Decision Programming Formulations for Optimal Asset Portfolio Management Under Uncertainty

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Abstract of the master's thesis

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Abstract
The management of physical infrastructure assets is crucial to the operation of modern societies. For example, environmental concerns and increasing competition means that decision makers need to consider multiple non-trivial causal relationships. Thus, mathematical modeling may bring substantial benefits to aid decision making.

Asset management problems are typically characterized by multiple objectives, multiple periods, uncertainty, disruptions and risk management. The main tool of this thesis is the Decision Programming framework combined with influence diagrams. The unit of analysis is a single asset, for which optimal asset management strategies are determined. The asset management strategies consider, for example, asset condition, asset performance, failures, planned maintenance and reactive maintenance. Furthermore, chance constraints are utilized to solve asset management problems under uncertainty.

Asset portfolio management problems are addressed with the use of a multi-level model in which assets are classified according to their types and categories. The portfolio problems are solved by combining optimal single asset maintenance strategies while considering the effects of, for example, diversification and joint probability distributions.

The conceptual framework developed in this thesis was also discussed with the Finnish Transport Infrastructure agency and it was considered to be relevant. All in all, the results of this thesis indicate that asset portfolio management based on the Decision Programming framework and influence diagrams is viable and effective.

Keywords Decision Programming, decision analysis, decision models, influence diagrams, asset portfolio management, uncertainty

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Sammandrag
Förvaltning av fysiska infrastrukturtillgångar är en central aspekt i ett fungerande modernt samhälle. Exempelvis innebär ökad miljöhänsyn och ökad konkurrens att beslutsfattare måste ta hänsyn till flera icke-trivial orsakssamband. Matematisk modellering kan därför ge betydliga fördelar för att underlätta beslutsfattandet.

Tillgångsförvaltningproblem kännetecknas vanligtvis av flera kriterier, flera perioder, osäkerhet, störningar och riskhantering. Viktigaste verktyget i denna avhandling är ”Decision Programming” ramverket i samband med så kallade inflytandediagram. Analysenheten är en enskild tillgång, för vilken optimala förvaltningsstrategier, i stället för enbart underhållsbeslut, bestäms. Strategierna tar i beaktande tillgångens skick, prestanda och planerat samt reaktivt underhåll.

För att lösa tillgångsförvaltningsproblem på en portfolienivå klassifieras tillgångar först enligt deras typ samt kategori. Portföljproblemen löses sedan genom att kombinera optimala förvaltningstrategier för enskilda tillgångar samtidigt som effekterna av t.ex. diversifiering och gemensamma sannolikhetsfördelningar beaktas.

Konceptuella ramverket som utvecklades i denna avhandling diskuterades med Trafikledsverket och ansågs vara relevant. I sin helhet visar resultaten att tillgångsförvaltning baserat på ”Decision Programming” och inflytandediagram är genomförbart och effektivt.

Nyckelord Decision Programming, beslutsanalys, beslustmodeller, inflytandediagram, tillgångsförvaltning, osäkerhet

Preface

I want to thank my advisor and supervisor, Professor Ahti Salo, for providing me with an interesting thesis as well as for his guidance and for continuously pushing my limits. Additionally, I would also like to thank my colleagues at the Systems Analysis Laboratory, Aalto University, for their valuable insights and comments.

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Leevi Olander

Contents

1 Introduction

Asset management is a challenge in both public and private organizations. Organizations have to, for example, decide what assets should be acquired, how assets are to be maintained, which assets should be discarded and how much to invest in research and development [2]. The goal of asset portfolio management is to optimize, for example, asset performance, asset availability and asset condition, while considering, for instance, budgetary and risk constraints.

A key concern in asset management is that the estimates for management decisions may be inaccurate. For example, revenue forecasts of capital investment projects are overestimated roughly 80% of the time [3]. Conversely, costs are often heavily underestimated [6]. For instance, in Norway average overruns in infrastructure projects have been reported to be as high as 84% with extremes of up to 500% [1]. Additional problems are caused by uncertainties in maintenance scheduling, maintenance costs and testing accuracy, for example.

Asset portfolio management is also affected by the choice of a time scale. Multi-period problems differ from single period problems in that they involve more variables, relationships and constraints. For example, instead of having a single budget, each period may have a separate budget, of which the remainder may or may not be carried over into the next period. Regardless of these complexities, the multi-period approach is more realistic than the single period. For example, the single period framework is not suitable for long term investors [25], nor is it prudent to disregard the value of communicating information between periods [9]. A multi-period model should contribute to a greater expected utility, due to potential adjustments during the planning period [9].

Another important aspect is the time value of resources. This is often solved by discounting future costs and benefits by a predetermined discount rate. While discounting monetary costs is usually uncontroversial, the same can not be said for discounting benefits [31]. A compelling argument against discounting future health benefits, for example, is that they can not be reinvested [31]. If similar benefits are to be discounted, then the discount rate should be modest [31]. Furthermore, it is often difficult to determine a proper discount rate for projects with high costs and high probabilities of failure. For these reasons, some models do not take the time value of resources into account. Yet this can lead to very different solutions, especially in problems with long time horizons.

Portfolios of assets are often evaluated by expected benefits and expected costs. In some cases, the expected benefits and expected costs are lumped together to a single objective [16]. For this approach to be generally viable, utility functions need to be elicited from decision makers. However, this elicitation process can be fraught with difficulty [4], because, for example, decision maker preferences may be subject to uncertainty [18] and they may be reluctant to state their preferences [17]. A more detailed approach is to treat the benefits and the costs as separate objectives, which means that the problem is first transformed into a multi objective optimization problem for which a set of efficient solutions is then determined. The benefit is that the decision maker can get a more holistic view of the problem and then choose the best solution according to his or her subjective preferences. Ideally, the decision maker can get a better understanding of the relationship between risk and return and also become more confident in the chosen solution.

However, modeling expected benefits and expected costs is often not enough, especially if one is dealing with physical assets that can be severely affected by disruptions. Disruption risks are either man-made or natural. Examples include, but are not limited to, earthquakes, floods, hurricanes, equipment breakdown, labor strikes or economic crises [30]. The impact of disruption risks are often much greater than that of operational risks [30]. Modeling of them has received attention in supply chain management [15], but, disruption risks have typically not been addressed in asset portfolio management.

Consider a case where an asset portfolio has a large number of similar assets, which can be divided into distinct categories, for example, icebreakers and passenger ships perform completely different tasks. Attempts to create a model that encompasses all the different categories at once could result in a relatively complex model, both conceptually and computationally. A simpler and more efficient approach is to optimize the allocation of a budget between the asset categories and to solve the optimal asset management decisions within each asset category. This effectively leads to a multi-level optimization problem.

The main research objective of this master's thesis is to develop models in support of asset portfolio management decisions through the use of influence diagrams and the Decision Programming framework [28]. Aspects that are considered include uncertainties, multiple time periods, the time value of resources, multiple objectives and disruption risks.

Thus, as the methodological approach we consider the introduction of influence diagrams for asset condition modeling and a multi-level optimization model that first solves all efficient maintenance strategies for every asset category. The optimal maintenance strategies are constructed from asset specific decision strategies. Thus, they are not fixed maintenance actions. The elaboration of these non-dominated strategies help decision makers to better understand the rationale behind the solutions. Finally, the more encompassing objective of the maintenance strategy of the entire portfolio is solved and numerical examples are presented.

2 Methodological Background

Asset portfolio management is increasingly important in modern societies, due to pressing environmental issues and tighter competition. This means that decision makers are charged to deal with bigger and more complex issues. Thus, multiple models and methodologies have been developed for asset portfolio management ranging from single asset maintenance strategies to systems of multiple interdependent assets [27]. In asset management it is important to understand the causes for, for example, maintenance backlogs [13].

The main contribution of this thesis is the explicit consideration of disruption risks and the assessment of their significance at the portfolio level. For example, the implications of an elevator that is broken and out of use can not accurately be modeled when only considering expected scenarios. In reality, disruptions often cascade into other non-pleasant events, for example, excessive loads to support functions. Thus, modeling disruptions, or at least considering them separately increases the practical application potential of the asset management model.

Feature	Category
Number of assets	Multiple
Number of components	Single
Asset diversity	Heterogeneous
Asset category	Multiple
Intervention option	Different among assets
Optimization method	Deterministic multi level MILP
Optimization objective	Multi-objective

Table 1: An overview of the asset management features of this thesis.

Table 1 positions the fundamental features of the asset portfolio management model of this thesis in the terminology of Petchrompo & Parlikad [27].

2.1 Decision Programming

Multi-period decision problems under uncertainty, such as asset management, can be modeled with influence diagrams (ID) [14] and Decision Programming [28]. A concrete example is the impact of risk mitigation strategies on system reliability [20]. In addition to the expected value, the solution of an influence diagram gives optimal strategies for each decision. This is useful when comparing and analyzing the strategies. It may also give the decision maker new information that shows in what ways previous strategies have been non-optimal and, more importantly, why this has been the case.

Influence diagrams are acyclic graphs $G = (N, A)$ whose nodes $N = C \cup D \cup V$ consist of chance nodes C that represent uncertain events associated with random variables, decision nodes D that correspond to decisions among discrete alternatives and value nodes V that represent consequences of scenarios. The number of chance nodes is $n_C = |C|$, the number of decision nodes is $n_D = |D|$ and the sum of these is $n = n_C + n_D$. The arcs A describe the dependencies between the nodes and the information set of a node $j \in N$ is $I(j) = \{i \in N \mid (i, j) \in A\}$, or in other words, the set of the direct predecessors of node j. [28]

In Decision Programming a path $s = (s_1, s_2, ..., s_n) \in S$ of length n is the sequence of states for all chance and decision nodes, with S denoting the set of all paths. Each of these nodes $j \in C \cup D$ has a finite set S_j of discrete states that depend on their possible information states $s_{I(j)} \in S_{I(j)} = \prod_{i \in I(j)} S_i$. This means that for each chance node $j \in C$ the states correspond to the realizations of the random variable X_j. This variable on the other hand, depends probabilistically on the states s_i of the nodes $i \in I(j)$ [28] through

$$\mathbb{P}(X_j = s_j \mid X_{I(j)} = s_{I(j)}), \forall s_j \in S_j, s_{I(j)} \in S_{I(j)}, j \in C. \tag{1}$$

Equation (1) describes the probability of observing a specific state for each chance node $j \in C$. The state of each decision node $j \in D$ each state $s_j \in S_j$ depends on the information state $s_{I(j)}$, but is otherwise deterministic. [28]

A decision strategy $Z = (Z_1, Z_2, ..., Z_{n_D}) \in \mathbb{Z}$ is a set of local decisions strategies for each decision node, where $\mathbb{Z}$ is the set of all decision strategies. A decision strategy $Z \in \mathbb{Z}$ is compatible with a path $s \in S$ if $Z_j(s_{I(j)}) = s_j, \forall Z_j \in \mathbb{Z}, j \in D$. This means that the probability that path s occurs for decision strategy Z which is compatible with s can be greater than or equal to zero, where as if Z is non-compatible with s the probability is always zero. [28]

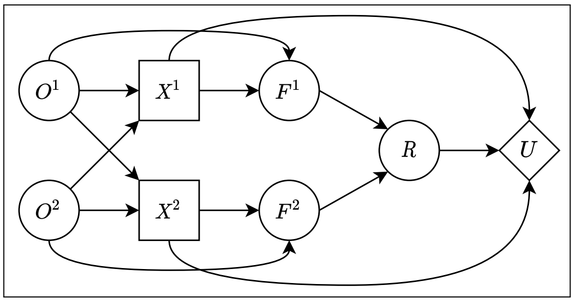

Figure 1: Example of an influence diagram.

Node	Explanation	Type	Information Set
O^1	Original state of generator 1	Chance	
O^2	Original state of generator 2	Chance	
X^1	Replacement decision for generator 1	Decision	O^1, O^2
X^2	Replacement decision for generator 2	Decision	O^1, O^2
F^1	Final state of generator 1	Chance	O^1, X^1
F^2	Final state of generator 2	Chance	O^2, X^2
R	Result	Chance	F^1, F^2
U	Utility	Value	X^1, X^2, R

Table 2: Information about the nodes of the influence diagram presented in Figure 1.

For a simple example, consider a factory in which electricity is provided by two separate generators. If either generator is on, the factory can operate profitably, otherwise a loss is incurred. The state of each generator is either operational or broken with respective probabilities. Either generator can be replaced for some cost to a generator that is working with a 100% probability. However, a generator does not have to be replaced.

This problem is represented by the influence diagram in Figure 1, in which decision nodes are represented with squares, chance nodes with circles and utility nodes with diamonds. The realizations of each node are indicated in lowercase, for example, X^2 refers to the node and x^2 refers to its realized state. Table 2 lists the nodes, their types and their information sets. Thus, the path is $s = (o^1, o^2, x^1, x^2, f^1, f^2, r)$.

Local strategy	**Node**	**Information state**	**Decision**
Z_1	X^1	*Off, off*	*Replace*
		Off, on	*Replace*
		On, off	*Replace*
		On, on	*Do not replace*
Z_2	X^2	*Off, off*	*Replace*
		Off, on	*Replace*
		On, off	*Replace*
		On, on	*Do not replace*

Table 3: An illustrative decision strategy with the information states and the decisions.

Node	**Value**
O^1	*On*
O^2	*On*
X^1	*Do not replace*
X^2	*Do not replace*
F^1	*On*
F^2	*On*
R	*On*

Table 4: An example of a path, its nodes and their values.

Node	**Value**
O^1	*On*
O^2	*On*
X^1	*Replace*
X^2	*Replace*
F^1	*Off*
F^2	*On*
R	*Off*

Table 5: An example of a path, its nodes and their values.

Table 3 specifies a decision strategy and Tables 4 and 5 list two paths. The decision strategy in Table 3 is compatible with the path in Table 4, but not with the path in Table 5. In other words, the path in Table 4 occurs with a probability greater than or equal to zero, while the path in Table 5 occurs with a probability of zero. The reason for the non-compatibility is twofold. First, F^1 cannot be *off* if the corresponding generator was replaced, which is always the case in this decision strategy, because $P(F^1 = \mathit{Off} \mid X^1 = \mathit{Replace}) = 0$. Second, the value of R cannot be *off*, if either F^1 or F^2 are *on*. This means that the probability of the second path occurring is zero, given the decision strategy. Even if a path has zero chance of occurring given a decision strategy, this does not mean that it necessarily applies for all decision strategies.

$$\max_{Z \in \mathbb{Z}} \sum_{s \in S} \pi(s) U(s) \tag{2}$$

$$\text{s.t.} \sum_{s_j \in S_j} z(s_j \mid s_{I(j)}) = 1 \qquad \forall j \in D, s_{I(j)} \in S_{I(j)} \tag{3}$$

$$0 \leq \pi(s) \leq p(s) \qquad \forall s \in S \tag{4}$$

$$\pi(s) \leq z(s_j \mid s_{I(j)}) \qquad \forall s \in S, j \in D \tag{5}$$

$$\pi(s) \geq p(s) + \sum_{j \in D} z(s_j \mid s_{I(j)}) - |D| \qquad \forall s \in S \tag{6}$$

$$z(s_j \mid s_{I(j)}) \in \{0, 1\} \qquad \forall j \in D, s_j \in S_j, s_{I(j)} \in S_{I(j)} \tag{7}$$

With Decision Programming [28] the corresponding optimization problem can be formulated and solved. The objective function (2) represents the expected utility, where $U(s)$ is the utility of path s encoded by the decision variables $z(s_j \mid s_{I(j)})$ and $\pi(s)$ is the probability of path s given the specific decision strategy Z. Constraint (3) ensures that one decision is made at each decision node for every possible information state. Constraint (4) bounds the probabilities of the scenario paths $s \in S$. Constraint (5) ensures that only compatible can paths have probabilities higher than zero. Constraint (6) ensures that the probabilities of active paths are set to their upper bound, but this constraint is not needed if the utilities of all possible paths $s \in S$ are greater than zero. Finally, constraint (7) enforces binary variables for the decisions. [28]

This general formulation can be written explicitly in therms of the chance events and decisions contained in the paths as follows. Consider the information states $s_{I(j)}$ for all decision nodes $j \in D$, they can be rewritten as $s_{I(j)} = (o^1, o^2)$. The set of possible states for each chance node $j \in C$ can also be written as $S_j = (\mathit{off}, \mathit{on})$. The utility depends only on x^1, x^2 and r. This leads to the following model:

$$
\begin{aligned}
\max_{Z \in \mathbb{Z}} \quad & \sum_{(o^1,o^2,x^1,x^2,f^1,f^2,r)} \pi(o^1,o^2,x^1,x^2,f^1,f^2,r)U(x^1,x^2,r) & \\
\text{s.t.} \quad & \sum_{x^1 \in (\text{off},\text{on})} z(x^1 \mid o^1,o^2) = 1 & \forall o^1 \in O^1, o^2 \in O^2 \\
& \sum_{x^2 \in (\text{off},\text{on})} z(x^2 \mid o^1,o^2) = 1 & \forall o^1 \in O^1, o^2 \in O^2 \\
& 0 \le \pi(o^1,o^2,x^1,x^2,f^1,f^2,r) \le p(o^1,o^2,x^1,x^2,f^1,f^2,r) & \forall s \in S \\
& \pi(o^1,o^2,x^1,x^2,f^1,f^2,r) \le z(x^1 \mid o^1,o^2) & \forall s \in S \\
& \pi(o^1,o^2,x^1,x^2,f^1,f^2,r) \le z(x^2 \mid o^1,o^2) & \forall s \in S \\
& \pi(o^1,o^2,x^1,x^2,f^1,f^2,r) \ge p(o^1,o^2,x^1,x^2,f^1,f^2,r) + & \forall s \in S \\
& \qquad z(x^1 \mid o^1,o^2) + z(x^2 \mid o^1,o^2) - 2 & \\
& z(x^1 \mid o^1,o^2) \in \{0,1\} & \forall x^1 \in X^1, o^i \in O^i \\
& z(x^2 \mid o^1,o^2) \in \{0,1\} & \forall x^2 \in X^2, o^i \in O^i \\
& s = (o^1,o^2,x^1,x^2,f^1,f^2,r) &
\end{aligned}
$$

2.2 Portfolio Optimization

While portfolio optimization is pervasive in the financial sector [21] it has been widely adopted elsewhere as well [27]. The ISO 55000:2014 standard defines "asset portfolio" as a set of assets that fall under the same asset management system [12]. However, this distinction is not entirely adequate, because asset diversity is also a key concern. For example, in finance stocks belong to the same category and are thus comparable [27], but this is not generally the case, because some asset categories may be incomparable in one or more ways. For example, how does one compare the utility of buildings and pavement? Buildings and pavements are constructed from different materials, have distinct maintenance demands and serve different purposes. This means that comparisons between different categories are not straightforward, as the preference functions concerning the perceived value of different assets may differ. This thesis considers a portfolio of assets containing several categories. This is referred to as a *multi-category portfolio* [27].

A *multi-category portfolio* usually leads to a multi-objective problem. This is more challenging than the consideration of a single objective, because different objectives are usually conflicting [34] and the preferences of decision makers are often subjective [24]. This means that there is no single optimal solution. To overcome this challenge the concept *Pareto optimality* has been introduced. Specifically, a solution to a multi-objective optimization problem is *Pareto optimal* if no objective can be improved without negatively affecting any other objective [27]. This means the entire set of *Pareto optimal* solutions is interesting to the decision maker. This set of solutions allows the decision maker to make a well-informed decision and thus the trade-offs between the different alternatives are easier to comprehend [27]. However, problems with multiple objectives can be cognitively demanding [27]. The problem can, in principle, be alleviated by collating some objectives. For example, while a distinction between maintenance costs and acquisition costs could be interesting, they are still comparable and, thus, to reduce the number of objectives one may have to collate them.

Another key aspect of portfolio optimization is *risk management*, which is especially true under disruptions. Decision makers may not be interested in only the risk profile of a single asset, but also in the risk profile of every asset category as well as the risk profile of the entire portfolio.

2.3 Multi Level Optimization

Large asset portfolio management problems with multiple objectives can be difficult to solve computationally. For example, the number of feasible decision strategies in a single influence diagram can grow fast [28]. If multiple assets were to be evaluated simultaneously, the problem space would grow even faster and quickly become unsolvable. This issue can be alleviated by building multi-level models and by only considering the set of *Pareto optimal* solutions from the lower level to solve the higher level problem. The optimum strategy for a portfolio is a combination of the lower level *Pareto optimal* solutions if no additional constraints apply. For example, the national power production strategy of the USA can, in principle, be divided into federal, state and county levels. The problem can be solved by first determining the *Pareto optimal* solutions for each county. Then using only these *Pareto optimal* solutions, the problem for each state is solved. Finally, the federal level problem is solved in the same fashion. In this case, the federal level problem would have been the highest level problem, with the state level problems being the next level and the county level problems being the lowest level problems. The multi-level approach significantly reduces the size of the problem space in most cases.

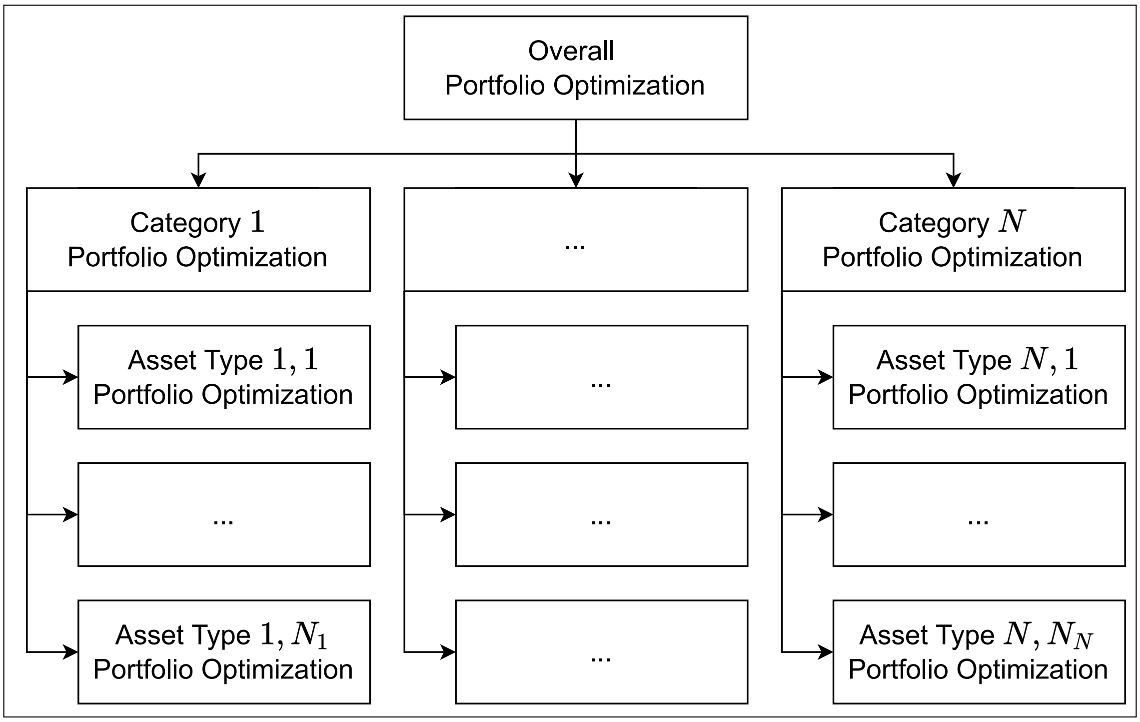

Figure 2: The structure of a multi-level model. Each category portfolio and each asset type portfolio may contain a variable number of assets.

The decision concerning the number and exact meaning of in a multi-level model is central. A natural division in the case of an asset portfolio management problem with multiple categories is to assess each asset category as a separate portfolio. Furthermore, individual assets within a category of the same type can be grouped together. This structure is in Figure 2. The resolution could be further increased by grouping assets of the same type together based on some other relevant attribute, such as initial condition or year of manufacture.

Consider an asset portfolio management problem with 240 assets, 8 asset categories each consisting of 3 asset types (these numbers approximately represent the portfolio of United States Coast Guard cutters [32]). If each asset type contains the same number of assets (=10) and there are 1000 decision strategy alternatives for each asset, then the exhaustive enumeration of all decision strategies and all assets would lead to $1000^{240} = 1 \times 10^{720}$ different strategies for the portfolio. Note that in this unrealistic case the decision strategies are different for all assets and thus symmetry would not be exploited.

However, ff only 5% of the solutions at each level in a corresponding multi-level problem are Pareto optimal, then enumeration over all alternatives would result in $(((1000 \times 5\%)^{10} \times 5\%)^{3} \times 5\%)^{8} \approx 1.32 \times 10^{366}$ strategies for the portfolio. This is only an indicative estimate, because in large problems the number of *Pareto optimal* solutions are likely to be less than 5% of all solutions. Still, the improvement is already substantial.

Conceptually, the higher level optimization problems may be simpler than those for a single asset. The influence diagram for the maintenance policy of a single asset may have millions of paths and thus millions of variables that need to be taken into account [28]. However, if the decision makers are, for example, interested in the expected cost of the portfolio, then the number of variables in the higher problem is a function of the number of assets as well as of the number of Pareto optimal decision strategies and not of the number of paths.

Decisions in influence diagrams are often choices from a set of decision alternatives and thus it is natural for them to be modeled with binary variables. However, portfolio models can in certain cases accommodate the added flexibility of modeling everything as continuous variables. This is the case in the Markowitz model [21] which assumes that fractional shares of assets can be purchased and sold, and rounding of the solution takes place only at the end. In practical terms, this could mean that instead of determining which assets a certain strategy is applied to, one can determine the percentage of assets the strategy is applied to.

While a multi-level model has many advantages, there are also limitations. One of the biggest is that certain interdependencies between assets are difficult if not impossible to accurately model, for example, constraints on shared maintenance capacity. Problems may also arise if one asset depends on multiple other assets as is the case in, for example, the relationship between aerial refueling tankers and fighter jets.

2.4 Optimization Methods

There are many optimization methods for solving asset management problems, for instance, mixed integer linear programming (MILP) and genetic algorithms. This thesis focuses on MILP, which is a deterministic optimization method [27]. A deterministic optimization method refers only to the methodology used to solve a problem, not to the problem itself. Thus, MILP formulation can also be built for stochastic problems.

Deterministic optimization methods have a long tested and tried history and tend to be highly reliable [27]. These methods are also supported by high performance software and can guarantee a globally optimal solution if given enough time for computation [27]. However, a key issue with modeling problems as MILPs is that the model must be linearized [27], which is not always possible.

All *Pareto optimal* solutions of discrete multi-objective optimization (MOO) problems for any number of objectives can be solved efficiently and exactly with the SAUGMECON method [35], which is an improvement on the AUGMECON [22] and AUGMECON2 [23] methods. All of these methods are based on the ϵ-constraint [8] method, which means that a MOO problem is transformed into a single objective optimization problem by choosing one of the objectives to be optimized and the other objectives to be transformed into constraints. The single objective optimization problem is then solved with increasingly tightened constraints until all Pareto efficient solutions have been solved.

A key issue with most optimization software (e.g. Gurobi Optimizer [7]) is that they can not handle strict inequalities. This means that strict inequalities must be transformed into non-strict inequalities with some tolerance level. The SAUGMECON method was originally designed for pure multi-objective integer optimization problems and thus, without loss in generality, the tolerance could be set to one. However, in this thesis the image space of the objectives can not be guaranteed to be integer valued and thus the tolerance level of the constraints is set to the same value as the feasibility tolerance of the Gurobi optimizer. In the SAUGMECON method the number of optimization models that have to be solved is not affected by the choice of the tolerance level, as long as it is small enough to guarantee that all efficient solutions can be solved. Thus, all *Pareto optimal* solutions that can be found with the Gurobi Optimizer will be found.

3 Decision Models for Asset Management

3.1 Management of a Single Asset

The unit of analysis in portfolio management problems is often a single asset. Even if the performance, cost structure and management strategy of a single asset may not be a key concern for a decision maker, it is still important for the formulation of the aggregate problem.

This section iteratively develops models for the maintenance strategy of a single asset, by first starting from a simple model and then adding additional modeling features to it. The implications of the changes are discussed and analyzed. Ultimately, the model will encompass uncertainties, multiple periods, the time value of resources, multiple objectives and disruption risks. The parameter values are illustrative.

3.1.1 Basic Model

Consider a case in which an asset manager is responsible for managing a given asset for four years. The condition of the asset can be either *poor* or *good* and it can worsen or improve during each period. The asset manager may maintain the asset for the cost of 100000 euros and thus potentially improve its state.

The initial condition of an asset is known and the probability with which the condition worsens for an unmaintained and *good* asset in a single period is 20%, while the probability that the condition worsens for a maintained *good* asset in a single time period is 5%. The state of an unmaintained and *poor* asset will remain *poor* with the probability 95% and the state of a maintained and *poor* asset will remain *poor* with the probability 10%.

An important goal in asset management is to optimize the performance of an asset. This thesis models the performance of an asset as a function of operational availability, which depends on the underlying condition as well as on the impact of maintenance actions. For example, if the asset is in a poor condition, then its performance is not optimal and applying a maintenance action will make the asset inoperable for the duration of the maintenance, which could have some small impact on the performance in that specific period. This thesis models performance as a relative percentage number, which means that a performance of 100% refers to an asset that performs at its practical maximum.

What is the optimal maintenance strategy for this problem when considering asset performance and cost?

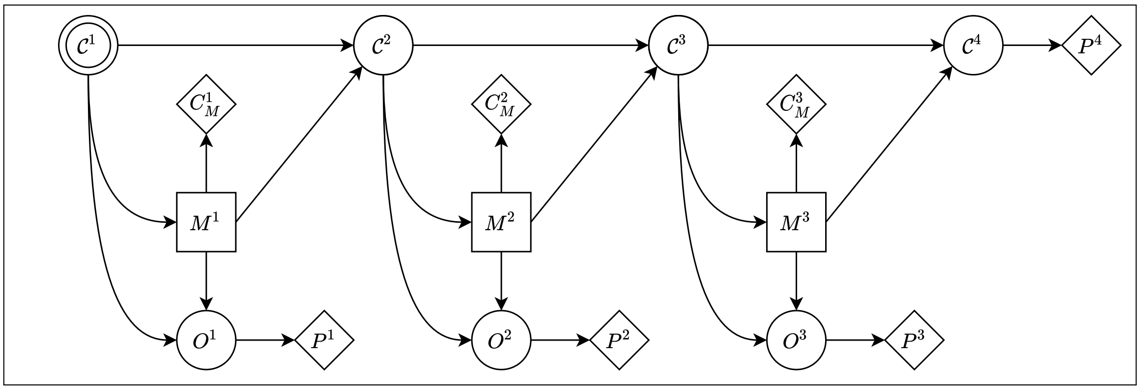

Figure 3: The limited memory influence diagram of the problem for four periods.

Figure 3 presents the influence diagram of this problem. In this case the influence diagram has been constrained to take into account only the most recent information and as such it can be referred to as a limited memory influence diagram (LIMID) [16].

The nodes $\mathcal{C}^t$, $t \in \{1, 2, 3, 4\}$ represent the condition of the asset at time t. Their realizations c^t are indexes to conditions on an ordinal scale $c^t \in \mathcal{C} = \{1, ..., n_{\mathcal{C}}\}$ so that $c^t = 1$ represents the worst possible condition and $c^t = n_{\mathcal{C}}$ represents the best condition. The nodes M^t, $t \in \{1, 2, 3\}$ represent the maintenance actions, the nodes O^t represent the operational availability, the nodes C_M^t, represent the maintenance costs and the nodes P^t represent the asset performance.

In this LIMID model, only the current asset condition affects decisions. For example, in a pure Markov decision process the decision whether to maintain a broken engine depends only on the engine's current condition and not on any previous information. However, this may not hold true for all problems, which means that the previous history may have to be taken into consideration. If the entire state history of an asset were to be taken into account for each decision, then the problem could efficiently be solved with dynamic programming. However, large influence diagrams may still make this computationally difficult [16].

The assumption of limited memory holds true in many cases, for example, multiple cooperating decision makers may only be able to partially share the reasoning behind his or her decisions [16]. Encoding a decision strategy that takes into account the entire state history of an asset may also be impractical due to the potentially large information state, especially if a multi-component system were to be considered. This is why, for example, the maintenance history of only the past five years may be taken into account when servicing a vehicle.

One advantage of approaching this problem as a LIMID rather than a fully connected ID is that the computational burden is reduced in the Decision Programming formulation. For example, in a fully connected ID problem the number of local decision strategies for node M^3 is $(2 \times 2 \times 2)^2 = 64$, while in the LIMID problem the corresponding number is only $2^2 = 4$. However, it is important to note that in general the *Pareto optimal* solutions may not be as good in the LIMID formulation as in corresponding fully connected ID formulations, due to the information loss [16].

The average performance and the total cost of the decision strategy are

$$P^{\text{tot}}(s) = \frac{p^1 + p^2 + p^3 + p^4}{4}, \tag{8}$$

$$C_M^{\text{tot}}(s) = c_M^1 + c_M^2 + c_M^3, \tag{9}$$

for the path s, where p^t is the realization of the node P^t and c_M^t is the realization of the node C_M^t.

$$\max_{Z \in \mathbb{Z}} \left\{ \sum_{s \in S} \pi(s) P^{\text{tot}}(s), -\sum_{s \in S} \pi(s) C_M^{\text{tot}}(s) \right\} \tag{10}$$

$$\text{s.t.} \sum_{m^t \in M} z(m^t \mid s_{I(M^t)}) = 1 \qquad \forall t \in T, s_{I(M^t)} \in S_{I(M^t)} \tag{11}$$

$$0 \leq \pi(s) \leq p(s) \qquad \forall s \in S \tag{12}$$

$$\pi(s) \leq z(m^t \mid s_{I(M^t)}) \qquad \forall s \in S, \forall t \in T, s_{I(M^t)} \in S_{I(M^t)} \tag{13}$$

$$\pi(s) \geq p(s) + \sum_t z(m^t \mid s_{I(M^t)}) - |D| \qquad \forall s \in S \tag{14}$$

$$s = (c^1, ..., c^T, m^1, ..., m^{T-1}, o^1, ..., o^{T-1}) \tag{15}$$

$$z(m^t \mid s_{I(M^t)}) \in \{0, 1\} \qquad \forall t \in T, s_{I(M^t)} \in S_{I(M^t)} \tag{16}$$

The optimization formulation of this problem, for any number of periods and regardless of whether the problem is modeled as a fully connected ID or a LIMID, is given by (10) - (16). The multi-objective function (10) maximizes the performance of the asset and minimizes the maintenance costs, Z is the decision strategy, $\mathbb{Z}$ is the set of all possible decision strategies, S is the set of all paths, s is a path and $\pi(s)$ is the probability of path s occurring for the decision strategy encoded in the variables $z(m^t \mid s_{I(M^t)})$. The maintenance actions are mutually exclusive, which means that only one decision can be with regards to every distinct information state. This is represented by (11), where $s_{I(X^t)}$ describes the information state for the decision node X^t. The path is represented by (15). The maintenance actions are binary and they are either completed or not, which is represented by (16). Constraints (12) - (14) place bounds on the path probabilities. However, this generalized formulation can be made more explicit and thus more interpretable for specific modeling parameters.

$$\max_{Z \in \mathbb{Z}} \left\{ \sum_{s \in S} \pi(s) P^{\text{tot}}(s), - \sum_{s \in S} \pi(s) C_M^{\text{tot}}(s) \right\} \tag{17}$$

$$\text{s.t.} \sum_{m^t \in M} z(m^t \mid c^t) = 1 \qquad \forall t \in T, c^t \in \mathcal{C}^t \tag{18}$$

$$0 \leq \pi(s) \leq p(s) \qquad \forall s \in S \tag{19}$$

$$\pi(s) \leq z(m^t \mid c^t) \qquad \forall s \in S, t \in T, c^t \in \mathcal{C}^t \tag{20}$$

$$\pi(s) \geq p(s) + \sum_t z(m^t \mid c^t) - |D| \qquad \forall s \in S \tag{21}$$

$$s = (c^1, ..., c^T, m^1, ..., m^{T-1}, o^1, ..., o^{T-1}) \tag{22}$$

$$z(m^t \mid c^t) \in \{0, 1\} \qquad \forall t \in T, c^t \in \mathcal{C}^t \tag{23}$$

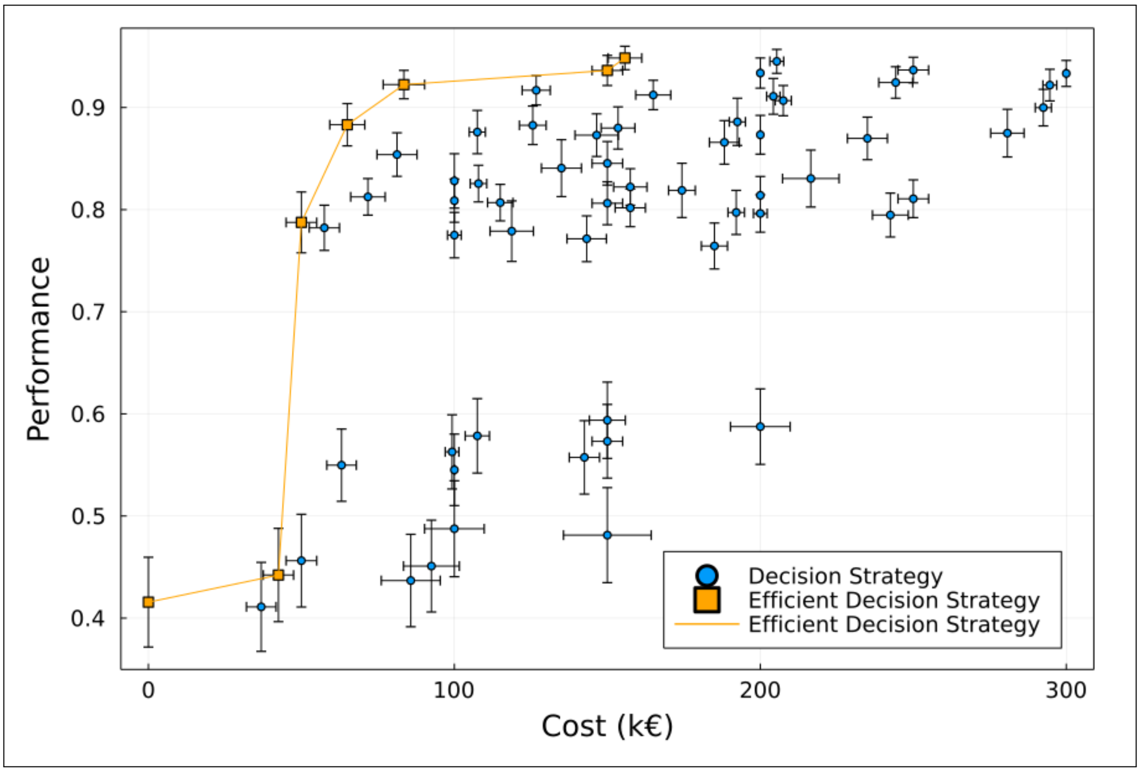

Figure 4: The efficient decision strategies and all other decision strategies.

Feature	Value
Number of feasible decision strategies	64
Number of efficient decision strategies	7
Number of paths	1024
Solution time (single solution)	0.049s

Table 6: The key features of the model for four periods. The solution time reports the average time to solve a single efficient decision strategy.

Equations (17) - (23) present the LIMID formulation of this problem for four periods. Figure 4 shows the performance and the maintenance costs and their respective scaled standard deviations for all feasible decision strategies. The error bars were scaled to 10% of the true standard deviation for the purposes of visualization. The efficient decision strategies were solved with the SAUGMECON method. Table 6 lists the key features of the model. Modeling the problem as a fully connected ID increases the computation time for a single efficient solution to 0.121 seconds, in other words, the required computation time would have been approximately 2.5 times longer. However, in this case the *Pareto optimal* frontier stays the same regardless of whether the problem is modeled as a fully connected ID or as a LIMID.

3.1.2 Maintenance With Uncertain Condition Estimation

If the asset manager does not know the condition of the asset with perfect accuracy, then decisions must be based on estimates. If the estimate can be obtained at no cost then they should always be obtained. In this case, the mathematical formulation of the optimization problem does not change, except for the information sets. Here, we assume that if the asset is in a poor condition, the test will indicate this with probability 80% and if the asset is in a good condition, the test will indicate this with probability 90%.

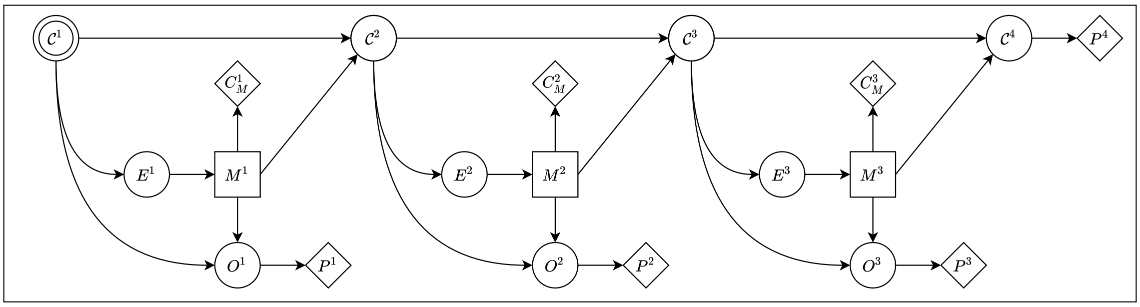

Figure 5: The limited memory influence diagram of the problem for four periods.

Figure 5 presents the LIMID of this problem. The nodes E^t, $t \in \{1, 2, 3\}$, represent estimates on the condition c^t. The mathematical optimization formulation has not changed, except for the path which is now $s = (c^1, ..., c^T, m^1, ..., m^{T-1}, e^1, ..., e^{T-1}, o^1, ..., o^{T-1})$, and is thus still represented by (10) - (16). However, the maintenance decision variables are now informed by the estimate of the condition, rather than the condition itself.

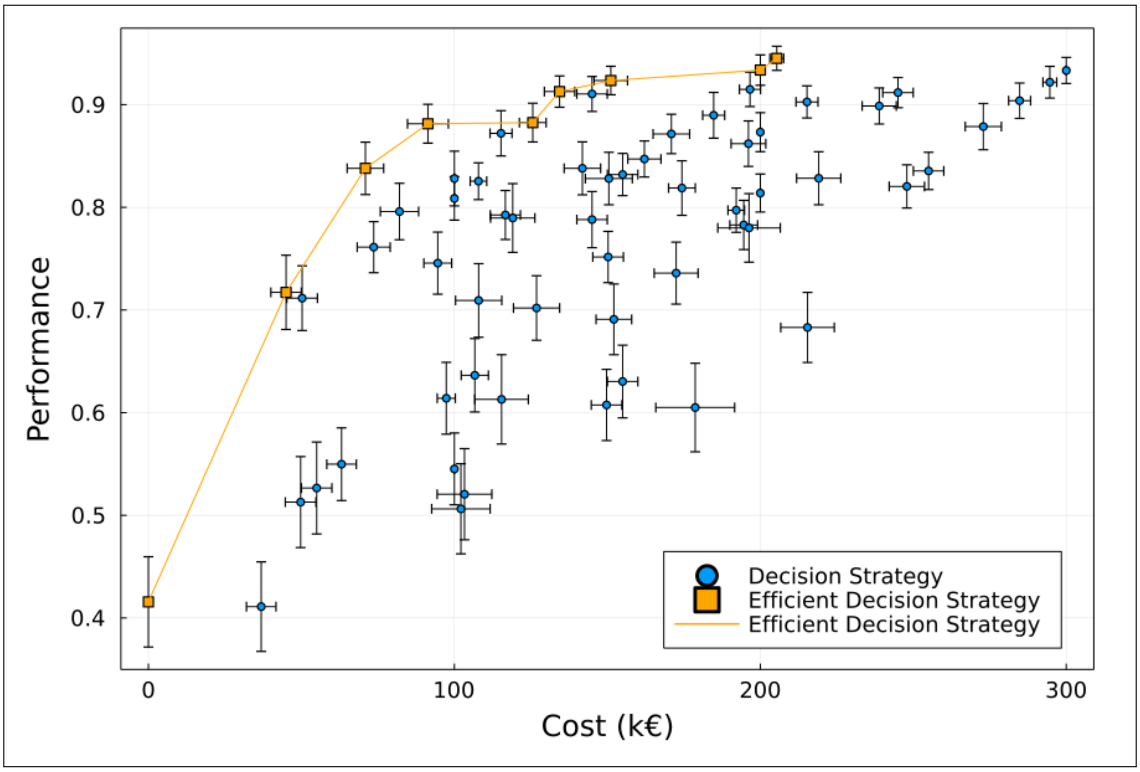

Figure 6: The efficient decision strategies and all other decision strategies.

Feature	Value
Number of feasible decision strategies	64
Number of efficient decision strategies	9
Number of paths	8192
Solution time (single solution)	0.068s

Table 7: The key features of the model for four periods. The solution time reports the average time to solve a single efficient decision strategy.

Figure 6 shows the performance and the maintenance costs and their respective scaled standard deviations for all feasible decision strategies. The error bars were scaled to 10% of the true standard deviation for the purposes of visualization. Table 7 lists the key features of the model. If the problem would have been modeled as a fully connected ID (no forgetting), then all decision strategies could not have been enumerated due to their large number. Furthermore, the number of efficient decision strategies would also have been greater. This means that in some cases it might be of value to model problems as fully connected influence diagrams, even if the computational burden might be greater.

3.1.3 Testing Decisions on Asset Condition

Estimates about the underlying condition may come at a cost [11] and can thus be viewed as a decision in its own right. In this case, it is not enough to optimize the maintenance strategies, but one must also deduce optimal testing strategies. This increases the complexity of the problem, but may make it more realistic. For purposes of illustration, let the cost for conducting a test be 1000.

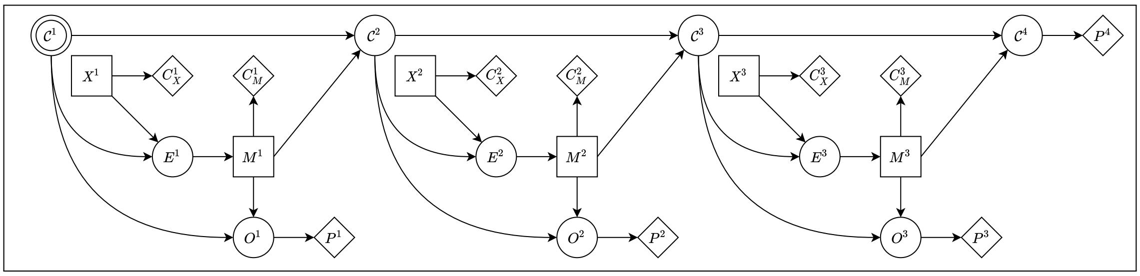

Figure 7: The limited memory influence diagram of the problem for four periods.

Figure 7 presents the LIMID of this problem. The nodes X^t, $t \in \{1, 2, 3\}$ represent the testing decisions and the nodes C_X^t represent the testing costs. The state space of the estimates must be amended, because choosing not to conduct a test will result in an unknown condition, in other words, the states of the estimates are that the asset is in a *poor*, *good* or *unknown* condition. Thus, the size of the information set for each of the maintenance decision node increases and so does the total number of decision strategies. The total testing costs for a path are represented by:

$$C_X^{\text{tot}}(s) = c_X^1 + c_X^2 + c_X^3. \tag{24}$$

$$\max_{Z \in \mathbb{Z}} \begin{aligned} &\Big\{ \sum_{s \in S} \pi(s) P^{\text{tot}}(s), \\ &- \sum_{s \in S} \pi(s) C_M^{\text{tot}}(s), \\ &- \sum_{s \in S} \pi(s) C_X^{\text{tot}}(s) \Big\} \end{aligned} \tag{25}$$

$$\text{s.t.} \sum_{m^t \in M} z(m^t \mid e^t) = 1 \qquad \forall t \in T, e^t \in E^t \tag{26}$$

$$\sum_{x^t \in X} z(x^t) = 1 \qquad \forall t \in T \tag{27}$$

$$0 \leq \pi(s) \leq p(s) \qquad \forall s \in S \tag{28}$$

$$\pi(s) \leq z(m^t \mid e^t) \qquad \forall s \in S, t \in T, e^t \in E^t \tag{29}$$

$$\pi(s) \leq z(x^t) \qquad \forall s \in S, t \in T \tag{30}$$

$$\pi(s) \geq p(s) + \sum_t z(m^t \mid e^t) + \sum_t z(x^t) - |D| \qquad \forall s \in S \tag{31}$$

$$\begin{aligned} s = (&c^1, ..., c^T, x^1, ..., x^{T-1}, e^1, ..., e^{T-1}, \\ &m^1, ..., m^{T-1}, o^1, ...o^{T-1}) \end{aligned} \tag{32}$$

$$z(m^t \mid e^t) \in \{0, 1\} \qquad \forall t \in T, e^t \in E^t \tag{33}$$

$$z(x^t) \in \{0, 1\} \qquad \forall t \in T \tag{34}$$

The minimization of the testing costs has been added as a new objective in the amended optimization model (25) - (34). Furthermore, constraint (27) states that only a single testing action can be conducted in each period and constraint (34) states that the testing actions are represented by binary variables.

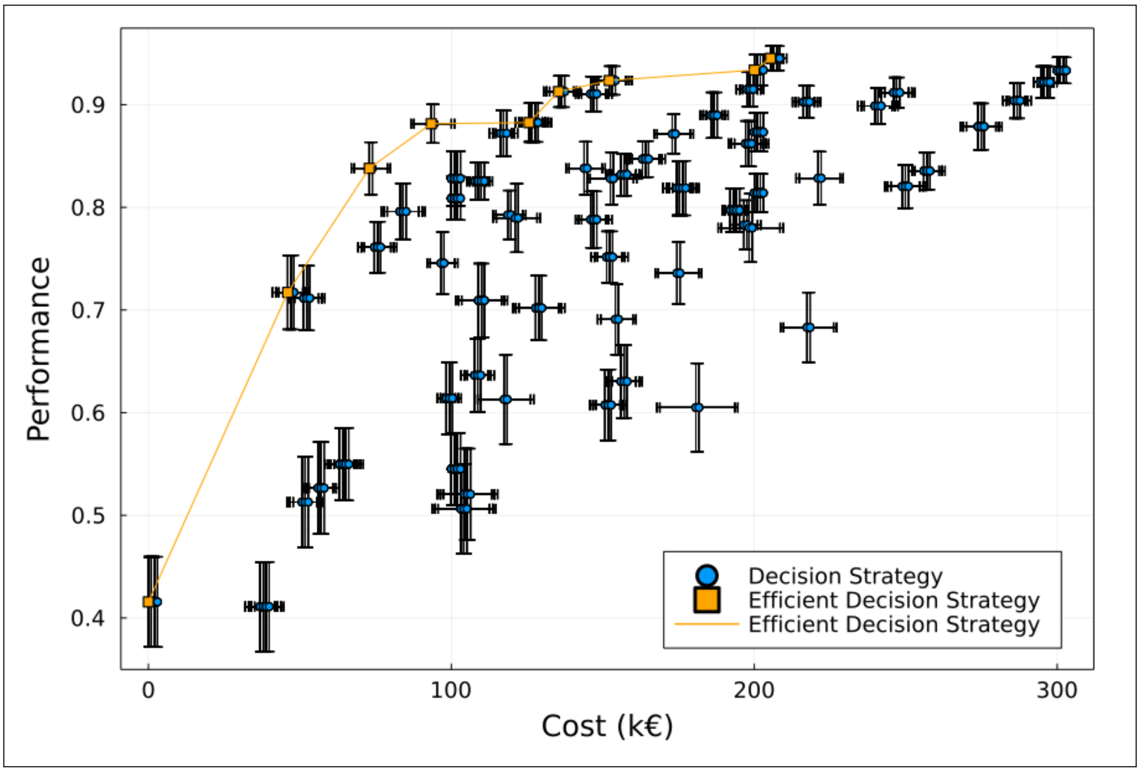

Figure 8: The efficient decision strategies as well as a sample of random decision strategies.

Feature	**Value**
Number of feasible decision strategies	4096
Number of efficient decision strategies	9
Number of paths	221184
Solution time (single solution)	0.524s

Table 8: The key features of the model for four periods. The solution time reports the average time to solve a single efficient decision strategy.

Figure 4 shows the performance and the costs and their respective scaled standard deviations for all efficient as well as for a random sample of decision strategies. The error bars were scaled to 10% of the true standard deviation for the purposes of visualization. The random sample of decision strategies consists of 1000 uniformly selected random decision strategies. The maintenance and testing costs were added for purposes of visualization. Table 8 lists the key features of the model. The efficient solutions do not differ significantly from the case where testing was not a decision. This is because the testing costs are low in comparison to the maintenance costs. In some asset management problems testing costs may even primarily depend on labor costs, rather than the number of tests conducted, which means that testing may not have to be modeled as decisions.

3.1.4 Disruptions and External Loads

The impacts of disruption risks on asset performance are often greater than those of operational risks [30]. Take an elevator in a poor condition as an example. As long as it is still able to operate, even if at a lower capacity than optimally, then the situation may be tolerable. However, the moment the elevator breaks down and becomes inoperable, the urgency to fix it increases significantly. For example, people may no longer be able to reach their apartments. Thus, disruptions give rise to failures that call for reactive maintenance and the reactions to these events are usually not covered by the prior maintenance schedule because they can be unpredictable.

The probability of a disruption depends typically on the asset condition, but also on external loads that range from man made to natural changes in circumstances. For example, the COVID-19 pandemic has significantly disrupted operations of organizations world wide. This means that some assets may have to be overutilized for a longer period of time. For illustrative purposes, assume that external load occurs with 5% probability, which results in the probability of a failure increasing for a *poor* asset from 20% to 30% and for a *good* asset to 10% to 20%. The reactive repair actions are assumed to revert the condition of an asset back to what it was before the failure and let its cost be 10000.

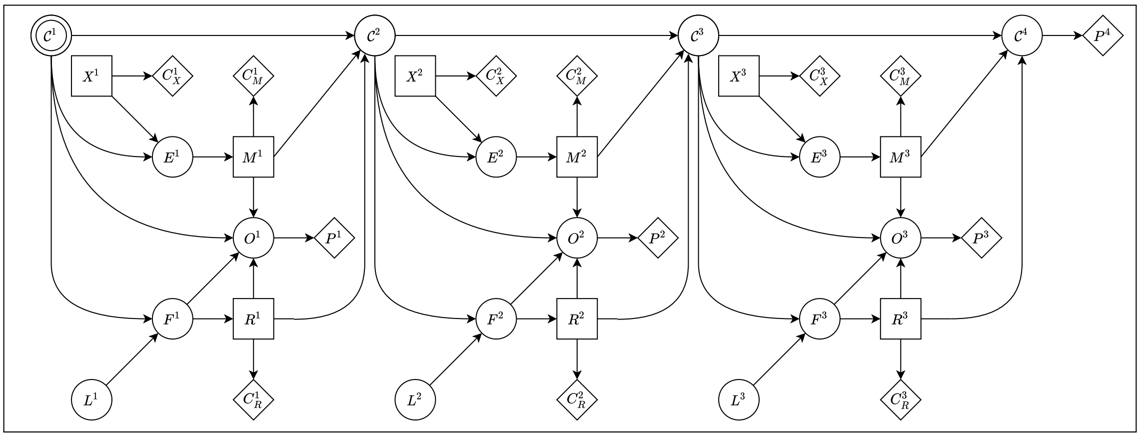

Figure 9: Influence diagram that entails all the features.

Figure 9 presents the LIMID for the problem. The nodes L^t, $t \in \{1, 2, 3\}$, represent the external load in each period, the nodes F^t represent the potential failures, the nodes R^t represent the repair maintenance and the nodes C_R^t represent repair costs. In addition to this, the information sets of nodes O^t and C^t, $t \in \{2, 3, 4\}$ were amended. Thus, the repair costs can be represented with

$$C_R^{\text{tot}}(s) = c_R^1 + c_R^2 + c_R^3. \tag{35}$$

The external loads can be dealt with in several ways. This thesis models its realization as a scenario $l = (l_1, l_2, ..., l^{T-1})$ with a given probability. This means that constraints may have to be generated for each scenario separately and that the objectives may need to also account for each scenario. However, because the worst case load scenario (when the load is at a maximum for each period) is the most constraining, the other scenarios may not have to be considered.

$$\max_{Z\in\mathbb{Z}} \begin{Bmatrix} \sum_{s\in S}\pi(s)P^{\text{tot}}(s), \\ -\sum_{s\in S}\pi(s)C_M^{\text{tot}}(s), \\ -\sum_{s\in S}\pi(s)C_X^{\text{tot}}(s), \\ -\sum_{s\in S}\pi(s)C_R^{\text{tot}}(s) \end{Bmatrix} \tag{36}$$

$$\text{s.t.} \sum_{m^t\in M} z(m^t \mid e^t) = 1 \qquad \forall t\in T, e^t\in E^t \tag{37}$$

$$\sum_{x^t\in X} z(x^t) = 1 \qquad \forall t\in T \tag{38}$$

$$\sum_{r^t\in R} z(r^t \mid f^t) = 1 \qquad \forall t\in T, f^t\in F^t \tag{39}$$

$$0 \le \pi(s) \le p(s) \qquad \forall s\in S \tag{40}$$

$$\pi(s) \le z(m^t \mid e^t) \qquad \forall s\in S, t\in T, e^t\in E^t \tag{41}$$

$$\pi(s) \le z(x^t) \qquad \forall s\in S, t\in T \tag{42}$$

$$\pi(s) \le z(r^t \mid f^t) \qquad \forall s\in S, t\in T, f^t\in F^t \tag{43}$$

$$\begin{aligned}\pi(s) \ge p(s) + \sum_t z(m^t \mid e^t) + \sum_t z(x^t) + \\ \sum_t z(r^t \mid f^t) - |D|\end{aligned} \qquad \forall s\in S \tag{44}$$

$$\begin{aligned} s = (&c^1, ..., c^T, l^1, ..., l^{T-1}, x^1, ..., x^{T-1}, \\ &e^1, ..., e^{T-1}, f^1, ..., f^{T-1}, r^1, ..., r^{T-1}, \\ &m^1, ..., m^{T-1}, o^1, ...o^{T-1}) \end{aligned} \tag{45}$$

$$z(m^t \mid e^t) \in \{0,1\} \qquad \forall t\in T, e^t\in E^t \tag{46}$$

$$z(x^t) \in \{0,1\} \qquad \forall t\in T \tag{47}$$

$$z(r^t \mid f^t) \in \{0,1\} \qquad \forall t\in T, f^t\in F^t \tag{48}$$

Equations (36) - (48) represent the revised optimization formulation. The repair costs were added as an objective to function (36). The load is included in the path, but separate objectives could have been added for each different load scenarios, however, this would make further analysis more complex. Furthermore, Equation (39) represents that exactly one repair action is taken for a given information set. Equation (44) was amended to take the repair action decision variables into account and Equation (48) states that the repair decision variables are binary.

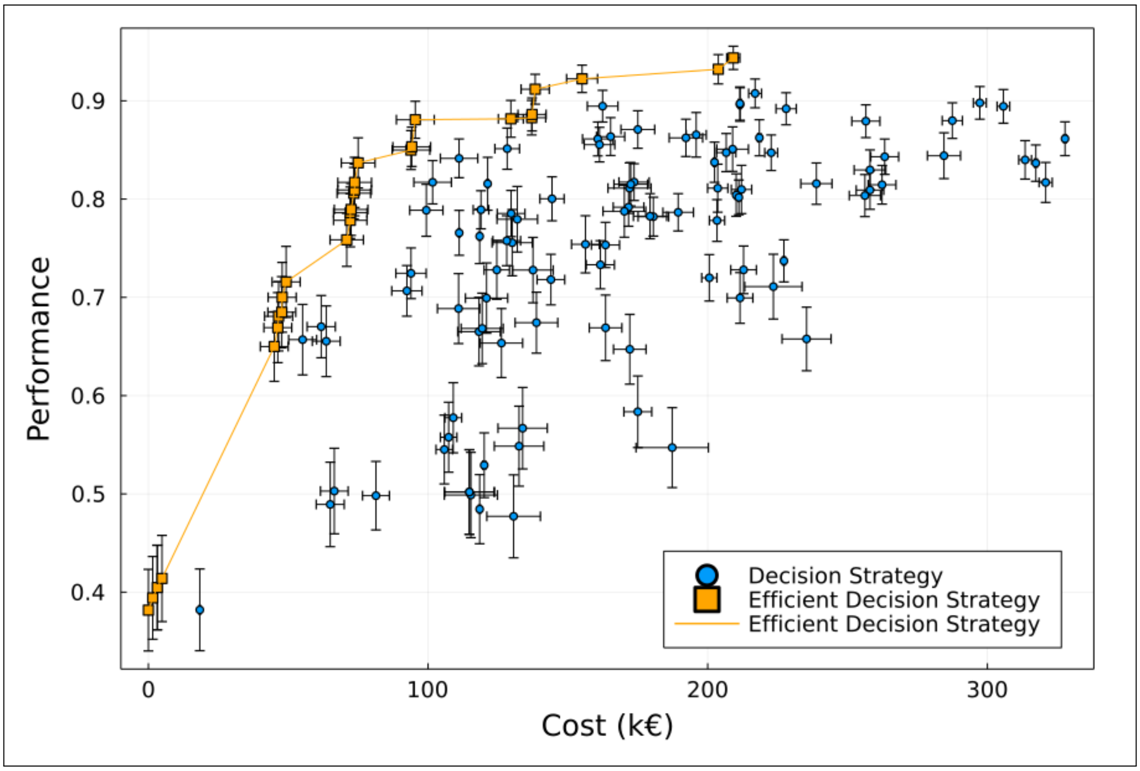

Figure 10: The efficient decision strategies as well as a sample of random decision strategies.

Feature	Value
Number of feasible decision strategies	262144
Number of efficient decision strategies	28
Number of paths	1.1×10^8
Solution time (single solution)	220.15s

Table 9: The key features of the model for four periods. The solution time reports the average time to solve a single efficient decision strategy.

Figure 4 shows the performance and the maintenance costs and their respective scaled standard deviations for all efficient as well as for a random sample of decision strategies. The error bars were scaled to 10% of the true standard deviation for the purposes of visualization. The random sample of decision strategies consists of 1000 uniformly selected random decision strategies. Table 9 lists the key features of the model. The number of paths and decision strategies are large. This is a problem because increasing the number of periods would further increase the time needed to solve a single *Pareto optimal* solution.

3.1.5 Discretization of Nodes in Influence Diagrams

Instead of using binary states the states can be modelled on an ordinal scale. For example, the conditions of an asset need not be limited to *poor* and *good*, but can be extended to, for example, *very poor*, *poor*, *good* and *very good*.

The decisions too can be instead of deciding between maintaining or not maintaining an asset, one could also determine the magnitude of the maintenance action. The different maintenance actions would have different costs and impacts.

Node type	Set of discrete states	Number of states
Condition	{ *very poor*, *poor*, *good*, *very good* }	4
Load	{ *none*, *minor*, *major* }	3
Estimate	{ *not announced*, *very poor*, *poor*, *good*, *very good* }	4
Failure	{ *none*, *failure* }	2
Operational availability	{ *very low*, *low*, *high*, *very high* }	4
Test action	{ *none*, *test* }	3
Maintenance action	{ *none*, *minor*, *major* }	3
Repair action	{ *none*, *repair* }	2

Table 10: The ordinal states of each node type.

The ordinal states for each node type are in Table 10. This inclusion of new states means that the number of parameters to be elicited grows, but the optimization formulation stays the same. The elicitation of this data may call for a considerate amount of effort, because, for example, the size of the information state for the operational availability is $4 \times 3 \times 2 \times 2 = 48$. This combined with 4 states means that 192 different cases have to be considered.

3.1.6 Reducing the Size of the Problem

In Decision Programming formulations of large influence diagrams with multiple states of nodes, the number of paths and decision strategies grows fast. This is especially true if there are multiple periods. For example, the number of paths for the influence diagram in Figure 9 with ordinal states is 3.8×10^{11}. Possible simplifications include the removal of decision nodes with a comparatively low impact, removing some uncertainty and merging chance and value nodes into a single value node [10].

If the generation of estimates on the underlying condition of an asset is relatively accurate, the implied uncertainties do not have too great of an impact on the optimal solution. For example, by visually comparing Figures 4 and 6 the Pareto efficient frontier is almost the same. Thus, to simplify the model, the estimate can be removed.

In this example, the cost of conducting a test is so small in comparison to the costs of maintenance (approximately 1%) that its effect on the Pareto optimal solutions is limited. Again, by visually comparing Figures 6 and 8, the Pareto efficient frontier is almost the same.

Finally, collapsing the relationship between the operational availability and performance does not change the expected value of the solutions [10]. This is because the performance has been modeled as value nodes that only depend on the operational availability. Thus, without affecting the solutions too much, this portion of the influence diagram can also be simplified.

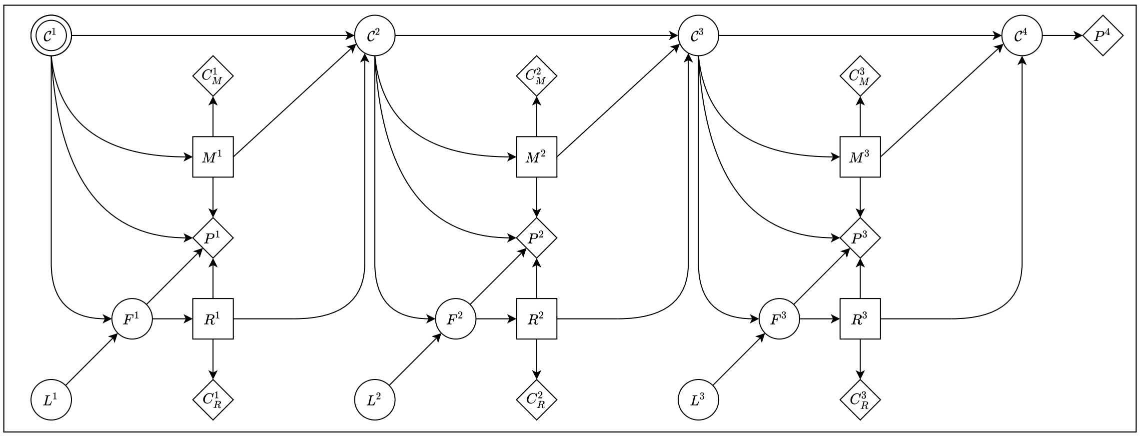

Figure 11: Simplified influence diagram of four periods for a single asset.

Figure 11 shows the simplified LIMID for four periods. The nodes X^t, E^t, O^t and C_R^t were removed, $t \in 1, 2, 3$. The information sets of nodes P^t and M^t were also changed. This means that the number of local decision strategies for node M^t was also decreased because $|E^t| > |C^t|$. All in all, the total number of paths comes down from 3.8×10^{11} to 1.1×10^8, which represents an improvement of several orders of magnitude. The total number of feasible decision strategies is also reduced from 2.7×10^8 to 3.4×10^7. The Decision Programming framework is more sensitive to the number of paths than to the number of total decision strategies and thus the benefits of the simplification are significant. However, the problem is still large and modeling more than a few periods is computationally difficult. The revised optimization formulation is then given by:

$$
\begin{aligned}
&\max_{Z\in\mathbb{Z}} \left\{ \begin{aligned} &\sum_{s\in S}\pi(s)P^{\text{tot}}(s), \\ &-\sum_{s\in S}\pi(s)C_M^{\text{tot}}(s), \\ &-\sum_{s\in S}\pi(s)C_R^{\text{tot}}(s) \end{aligned} \right\} && && (49) \\
&\text{s.t.} \sum_{m^t\in M} z(m^t \mid c^t) = 1 && \forall t\in T, c^t\in\mathcal{C}^t && (50) \\
&\sum_{r^t\in R} z(r^t \mid f^t) = 1 && \forall t\in T, f^t\in F^t && (51) \\
&0 \le \pi(s) \le p(s) && \forall s\in S && (52) \\
&\pi(s) \le z(m^t \mid c^t) && \forall s\in S, t\in T, c^t\in\mathcal{C}^t && (53) \\
&\pi(s) \le z(r^t \mid f^t) && \forall s\in S, t\in T, f^t\in F^t && (54) \\
&\pi(s) \ge p(s) + \sum_t z(m^t \mid c^t) + \sum_t z(r^t \mid f^t) - |D| && \forall s\in S && (55) \\
&s = (c^1,...,c^T, l^1,...,l^{T-1}, f^1,...,f^{T-1}, r^1,...,r^{T-1}, m^1,...,m^{T-1}) && && (56) \\
&z(m^t \mid c^t) \in \{0,1\} && \forall t\in T, c^t\in\mathcal{C}^t && (57) \\
&z(r^t \mid f^t) \in \{0,1\} && \forall t\in T, f^t\in F^t && (58)
\end{aligned}
$$

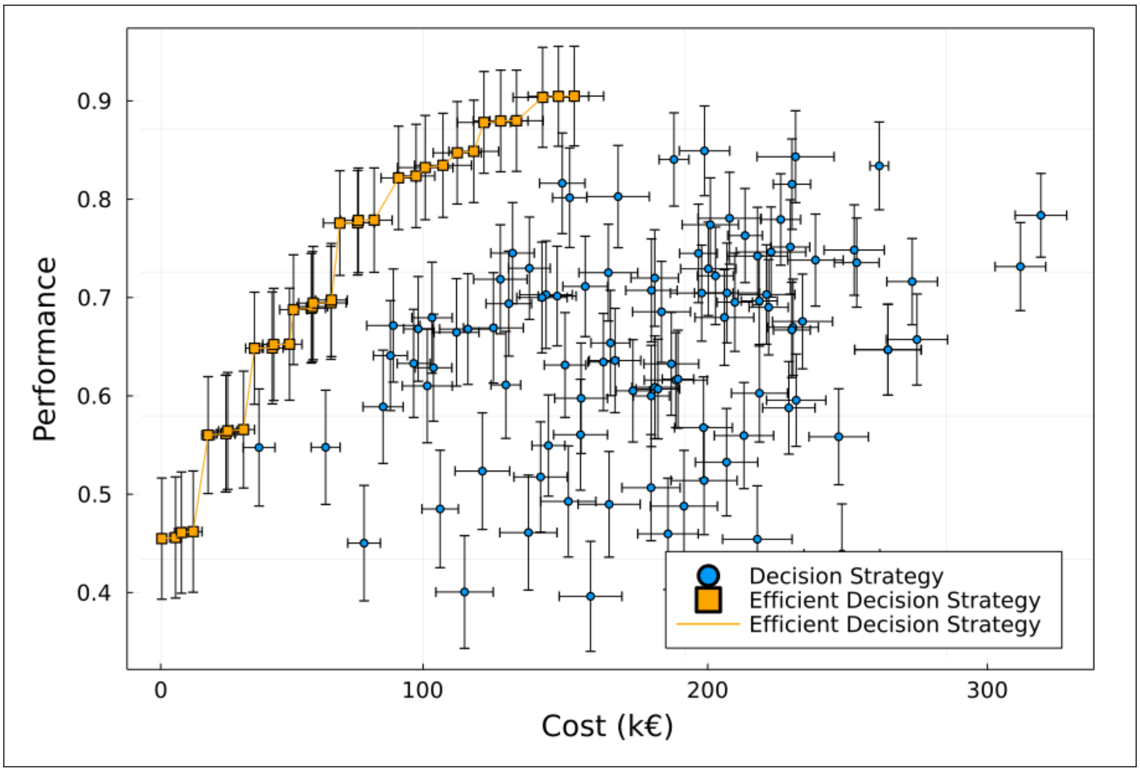

Figure 12: The efficient decision strategies as well as a sample of random decision strategies.

Feature	Value
Number of feasible decision strategies	3.4×10^7
Number of efficient decision strategies	34
Number of paths	1.1×10^8
Solution time (single solution)	220.15s

Table 11: The key features of the model for four periods. The solution reports the average time to solve a single efficient decision strategy.

Figure 12 shows the performance and the maintenance costs and their respective scaled standard deviations for all efficient as well as for a random sample of decision strategies. The error bars were scaled to 10% of the true standard deviation for the purposes of visualization. The random sample of decision strategies consists of 1000 uniformly selected random decision strategies. Table 11 lists the key features of the model. Solving a single efficient solution takes over 1000 seconds, which means that increasing the expanding the problem for more than a few periods is computationally intractable.

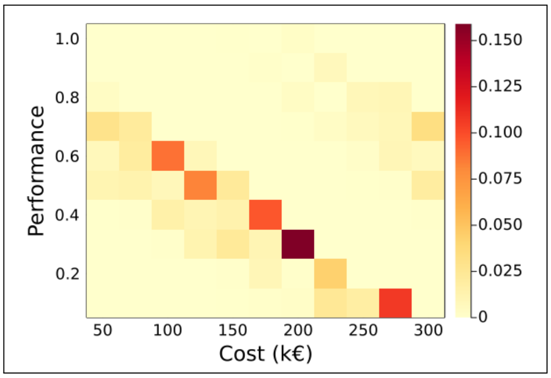

Figure 13: Illustrative example of a joint probability distribution of costs and performance for a decision strategy.

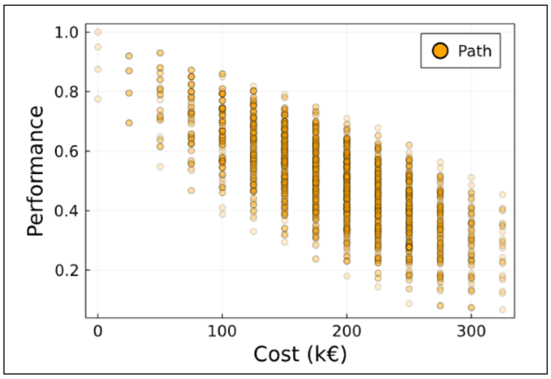

Figure 14: Illustrative example of the realizations of every path for a decision strategy. The darkness indicates how frequent a specific value is.

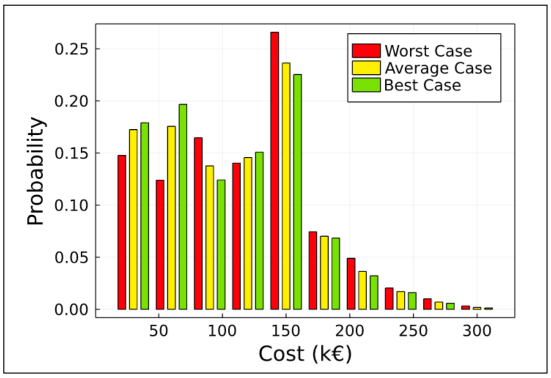

Figure 15: Illustrative example of a cost distribution for a decision strategy.

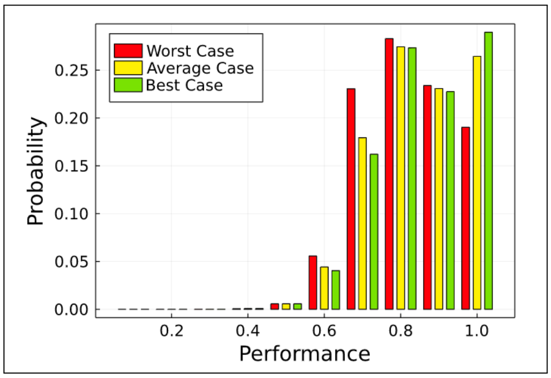

Figure 16: Illustrative example of a performance distribution for a decision strategy.

Figure 13 shows the joint probability distribution of costs and performance for a single efficient decision strategy. The resolution of the performance axis was rounded to 10% for purposes of visualization. The cost and performance of every path are shown in Figure 14. The performance declines as the cost increases for two reasons. First, it may not be optimal to maintain an asset that is already in good enough a condition, which means that maintenance and repair costs are typically not caused by high performing assets. Second, maintaining or repairing an asset has a slight negative impact on its operational availability, because the asset is out of operation for the duration of the maintenance or repair. This means that if the cost is higher than zero, the performance can never be 100% in that period. However, the performance can be 100% in the next period.

Furthermore, Figures 15 and 16 show the cost and performance distribution, respectively. The distributions are shown for the average, the worst and the best load scenario. A higher load both raises costs and decreases performance. None of the distributions can accurately be represented by their respective expected values. This means that analyzing the tails could be of value for a decision maker.

3.1.7 Discounting, Chance Constraints and Stochastic Dominance

The time value of resources is important in asset management. It can be taken into account with a discount rate that is applied appropriately to costs and benefits. However, discounting benefits may be controversial in some settings [31]. The discount rate should, in principle, be based on some sort of an opportunity cost, but as discussed in the introduction, determining this rate is not straightforward. The effect of this revision to the problem is unlikely to change the optimal strategies too much if the discount rate is small enough, due to its relatively low impact. However, in some cases discounting will model the preferences of a decision maker more accurately. Taking the time value of resources into account means that the cost functions are modified with the discount factor d and thus

$$C_M^{\text{tot}}(s) = \sum_{t}^{T-1} c_M^t d^t, \tag{59}$$

$$C_R^{\text{tot}}(s) = \sum_{t}^{T-1} c_R^t d^t. \tag{60}$$

Here d^t refers to the discount factor for the time interval from 0 to t and not necessarily to just raising a specific rate to a specific power. This generalization allows for extra flexibility as well as the case of a constant discount rate.

Another important aspect of asset management is risk measurement and management. Many approaches exist for these purposes. For example, Value at Risk (VaR) is an indication of the lower bound for a loss at a given probability level, however, it does not assess the actual magnitude of the loss [5]. Conditional Value at Risk (CVaR) estimates the expected loss at a given probability level [29]. Another approach is to model risk by using chance constraints [33], which share certain properties with VaR [29].

Different risk measures have their respective advantages and disadvantages. For example, CVaR and chance constraints are well suited for optimization problems, but VaR is often easier to interpret than CVaR. Thus, there is no universally correct methodology for all problems. This thesis focuses on modeling risk with chance constraints, but alternative approaches could be just as viable.

In Decision Programming, chance constraints can be modeled by associating a binary parameter with each path s, indicating whether the realized consequences of the value nodes of that path exceeds a given threshold. For example,

$$\Lambda^P_{\theta^P}(s) = \begin{cases} 1 & \text{if } P^{\text{tot}}(s) \geq \theta^P \\ 0 & \text{else} \end{cases} \tag{61}$$

represents binary variables associated with each path s that indicate whether the performance P exceeds the threshold level θ^P. Similar parameters can also be created for the other objectives. Thus, the probability with which the performance exceeds the predetermined threshold level θ^P can be constrained to exceed a predetermined probability level Θ^P through

$$\sum_{s \in S} \pi(s) \Lambda^P_{\theta^P}(s) \geq \Theta^P. \tag{62}$$

Chance constraints can also be added separately for each load level, however, in this case greater loads generally lead to worse performance and higher costs. Thus, the constraint for the worst load scenario may be the only truly constraining constraint. This is because the constraints for the worst case scenario are otherwise similar as the constraints for the other scenarios, except that they are stricter.

Furthermore, there is value in analyzing all first order stochastically non-dominated (FSD) solutions rather than just all *Pareto optimal* solutions with regard to the expected values for the relevant multiple criteria. The chance constraints remove some potential solution alternatives in the case where expected values are considered, however, this is still not entirely comparable with determining all FSD solutions. Salo et al. present an iterative algorithm to determine all FSD decision strategies for Decision Programming formulations [28]. Alternatively, if the number of decision strategies is relatively small, then all strategies can be exhaustively enumerated and then compared.

3.1.8 Alternative Multi-Period Approach

Extending an influence diagram over multiple similar periods leads to a large number of paths, as shown in Section 3.1.5. In other words, this approach does not scale up well for larger problems. An alternative approach that scales better is to solve all the efficient strategies for a single period and then combine these efficient strategies for multiple periods. In this case, the multi-period problem can be modeled with a dynamic programming approach, in which the problem is solved from the first period and then combining strategies until the desired period is reached.

However, if asset management is modeled with multiple objectives, then rather than having a single explicitly best solution at each time period, a set of efficient solutions must potentially be considered. This means that the the set of non-dominated solutions will still grow with this approach, but not as rapidly as when extending an influence diagram with Decision Programming. This is because extending an influence diagram will inevitably generate a large number of paths, while the dynamic approach will only consider the active paths for each efficient decision strategy.

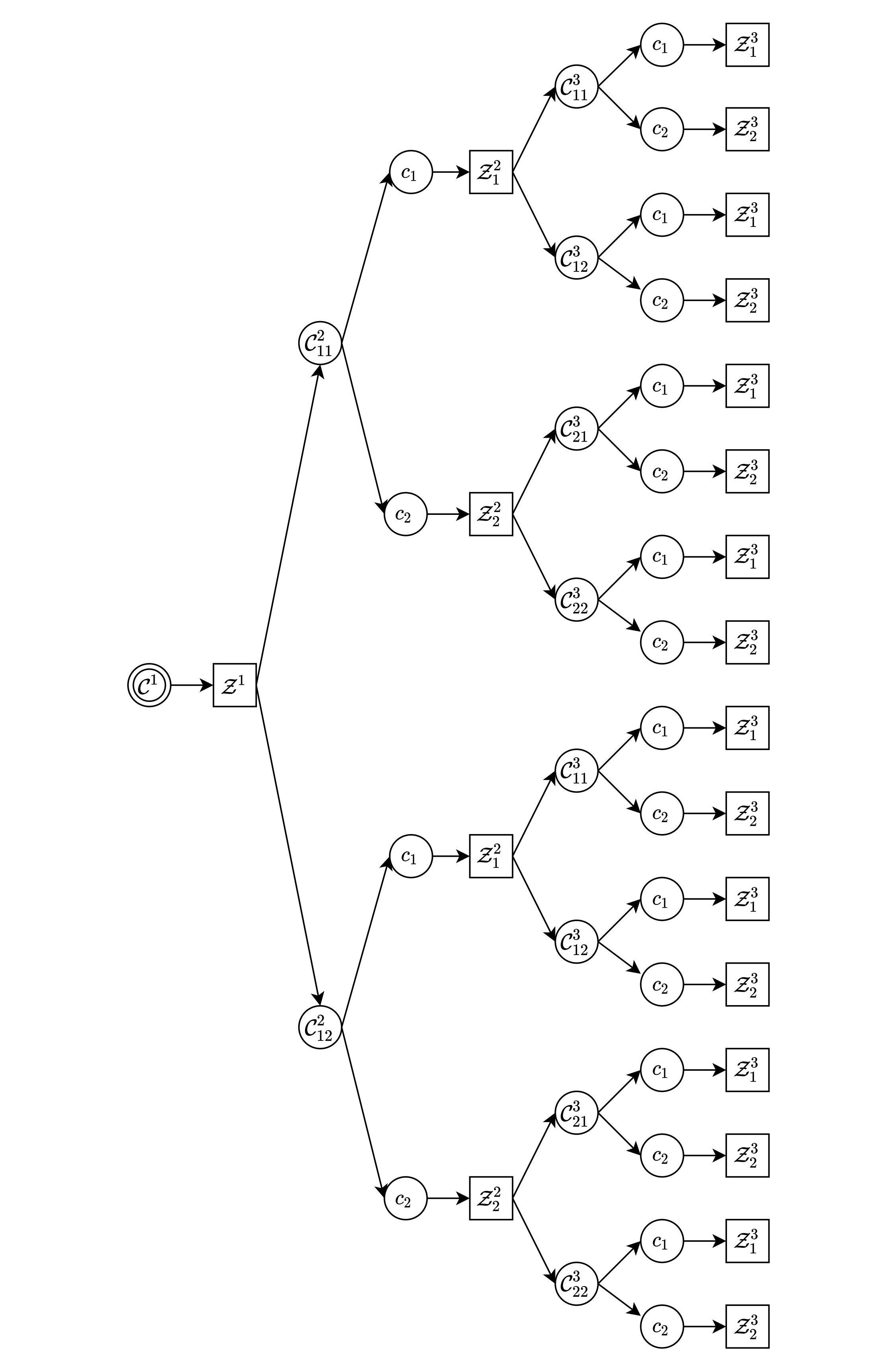

Figure 17: A decision tree representing how single period strategies are combined to form a multi-period strategy.

Figure 17 shows the decision tree of the dynamic approach. For the sake of visualization, there are two asset conditions and two efficient single period decision strategies. This is a simplification that may generally not hold true as the number of efficient decision strategies may vary as a function of the underlying condition. The node $\mathcal{C}^1$ refers to the known initial condition distribution of a single asset. The nodes $\mathcal{Z}_i^t$ refers to a decision node where one efficient single period decision strategy is chosen, conditional on the previous condition i. Furthermore, the nodes $\mathcal{C}_{ij}^t$ represent the realization of the asset condition distribution for the previous condition i and for the jth efficient decision strategy. The nodes c_i refer to the set of potential asset conditions. However, not all strategy combinations are necessarily non-dominated. This can be checked with

$$\begin{aligned}\Psi : \mathcal{P}(\mathbb{Z}) \mapsto \mathcal{P}(\mathbb{Z}) = \{Z \in \mathbb{Z} : \nexists Z' \in \mathbb{Z} :& (\forall \theta^o, \mathbb{E}[\Lambda^o_{\theta^o} \mid Z'] \geq \mathbb{E}[\Lambda^o_{\theta^o} \mid Z]), \\ & (\exists \theta^o, \mathbb{E}[\Lambda^o_{\theta^o} \mid Z'] > \mathbb{E}[\Lambda^o_{\theta^o} \mid Z]) \\ & \forall o \in \mathcal{O}\},\end{aligned} \tag{63}$$

which returns all non-dominated decision strategies in the sense of first order stochastic dominance, where $\mathbb{Z}$ is a set of decision strategies, $\mathcal{P}(\mathbb{Z})$ is the power set of $\mathbb{Z}$, the expectation $\mathbb{E}[\Lambda^o_{\theta^o} \mid Z]$ is the probability of reaching or exceeding the threshold θ^o for objective o and $\mathcal{O}$ is the set of all objectives to be maximized.

Furthermore, let $\Xi(t)$ be the set of all efficient single period decision strategies in period t. In other words, this set represents the solutions to influence diagrams in section 3.1.5. Then, the set of efficient decision strategies for periods spanning 1 through t is

$$\Phi(1, ..., t) = \begin{cases} \Xi(t), & t = 1 \\ \Psi(\Phi(1, ..., t-1) \times \Xi(t)), & t \geq 2. \end{cases} \tag{64}$$

The key implication is that for a decision strategy Z to be non-dominated all its local decision strategies $Z_i, i \in D$ must also be locally non-dominated. Because if Z_i is locally dominated, it can be replaced with another local decision strategy without adversely affecting any of the objectives. However, a combination of non-dominated local decision strategies does not necessarily mean that a global decision strategy would also be non-dominated. In other words, if the objectives are additive over the periods, then non-dominated local decision strategies is a necessary condition for a global strategy to be non-dominated, but not a sufficient condition. By combining multiple single period strategies the number of periods that can be considered is significantly more than just by extending an influence diagram, especially if the period specific chance constraints are strict enough to discard many decision strategy alternatives.

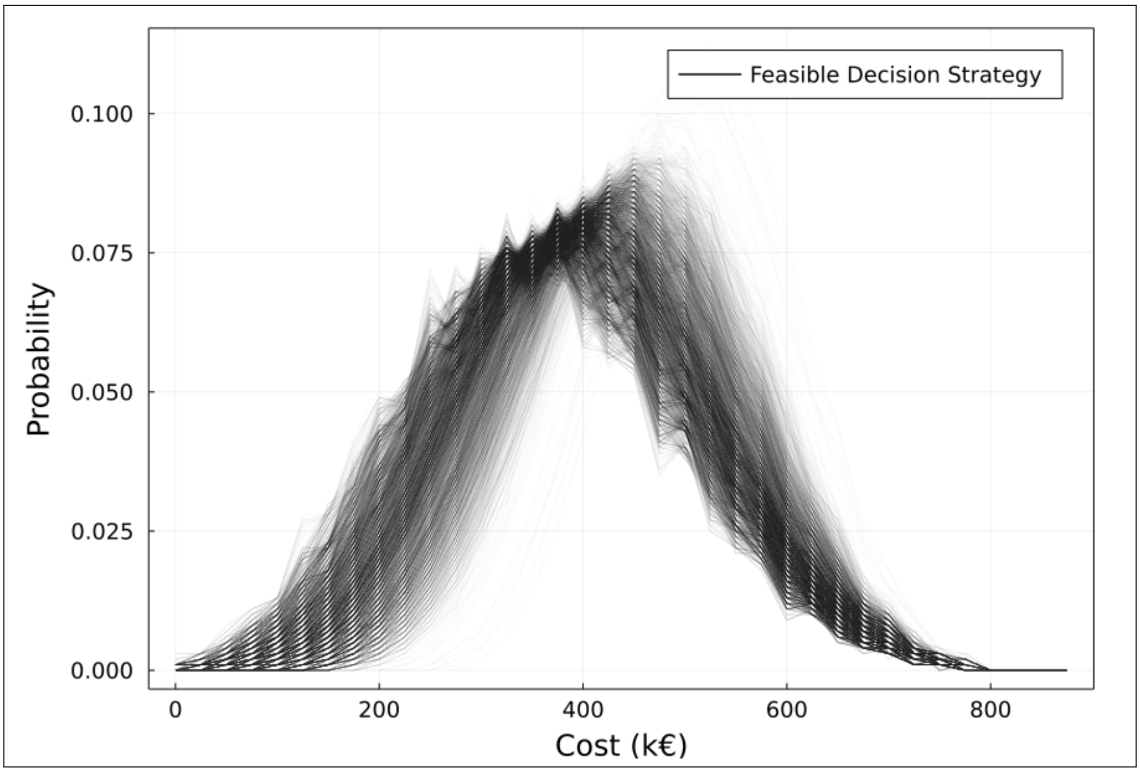

Figure 18: The cost distribution of the first order stochastically non-dominated strategies for seven periods. The darkness indicates how frequent a specific value is.

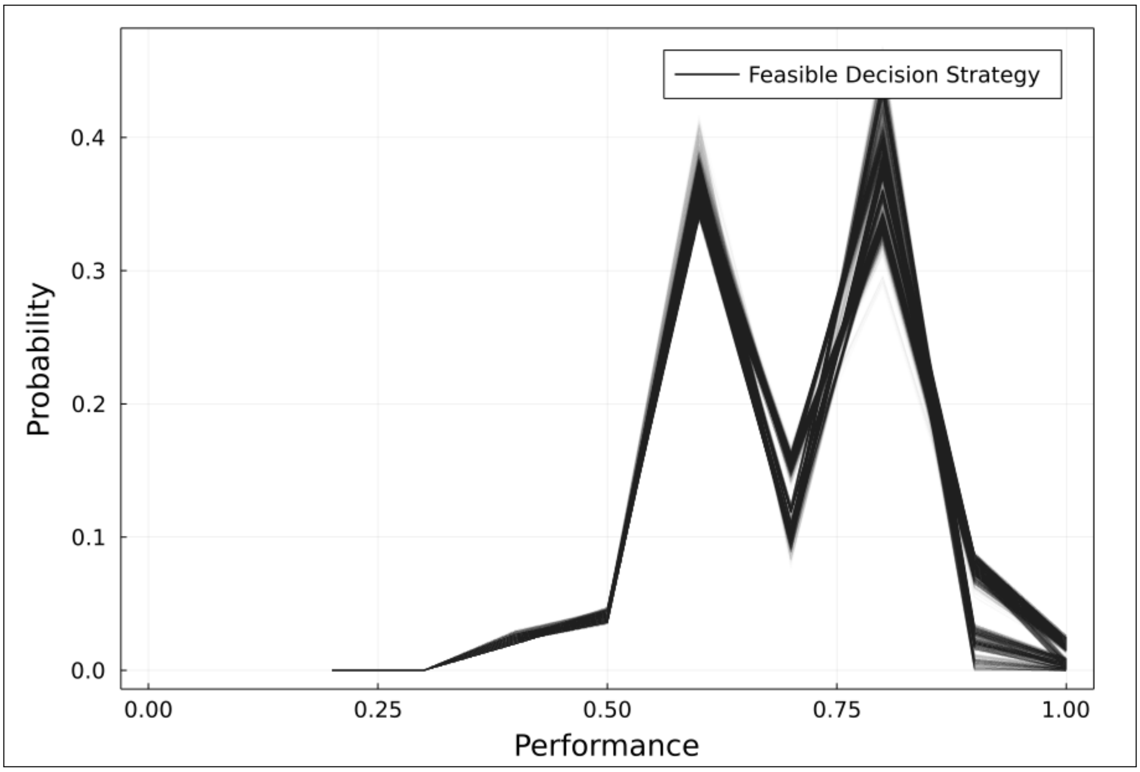

Figure 19: The performance distribution of the first order stochastically non-dominated strategies for seven periods. The darkness indicates how frequent a specific value is.

For a seven period model, determined with $\Phi(1, ..., 7)$, there are 5145 feasible non-dominated decision strategies with the parameter values from Section 3.1. The strategies were also subject to chance constraints on the single period costs performance. The single period cost is not allowed to exceed 100000 with a probability higher than 25% and the performance can not be lower than 40% with a probability higher than 10%. The cost distributions are in Figure 18 and the performance distributions are in Figure 19. Many of the feasible decision strategies are similar to each other. This is caused by the strictness of first order stochastic dominance, because even the slightest difference in on of the threshold levels may make a strategy non-dominated. Using some other method to determine dominance, for example, second order stochastic dominance, could significantly lower the number of non-dominated solution alternatives. In asset portfolio management problems small differences between decision strategy alternatives may not bring extra value to a decision maker, at least when considering the extra computational effort that they entail. Thus some of the decision strategies may have to be discarded.

To limit the number of strategies they can, for example, be clustered according to their objectives with the k-means clustering method [19]. This means that the entire set of non-dominated strategies can be used to determine a representative sample of decision strategies without losing too much accuracy. For example, if the number of decision strategies that a decision maker wants to consider is ten, then the k-means method can be used to calculate ten alternatives. Regardless, the number of considered decision strategies can in most cases be decreased significantly from the set of all first order stochastically non-dominated strategies without affecting the results too much.

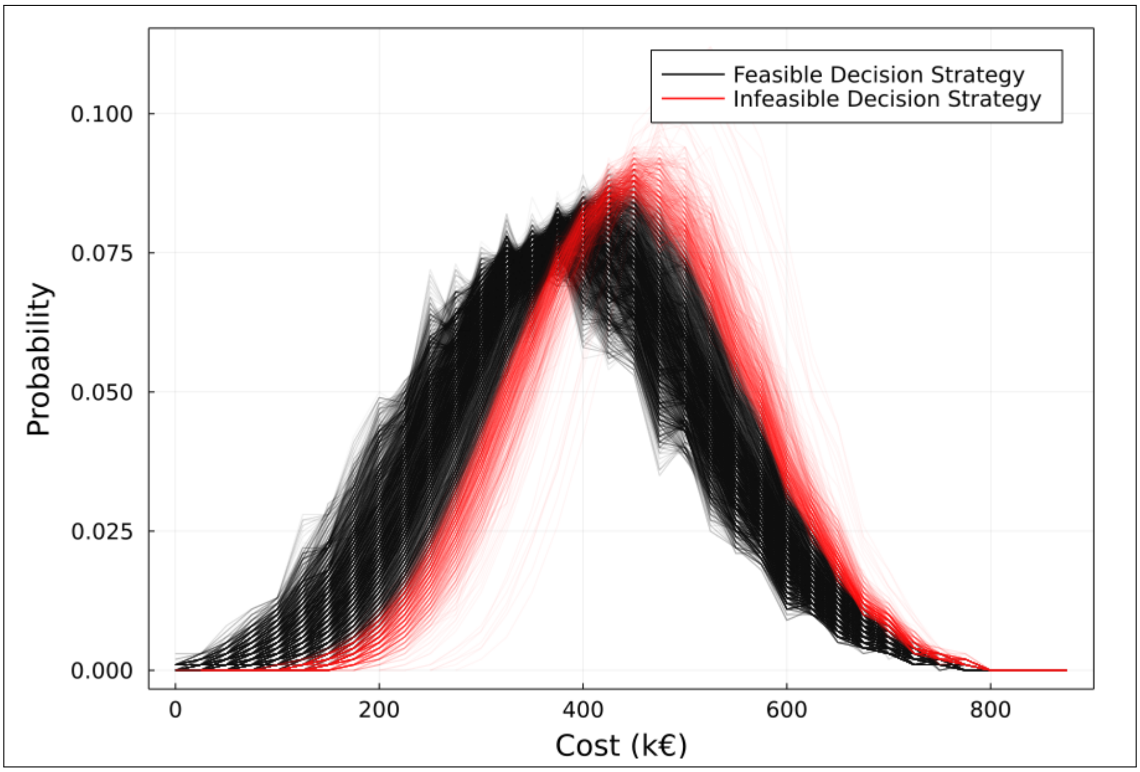

Figure 20: The cost distribution of the feasible and infeasible first order stochastically non-dominated strategies for seven periods. The darkness indicates how frequent a specific value is.

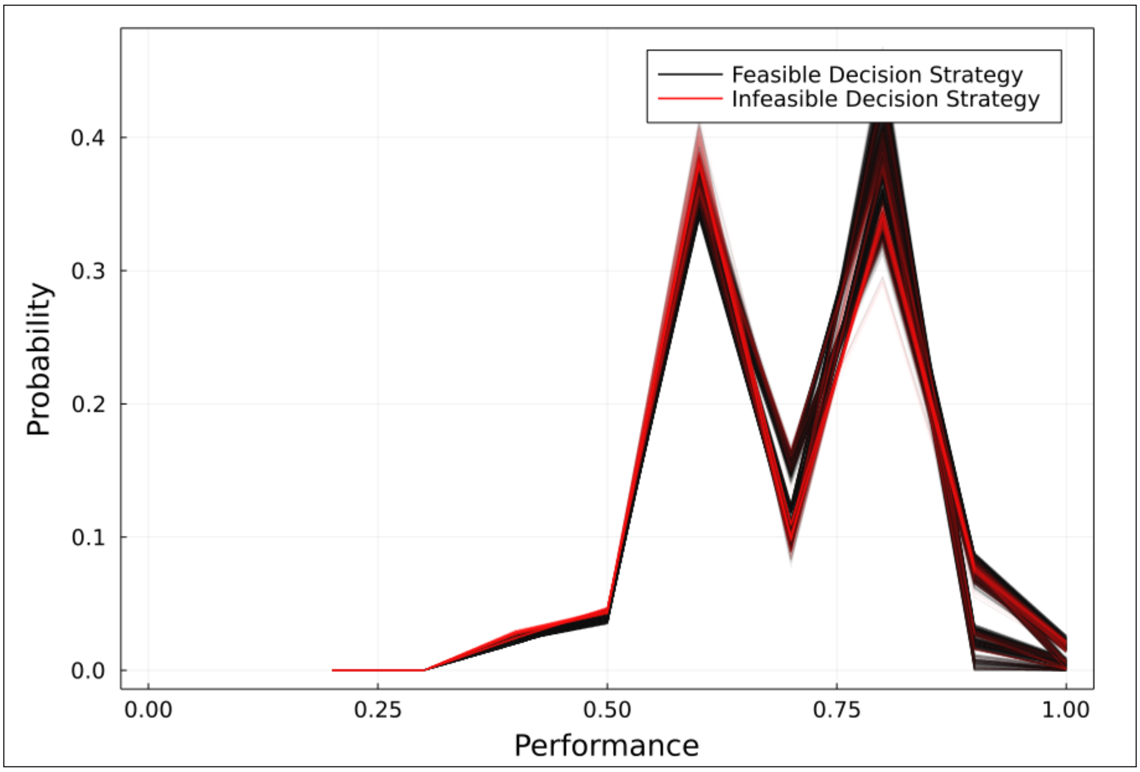

Figure 21: The performance distribution of the feasible and infeasible first order stochastically non-dominated strategies for seven periods. The darkness indicates how frequent a specific value is.

Furthermore, the decision maker can also determine chance constraints that span multiple periods. For example, the probability that total costs exceed 600000 can be constrained to less than 10% and the probability that the total performance is lower than 50% can be constrained to be less than 7%. This means that all the red decision strategies in Figures 20 and 21 become infeasible. In total 816 of the previously feasible and non-dominated decision strategies become infeasible with these chance constraints.

3.2 Management of an Asset Type Portfolio

An asset type portfolio is a portfolio of assets that all share the same type and thus their performance on all objectives are comparable. The goal is to determine a maintenance strategy for each asset. One way to model this problem is to duplicate the

Decision Programming formulation for all the assets that are to be managed. However, this will cause the numerical complexity of the problem to increase considerably. To solve the problem efficiently, symmetry can be utilized because the assets are comparable. Thus, instead of determining which asset a specific maintenance strategy is applied to, the number of assets to which a specific maintenance strategy is to be determined. In other words, the goal is to solve

$$a_1 + a_2 + ... + a_k = m, \tag{65}$$

$$a_i \in \{0, 1, ..., m\}, \tag{66}$$

in which a_i is a non-negative integer that represents the number of assets to which maintenance strategy i is applied to and m is the total number of assets. The number of solutions to this problem is $\binom{m+k-1}{k-1}$ [26].

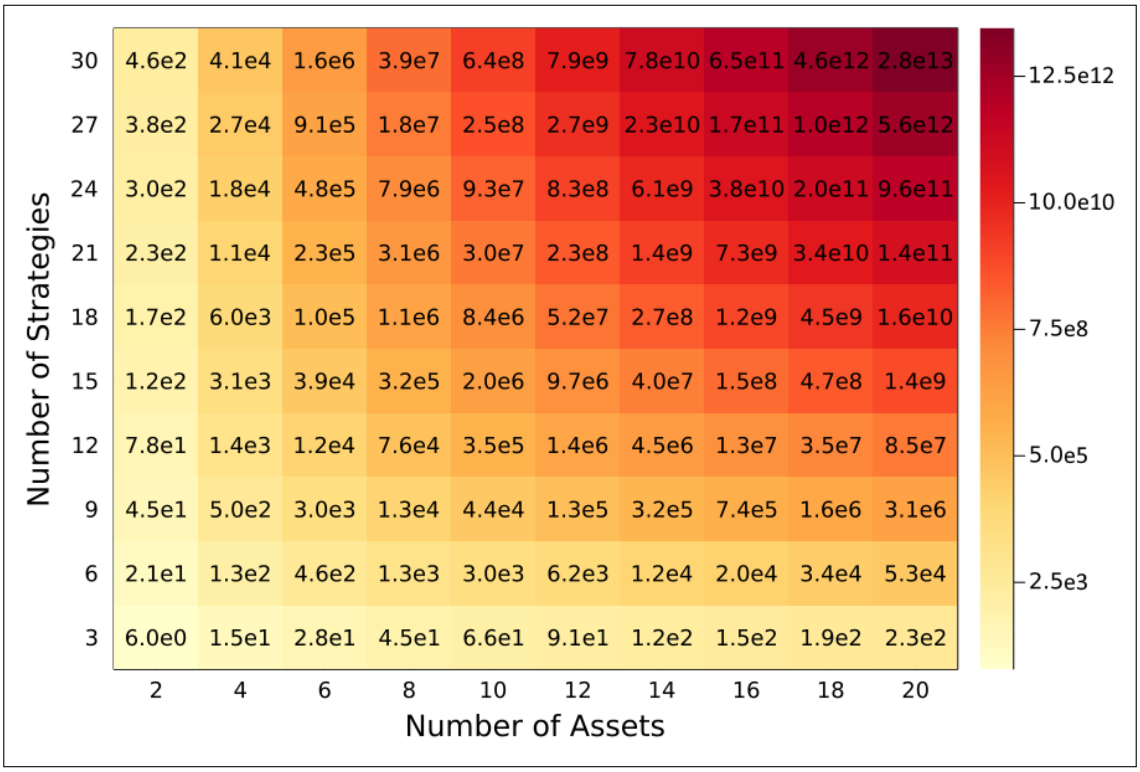

Figure 22: The number of solutions as a function of the number of assets and strategies.

Figure 22 shows the number of solutions to the problem in (65). The number of solutions does not grow too fast, for example, when the number of assets is 20 and the number of strategies is 30, the total number of portfolio strategies is approximately 2.8×10^{13} (top right corner in figure 22). This means that enumerating over all alternatives is possible with a standard computer. However, enumerating over all alternatives becomes intractable if the number of assets or strategies are significantly higher. In this case one can instead determine what percentage of assets a maintenance strategy is applied to. By choosing an appropriate level of accuracy, for example, 10%, the computational complexity can be adjusted. Another benefit is that an enumerative approach does not impose limitations on modeling and, for example, joint probability distributions are easier to model than with a MILP formulation.

procedure NDATP($\mathbb{Z}$, m)
  $\mathbb{Z}^* \leftarrow \emptyset$
  for $a_1 \in \{0, ..., m\}$ **do**
    ...
    for $a_{k-1} \in \{0, ..., m - a_1 - ... - a_{k-2}\}$ **do**
      $a_k \leftarrow m - a_1 - ... - a_{k-1}$
      $\mathbb{Z}^* \leftarrow \mathbb{Z}^* \cup (\underbrace{\mathbb{Z}_1, ..., \mathbb{Z}_1}_{a_1}, \underbrace{\mathbb{Z}_2, ..., \mathbb{Z}_2}_{a_2}, ..., \underbrace{\mathbb{Z}_k, ..., \mathbb{Z}_k}_{a_k})$
  return $\Psi(\mathbb{Z}^*)$

Algorithm 1: The pseudocode of an algorithm that determines all first order stochastically non-dominated asset type portfolios.

Algorithm 1 takes as input a set of decision strategies and the number of assets, then the algorithm determines all first order stochastically non-dominated asset type portfolios. The assets in the returned portfolios are assumed to be identical, which means that the order of the decision strategies in the non-dominated asset type portfolios can be freely determined without affecting any of the objectives. In practice this means that the algorithm only determines the number of assets a decision strategy is applied to and that a decision maker must choose the specific assets to which the decision strategies are applied to.

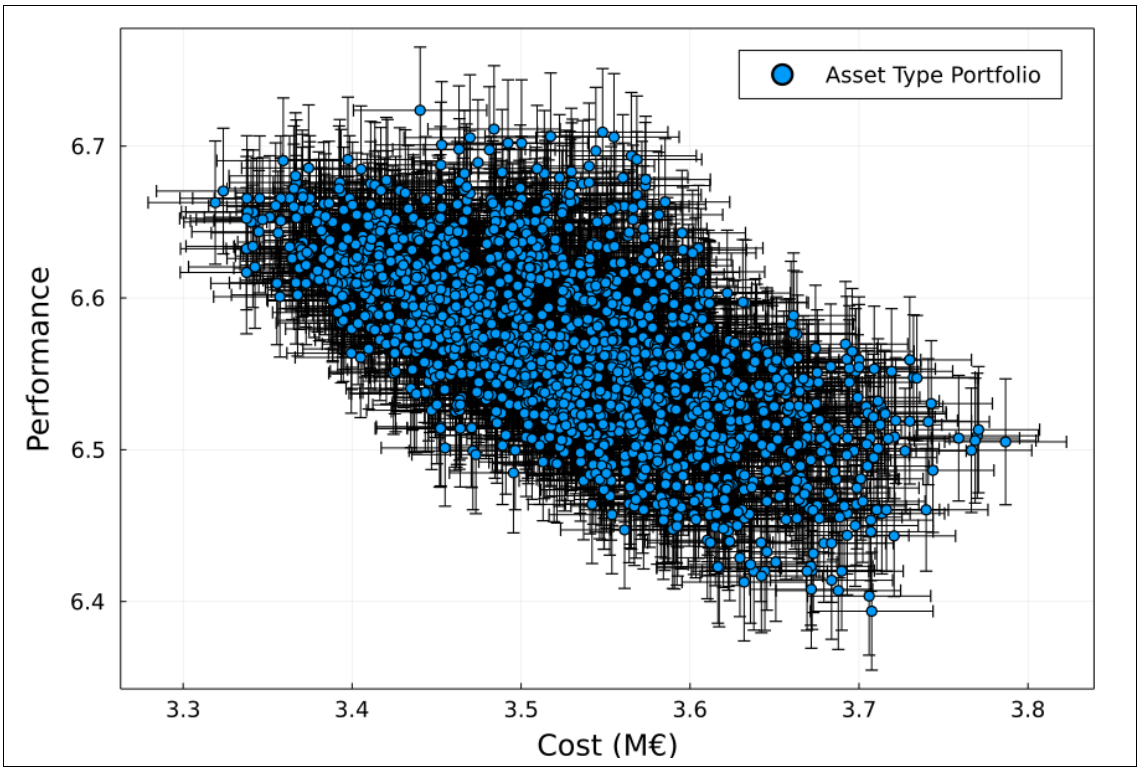

Figure 23: All first order stochastically non-dominated portfolios.

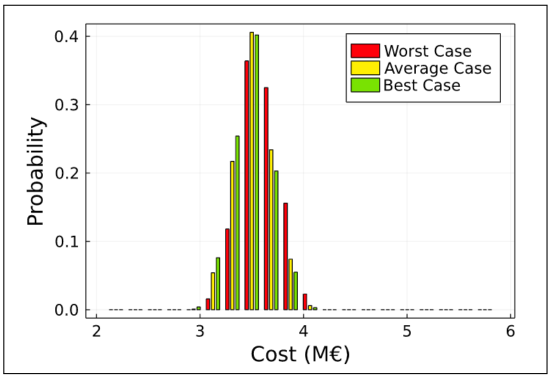

Figure 24: The cost distribution for an asset type portfolio.

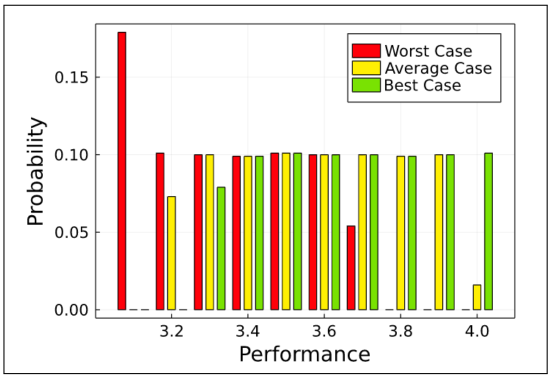

Figure 25: The performance distribution for an asset type portfolio.

For a portfolio of ten assets with the same parameter values as in Section 3.1 and a set of ten representative efficient decision strategies from Section 3.1.8, then there are 1715 first order stochastically non-dominated portfolios in Figure 23. The error bars represent the standard deviation scaled to 10% for purposes of visualization. The portfolio cost and performance distributions for one decision strategy are in

Figures 24 and 25. The external load that the assets are subject to is correlated and thus this also affects the costs and performances. The impact of the load on the cost distribution is less significant than its impact on the performance. This is because the load directly affects the failure rate and because the cost of the repair actions is less than the cost of maintenance actions. However, the load directly affects the performance (see the ID in Figure 11) and thus its impact is more significant.

In accordance with the law of large numbers, the portfolio distribution for both costs and performance are better centered around their respective means than in the case with only one asset. This means that it may be viable to model the portfolio objectives with expected values if the number of assets is large.

3.3 Management of an Asset Category Portfolio

An asset category portfolio is a portfolio of assets that all share the same functionality, but are not necessarily of the same type. This means that the relative performance of each asset type may be different. Which means that the performances of all asset types are scaled so that the asset type with the highest performance is set to 100% and the performance of the other asset types are scaled accordingly. The scaling bounds can be selected arbitrarily, but the lower bound should not be set to 0%. Because then the contribution of certain asset types would be zero, even if it would not be true in practice. The scaling is only applied to the performance and not to the costs.

The asset category management problem can be solved by selecting an efficient portfolio of each asset type with, for example, exhaustive enumeration or with a genetic algorithm and then determining all efficient portfolios. Typically the number of asset types in a single category is relatively small, for example, the USCG has three asset types in the patrol boat category and two asset types in the medium endurance cutter category.

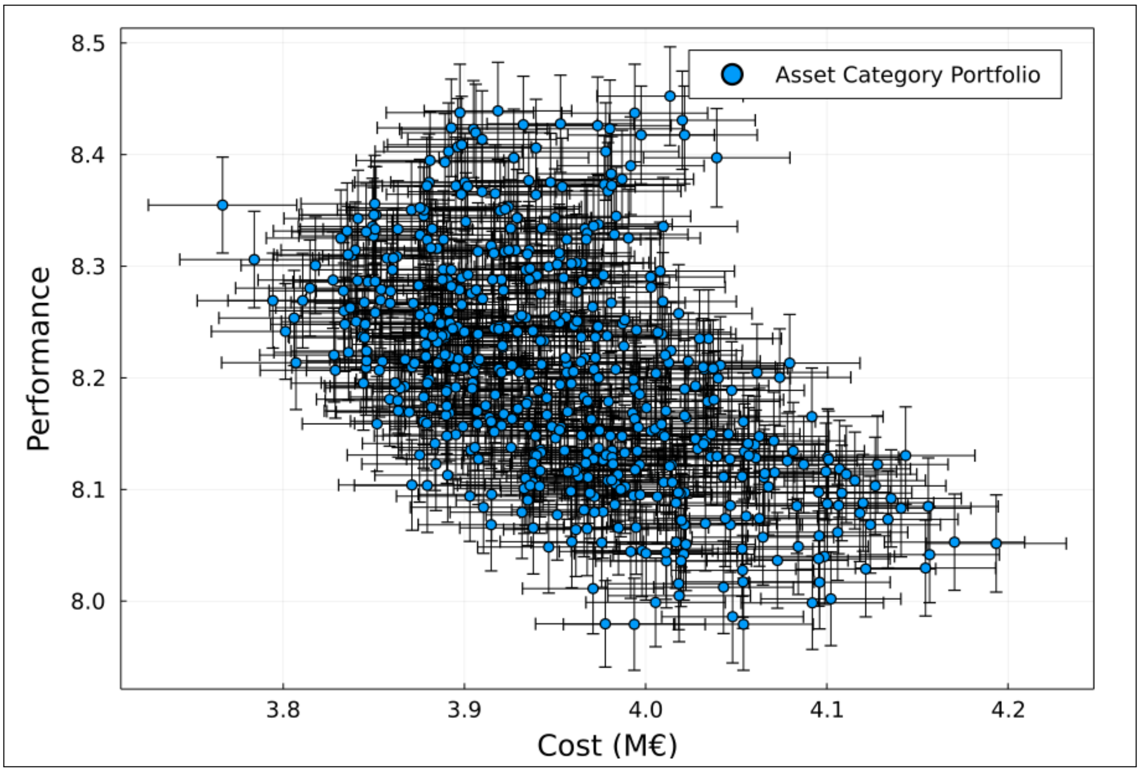

Figure 26: All first order stochastically non-dominated asset category portfolios.

For illustrative purposes consider an asset category portfolio that consists of two asset types. Assume that there are ten assets with the same parameter values as in Section 3.1 and assume that there are four assets with costs being 30% lower and performance being 10% lower. Furthermore, assume that a representative sample of 50 portfolios are selected with the k-means method for both asset types to improve on computational performance, then the number of first order stochastically non-dominated asset category portfolios is 578, which are presented in Figure 26. The error bars represent the standard deviation scaled to 10% for purposes of visualization. The relative standard deviation of costs is decreased on average by approximately 5% and the relative standard deviation of performance is decreased on average by approximately 15% when comparing the asset category portfolio to either asset type portfolio. This is an expected result, because diversification yields benefits when managing multiple assets that are not perfectly correlated with each other.

3.4 Management of a Multi-Category Portfolio

A multi-category portfolio is a portfolio of assets of which some have distinct functionalities. In other words, all assets do not belong to the same category and multiple different and non-comparable performance objectives must be considered. For example, for the USCG the different categories are: long range enforcers, icebreakers, buoy and construction tenders, medium endurance cutters and patrol boats. The distinction of assets into different categories is especially important if the performance of one category is susceptible to disruptions that may interfere with the performance of the other categories. For example, the USCG manages only three icebreakers [32], which means that the overall mission of the USCG may be critically impeded if all three break down at once. Even if it may be difficult to compare the performances of the different categories, comparison of costs is relatively straightforward, which is why it is import to consider all asset categories simultaneously.

Conceptually this problem is the same as in the asset category management problem and is solved by selecting one efficient portfolio for each asset category. In other words, the problem can be solved with exhaustive enumeration or with a genetic algorithm to determine all efficient multi-category portfolios.

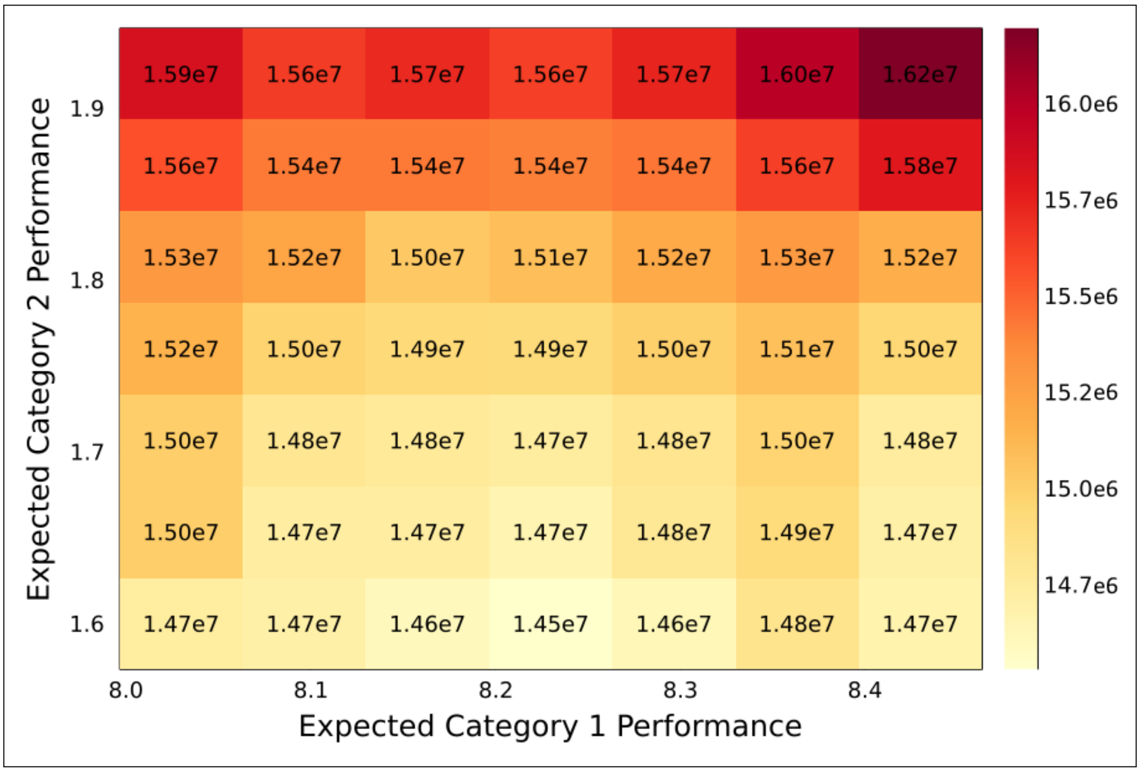

Figure 27: Total portfolio cost, indicated by the color bar, as a function of category performances.

For illustrative purposes consider that there are two asset categories, of which one is described in Section 3.3. Assume that the other asset category consists of two assets, with 400% higher costs and otherwise similar characteristics as in Section 3.1. Furthermore, assume that a representative sample of 50 portfolios are selected with the k-means method for both asset types to improve on computational performance. Then the number of first order stochastically non-dominated multi-category portfolios is 2116. The lowest cost for expected performances levels for both categories are in Figure 27. Each portfolio is in reality represented by underlying probability distributions, however, the expected performances are used for the purpose of visualization. This is also the reason for why, for example, the portfolio with expected performances of 8.05 with respect to the first category and 1.55 with respect to the second category and with a total cost of 15.0×10^6 may seem to be dominated even if it is not.

The percentage of feasible portfolios that are not dominated is approximately 85%. In the asset category portfolio problem the percentage of feasible portfolios that are not dominated is approximately 23%. The difference is caused by the added performance objectives, which generally results in more non-dominated solutions. Nonetheless, some feasible multi-category portfolios are dominated because the costs are additive and multiple categories do generally bring benefits in the form of diversification. Therefore, it may be of value for decision makers to consider a multi-category portfolio rather than to just focus on multiple single category portfolios independently. Thus, a decision maker can make better informed decisions and he is also presented with information that shows how what budget must be spent to achieve his performance target levels. For example, Figure 27 shows that it is on average more costly to increase the performance in category two than it is to increase it in category one.

4 Conclusions

This thesis studies the applicability of the Decision Programming framework and influence diagrams to asset portfolio management problems. This thesis iteratively builds an illustrative decision model for a single asset for multiple periods and multiple objectives under uncertainty and disruption risks. The decision model is then used to derive a maintenance strategy. The maintenance strategies are then used to construct portfolio level maintenance strategies in a multi-level model. Ultimately, a general framework for tackling asset portfolio management problems is created.

The main contribution of this thesis is the exploration of how disruption risks can be modeled with Decision Programming. Furthermore, a method with which multi-period maintenance strategies can be constructed from single period models was developed. This approach scales better than just extending an influence diagram and thus more complex multi-period problems can potentially be modeled with Decision Programming.

A key limitation of this thesis is that the methodology is only tested on illustrative data and that all assumptions may not apply. For example, this thesis assumes that the costs of testing the condition of an asset are low compared to the other costs, but this may not hold true in all cases. Another limitation is that the decisions in the Decision Programming models are limited to only the most current information, in other words the assumption of no forgetting is not considered. This may be a central aspect in asset management problems in which the assets do not deteriorate based on a Markovian process. Furthermore, all aspects of the decision models in this thesis are discretized. This means that the discretization resolution may have a significant impact on the solution. Incomplete information about maintenance or repair costs was also not modeled, which may in some cases impact the results [18].

The conceptual framework was also discussed with representatives of the Finnish Transport Infrastructure Agency and the aspects considered by this thesis were considered to be relevant, especially the use of influence diagrams. All in all, the conceptual asset portfolio management framework developed in this thesis in combination with the Decision Programming framework is a general methodology that is applicable to a vast number of asset portfolio management problems.

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