \mathcal{T}$, where $v_1, v_n \in V^{\mathrm{T}}$ and $v_1 \neq v_n$, for which transportation is to be enabled. We denote with $\mathcal{T}$ the set of all such terminal pairs of the transportation network G. Let a path from node $v_1 \in V^{\mathrm{T}}$ to node $v_n \in V^{\mathrm{T}}$ in the network be a sequence of distinct nodes, which enables transportation between terminal nodes v_1 and v_n such that there exists edges between consecutive nodes of the sequence. The formalization of this concept is below in Definition 3.1.

Definition 3.1. *A path is a sequence of distinct nodes $\pi = (v_1, v_2, \ldots, v_{n-1}, v_n)$ in a transportation network $G = (V, E)$ such that*

$$(v_1, v_2), \ldots, (v_{n-1}, v_n) \in E \wedge v_1, v_n \in V^T \wedge v_2, \ldots, v_{n-1} \in V^S.$$

The set of all paths in the transportation network is $\mathcal{P}$. The set of paths that enable transportation for terminal pair t is $\mathcal{P}^t = \{\pi_1^t, \ldots, \pi_s^t\} \subseteq \mathcal{P}$, where s is the number of those paths. Figure 1 presents a path $\pi = (v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8)$ for the terminal pair $t = (v_1, v_8)$ in an illustrative transportation network.

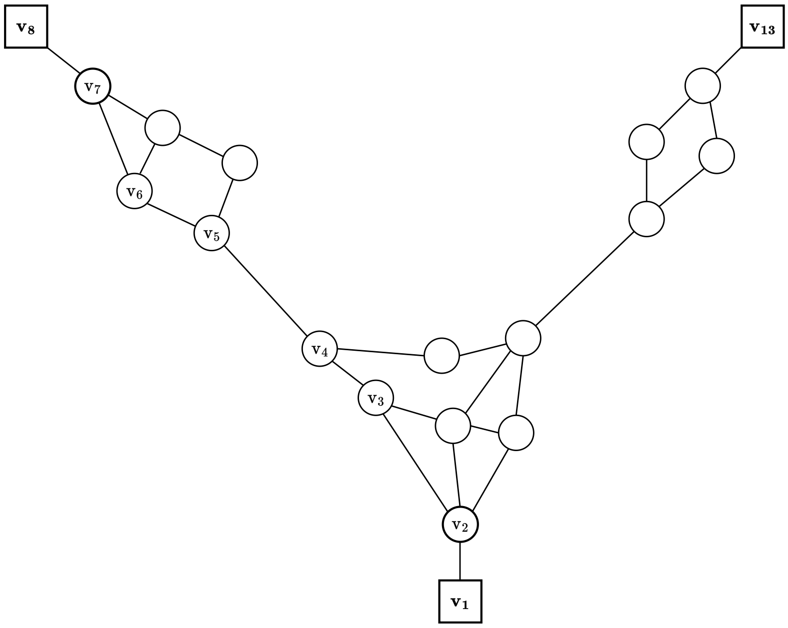

Figure 1: A path from v_1 to v_8 in an illustrative transportation network.

The traffic volume (e.g., people or goods) between the terminal pairs is represented with the vector $f = (f^t)_{t \in \mathcal{T}}$, where $f^t \in \mathbb{R}_+$ denotes the traffic volume corresponding to terminal pair $t \in \mathcal{T}$.

In this thesis, we consider transportation networks that can be partitioned into $k \in \mathbb{Z}_+$ subnetworks $G_1, \ldots, G_k$. These subnetworks, for example, may represent railway stations in a railway network, where the nodes of the subnetwork are railway switches or other railway infrastructure that enables transportation in the network. The terminal nodes and the edges connected to them are added to each subnetwork to model traffic flow across multiple subnetworks. Figure 2 shows an illustrative network that is partitioned into three subnetworks, where the terminal nodes v_9, v_{10}, v_{11}, and v_{12} were added.

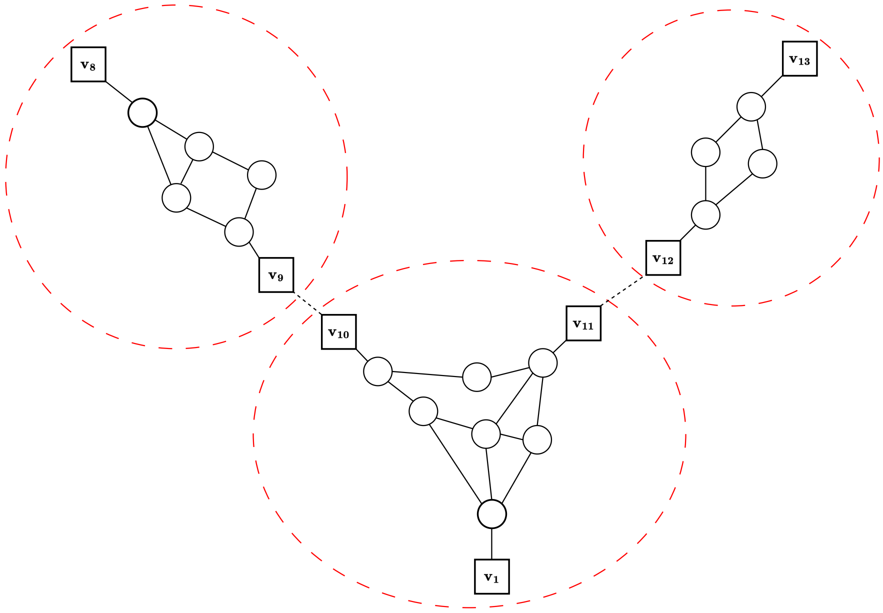

Figure 2: An illustrative transportation network partitioned into three subnetworks.

Each subnetwork $G_j = (V_j, E_j)$ consists of a set of nodes $V_j = V_j^{\mathrm{S}} \cup V_j^{\mathrm{T}}$, where $V_j^{\mathrm{S}} \subseteq V^{\mathrm{S}}$ denotes a subset of the physical nodes of the transportation network G, and V_j^{T} is the set of terminal nodes of the jth subnetwork. Similarly, the set of edges of the subnetwork is $E_j = E_j^{\mathrm{S}} \cup E_j^{\mathrm{T}}$, where $E_j^{\mathrm{S}} = \{(v_1, v_2) \mid v_1, v_2 \in V_j^{\mathrm{S}} \wedge (v_1, v_2) \in E\} \subseteq E$ is a subset of the original edges and $E_j^{\mathrm{T}} \subseteq V_j^{\mathrm{S}} \times V_j^{\mathrm{T}}$ denotes the set of edges connecting nodes $v \in V_j^{\mathrm{S}}$ to terminal nodes $v \in V_j^{\mathrm{T}}$. Lastly, $N_j = |V_j^{\mathrm{S}}|$ denotes the number of physical nodes in the jth subnetwork.

The set of terminal pairs of the jth subnetwork is $\mathcal{T}_j \subset V_j^{\mathrm{T}} \times V_j^{\mathrm{T}}$. The vector of traffic volumes of the terminal pairs in subnetwork G_j is $f_j = (f_j^t)_{t \in \mathcal{T}_j}$.

3.2 Disruptions in Transportation Networks

In this thesis, the nodes of the transportation network may get disrupted, while the edges may not. We assume that the physical nodes $v \in V^{\mathrm{S}}$ of the network G are either operational or disrupted. We denote with $O(v) = x_v$ the operational status of node $v \in V^{\mathrm{S}}$, where $x_v = 1$ denotes that it is operational and $x_v = 0$ that it is disrupted. The binary state vector $x = (x_1, \ldots, x_N) \in \{0, 1\}^N$ represents the operational status of all physical nodes $v \in V^{\mathrm{S}}$ of the transportation network. The terminal nodes cannot be disrupted, since they are not physical.

Disrupted nodes in the network may impact the operational status of the paths in the network. A path π is operational, which is denoted by $O(\pi) = 1$, if and only if all of the nodes in π are operational. Thus $O(\pi) = 1 \Leftrightarrow \bigwedge_{v\in\pi}[O(v) = 1]$. Conversely, the path π is disrupted, which is denoted by $O(\pi) = 0$, if at least one of its nodes is disrupted, which is equivalent to the event $\bigvee_{v\in\pi}[O(v) = 0]$.

Similarly a terminal pair t is operational, which is denoted by $O(t) = 1$, if at least one path in $\mathcal{P}^t$ is operational, therefore $O(t) = 1 \Leftrightarrow \bigvee_{\pi\in\mathcal{P}^t}[O(\pi) = 1]$. Conversely, it is disrupted if and only if all paths in $\mathcal{P}^t$ are disrupted, which is denoted by $O(t) = 0 \Leftrightarrow \bigwedge_{\pi\in\mathcal{P}^t}[O(\pi) = 0]$.

3.3 Portfolios of Reinforcement Actions in Subnetworks

Reinforcement actions reduce the disruption probabilities of the nodes. A portfolio is a combination of reinforcement actions, which for the jth subnetwork is represented by a binary vector $q_j = (q_{j,v})_{v\in V_j^{\mathrm{S}}} \in \{0,1\}^{N_j} = \mathcal{Q}_j$, where $q_{j,v} = 0$ indicates that node $v \in V_j^{\mathrm{S}}$ is not reinforced in q_j and $q_{j,v} = 1$ indicates that it is. Additionally, $\mathcal{Q}_j$ denotes the set of all possible portfolios for the jth subnetwork.

Each reinforcement action has a cost vector $c_v = (c_{v,1}, \ldots, c_{v,r}) \in \mathbb{R}_+^r$ associated with it, where r is the number of different types of required resources and $c_{v,i}$ indicates how much of the ith resource type the reinforcement of node $v \in V_j^{\mathrm{S}}$ requires. Additionally, we assume that the DM has limited resources at their disposal, which is represented with the budget vector $b = (b_1, \ldots, b_r) \in \mathbb{R}_+^r$, where $b_i \in \mathbb{R}_+$ indicates the units of the ith resource type the DM has at their disposal. In this thesis, the cost vector of a portfolio q_j is

$$C(q_j) = \sum_{v\in V_j^{\mathrm{S}}} c_v q_{j,v} \in \mathbb{R}_+^r. \tag{1}$$

Those portfolios, which do not exceed the limited amount of resources, are feasible portfolios, as characterized by Definition 3.2.

Definition 3.2. *Portfolio q_j is feasible if and only if*

$$C(q_j) \leq b,$$

where $\leq$ denotes the componentwise less than or equal to operator. The set of all feasible portfolios for the subnetwork G_j is

$$\mathcal{Q}_j^F = \{q_j \in \mathcal{Q}_j \mid C(q_j) \leq b\} \subseteq \mathcal{Q}_j.$$

3.4 Probabilities of Operational Statuses in Subnetworks

The probability that the node $v \in V_j^{\mathrm{S}}$ is disrupted, given that portfolio q_j has been implemented, is

$$\mathbb{P}[O(v) = 0 \mid q_j] = \alpha_v - \delta_v q_{j,v}, \tag{2}$$

where $\alpha_v \in (0, 1]$ is the disruption probability of node v without reinforcement and $\delta_v \in (0, \alpha_v]$ is the extent to which the reinforcement of node $v \in V_j^{\mathrm{S}}$ reduces its disruption probability. The probability that a path π belonging to the jth subnetwork is operational, given that portfolio q_j has been implemented is

$$\mathbb{P}[O(\pi) = 1 \mid q_j] = \mathbb{P}\left[\bigwedge_{v\in\pi}[O(v) = 1] \,\middle|\, q_j\right] = \prod_{v\in\pi}\mathbb{P}[O(v) = 1 \mid q_j]. \tag{3}$$

The probability that the terminal pair $t \in \mathcal{T}_j$ is operational, given that portfolio q_j has been implemented, is the terminal pair reliability, as defined by Yoo and Deo [11], and is given by

$$\mathbb{P}[O(t) = 1 \mid q_j] = \mathbb{P}\left[\bigvee_{\pi\in\mathcal{P}^t}[O(\pi) = 1] \,\middle|\, q_j\right]. \tag{4}$$

This probability can be computed using, for example, the modified Dotson algorithm presented by Yoo and Deo [11], which considers edge disruptions instead of node disruptions. We adapt their algorithm to node disruptions. Consider an alternative form for terminal pair reliability

$$\mathbb{P}[O(t) = 1 \mid q_j] = \sum_{i=1}^{s}\mathbb{P}\left[O(\pi_i^t) = 1 \,\middle|\, A_{i-1}, q_j\right]\cdot\mathbb{P}[A_{i-1} \,|\, q_j], \tag{5}$$

where we denote the event $A_k = \bigwedge_{m=1}^{k}[O(\pi_m^t) = 0],\ k = 0, \dots, s$, with A_0 denoting the sure event and $s = |\mathcal{P}^t|$. Proof for this formula is in Appendix A. This probability for terminal pair $t \in \mathcal{T}_j$ may be computed with Algorithm 1. Below $\pi_m^t[k]$ denotes the kth element in the path π_m^t.

Algorithm 1 Terminal pair reliability

Output: R_t

$R_t \leftarrow 0$
$X_0 \leftarrow ()$
$Y_0 \leftarrow ()$
$W \leftarrow \text{Queue}()$
$W.\text{push}((X_0, Y_0))$
$S \leftarrow \{(X_0, Y_0)\}$
while W is not empty **do**
  $X, Y \leftarrow W.\text{pop}()$
  for $m \leftarrow 1$ to s **do**
    if $\forall v \in \pi_m^t : v \notin Y$ **then**
      for $k \leftarrow 2$ to $|\pi_m^t| - 1$ **do**
        $v \leftarrow \pi_m^t[k]$
        if $v \notin X$ **then**
          Append v to X
        end if
      end for
      $R_t \leftarrow R_t + \prod_{v \in X} \mathbb{P}[O(v) = 1 \mid q_j] \cdot \prod_{v \in Y} \mathbb{P}[O(v) = 0 \mid q_j]$
      for $k \leftarrow 2$ to $|\pi_m^t| - 1$ **do**
        $X' \leftarrow X$
        for $l \leftarrow k$ to $|\pi_m^t| - 1$ **do**
          Remove $\pi_m^t[l]$ from X'
        end for
        $Y' \leftarrow Y$
        Append $\pi_m^t[k]$ to Y'
        if $(X', Y') \notin S$ **then**
          $W.\text{push}((X', Y'))$
          $S \leftarrow S \cup \{(X', Y')\}$
        end if
      end for
      break
    end if
  end for
end while
return R_t

First, in Steps 1–5, the terminal pair reliability R_t is initialized to zero, the list of operational used nodes X_0 and the list of disrupted nodes Y_0 are initialized to be empty lists, a queue W is initialized to contain only the tuple (X_0, Y_0) and the tuple is also added to the set of visited tuples S, which is used to track which tuples have been explored to not explore them again. After that, the algorithm proceeds to the loop. The first element (X, Y) is popped from the queue in Step 8. Then the list of paths is scanned through until we find a path that is not disrupted. If such path π^t_m is found, X is modified by adding each physical node v in the path π^t_m to X, which can be done by excluding the first and last node of the path from the loop since by definition they are terminal nodes while the rest are physical nodes. Now, in Step 16, the terminal pair reliability is incremented by the probability of all nodes in X being operational and all nodes in Y being disrupted. Then, in Steps 17–28, the physical nodes of the found path are looped through, where in the kth iteration in Step 18, we copy X to X' and then in, Steps 19–21, the nodes $\pi^t_m[k], \dots, \pi^t_m[|L|]$ are removed from X', where $L = |\pi^t_m|$ is the length of the path π^t_m. In Step 22, Y is copied to be Y' and then $\pi^t_m[k]$ is appended to it in Step 23. If the tuple (X', Y') has not been explored yet, it is added to the queue and marked as explored in Steps 24–27. After that, the algorithm breaks to not explore further operational paths and continues to the next element in the queue. The algorithm terminates once the queue becomes empty and then returns the computed terminal pair reliability R_t in Step 33. The algorithm explores all mutually exclusive events where at least one path is operational, ensuring each operational scenario is counted toward the terminal pair reliability exactly once.

The following example illustrates Algorithm 1. Consider the illustrative subnetwork in Figure 3 with five physical nodes v_5, v_6, v_7, v_{14}, and v_{15}, which all have a disruption probability of 0.2. There are two terminal nodes v_8 and v_9, which form the only terminal pair $t = (v_8, v_9)$. There are four paths corresponding to the terminal pair t: $\pi^t_1 = (v_8, v_7, v_6, v_5, v_9)$, $\pi^t_2 = (v_8, v_7, v_{14}, v_{15}, v_5, v_9)$, $\pi^t_3 = (v_8, v_7, v_6, v_{14}, v_{15}, v_5, v_9)$, and $\pi^t_4 = (v_8, v_7, v_{14}, v_6, v_5, v_9)$.

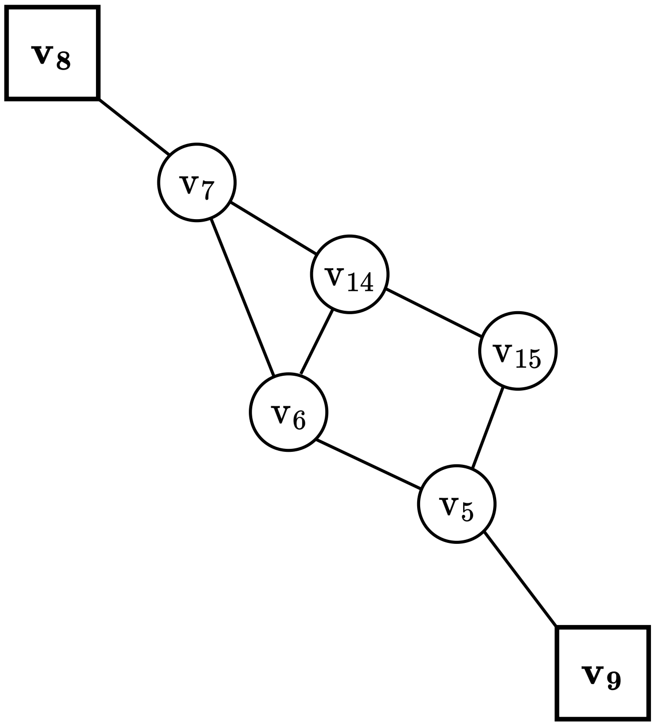

Figure 3: An illustrative transportation network.

At first, all nodes can be in either state so $(X_0, Y_0) = ((), ())$. The path $\pi_1^t = (v_8, v_7, v_6, v_5, v_9)$ is operational, so X becomes (v_7, v_6, v_5). Now we increment the terminal pair reliability by $\prod_{v \in X} \mathbb{P}[O(v) = 1] \cdot \prod_{v \in Y} \mathbb{P}[O(v) = 0] = 0.8^3 \cdot 1$. Then we disrupt the path π_1^t. We add the three following tuples to the queue: $((), (v_7))$, $((v_7), (v_6))$, and $((v_7, v_6), (v_5))$. In the next iteration $(X, Y) = ((), (v_7))$, but in this case there are no operational paths, so we proceed to the next iteration, where now $(X, Y) = ((v_7), (v_6))$. Now the path π_2^t is operational, so X becomes $(v_7, v_{14}, v_{15}, v_5)$ and Y is still (v_6). Now we increment the terminal pair reliability by $\prod_{v \in X} \mathbb{P}[O(v) = 1] \cdot \prod_{v \in Y} \mathbb{P}[O(v) = 0] = 0.8^4 \cdot 0.2$. Disrupting this path yields us events, which we have already explored, so they are not added to the queue. The last element in the queue is $(X, Y) = ((v_7, v_6), (v_5))$. And in this case there are no operational paths. Since the queue is now empty, the algorithm stops and returns the terminal pair reliability $R_t = 0.8^3 + 0.8^4 \cdot 0.2 = \frac{1856}{3125}$.

3.5 Subnetwork Optimization Model

In this thesis, the objective function is to maximize the enabled traffic volume by expectation between the terminal pairs $\mathcal{T}_j$ for each subnetwork G_j while minimizing the cost vector of the implemented portfolio of reinforcement actions. The problem for the subnetwork G_j is formulated as the following multi-objective optimization problem with $1 + r$ objectives

$$\max_{q_j \in Q_j^{\mathrm{F}}} \begin{bmatrix} \sum_{t \in \mathcal{T}_j} f_j^t \cdot \mathbb{P}[O(t) = 1 \mid q_j] \\ -\sum_{v \in V_j^{\mathrm{S}}} c_v q_{j,v} \end{bmatrix}. \tag{6}$$

Solving the multi-objective optimization problem in Equation (6) requires that the set of cost-efficient portfolios is computed. The concept of cost-efficiency of portfolios is defined in Definition 3.3.

Definition 3.3 (Cost-efficient portfolios)**.** *A portfolio $q'_j \in Q_j^F$ is cost-efficient if there does not exist $q''_j \in Q_j^F$ for which*

$$\sum_{t \in \mathcal{T}_j} f_j^t \cdot \mathbb{P}[O(t) = 1 \mid q'_j] \le \sum_{t \in \mathcal{T}_j} f_j^t \cdot \mathbb{P}[O(t) = 1 \mid q''_j] \tag{7}$$

$$C(q''_j) \le C(q'_j), \tag{8}$$

with at least one strict inequality either in Equation (7) or for at least one resource type in Equation (8). If such portfolio q''_j exists, then q''_j dominates q'_j, denoted with $q''_j \succ q'_j$. The set of cost-efficient portfolios of subnetwork G_j is denoted with Q_j^{CE}.

The sets of cost-efficient portfolios $Q_1^{\mathrm{CE}}, \ldots, Q_k^{\mathrm{CE}}$ can be computed, for example, by adapting the procedure presented by de la Barra et al. [31] for each instance of the optimization problem presented in Equation (6).

3.6 Hierarchical Model

We propose the following hierarchical optimization model in Equations (9a)–(9c). In this model, the optimization model in Section 3.5 is first solved for each subnetwork G_j of the transportation network G. We assume that the DM seeks to maximize the expected enabled traffic volume between the terminal pairs $\mathcal{T}$ of the transportation network G and minimize the cost of the chosen combination of portfolios subject to the constraint of the budget vector b. This problem is formulated as the optimization problem

$$\max_{q_1,\ldots,q_k} \left[\sum_{t\in\mathcal{T}} f^t \cdot \mathbb{P}[O(t) = 1 \mid q_1, \ldots, q_k] - \sum_{j=1}^{k} C(q_j) \right], \tag{9a}$$

$$\text{s.t.} \sum_{j=1}^{k} C(q_j) \leq b, \tag{9b}$$

$$q_j \in Q_j^{\mathrm{CE}}, \ \forall j \in \{1, \ldots, k\}. \tag{9c}$$

The constraint (9b) ensures that the total cost of the selected portfolios $q_1, \ldots, q_k$ does not exceed the budget vector b for any resource type. The constraint (9c) ensures that the selected portfolios are cost-efficient in their corresponding subnetwork.

We propose the Algorithm 2 to solve the multi-objective optimization problem in Equations (9a) – (9b). For simplicity denote by $Q_j = (q_1, \ldots, q_j), \forall j \in \{2, \ldots, k\}$ a combined portfolio, where $q_i \in Q_i^{\mathrm{CE}}, \ \forall i \in \{1, \ldots, j\}$.

Algorithm 2 Cost-efficient combined portfolios

Output: Q^{CE}

1: Compute Q_j^{CE}, for all $j \in \{1, \ldots, k\}$
2: $Q_1^* \leftarrow Q_1^{\mathrm{CE}}$
3: **for** $j \leftarrow 2$ to k **do**
4: $\quad Q_j \leftarrow \{(q_1, \ldots, q_{j-1}, q_j) \mid (q_1, \ldots, q_{j-1}) \in Q_{j-1}^* \wedge q_j \in Q_j^{\mathrm{CE}}\}$
5: $\quad Q_j \leftarrow \{(q_1, \ldots, q_j) \in Q_j \mid \sum_{i=1}^{j} C(q_i) \leq b\}$
6: $\quad$ Compute $\sum_{t\in\mathcal{T}} f^t \cdot \mathbb{P}[O(t) = 1 \mid q_1, \ldots, q_j]$ for all $(q_1, \ldots, q_j) \in Q_j$
7: $\quad Q_{j-1}^{\mathrm{D}} \leftarrow \{Q' \in Q_{j-1}^* \mid \exists Q'' \in Q_j : Q'' \succ Q'\}$
8: $\quad Q_{j-1} \leftarrow \{(q_1, \ldots, q_{j-1}, \bar{0}) \mid (q_1, \ldots, q_{j-1}) \in Q_{j-1}^* \setminus Q_{j-1}^{\mathrm{D}}\}$
9: $\quad Q_j^{\mathrm{D}} \leftarrow \{Q' \in Q_j \mid \exists Q'' \in Q_{j-1} : Q'' \succ Q'\}$
10: $\quad Q_j \leftarrow Q_j \setminus Q_j^{\mathrm{D}}$
11: $\quad Q_j \leftarrow Q_j \cup Q_{j-1}$
12: $\quad Q_j^* \leftarrow Q_j \setminus \{Q' \in Q_j \mid \exists Q'' \in Q_j : Q'' \succ Q'\}$
13: **end for**
14: $Q^{\mathrm{CE}} \leftarrow Q_k^*$
15: **return** Q^{CE}

In Step 1, the sets of cost-efficient portfolios are computed for each subnetwork using the algorithm in de la Barra et al. [31]. First, Q_1^* is initialized with the cost-efficient portfolios of the first subnetwork. Then, in Steps 3–13, the remaining subnetworks are examined one by one. In Step 4, Q_j is initialized with all of those combined portfolios $(q_1, \ldots, q_{j-1}, q_j)$ for which $(q_1, \ldots, q_{j-1})$ belongs to the previously found set of cost-efficient combined portfolios Q^*_{j-1} and q_j belongs to the set of cost-efficient portfolios of the jth subnetwork. In Step 5, the infeasible combined portfolios from Q_j are removed, and after that, in Step 6, the expected enabled traffic volume is computed for each combined portfolio in Q_j. In Steps 7–8, those previously found combined portfolios in Q^*_{j-1} which are dominated by combined portfolios in Q_j are filtered out. The remaining combined portfolios are padded with the trivially cost-efficient portfolio, the zero vector $\bar{0} \in Q_j^{\mathrm{CE}}$, to keep the dimensions consistent. Conversely, in Steps 9–10, the dominated combined portfolios in Q_j are filtered out. In Step 11, the combined portfolios in Q_{j-1} are added to the set Q_j. Then, in Step 12, the combined portfolios in Q_j that are dominated by combined portfolios in the same set are filtered out. In Step 14, Q^*_k is saved as the set of cost-efficient combined portfolios, which is then returned in Step 15.

4 Application to Ten Railway Stations in Finland

To illustrate the hierarchical optimization model, consider the transportation network comprising ten stations surrounding the Siilinjärvi train station in Northern Savonia, Finland. We seek to compute the cost-efficient portfolios of reinforcement actions for the railway switches, which are mechanical devices that enable a train to switch from one railway track to another in the network. The network consisting of these ten stations is shown in Figure 4 as a graph. Red dots represent physical nodes, most of which are railway switches. There are other types of nodes in the network, such as buffer stops, which we do not consider here. The blue lines represent the railway track connecting the physical nodes, and each black square represents a terminal node, which acts as a connection to other parts of the Finnish railway network and is used to model transportation to or from outside this part of the network. Terminal nodes of the subnetworks are omitted for clarity. Additionally, the subnetworks have been circled with black dotted lines and labeled with their abbreviations. The ten stations, their respective number of physical nodes, and terminal pairs are presented in Table 1.

Table 1: The ten stations of the network.

Abbreviation	Station name	Physical nodes	Terminal pairs
TE	Taipale	2	2
LNA	Lapinlahti	8	2
APT	Alapitkä	6	2
SIJ	Siilinjärvi	37	3
SKM	Sänkimäki	4	2
KNH	Kinahmi	4	2
JKI	Juankoski	4	2
TOI	Toivala	4	2
SOR	Sorsasalo	4	2
KUO	Kuopio	42	2
	Total	115	33

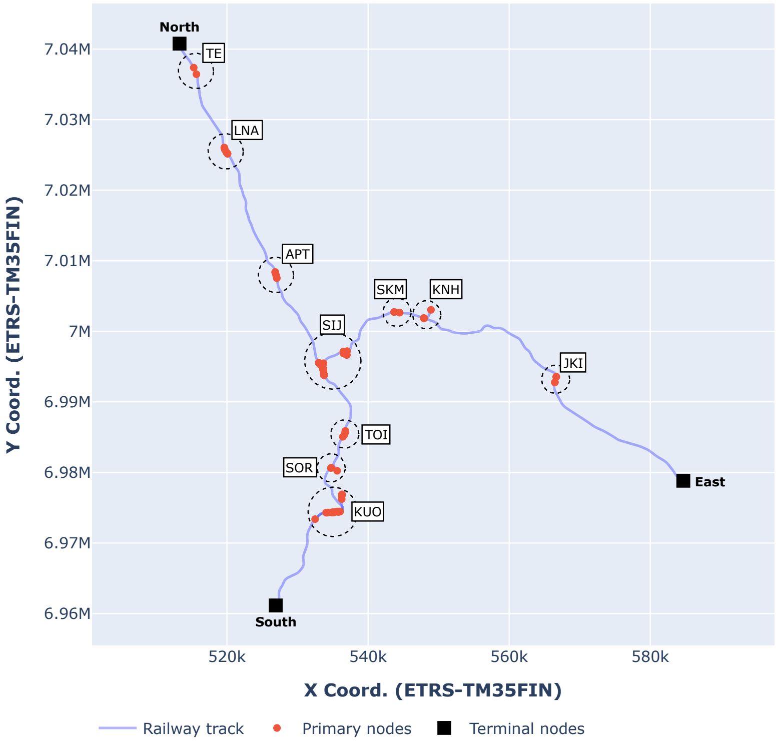

Figure 4: Transportation network including ten stations.

Terminal pairs of the network are pairs of terminal nodes for which traffic is to be enabled. Since the graph is undirected, we do not consider traffic direction separately. This reduction via symmetry gives a total of 36 terminal pairs listed in Table B1 with their corresponding yearly traffic volumes, measured in the number of trains, for 2024 (data from Fintraffic [41]), which are visualized as the heatmap in Figure 5.

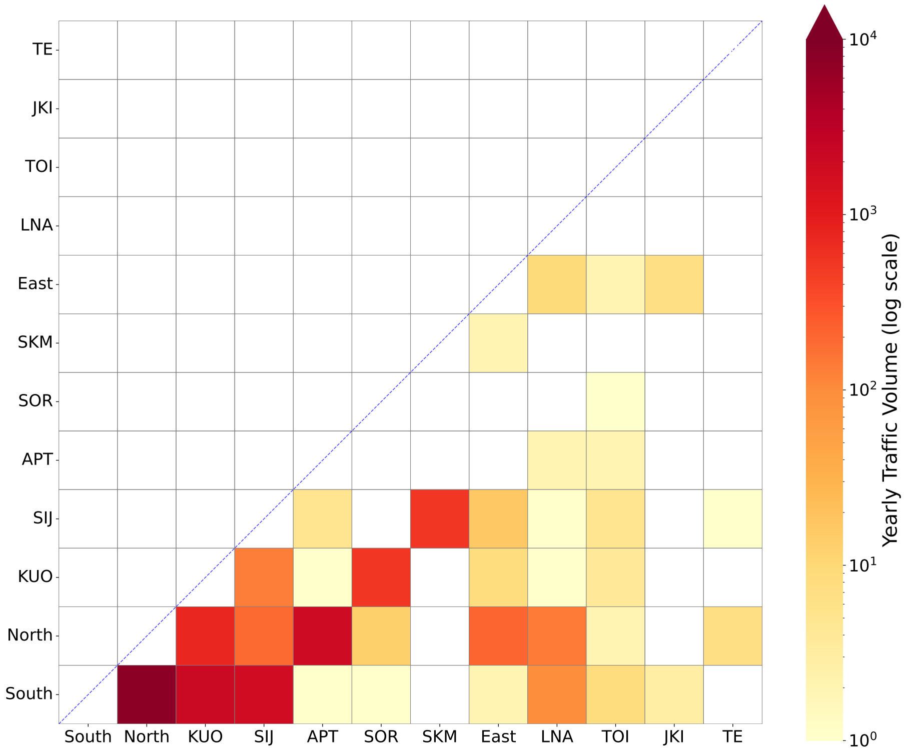

Figure 5: Yearly number of trains of the network.

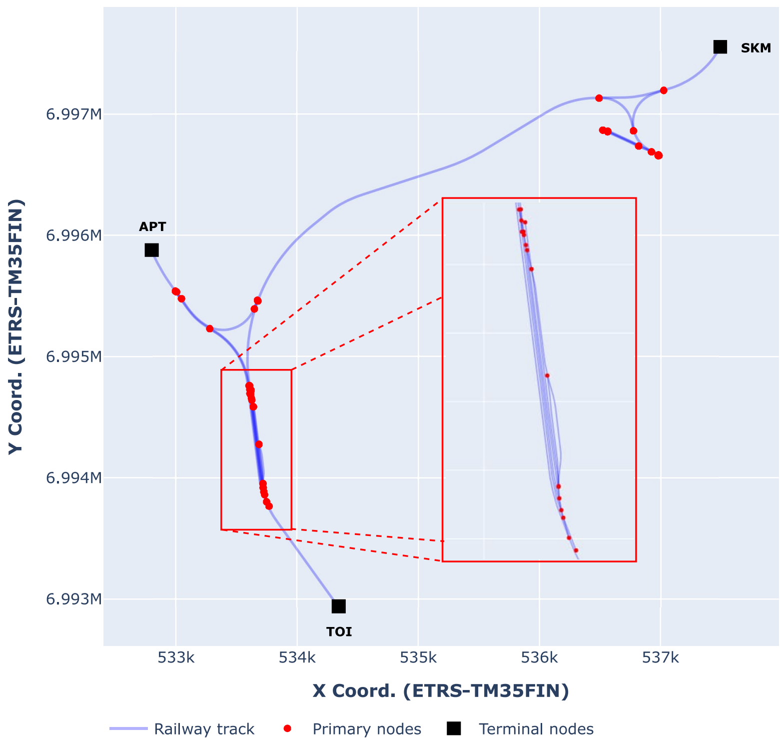

Figure 6: Siilinjärvi station.

Figure 6 presents the Siilinjärvi station as an undirected graph. Each red dot represents a physical node, most of which are railway switches. The black squares represent the terminal nodes, which act as connections to other stations. The edges of the graph are shown as blue lines, which represent the railway tracks connecting the switches. Yearly traffic volumes for the Siilinjärvi station are presented in Table 2, measured in the number of trains. These traffic volumes were derived for all terminal pairs of all ten stations from the traffic volume for the whole network in Table B1. For example, trains going from South to North in the network are included in all traffic volumes of those terminal pairs that belong to any path corresponding to the pair (South, North) in all subnetworks.

Traffic that originates or terminates within a subnetwork is modelled as beginning or ending at the closest terminal node of the adjacent subnetwork. For example, traffic corresponding to the terminal pair (South, SIJ) is modelled as: South $\rightarrow$ SOR in subnetwork KUO, then KUO $\rightarrow$ TOI in subnetwork SOR, and finally SOR $\rightarrow$ SIJ in subnetwork TOI. This volume is therefore not considered in the subnetwork SIJ itself.

Table 2: Yearly number of trains for the Siilinjärvi station.

Terminal pair	**Yearly number of trains**
(APT, TOI)	8498
(APT, SKM)	221
(SKM, TOI)	15

Due to the physical limitations of trains, we consider only paths with a maximum turning angle of 90 degrees or less. For example, consider the illustrative transportation network in Figure 7, where the feasible paths corresponding to the terminal pair (v_1, v_2) are in bold.

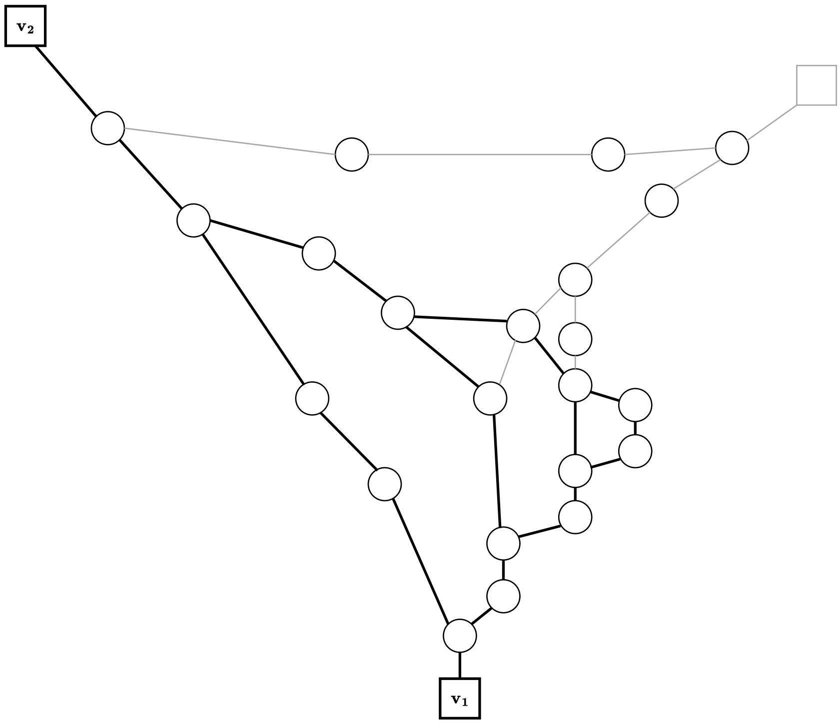

Figure 7: Feasible paths for the terminal pair (v_1, v_2) in an illustrative network.

Disruptions to railway switches may occur, for instance, due to the deterioration of their parts or extreme weather conditions. For illustrative purposes, we assume that all switches $v \in V_j^{\mathrm{S}}$ are identical and have an uniform disruption probability $\mathbb{P}[O(v) = 0 \mid q_j] = 0.01 - 0.005 \cdot q_{j,v}$, and each switch can be reinforced, which lowers its disruption probability from 0.01 to 0.005. Additionally, we assume that there is only one type of resource, and that reinforcing each switch costs one unit of that resource.

4.1 Cost-Efficient Portfolios for the Subnetworks

The number of cost-efficient portfolios for each subnetwork is presented in Table 3. The computations took approximately one minute to solve all ten optimization problems using the procedure presented by de la Barra et al. [31]. Most subnetworks have a small number of railway switches that can be reinforced, which yields a small number of possible portfolios to consider. The two larger subnetworks, Siilinjärvi and Kuopio, have a large number of possible portfolios to consider.

Table 3: Number of cost-efficient portfolios for each subnetwork.

Station	Number of cost-efficient portfolios
TE	3
LNA	3
APT	3
SIJ	24
SKM	3
KNH	2
JKI	3
TOI	3
SOR	2
KUO	22

Figure 8 presents the cost-efficient portfolios of Siilinjärvi. The most costly cost-efficient portfolio reinforces only 15 switches, which is less than the number of reinforcement actions available. This is explained by our choice of only modelling the traffic going through the subnetwork and assuming that it is enough for the train to get to any node of the subnetwork for it to be considered enabled traffic. Figure 9 presents the cost-efficient portfolios of Kuopio. In both of these two subnetworks, the expected enabled traffic volume increases marginally, which is to be expected due to the low disruption probability of railway switches. Cost-efficient portfolios are not presented for the rest of the subnetworks, because there are relatively few cost-efficient portfolios for them.

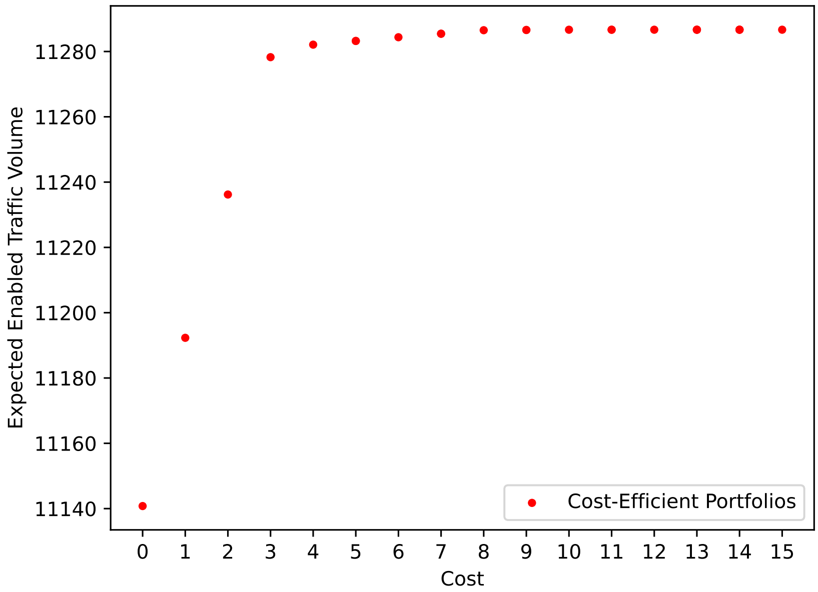

Figure 8: Cost-efficient portfolios for Siilinjärvi.

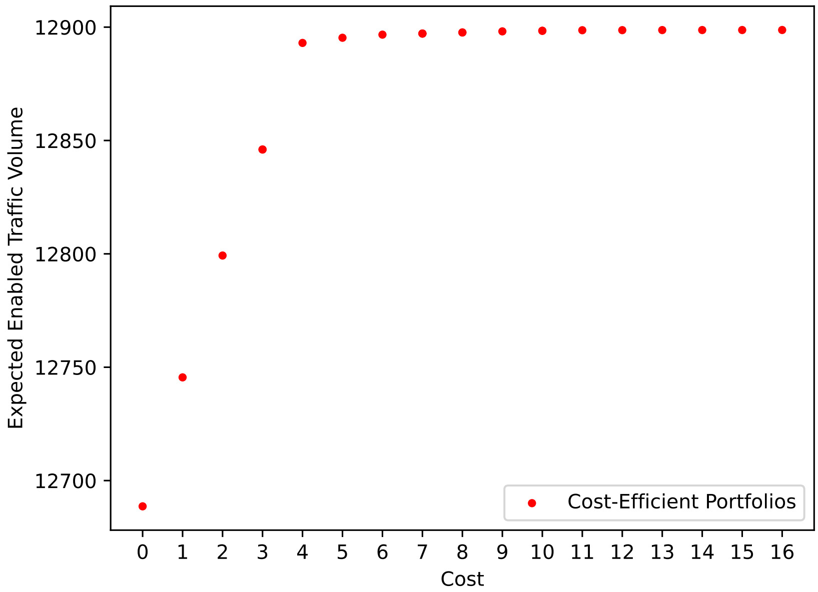

Figure 9: Cost-efficient combined portfolios for Kuopio.

4.2 Cost-Efficient Combined Portfolios

In total, there are 1,539,648 combined portfolios obtained via combining the cost-efficient portfolios of the subnetworks. Identifying the cost-efficient combined portfolios took a little over one minute with a modern desktop CPU using Algorithm 2. In total, there are 64 cost-efficient combined portfolios for which the expected enabled traffic volumes and associated costs are shown in Figure 10. Additionally, a random sample of 855 portfolios is presented in Figure 10 with black dots. The increase in expected enabled traffic volume begins to diminish for portfolios that reinforce more than 15 switches. There is an increase of around 5.9% in expected enabled traffic volume for the cost-efficient portfolios that reinforces 45 switches when compared to the baseline, where no switches are reinforced.

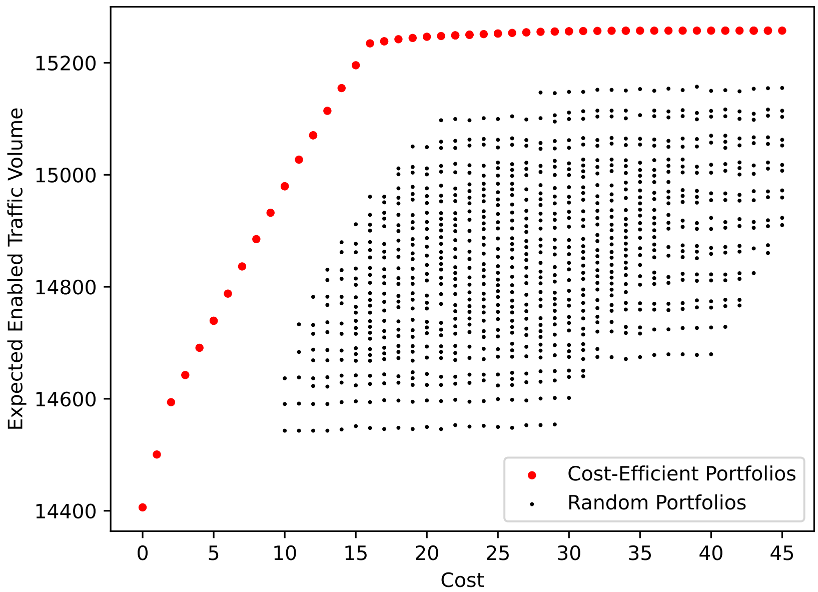

Figure 10: Cost-efficient combined portfolios for the whole network.

4.3 Selecting Switches to Reinforce

A rational DM should choose one of the cost-efficient combined portfolios at a budget level of their choice, but this is not straightforward when there are multiple alternatives. One approach to help select reinforcement actions is to study the composition of cost-efficient combined portfolios using the *core index* as proposed in Liesiö et al. [28]. The core index of the reinforcement of node $v \in V_j^S$ belonging to the

jth subnetwork for a budget level $\beta \in \mathbb{Z}_+^r$ is defined as the relative share of those cost-efficient combined portfolios, which have a cost equal to β, where node v is reinforced. We denote those cost-efficient combined portfolios, which have a cost of β with $\mathcal{Q}^{\mathrm{CE}}(\beta) = \{Q \in \mathcal{Q}^{\mathrm{CE}} \mid C(Q) = \beta\} \subset \mathcal{Q}^{\mathrm{CE}}$. Thus, the core index of the reinforcement of the railway switch $v \in V_j^{\mathrm{S}}$ for a budget level β is

$$\mathrm{CI}_j(v, \beta) = \frac{|\{(q_1, \dots, q_k) \in \mathcal{Q}^{\mathrm{CE}}(\beta) \mid q_{j,v} = 1\}|}{|\mathcal{Q}^{\mathrm{CE}}(\beta)|} \in [0, 1]. \tag{10}$$

A core index equal to 1 indicates that the reinforcement of node $v \in V_j^{\mathrm{S}}$ belongs to all cost-efficient combined portfolios for the budget level β, and therefore it can safely be recommended to the DM at the budget level β. Conversely, if the core index of the reinforcement of node v is equal to 0 in the same scenario, the corresponding reinforcement action can be disregarded, as it is not present in any cost-efficient combined portfolio at this budget level. For those nodes for which $0 < \mathrm{CI}_j(v, \beta) < 1$, one can not draw similar conclusions. The core indexes of railway switch reinforcement actions for all budget levels $\beta \in \{1, \dots, 45\}$ are in Figure 11, where those reinforcement actions, which were not present in any cost-efficient combined portfolios, were omitted. In total, there are 45 switches, which were present in at least one cost-efficient combined portfolio.

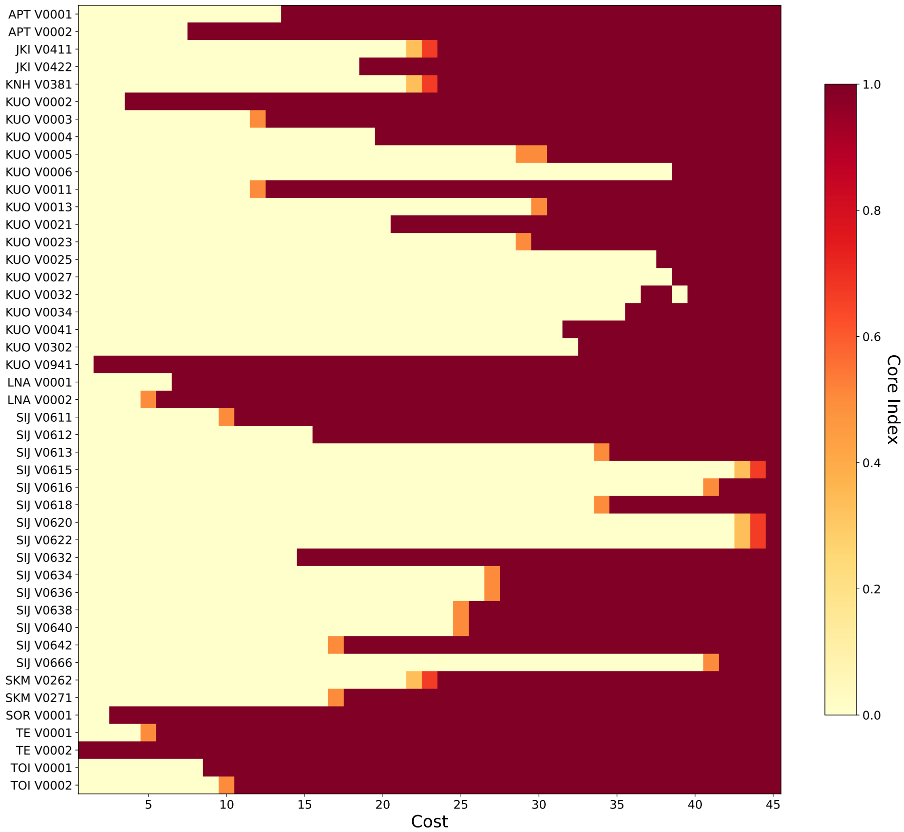

Figure 11: Core indexes of railway switches.

Due to the delegation of responsibilities in the maintenance of the subnetworks, it may not be required to recommend certain reinforcement actions, but to just allocate resources to the reinforcement of each subnetwork and let their respective DMs decide which reinforcement actions to implement. Let us devise a budget allocation plan based on the cost-efficient combined portfolios by examining the share of budget allocated to the subnetworks for all budget levels $\beta \in \{1, ..., 45\}$. The average relative shares of the budget allocated to the reinforcement of each subnetwork for all cost-efficient combined portfolios are presented in Figure 12.

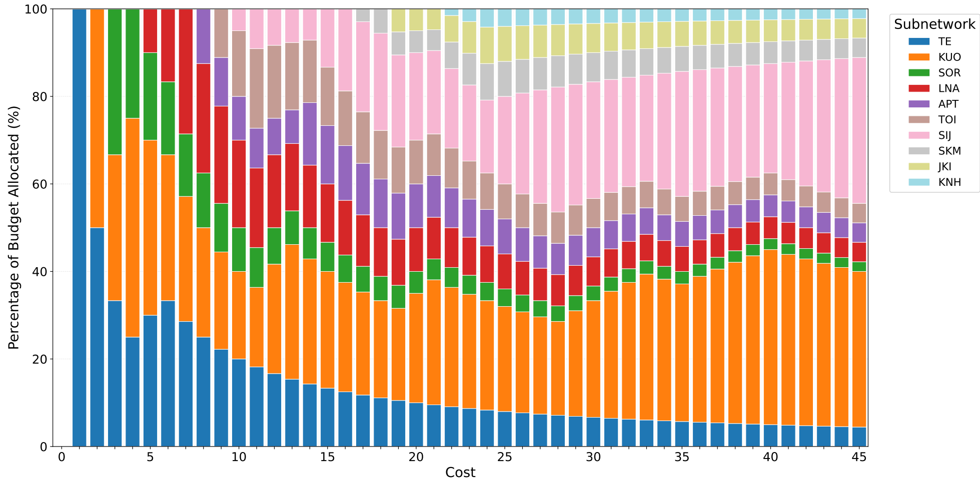

Figure 12: Relative share of budget allocated towards each subnetwork for all cost-efficient combined portfolios.

The budget allocation suggests heuristics to follow when suggesting how to allocate the budget to the different subnetworks. At budget levels less than 10 units, TE and KUO receive the majority of resources, which reflects their critical nature for enabling traffic flow in the network. The allocation of resources becomes more diversified across all subnetworks for the cost-efficient combined portfolios with a higher cost.

4.4 Sensitivity Analysis

Because the parameters may involve inaccuracies, sensitivity analysis should be conducted to study the robustness of the solutions subject to changes in the parameters. We conduct the sensitivity analysis on the traffic volumes, but not on the other parameters like disruption probabilities. This choice is further discussed in Section 5. One approach, for example, includes deriving some confidence intervals I_t for each traffic volume. With those, we can construct the corresponding uncertainty set $\mathcal{D}^f = \bigtimes_{t\in\mathcal{T}} I_t$ as the Cartesian product of the confidence intervals I_t. Since the expected traffic volume is a linear combination of the terminal pair reliabilities with the traffic volumes as the weights, and the uncertainty set is convex, it is sufficient to study the extreme points of the uncertainty set denoted with $\mathcal{D}^f_{\text{ext}}$ (see e.g., Liesiö et al. [29]). The uncertainty set is a hyperrectangle of dimension $|\mathcal{T}| = 36$ and it therefore has $2^{36} \approx 6.8 \cdot 10^{10}$ extreme points, making this approach computationally intractable.

A more straightforward approach is to study the effect of variation in each traffic volume separately, while keeping others fixed. For illustration, suppose that each traffic volume f_t varies by $\pm 10\%$. The results of this analysis are presented in the tornado diagram in Figure 13, where the resulting difference in total expected enabled traffic volume from the baseline is presented for one cost-efficient combined portfolio with a cost of 45 units presented in Appendix C. The expected enabled traffic volume changes the most when the largest traffic volume is varied.

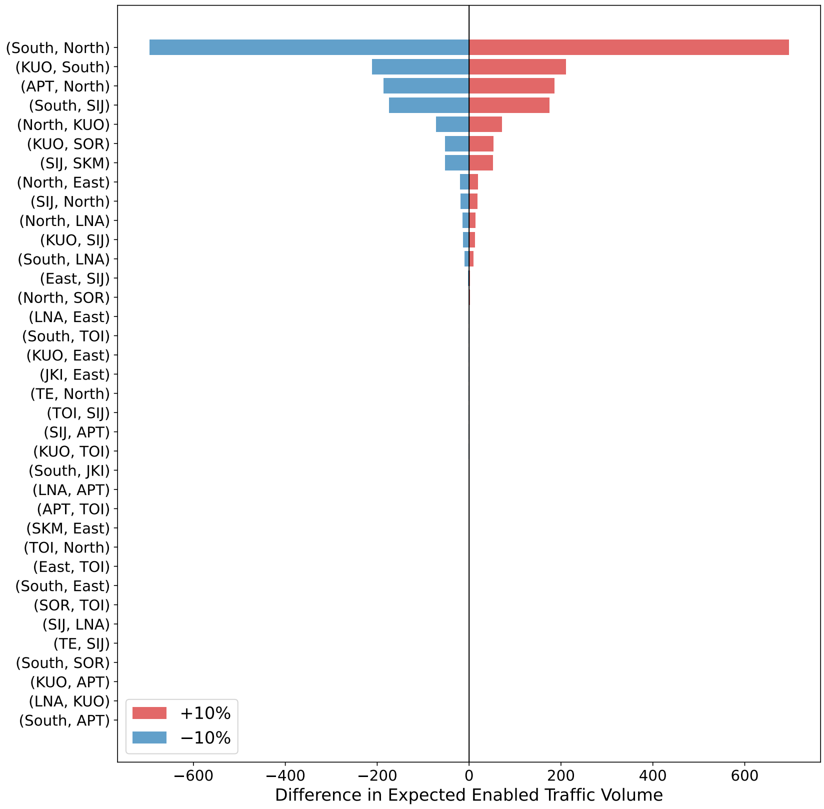

Figure 13: Sensitivity of each traffic volume.

5 Discussion

This thesis has developed a hierarchical portfolio optimization model for identifying cost-efficient portfolios of reinforcement actions in large transportation networks. The results from the case study indicate that the proposed hierarchical approach can reduce computation time while still being helpful in computing cost-efficient solutions. This is possible by considering only cost-efficient portfolios for each subnetwork separately and then combining them to compute cost-efficient combined portfolios for the entire transportation network.

The limitations of the model may limit its applicability; for example, the model may not be suitable for transportation networks that cannot be partitioned into subnetworks. Furthermore, while the algorithm proposed for solving the model is computationally efficient, it does not necessarily guarantee optimal solutions.

A potential extension to the work here is to consider also directed graphs, which may better represent transportation networks with asymmetric traffic flows. Additionally, one could also take edge disruptions into account, which would allow the framework to be extended to transportation networks, which may have both types of disruptions. Modelling common-cause disruptions could also be beneficial when seeking to model transportation networks in more detail. Interdependencies and cost synergies between the reinforcement actions could also be incorporated. To further decrease the computation time, one could utilize parallel computing to solve the subnetwork optimization models. This would enable the application of this model to even larger networks.

In the presented case study, given more computational power and fewer uncertain parameters, sensitivity analysis utilizing uncertainty sets could be computationally tractable. This could support more robust decision-making. We did not conduct sensitivity analysis on the disruption probabilities since it was assumed that the disruption probabilities in the case study were uniform; conducting sensitivity analysis on them would not necessitate a change in the composition of cost-efficient portfolios, as discussed in de la Barra et al. [31]. A similar conclusion can be made for the sensitivity of the reinforcement action costs. However, if these assumptions of uniformity in the disruption probabilities and reinforcement action costs do not hold, sensitivity analysis on them should be conducted to support robust decision-making.

Overall, the results from the case study indicate that hierarchical portfolio optimization is a promising approach for supporting decision-making in the reinforcement of large transportation networks. The approach seems to be computationally tractable, allowing the study of larger transportation networks than what would otherwise be possible. While the model has limitations, it provides a strong foundation for building more scalable and realistic models for the reinforcement of transportation networks.

6 Conclusions

The objective of this thesis was to develop a hierarchical portfolio optimization model to identify cost-efficient portfolios of reinforcement actions in large transportation networks. Motivated by the computational challenges related to large transportation networks, the proposed approach solves smaller subnetwork-level optimization problems that are combined into solutions for the entire transportation network. Additionally, this approach may represent the delegation of real-world administrative responsibilities better than approaches with a single decision maker.

The applicability of the proposed hierarchical portfolio optimization model was demonstrated with a case study involving ten railway stations in Finland. The results from this case study indicate that the hierarchical approach offers significant computational advantages. The conducted sensitivity analysis demonstrated robustness of the cost-efficient combined portfolios against uncertainty in the parameters of the model.

This thesis addresses a pressing gap in the reliability engineering of transportation networks: the scalability of portfolio optimization for reinforcing large networks. The proposed hierarchical portfolio optimization model provides a middle ground between optimality of solutions and computational tractability. Furthermore, by structuring the optimization model based on the geographical decomposition, the results of the model can be readily interpreted and presented to decision makers.

Future research directions include the relaxation of the simplifying assumptions regarding, for example, the independence of disruptions and uniform parameter values for the probabilities and costs. Extending the model to directed transportation networks could further enhance the applicability of the proposed hierarchical approach. In conclusion, this thesis demonstrates that hierarchical portfolio optimization is a promising approach for supporting decision-making in the reinforcement of transportation networks, offering the scalability to analyze larger transportation networks than what would otherwise be possible.

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A Proof of Terminal Pair Reliability Formula

Proposition A.1. *Let $\mathcal{P}^t = \{\pi_1^t, \ldots, \pi_s^t\}$ be an ordered set of paths connecting a terminal pair $t \in \mathcal{T}_j$. Let*

$$A_k = \bigwedge_{m=1}^{k} [O(\pi_m^t) = 0], \quad k = 0, \ldots, s,$$

be the event that the first k paths are disrupted, where A_0 denotes the sure event. Then,

$$\mathbb{P}[O(t) = 1 \mid q_j] = \sum_{i=1}^{s} \mathbb{P}\big[O(\pi_i^t) = 1 \mid A_{i-1}, q_j\big] \cdot \mathbb{P}[A_{i-1} \mid q_j]. \tag{A1}$$

Proof. By definition, the terminal pair t is operational if and only if at least one path connecting the terminals is operational. Thus,

$$\mathbb{P}[O(t) = 1 \mid q_j] = \mathbb{P}\left[\bigvee_{i=1}^{s} [O(\pi_i^t) = 1] \,\middle|\, q_j\right]. \tag{A2}$$

Let

$$E_i = [O(\pi_i^t) = 1] \wedge A_{i-1}, \quad i = 1, \ldots, s,$$

be the event that the ith path is the first operational path. Note that these events $\{E_i\}_{i=1}^s$ are mutually exclusive and collectively exhaustive w.r.t. the event that at least one path is operational, and therefore

$$\bigvee_{i=1}^{s} E_i = \bigvee_{i=1}^{s} [O(\pi_i^t) = 1],$$

from which it follows that

$$\mathbb{P}[O(t) = 1 \mid q_j] = \sum_{i=1}^{s} \mathbb{P}[E_i \mid q_j]. \tag{A3}$$

Using the definition of conditional probability, each term can be written as

$$\mathbb{P}[E_i \mid q_j] = \mathbb{P}\big[O(\pi_i^t) = 1 \mid A_{i-1}, q_j\big] \cdot \mathbb{P}[A_{i-1} \mid q_j],$$

which can be substituted into the sum in Equation A3 to complete the proof. □

B Detailed Traffic Volume Table

Table B1: Yearly number of trains in the network.

Terminal pair	Yearly number of trains
(South, North)	7614
(KUO, South)	2116
(APT, North)	1917
(South, SIJ)	1824
(North, KUO)	766
(KUO, SOR)	528
(SIJ, SKM)	525
(North, East)	212
(SIJ, North)	190
(North, LNA)	143
(KUO, SIJ)	133
(South, LNA)	98
(East, SIJ)	16
(North, SOR)	13
(LNA, East)	9
(KUO, East)	8
(South, TOI)	8
(JKI, East)	7
(TE, North)	7
(SIJ, APT)	5
(TOI, SIJ)	5
(KUO, TOI)	4
(South, JKI)	3
(SKM, East)	2
(South, East)	2
(East, TOI)	2
(APT, TOI)	2
(LNA, APT)	2
(TOI, North)	2

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Terminal pair	Yearly number of trains
(LNA, KUO)	1
(SIJ, LNA)	1
(TE, SIJ)	1
(South, APT)	1
(South, SOR)	1
(SOR, TOI)	1
(KUO, APT)	1
Total	16,170

C Example Cost-Efficient Combined Portfolio

Table C1: Example cost-efficient combined portfolio.

Subnetwork	Reinforced nodes
TE	TE V0001, TE V0002
LNA	LNA V0001, LNA V0002
APT	APT V0001, APT V0002
SIJ	SIJ V0611, SIJ V0612, SIJ V0613, SIJ V0615, SIJ V0616, SIJ V0618, SIJ V0620, SIJ V0622, SIJ V0632, SIJ V0634, SIJ V0636, SIJ V0638, SIJ V0640, SIJ V0642, SIJ V0666
SKM	SKM V0262, SKM V0271
KNH	KNH V0381
JKI	JKI V0411, JKI V0422
TOI	TOI V0001, TOI V0002
SOR	SOR V0001
KUO	KUO V0002, KUO V0003, KUO V0004, KUO V0005, KUO V0006, KUO V0011, KUO V0013, KUO V0021, KUO V0023, KUO V0025, KUO V0027, KUO V0032, KUO V0034, KUO V0041, KUO V0302, KUO V0941
Total	**45 switches**