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# Macroeconomic Calibration Methods in Loss Given Default Modelling

**Andrea Lyly**

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**Author** Andrea Lyly

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**Supervisor** Prof. Ahti Salo

**Advisor** M.Sc. Juuso-Markus Jeskanen

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**Abstract**

Financial institutions are required to perform credit loss provisioning under the International Financial Reporting Standard 9 (IFRS 9) accounting standard by calculating expected credit losses (ECL). ECL is calculated using three risk parameters: the probability of default (PD), loss given default (LGD) and exposure at default (EAD). This thesis focuses on LGD, which describes the percentage loss when the counterparty goes into default. The objective of the thesis is to develop a framework for calibrating internal-ratings based LGD models for non-defaulted exposures to IFRS 9 ECL calculations.

The calibration of the LGD model to IFRS 9 realized loss levels is discussed, and two methods for incorporating point-in-time macroeconomic scenario adjustments are presented. The first method models the loss rate time series with ordinary least squares (OLS) regression, and compares the loss rate scenario forecasts to the long-run average losses to obtain scenario scalars, which are used to directly adjust the LGD estimates. The second method models the growth rates of collateral values using OLS and expert judgement. The collateral values are typically used as input for the LGD model, because LGD is highly dependent on the ratio of the exposure and collateral values. The collateral values are adjusted for macroeconomic scenarios to indirectly adjust the LGD estimates.

The framework is tested using simulated data sets that aim to describe the behaviour of a real residential mortgage portfolio. The results discuss how the presented models can be developed and incorporated into ECL calculations. The loss rate model adjustments perform well when the loss rates are influenced by macroeconomic factors. If the loss rate model is not applicable, the collateral value adjustments can be used instead. The effectiveness of the latter approach, however, depends on the underlying LGD model structure and the performance of collateral value forecasts. In summary, the framework can be used for calibrating LGD models into real ECL applications, and it can be also extended for other portfolios, defaulted exposures, and alternative modelling techniques.

**Keywords** Loss given default, international financial reporting standard 9, calibration, regression, macroeconomic scenarios

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**Tiivistelmä**

Rahoituslaitosten on tehtävä luottotappiovarauksia laskemalla odotettuja luottotappioita (ECL) International Financial Reporting Standard 9 (IFRS 9) tilinpäätösstandardin mukaisesti. ECL lasketaan käyttämällä kolmea riskiparametria: maksukyvyttömyyden todennäköisyyttä (PD), tappio-osuutta (LGD) ja vastuun määrää maksukyvyttömyyden hetkellä (EAD). Tämä opinnäytetyö keskittyy LGD-parametriin, joka kuvaa prosentuaalista tappiota, kun vastapuolesta tulee maksukyvytön. Opinnäytetyön tavoitteena on kehittää viitekehys, jossa terveiden vastuiden sisäisiin luottoluokituksiin tarkoitettuja LGD-malleja kalibroidaan IFRS 9 ECL-laskentoihin, missä hyödynnetään makrotaloudellisia skenaarioita.

Viitekehyksessä LGD-malli kalibroidaan vastaamaan IFRS 9:n mukaisia luottotappioita, ja kahta menetelmää tutkitaan makrotaloudellisten skenaarioden sisällyttämisestä LGD-estimaatteihin. Ensimmäisessä menetelmässä tappio-osuus aikasarjaa mallinnetaan lineaarisella regressiolla, ja mallin tuottamia skenaarioita verrataan pitkän aikavälin tappio-osuuksiin, joista saadaan kertoimet eri skenaarioille. Kertoimilla korjataan LGD-estimaatteja suoraan. Toisessa menetelmässä mallinnetaan vakuusarvojen kasvuasteita regressiolla ja asiantuntijaperusteisesti. Vakuusarvot toimivat tyypillisesti LGD-mallien syötteenä, koska LGD on vahvasti riippuvainen vastuun määrän ja vakuusarvojen suhteesta. Vakuusarvoja korjataan erilaisiin skenaarioihin, joka näkyy epäsuorasti LGD-estimaateissa.

Viitekehystä testataan käyttämällä simuloitua dataa, joka pyrkii kuvaamaan todellista asuntolainasalkun käyttäytymistä. Tuloksissa kuvataan esitettyjen mallien kehitys ja soveltaminen ECL-laskennoissa. LGD-estimaattien korjaukset tappio-osuus aikasarjamallin avulla toimivat hyvin, kun tappioista löytyy riippuvuus makrotalousellisiin tekijöihin. Vakuusarvojen korjauksia voidaan käyttää, mikäli toimivaa aikasarjamallia tappio-osuudelle ei löydy. Vakuusarvojen korjausten tehokkuus kuitenkin riippuu LGD-mallin rakenteesta ja vakuusennusteiden toimivuudesta. Yhteenvetona voidaan todeta, että viitekehystä voidaan käyttää LGD-mallien kalibroimiseen ECL laskentoihin, ja jatkokehittää kattamaan erilaisia lainasalkkuja, maksukyvyttömiä vastuita sekä erilaisia menetelmiä.

**Avainsanat** Tappio-osuus, IFRS 9, kalibrointi, regressio, makrotaloudelliset skenaariot

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Vantaa, February 5, 2024

Andrea Lyly

# Contents

## Abbreviations and Acronyms

| | |
|---|---|
| ACF | Autocorrelation function |
| AIC | Akaike Information Criterion |
| BCBS | Basel Committee of Banking Supervision |
| BG | Breusch–Godfrey (test) |
| CAL | Calibration sample |
| CPI | Consumer price index |
| EAD | Exposure at default |
| EBA | European Banking Authority |
| ECL | Expected credit loss |
| EIR | Effective interest rate |
| EUR12 | 12-month Euribor |
| Euribor | Euro Interbank Offered Rate |
| GDP | Gross domestic product |
| HPI | House price index |
| IASB | International Accounting Standards Board |
| IFRS 9 | International Financial Reporting Standard 9 |
| IRB | Internal ratings-based (approach) |
| IS | In-sample |
| LGD | Loss given default |
| LRA | Long-run average |
| LTV | Loan-to-value |
| MAE | Mean absolute error |
| OLS | Ordinary least squares |
| OOS | Out-of-sample |
| OOT | Out-of-time |
| PD | Probability of default |
| PIT | Point-in-time |
| RMSE | Root mean squared error |
| RR | Recovery rate |
| SA | Standardized approach |
| SR | Survival rate |
| TTC | Through-the-cycle |
| VIF | Variance inflation factor |

# 1 Introduction

Granting loans is core business for banks. Similarly as making investments into stocks or bonds, lending money can be viewed as a long- or short-term investment, which relies on the customers ability to pay back the borrowed amount with interest. Lending exposes banks to credit risk, which is a type of risk where the bank might not receive payments on the made investments because the counterparty goes into default (McNeil et al., 2015). Default events can result into small or large credit losses depending on the recovery process. Hence, banks need to perform credit loss provisioning in their financial reporting by creating a capital reserve to cover the losses.

Credit loss provisioning has previously been practiced under the *IAS 39: Financial Instruments: Recognition and Measurement* (IAS 39) standard adopted by the International Accounting Standards Board (IASB) in 2001 (IASB, 2014). The IAS 39 standard used the *incurred loss model* in provisioning, which allowed to recognize and provision for credit losses only when there was objective evidence of losses being incurred on the balance sheet date, and it was not allowed to include expected future losses (Novotny-Farkas, 2016). The financial crisis revealed that the incurred loss model under the IAS 39 was a weakness in the global financial system, as the delayed recognition of credit losses was seen to have hidden systemic costs (Gubareva, 2021). To combat these issues, the IASB issued a new standard called *International Financial Reporting Standard 9* (IFRS 9) in 2014 to replace the preceding IAS 39. The new standard was fully adopted in January 2018. IFRS 9 establishes principles that financial institutions and banks can use in their financial reporting regarding assets and liabilities (IASB, 2014). The novelty in terms of credit risk and provisioning was to replace the incurred loss model with a new *expected loss model*, which quantifies expected credit loss (ECL) over the life of the financial instruments using forward-looking estimates (Miu and Ozdemir, 2017).

The calculation of ECL is based on three risk parameters: Probability of Default (PD), Loss Given Default (LGD) and Exposure at Default (EAD) (Gubareva, 2021, Miu and Ozdemir, 2017). The same risk parameters are also used in the regulatory capital calculations under the advanced internal ratings-based approach (A-IRB) (Miu and Ozdemir, 2017, Stephanou and Mendoza, 2005), which was introduced by the Basel Committee of Banking Supervision (BCBS) in the Basel II accord (BCBS, 2006).

This thesis focuses on the LGD risk parameter, which is the percentage amount of loss from the exposure at default that is determined from the received discounted payments from the default recovery process (Kellner et al., 2022). The general predictability of LGD is usually challenging due to complex and long recovery processes, and distributions with typically high probability masses around zero and one values (Kellner et al., 2022). Many modelling approaches for LGD under A-IRB approach have been studied, which include regression and multi-stage models (Tanoue et al., 2017), survival analysis models (Joubert et al., 2018, Witzany et al., 2010), and more advanced models such as neural networks and support vector machines (Loterman et al., 2012, Qi and Zhao, 2011). In practice, models such as linear or logistic regression are usually employed due to their simplicity and ease of interpretation.

Appropriate risk quantification is further ensured by calibrating the LGD model estimates to correspond long-run average LGD (EBA, 2017). Furthermore, the A-IRB models tend to follow though-the-cycle (TTC) rating philosophy, i.e., they are unconditional to the macroeconomic environment (Miu and Ozdemir, 2017). However, the average levels of LGD can in reality vary through time depending on the macroeconomic circumstances. Therefore, the IFRS 9 standard calls for point-in-time (PIT) risk rating philosophy, meaning that estimates should be sensitive and conditioned to the expected macroeconomic environment (Miu and Ozdemir, 2017). The TTC and PIT ratings usually differ in the stability of the risk quantification over time such that the PIT ratings tend to be more volatile and cyclical (Novotny-Farkas, 2016).

In IFRS 9, there are no specific or strict methodologies for calculating ECL, and thus, financial institutions often choose to adapt already developed A-IRB models into the IFRS 9 ECL framework (Gubareva, 2021). This approach aims to be consistent with the risk management operations, and in terms of resources in model development and maintenance it aims to reduce workload (Miu and Ozdemir, 2017). The alternative approach would be to develop entirely new IFRS 9 risk parameter models. The former, while being lighter, requires identifying the most crucial methodological differences between A-IRB and IFRS 9 and applying adjustments in correct and consistent manner to obtain proper PIT estimates. Incorporating macroeconomic conditions to the IFRS 9 LGD models via PIT adjustment techniques are not widely present in literature, but still there are studies that cover the most reasonable methodologies.

The first PIT adjustment method is to directly adjust the calibrated LGD estimates, i.e., adjusting the outputs of the model. An example was presented in Joubert et al. (2021), where a separate error correction model (ECM) was applied on top of the developed survival analysis based LGD model. The ECM was fitted to the average LGD estimate time series with exogenous macroeconomic factors. The ECM outputs were then used to calculate adjustment scalars for different macroeconomic scenarios, which were used to adjust individual LGD estimates. Joubert et al. (2021) also proposed to study the use of regression models with time series errors for the same application.

The second method is to incorporate macroeconomic conditions into the risk drivers (explanatory variables) used in the LGD model, i.e., adjusting the inputs of the model. An example was presented in Miu and Ozdemir (2017), where LGD was estimated by applying a regression model for forecasting annualized growth rates of collateral values. This is an intuitive approach, because collateral values can be assumed to follow the macroeconomic conditions. However, the research of Miu and Ozdemir (2017) used a relatively simple model, where it was assumed that the total recovery rate could be approximated in terms of collateral value and the exposure at default. Thus, there is room for development by generalizing the methodology for regression type of models, and also to alternative risk drivers.

Both adjustment types can be modelled with similar methodologies which take into account time series theory. The objective is to model how the observed loss rate time series and the growth rates of risk drivers depend on the macroeconomic conditions. The main modelling method that will be considered is ordinary least squares (OLS) regression. This is a simple method as it is easy to interpret and it enables the use of different model configurations by including the effects of multiple macroeconomic factors with or without lags using different time series variable transformation techniques. Furthermore, it can be extended to a regression with time series errors model if the error terms exhibit autocorrelated behaviour, see, e.g., Tsay (1984).

This thesis presents a framework for converting an A-IRB LGD model into the IFRS 9 expected credit loss regime. The framework describes methodology for calibrating the LGD estimates to the IFRS 9 LGD realization level for proper risk quantification. This thesis assumes that the risk differentiation obtained from the A-IRB model can be applied for IFRS 9 purposes as well to keep synergy between the two, and there is no need to re-estimate the model. The main topic in the framework is to present PIT adjustment methods to obtain forward-looking LGD estimates for the ECL calculations. This consists of identifying the impact of macroeconomic factors to the LGD realizations and risk drivers using OLS time series modelling and applying the developed macroeconomic models to adjust the LGD estimates. Furthermore, the PIT adjusted LGD model is evaluated by testing the model performance on separate testing data. Additionally, the models are backscored to historical data to see how responsive the models are to macroeconomic fluctuations. The LGD modelling and PIT adjustment framework focuses on non-defaulted secured residential mortgages. The consideration of defaulted assets and other types of loans is left for future research although similar methods can be employed for those as well.

As a restriction, the model development and analysis is conducted on simulated data sets, because actual data cannot be used in this thesis due to data privacy policies. Via simulated data, the results are also easier to be generalized for alternative studies. Simulation introduces the possibility to test different types of macroeconomic structures w.r.t risk drivers and LGD to give better understanding how the different PIT adjustment techniques work, and how the performances depend on the macroeconomic conditions. However, it is noted that simulations can cause bias in the modelling and interpretation of the results, and it is not an accurate representation of reality. The simulation algorithm is based on simulating LGD realizations and risk drivers via Gaussian copulas as described in McNeil et al. (2015), and incorporating risk driver distribution changes according to macroeconomic factors which are sourced from Statistics Finland.

The thesis is structured as follows: Section 2 reviews the expected credit loss framework and discusses the definition of loss given default along with background on used risk drivers and modelling techniques. Section 3 presents the point-in-time adjustment framework, which comprises of calibrating the A-IRB LGD model to IFRS 9 loss levels, OLS time series modelling for loss rates, developing a risk driver model for collateral values, and the evaluation of the PIT adjustments. It also describes the simulation algorithm and macroeconomic data. Section 4 presents detailed analysis of the results and shows examples on how the methods can be used. Section 5 provides discussion and conclusions.

# 2 Background

## 2.1 Expected Credit Loss Framework

### 2.1.1 Stage Allocation

The new expected loss model in IFRS 9 introduces a three-stage impairment framework to recognize ECL, where assets are allocated into three different stages according to the changes in the assets credit quality since the initial recognition (Beerbaum and Ahmad, 2015). The stages are illustrated in Figure 1.

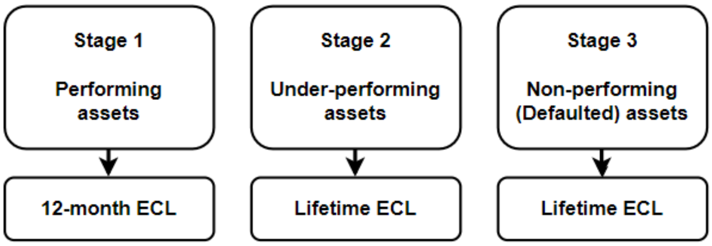


**Figure 1:** The IFRS 9 asset stages according to Beerbaum and Ahmad (2015).

Stage 1 includes assets that have no significant increase in credit risk observed up to the reporting date compared to the origination date, or they have low credit risk in general at the reporting date (Novotny-Farkas, 2016). The stage 1 assets can be named as "performing", and the financial institution can assume to obtain an adequate compensation to the risk it has taken (Gubareva, 2021). For these assets the ECL is recognized on 12-month horizon, meaning expected credit losses that can occur from possible defaults up to 12-months after the reporting date (Novotny-Farkas, 2016). In stage 1 assets the interest revenue is calculated using the assets gross carrying amount (Beerbaum and Ahmad, 2015).

In case significant increase in credit risk is identified, but no objective evidence of impairment is observed, then assets are moved to stage 2 (Novotny-Farkas, 2016). In this stage assets are called "under-performing", but are not considered as defaulted. However, the risk related to holding the assets is assumed not to be anymore compensated by the banks generated proceeds, and thus, lifetime ECL is recognized (Gubareva, 2021). Lifetime ECL stands for expected credit losses which are calculated on the entire remaining lifetime of the assets. Lifetime ECL aims to reflect the present value of possible default losses that can occur during all periods in the assets remaining lifetime, and it is essentially a weighted average over all periods with probabilities of default as weights (Gubareva, 2021). As in stage 1, the interest revenue is calculated on the gross carrying amount of the asset (Beerbaum and Ahmad, 2015).

Lastly, the stage 3 assets in the three-stage approach comprise of assets where objective evidence of credit impairment exists at reporting date (Novotny-Farkas, 2016). Hence, these assets are called "non-performing" or "defaulted". For these assets, lifetime ECL is recognized as in stage 2. Compared to stage 1 and 2, however, the interest revenue is calculated on the net carrying amount of the asset (Beerbaum and Ahmad, 2015).

The IFRS 9 staging approach aims to have forward-looking and timely assessment of potential losses, which was lacking in the IAS 39 standard. For example, the stage 3 assets in IFRS 9 can be interpreted in similar way as in IAS 39, but now the stage 1 and 2 exposures replace those that would be assessed together for impairment under the IAS 39, and hence, the recognition of lifetime ECL will now occur earlier due to the stage 2 allocations (Novotny-Farkas, 2016). In this thesis the focus is on the stage 1 and 2 assets, as both represent non-defaulted assets, and can essentially be modeled using the Basel A-IRB models for performing exposures.

### 2.1.2 Expected Credit Loss

The expected credit loss in IFRS 9 framework is calculated using the probability of default (PD), loss given default (LGD) and exposure at default (EAD) risk parameters, and the general formula for ECL can be expressed as follows:

$$\mathrm{ECL} = \mathrm{PD} \times \mathrm{LGD} \times \mathrm{EAD}. \tag{1}$$

Thus, the ECL formula is a way to quantify the underlying credit risk, the expected value of losses, where PD can be interpreted as the probability of the event of interest (default) and LGD $\times$ EAD can be interpreted as the consequence of the event. For the complementary event, i.e., no default, the losses can be assumed to be zero.

The ECL formulation is considered in detail. Let $t \in \{1, 2, \ldots, T_M\}$ be a time interval that reflects monthly, quarterly or yearly frequencies with respect to a reporting date, i.e., ECL calculation date. The last time point $T_M$ is determined for each asset with respect to the stage allocation and the remaining maturity, i.e., $T_M$ is at maximum 12-months for stage 1 assets, and the remaining lifetime of the assets in stages 2 and 3 (Joubert et al., 2021). Note that the frequency of the forward-looking calculations can vary depending on how the financial institution seeks to address expected credit losses. In the context of converting A-IRB models to IFRS 9 format, it may be convenient to use yearly based ECL calculations as the Basel regulatory capital is calculated on one-year horizon (Miu and Ozdemir, 2017).

Consider the PD risk parameter. The PD gives the probability that an asset will be in default at some point in the upcoming time horizon. For stage 1 assets, for example, the PD gives the probability that the asset will be in default in the next 12-months after the current estimation moment. For stage 2 assets, the PD gives the probability that the asset defaults at some point during its remaining lifetime. For stage 3 assets the PD equals to one, because there is no more uncertainty regarding the default event. A default essentially means that the asset is not able to satisfy its obligations to repay the loan. In IFRS 9 there is no strict definition of default, but the definition should be aligned and consistent with the definition used for internal credit risk management, see, e.g., IASB (2014) par. B5.5.37.

The PD for the entire time horizon may also be characterized in terms of individual time intervals using marginal PD:s. A marginal PD gives the probability that the asset survives to time $t - 1$ and then defaults in time period $t$ (Joubert et al., 2021). This illustrates the term structure of credit risk, which can be defined as the behaviour of credit spread on the varying maturity of assets (Fons, 1994). Mathematically, the marginal PD can be expressed as

$$\mathrm{PD}(t-1,t) = \mathrm{PD}(t)\cdot\mathrm{SR}(t-1),$$

where $\mathrm{PD}(t)$ is the probability of default in the time interval $t$ and $\mathrm{SR}(t-1)$ is the survival rate at time interval $t-1$ (Gubareva, 2021). The marginal PD can equivalently be expressed in terms of the cumulative probability of default $\mathrm{CPD}(t)$ as is described in Gubareva (2021), i.e., $\mathrm{PD}(t-1,t) = \mathrm{CPD}(t) - \mathrm{CPD}(t-1)$.

Next, the LGD and EAD risk parameters are considered. In contrast to PD, these do not exhibit a probabilistic term structure. Instead, LGD is measured over the lifetime of the loan exposure (BCBS, 2015), which essentially means that the LGD estimate is always related to the entire recovery process. Thus, the LGD can be expressed just as an expected value of a percentage loss, when the asset defaults in time interval $t$. This can be noted as $\mathrm{LGD}(t)$. The LGD parameter is described more in detail in Section 2.2.

The EAD depends only on the amount of exposure at the time of default, which can be deterministic according to a pre-defined payment plan, which is common for, e.g., housing loans. There can also be stochastic variation in EAD if the loan has limit that can be withdrawn or that pre-payments occur, see, e.g, Miu and Ozdemir (2017). The EAD can thus be expressed as $\mathrm{EAD}(t) = \mathrm{Exposure}(t)$ in deterministic case, and $\mathrm{EAD}(t) = E[\mathrm{Exposure}(t)]$ in stochastic case.

The IFRS 9 expected credit loss model requires the use of the effective interest rate (EIR) for discounting future cash flow recoveries (Beerbaum and Ahmad, 2015). The EIR is either the effective interest rate determined at the initial recognition of the asset, or the current effective interest rate if the instrument has varying interest rate (IASB, 2014, par. B5.5.44.). Further, the ECL is discounted to the reporting date and not some other date like expected default date (IASB, 2014, par. B5.5.44.). The idea of the discounting is to address the time value of money in the ECL calculation, i.e., the cash flows occurring in the future are not of same value as currently obtained cash flows.

Finally, the ECL formula can be formalized in similar way as is done, for example, in Schutte et al. (2020), Gubareva (2021) and Joubert et al. (2021). Consider a portfolio $P = P_1 \cup P_2$, which consists of both performing assets $P_1$ (stage 1) and under-performing assets $P_2$ (stage 2). The ECL for an asset $i \in P$ is given by

$$\mathrm{ECL}_i = \sum_{t=1}^{T_{M,i}} \frac{\mathrm{PD}_i(t)\cdot\mathrm{SR}_i(t-1)\cdot\mathrm{LGD}_i(t)\cdot\mathrm{EAD}_i(t)}{(1+\mathrm{EIR}_i)^t}, \tag{2}$$

where $\mathrm{EIR}_i$ is the effective interest rate of the asset $i$, and $T_{M,i}$ is the maximum time of ECL calculation depending if the asset $i$ is in stage 1 or 2.

The formula (2) applies mainly for stage 1 and stage 2 assets, which are not defaulted. When an asset is in default, then the PD becomes one. The only uncertainty that lies in the recovery process is the amount of cash flows the institution is going to recover. Thus, the expected losses are calculated w.r.t the exposure amount which is outstanding at the reference date.

### 2.1.3 Comparison to Basel Accord

The main purpose of the IFRS 9 ECL framework is credit loss provisioning, which aims to cover expected losses. The ECL is assessed at each reporting date and the projected values of ECL are recognized in the profit and loss statement (Temim, 2019). Usually, when banks are subject to IFRS 9 they are also subject to the Basel III Accord capital requirements (Temim, 2019).

The Basel framework aims for credit measurements to calculate regulatory capital, which is the minimum capital requirement for banks to hold as buffer (Tobback et al., 2014) in order to cover for unexpected losses (Temim, 2019). The illustration of expected and unexpected losses is shown in Figure 2.

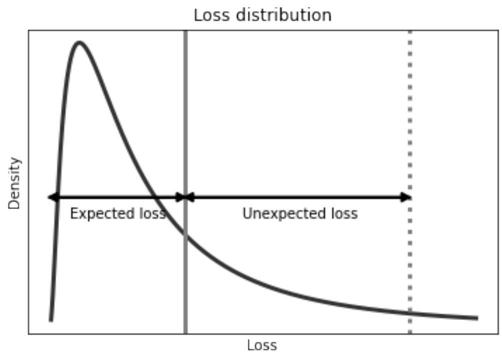


**Figure 2:** An illustration of the credit loss distribution and how expected and unexpected losses are seen.

The requirements are defined by the Basel Committee of Banking Supervision (BCBS), and the regulatory capital is calculated based on risk weighted assets (RWA) such that the total regulatory capital must be no lower than 8% of the RWAs (BCBS, 2006, King and Tarbert, 2011, Stephanou and Mendoza, 2005). The regulatory capital calculations are currently guided through the third Basel Accord (Basel III), see, e.g., BCBS (2011). Basel III was taken into full implementation in January 2023 in EU institutions.

The Basel accord provides two approaches to calculate regulatory capital: the standardized approach (SA) and the internal ratings-based approach (IRB). In the SA the banks use risk weights given by external credit assessments, while in IRB the banks can use internal models for RWA calculations (Temim, 2019). The IRB-approach can further be divided into two alternatives: "foundation" and "advanced". The differences between these is that in the foundation approach institutions can use internal models only for PD calculations, while in advanced approach all PD, LGD and EAD risk parameters can be calculated using internal models (Temim, 2019). Hence, for the rest of the thesis when IRB is mentioned it always indicates the advanced approach.

In terms of LGD models, IRB and IFRS 9 have some fundamental differences which are described next according to Temim (2019). First, the intention of the LGD estimate in the Basel IRB case is a "downturn" estimate that reflects adverse economic conditions, while in IFRS 9 the estimate reflects “current”, best estimate and forward-looking economic conditions. Second, in terms of collection costs, both direct and indirect costs associated with the collection of the exposure are considered in Basel, but in IFRS 9 only directly attributable costs to the collection of recoveries are considered. Third, Basel and IFRS 9 discount cash flows differently, see Section 2.2.1 for details. Lastly, in IFRS 9 there is no requirement on the data period used for modelling purposes, but Basel requires a minimum of five years of data to be used in modelling for retail exposures and seven years for sovereign, corporate and bank exposures.

In terms of rating philosophies the Basel and IFRS 9 have a different view. Point-in-time (PIT) rating philosophy is an essential concept in the IFRS 9 ECL framework. The main target of PIT philosophy is to capture the conditions of the current and expected macroeconomic environment and reflect them in risk parameter calculations. The through-the-cycle (TTC) rating philosophy aims in stable in risk parameter calculations regardless of the macroeconomic conditions. A hybrid approach is a combination of PIT and TTC rating philosophies, where current economic conditions are incorporated in long-run calibrated risk parameters (Novotny-Farkas, 2016). An illustration of rating philosophies for LGD, similarly as for PD in Novotny-Farkas (2016), is shown in Figure 3.

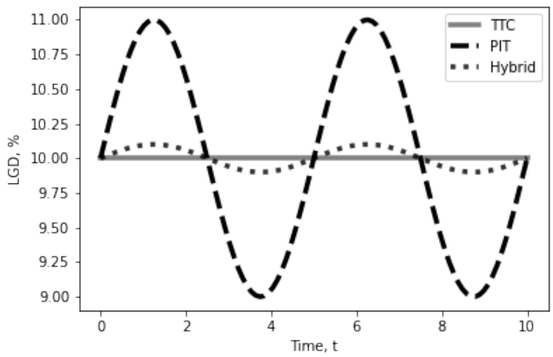


**Figure 3:** Example of LGD cyclicality between the rating philosophies.

There is no specific or strict quantitative definition on what is considered PIT, TTC or hybrid. Rather, it is a combination of testing and empirical evaluation of the risk parameter models to assess if the models are PIT or TTC. Typically, TTC models are stable over time and overestimation is seen during good economic cycles while underestimation is seen in bad economic cycles. PIT models on the contrary react to the fluctuations in the economic cycle, and hence, over time there should be less over- or underestimation. As seen in Figure 3 the PIT estimate goes from high to low while the TTC estimate is stable over time.

With these differences between Basel and IFRS 9 in mind it is possible to use the Basel models for IFRS 9 purposes. The requirement is that significant adjustments are made to the regulatory components like indirect costs, PIT corrections for the TTC estimates, and adjusting the estimates to be either 12-month or lifetime (Temim, 2019).

### 2.1.4 Scenarios

Scenario analysis is an important part of the IFRS 9 ECL framework, which aims to incorporate macroeconomic scenarios into the ECL calculations. As Miu and Ozdemir (2017) describe, it is common practice to incorporate macroeconomic conditions into the risk parameters via probabilistic measures in order to quantify the "expected economic outlook". In IASB (2014) par. 5.5.17(a) it is stated that expected credit loss estimation should reflect unbiased and probability-weighted amounts evaluated with a range of possible outcomes. In the most recent EBA (2023) IFRS 9 monitoring report, emphasis was put on the incorporation of macroeconomic scenarios, especially for PD and LGD.

Consider the simple notation for the ECL formula in (1), and suppose that each risk parameter PD, LGD and EAD is conditional to a macroeconomic factor $M$. Moreover, let there be a set of scenarios $S = \{1, 2, \dots, N_S\}$, where $N_S$ is the total number of scenarios. The macroeconomic factor can obtain a different values $M = m_s$ for all $s \in S$. The scenarios are assigned with probabilities $p(s) \in [0, 1]$ for all $s \in S$ such that $\sum_{s \in S} p(s) = 1$. The scenario based ECL can be written as the probability weighted sum

$$\mathrm{ECL} = \sum_{s \in S} p(s) \cdot \mathrm{PD}(M = m_s) \cdot \mathrm{LGD}(M = m_s) \cdot \mathrm{EAD}(M = m_s),$$

where $\mathrm{PD}(M = m_s)$, $\mathrm{LGD}(M = m_s)$ and $\mathrm{EAD}(M = m_s)$ are the risk parameter values conditional on the macroeconomic factor $M = m_s$. There can be more than one macroeconomic factors that influence the PD, LGD and EAD risk parameters, and they can also be different with respect to the risk parameters.

One must be aware of potential non-linear effects in terms of risk parameters and macroeconomic conditions. For example, calculating the LGD conditional to the expected value of the macroeconomic factor $E[m] = \sum_{s \in S} p(s) \cdot m_s$ can lead to the following inequality: $\mathrm{LGD}(M = E[m]) \neq \sum_{s \in S} p(s) \cdot \mathrm{LGD}(M = m_s)$, because the conditional risk parameters are not necessarily linear functions (Miu and Ozdemir, 2017).

The IFRS 9 standard does not specify the number of scenarios to be evaluated. However, in accordance to par. 5.5.18 and par. B5.5.42 in IASB (2014), the institutions must consider both the possibilities that credit losses occur or do not occur even if the possibility for credit losses is low, and therefore, there should be at least two different scenarios if it is identified that scenarios can impact the timing and amounts of cash flows. One way to specify scenarios is to evaluate three distinct scenarios: baseline (expected scenario), strong (optimistic scenario) and weak (pessimistic scenario) (Joubert et al., 2021). These cover the full range of bad, expected and good outcomes.

## 2.2 Loss Given Default

### 2.2.1 Definition

Loss given default is defined as the percentage amount of loss from the exposure at default which is determined from the recovered cash flows in the recovery process

(Kellner et al., 2022). The recovery process is assessed individually for all defaulted assets, and these processes can be very complex. Furthermore, the realized LGD can only be calculated based on recovery processes of defaulted assets (Witzany et al., 2010), because only then the recovered cash flow information is available. This creates the conditional relationship to PD.

The recovery processes can end in different ways, i.e., they become complete. For example, an asset can "cure" from the default by going back to performing stage after remedying actions. In these cases usually no collection actions need to be taken, and the realized LGD values are generally low. In most severe cases very few cash flows are recovered, and the collection and liquidation process of the collateral might not cover all of the outstanding net exposures. Hence, these unsecured exposures are eventually written-off, potentially causing larger LGD realizations.

Some recovery processes can last for extremely long times, even many years. Thus, in EBA (2017) par. 156, it is stated that institutions should define a maximum length for the recovery process. Once the recovery process length reaches the maximum length, then the institution considers the recovery process as complete, and does not expect any further cash flows. Recovery processes that are not completed with some criteria such as cure, write-off or reaching the maximum recovery length are considered as incomplete. In the IFRS 9 context, however, application of maximum recovery process length can introduce conservatism to loss calculations, which is not in line with the IFRS 9 view on unbiased estimation of losses IASB (2014) par. 5.5.17. Hence, the maximum recovery process length is not be required in IFRS 9, but it could be acceptable if it is deemed that the maximum length is sufficiently long such that it takes nearly all cash flows into account without biasing loss levels, because the assumption is that most recent and unbiased information yields more accurate calibration.

The realized LGD calculation for a defaulted asset can be expressed as follows. Let the recovery process of a defaulted asset have a length $K \in (0, T_{max}]$, where $T_{max}$ is the maximum length of the recovery process. Moreover, consider a partition $0 < t_0 < t_1 < \cdots < t_k < \cdots < t_K < T_{max}$, where $t_0$ is the time of default (González et al., 2018). Next, let $CF_{t_k}$ be the recovered net cash flow amount at time $t_k$. The net cash flows must be discounted with a proper interest rate $e$ with respect to a reference point, e.g., the time of default $t_0$ to calculate the realized loss from the full recovery process. Thus, according to Witzany et al. (2010), the discounted cash flow $DCF$ at time $t_k$ is calculated by

$$DCF_{t_k} = \frac{1}{(1+e)^{t_k}} CF_{t_k}.$$

The total discounted cash flow $TDCF$ is obtained by summing all discounted cash flows after the default moment $t_0$ by

$$TDCF = \sum_{k=1}^{K} DCF_{t_k}.$$

According to Witzany et al. (2010) the total recovery rate (RR) of the recovery process can be defined in terms of the total discounted cash flow and the EAD amount by

$$\mathrm{RR} = \frac{TDCF}{\mathrm{EAD}}.$$

Finally, the realized LGD value can be viewed as the complementary value of the recovery rate (Witzany et al., 2010). Hence, the realized LGD of the entire recovery process is

$$\mathrm{LGD} = 1 - \mathrm{RR}.$$

The logic of calculating the realized LGD for the entire recovery process can also be generalized in a way that the realized LGD is calculated with respect to a time point $t_j > t_0$, which is used for modelling how much losses can still occur after an asset has been in default up to $t_j$. The new calculation is done by discounting the cash flows obtained at $t_k$ with respect to the time point $t_j$, where $t_j < t_k$. The total cash flow is calculated by summing all discounted cash flows after $t_j$. Additionally, instead of using the EAD amount to calculate the recovery rate, the exposure at reference (EAR) is used. This is defined similarly as in González et al. (2018) via EAD and realized cash flows by $\mathrm{EAR}_{t_j} = \mathrm{EAD} - \sum_{i=1}^{j} CF_{t_i}$.

To clarify the use of the realized LGD calculations, for performing and under-performing (stage 1 and 2) assets, the dependent variable used in LGD modelling is the realized LGD calculated in terms of the default date, which takes into account the entire recovery process. For LGD modelling of defaulted assets (stage 3) the dependent variable is the realized LGD with respect to the reference date in the default period, i.e., the loss is assessed in terms of the current outstanding exposure where already realized cash flows are taken into account.

For incomplete defaulted cases, it is not possible to calculate the final realized LGD that reflects the outcome of the entire recovery process. Instead, it is only possible to calculate the LGD in terms of the so-far recovered cash flows using the same calculation logic described above.

Generally, the realized LGD can be assumed to lie in the interval $[0, 1]$, i.e., there are very small losses or the entire exposure amount is lost. However, in practice, the phenomenon of LGD is much more complicated and the realized LGD is not restricted between zero and one. This can be due to different sizes of exposures, cost components, and the effects of the discounting rate can cause extreme realized LGD observations to show up in the data, i.e., values significantly above one or below zero. Figure 4 shows an example visualization of a potential realized LGD distribution that is restricted between zero and one. The distribution is obtained by sampling from a Beta-distribution with parameters $\alpha = 0.2$ and $\beta = 0.5$. The key takeaway is the bimodal effect in the distribution, where most of the probability mass is located around zero and one values.

Cash flows can be defined in various ways, depending on the processes of the institution as well as the underlying modelling regime, e.g., A-IRB or IFRS 9. For example, Joubert et al. (2021) uses cash flows defined as the difference between the assets current exposure and the assets exposure in the previous month, adding the fees and interest, and subtracting the written-off amount. In Tanoue et al. (2017) the cash

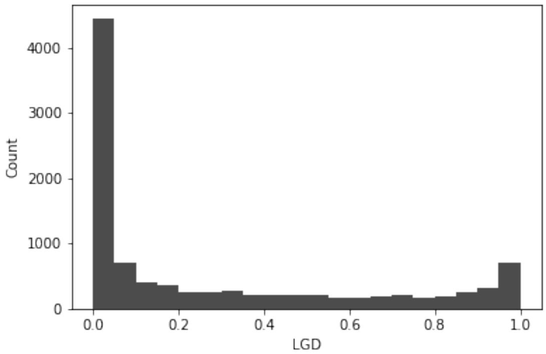


**Figure 4:** Example of a realized LGD distribution.

flows are defined as just a single amount, the total write-off amount of the recovery process. Qi and Yang (2009) define total cash flows as the sum of accrued interest, foreclosure expenses, property maintenance expenses and net recoveries.

In EBA (2017) par. 144 it is stated that institutions should include all direct and indirect costs related to the recovery process. According to EBA (2017) par. 145, direct costs should be directly traceable to the collection process of an exposure, such as costs of collection services, legal costs, or the cost of hedges and insurances. Moreover, EBA (2017) par. 146, states that indirect costs include overheads, costs related to the running of the recovery process, costs that are related to the collection procedures but that cannot be traceable to a specific exposure, and overall costs of collection services that are not part of direct costs. As indirect costs are not traceable to individual exposures, the IFRS 9 cash flow calculations will exclude these (Miu and Ozdemir, 2017).

The discounting rate is an essential part of the realized LGD calculation to address the time value of money, and therefore, it can have a large impact on the LGD realizations. In the Basel IRB approach the LGD is viewed as an "economic loss", while in IFRS 9 the LGD is viewed as an "accounting loss" (Miu and Ozdemir, 2017). Hence, the discounting approach is different.

The Basel discounting rate is composed of the primary inter-bank offered rate with respect to the moment of default, and an add-on of 5%-points, see, e.g., EBA (2017) par. 143. The primary inter-bank offered rate should be the 3-month Euribor (Euro Interbank Offered Rate) or some other comparable liquid interest rate, see, e.g., EBA (2017) par. 143. In the consultation paper EBA (2016) par. 122 it is discussed that the primary inter-bank offered rate can also be the 12-month Euribor, and that the 5%-add-on describes the average level of risk premium. Thus, the Basel discounting rate is

$$d_{Basel} = \frac{1}{1 + \text{PIB} + 5\%},$$

where PIB is the primary inter-bank offered rate.

In IFRS 9 the discounting rate should be based on the effective interest rate of the asset, which is either the effective interest rate at the moment of origination or the current effective interest rate for assets with varying interest rate (IASB, 2014, par. B5.5.44.). The effective interest rate is essentially the reference rate plus the margin determined by the bank. The reference rate can be for example the 12-month Euribor. Depending on the reference rate, the effective interest rate is revised from time to time. For example, in the case of 12-month Euribor the revision is done every 12-months. The interest rate margin is determined by the bank via pricing policies. There can be many ways to do it. For example, according to Witzany et al. (2010) the interest rate margin should cover the expected loss PD × LGD besides other components such as administrative costs or minimum profits. The IFRS 9 discounting rate can be expressed as

$$d_{IFRS9} = \frac{1}{1 + \mathrm{EIR}},$$

where EIR = REF + IM is the effective interest rate calculated as a sum of the reference rate (REF) and the interest rate margin (IM).

### 2.2.2 Risk Drivers

Loss given default can be characterized and modelled via a set of explanatory variables called risk drivers. Qi and Yang (2009) express that commonly known characteristics that impact LGD include information from contract information, borrower information, industry conditions and types, and macroeconomic factors. In retail loans, Matuszyk et al. (2010) describe that LGD is driven by a mixture of uncertainty about the customers willingness and ability to pay and the decisions made by the institution regarding collection processes.

One common risk driver that has been identified to impact LGD in secured retail portfolios is loan-to-value (LTV), see, e.g., Qi and Yang (2009). The LTV is a financial ratio that describes the proportion between the loan and collateral value. The collateral value can be defined in many ways, either taking only into account the collateral value of residential properties which could be reasonable for residential loans portfolios or a combinations of different values that can be deemed as collateral. The exposure amount could also be measure according to the loan origination or the reporting date. The location of the collateral may drive the level of losses as well, because it can affect the collateral valuation.

Qi and Yang (2009) define LTV in two ways. The first definition regards original LTV which is calculated as the ratio of the original loan amount and original property value. The second definition regards current-LTV which is calculated as the ratio of unpaid balance at default and the broker's opinion of the collateral value at default adjusted with house price index. Qi and Yang (2009) identified that the current-LTV has significant impact on LGD.

Other potentially important risk drivers can be, for example, arrears related risk drivers and the age of the loan until defaulting (Matuszyk et al., 2010). Moreover, information on the borrower such as the wealth, education and income can explain the level of losses (Qi and Yang, 2009).

Macroeconomic factors can have large impact on LGD as well. For example, Tobback et al. (2014) obtain improved performance by including macroeconomic factors in the LGD model, which were in line with business cycle intuition and other studies. Also Bellotti and Crook (2012) obtain improved predictive power when macroeconomic factors were included in the modelling data. Moreover, the current-LTV in Qi and Yang (2009) shows that the house price index corrected LTV explains the LGD better.

In summary the LGD is impacted by a mixture of risk drivers and macroeconomic factors. Out of all possible information the institution has available there can be a set of risk drivers that are proven to explain LGD in the best way according to some modelling techniques. Moreover, there can be a set of macroeconomic factors that impact LGD directly or indirectly in economic cycles, while the macroeconomic factors can also affect risk drivers such as collateral values.

### 2.2.3 Modelling Techniques

The common approach to model LGD is to use parametric linear models due to their simplicity and ease of interpretation. Non-parametric models like regression trees are used as well, because they are able to find non-linear effects while still being explainable. Multi-stage modelling, where a combination of modelling techniques such as logistic and linear regression models is used to tackle bimodal LGD distributions to boost predictive performance while being easy to understand (Tanoue et al., 2017). More complex non-linear models like neural networks and support vector machines have been tested in benchmark studies, and they have shown to give significantly better performances compared to traditional methods (Loterman et al., 2012). However, complex non-linear models are often “black boxes”, meaning that their outputs are hard to explain, if they are explainable at all. Credit risk models are required to be explainable as they are used as tools for, e.g., risk management, business decisions and setting risk limits.

One common modelling technique is the ordinary least squares (OLS) linear regression model due to its simplicity and relatively good performance compared to more complex models. However, the OLS model assumes linear relationship between the dependent and independent variables, which can be a strong assumption. Moreover, the normality assumption of the error terms in OLS modelling can be violated as the LGD distribution is often not normally distributed (Bellotti and Crook, 2012). This is not, however, a major problem as the non-normality of the error terms mainly affects the estimators for standard errors, which affect, e.g., confidence intervals and hypothesis tests (Chatterjee and Simonoff, 2013). Bootstrapping methods can be used to cope with this challenge (Bellotti and Crook, 2012).

Distribution transformations prior to OLS estimation can be performed to tackle this issue of bimodal distributions, for example. One can apply, e.g., beta-distribution or Box-Cox transformation to the LGD prior to estimating the OLS coefficients (Loterman et al., 2012). However, Loterman et al. (2012) found that these types of transformation perform consistently worse than OLS, and the reason was argued to be that these approaches also have trouble dealing with point densities in LGD and the transformations can introduce bias. Risk driver transformation techniques can also be used to boost OLS performance. For example, Matuszyk et al. (2010) shows that a weights of evidence binning approach with linear regression improves predictive performance compared to standard linear regression. OLS has also an advantage that the values of LGD are not needed to be restricted on any particular interval, e.g., $[0, 1]$, which is not feasible for real LGD distributions. For example, the beta-transformation OLS approach requires LGD values to be in $(0, 1)$, which requires clipping of the LGD distribution, potentially losing information.

A disadvantage in OLS and other statistical models is that they require the use of complete recovery process information, i.e., the models are always fitted to realized LGD values which are obtained from complete recovery processes. Therefore, incomplete defaults cannot be used in model fitting as the currently-realized LGD values for those defaults can bias the model. However, this topic is tackled in model calibration where LGD values for incomplete defaults need to be estimated.

One technique that is able to take into account also incomplete recovery processes in model estimation is the survival analysis approach, see, e.g., Witzany et al. (2010), Joubert et al. (2018), Joubert et al. (2021). This technique can be extremely effective for LGD modelling with small data sets where all available information about cash flow recoveries are crucial. Witzany et al. (2010) show that the survival analysis approach produce better predictive performance compared to OLS using a fairly small data set of 4000 observations. However, the modelling approach in Witzany et al. (2010) has its pitfalls as strong assumptions for cash flows must be made. For example, the cash flows need to be non-negative and the total cash flows never exceed the exposure at default amount. Some aspects to tackle these assumptions were presented in Joubert et al. (2018).

Overall, the general consensus in the LGD modelling literature is that the model performances are typically quite poor, having coefficient of determination ($R^2$) values around 10% (Witzany et al., 2010), but depending on the modelling data or technique used the $R^2$ values can range from 4% to 43% as was shown in the benchmark study of Loterman et al. (2012). Loterman et al. further shows that non-linear models significantly outperform linear models, suggesting that the relationship between LGD and risk drivers may be non-linear. All in all the LGD methodology literature indicates that LGD is an exceptionally complex phenomenon to model.

# 3 Methods

## 3.1 Loss Given Default Model

This section presents a summary of the LGD model development. The model development process of an IRB LGD model generally consists of two phases: the model estimation for risk differentiation purposes and model calibration for risk quantification purposes (EBA, 2017). These include defining training and testing data, estimating the model parameters, testing the model performance, and finally calibrating the model to long-run average loss levels.

Data from defaulted observations is required with which the full recovery process of the default is observed. For modelling LGD of non-defaulted assets, the realized LGD is calculated with respect to the default date and EAD. In EBA (2017) p. 28 and p. 30 it is stated that the information about risk drivers should be consistent with the LGD estimation and the application of estimates, and therefore, for non-defaulted exposures the institution should use risk driver values before the moment of default that is, e.g., within one year before the default.

The data is typically split into three samples: in-sample (IS), out-of-sample (OOS), and out-of-time (OOT). The IS sample is used for estimating the model parameters, while the OOS and OOT samples are used for testing the model performance. The OOT sample includes defaults that are resolved after a specified time period (Tanoue et al., 2017). The IS and OOS samples include defaults that are resolved before the specified OOT time period. The IS and OOS samples can be split randomly with, e.g., a 80%-20% ratio, respectively. Furthermore, an additional calibration sample (CAL) is defined, which is used for the model calibration to long-run average. According to EBA (2017) par. 147 the calibration sample should be a historical observation period, as broad as possible, which includes different economic circumstances.

In this thesis the LGD is modelled using linear regression with ordinary least squares (OLS) estimation as it has been found to be a generally well performing model (Bellotti and Crook, 2012). The LGD model is described in a summarizing manner as the main focus of this thesis will be on the point-in-time adjustments in Section 3.2.

The linear regression model is expressed as

$$y_i = \beta_0 + \beta_1 x_{1i} + \ldots \beta_p x_{pi} + \epsilon_i,$$

where $i \in \{1, \ldots, N\}$ is the observation (defaulted asset), $y_i$ is the dependent variable (realized LGD), $x_{1i}, x_{2i}, \ldots, x_{pi}$ are the independent variables (risk drivers), $\beta_0, \beta_1, \ldots, \beta_p$ are unknown model parameters (coefficients), $p$ is the number of risk drivers, and $\epsilon_i$ is a random error term (Chatterjee and Simonoff, 2013). Using matrix notation, the linear regression model can be expressed as

$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}, \tag{3}$$

where the elements are given by

$$\boldsymbol{X} = \begin{pmatrix} 1 & x_{11} & \cdots & x_{p1} \\ \vdots & \vdots & & \vdots \\ 1 & x_{1N} & \cdots & x_{pN} \end{pmatrix}, \boldsymbol{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_N \end{pmatrix}, \boldsymbol{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_p \end{pmatrix}, \boldsymbol{\epsilon} = \begin{pmatrix} \epsilon_1 \\ \vdots \\ \epsilon_N \end{pmatrix}$$

(Chatterjee and Simonoff, 2013). The coefficients $\boldsymbol{\beta}$ are estimated by minimizing the sum of squared errors, i.e.,

$$\widehat{\boldsymbol{\beta}} = \arg\min_{\beta} \sum_{i=1}^{N} \epsilon_i^2.$$

The solution to this problem is given by

$$\widehat{\boldsymbol{\beta}} = (\boldsymbol{X}^\top \boldsymbol{X})^{-1} \boldsymbol{X}^\top \boldsymbol{y}. \tag{4}$$

The fitted values can be obtained by $\widehat{\boldsymbol{y}} = \boldsymbol{X}\widehat{\boldsymbol{\beta}}$, and the model residuals are obtained by $\widehat{\boldsymbol{\epsilon}} = \boldsymbol{y} - \widehat{\boldsymbol{y}}$. The estimates $\widehat{\boldsymbol{y}}$ are called OLS estimates as they minimize the sum of squared residuals (Chatterjee and Simonoff, 2013).

Before the parameter estimation, the variables $\boldsymbol{X}$ should be be pre-processed and transformed in an appropriate way. In this thesis only continuous and binary risk drivers are considered, and it is assumed that no missing values are present in the data. The continuous variables are standardized using the training sample (IS) mean and standard deviation as was done in Loterman et al. (2012). Specifically, the continuous variables are standardized by

$$X_j^* = \frac{X_j - \mu_j}{\sigma_j},$$

where $X_j \in \boldsymbol{X}$ is a continuous variable $j$, $\mu_j$ is the mean value of $X_j$ and $\sigma_j$ is the standard deviation of $X_j$. The binary variables are labeled with zeros and ones.

The model can be fitted directly to the LGD values or to the recovery rate (RR) which is the complement of LGD. For example, in Bellotti and Crook (2012) and Yao et al. (2017) modelling was conducted to the recovery rate rather than to the LGD directly. In this thesis, the LGD model is specified as $\text{LGD} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$. After estimating the model parameters according to (4), the continuous LGD estimates are given by $\widehat{\text{LGD}} = \boldsymbol{X}\widehat{\boldsymbol{\beta}}$. To avoid negative values or excessively large values above one, the estimates can be bounded between zero and one by $\min[0, \max[\widehat{\text{LGD}}, 1]]$ following Yashkir and Yashkir (2013).

The EBA (2017) par. 161 states that institutions should calibrate the LGD estimates to correspond the long-run average (LRA) loss rates such that the calibration is done either on grade level, or on calibration segment level for direct LGD estimates. In the case of direct estimates, the institutions are required to compare the average LGD estimates to the calculated LRA LGD, and to correct the individual estimates for application portfolio accordingly based on, e.g., a scaling factor (EBA, 2017, par. 161). Moreover, according to EBA (2017) par. 149 and 153, the institutions should use information from all defaults that fall into the historical observation period such that relevant information from incomplete defaults is included in a conservative way. The methods for considering incomplete cases is left outside the scope of this thesis, and the illustrative effect of adjusting incomplete cases is obtained via simulation.

## 3.2 Point-in-Time Adjustment Framework

The framework for calibrating the LGD model for IFRS 9 using point-in-time (PIT) adjustments consists of the following steps: 1) calibrating the IRB LGD model to IFRS 9 realized loss level after applying appropriate changes to data, 2) developing a macroeconomic module for loss rates or risk drivers to adjust the LGD estimates for different macroeconomic scenarios, and 3) testing the performance and PIT appropriateness of the model. An overview of the framework is shown in Figure 5.

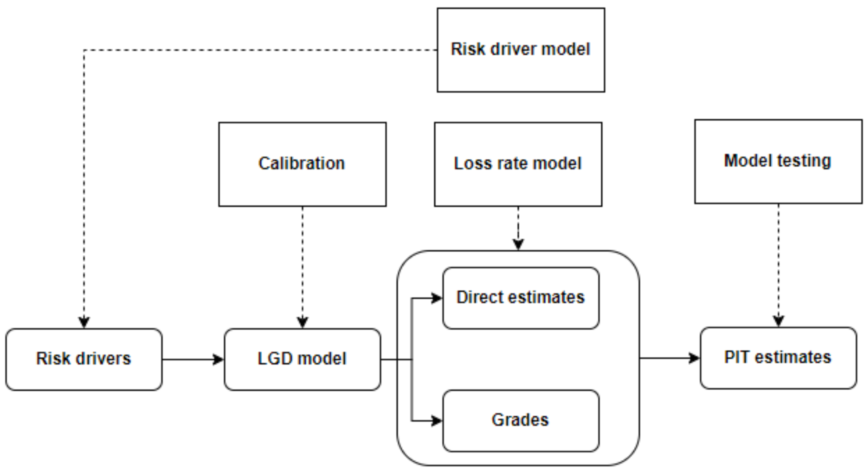


**Figure 5:** Overview of the IFRS 9 point-in-time adjustment framework.

### 3.2.1 Estimation Logic

Prior to the calibration and PIT adjustments it is convenient to understand how the LGD estimates are applied for stage 1 and stage 2 assets in ECL calculations. Suppose that the reporting date of the ECL calculations is $t_R$ and the LGD estimates for time points $t_R + \tau$ are given by a function $\mathrm{LGD}_{i,t_R+\tau} = f(X_{i,t_R+\tau-1})$ for all $\tau = 1, \ldots, T_{M,i}$ where $T_{M,i}$ is the maturity of the asset $i$ (e.g., in years), and $f(\cdot)$ is a parametric or non-parametric function that maps a given set of risk driver information $X_{i,t_R+\tau-1}$ projected to $t_R + \tau - 1$ into a real valued LGD estimate for the time point $t_R + \tau$. This is shown in Figure 6.

Note that the function $f(\cdot)$ produces the LGD estimate for $t_R + \tau$ according to the risk driver information at $t_R + \tau - 1$. As described in Section 3.1, the risk driver information should be obtained prior to the default such that the model estimation and the application of the estimates are aligned. Hence, for stage 2 calculations it is important that the risk drivers are projected to the future time points of the maturity.

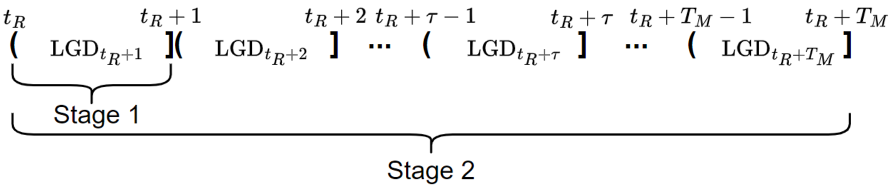


**Figure 6:** The stage 1 and stage 2 LGD estimate visualization on yearly frequency with respect to the reporting date $t_R$ and the maturity $T_M$.

The estimation or approximation of risk driver values requires both qualitative and quantitative assessment. For example, some risk drivers can be assumed to be constant for the remaining lifetime of the asset. Such risk drivers are, for example, the location of the collateral or some more complex payment behaviour characteristic type variable, e.g., the number of arrears, which can be hard to forecast. For some risk drivers, e.g., residential collateral values, a macroeconomic factor based forecast model is possible to be incorporated. A collateral forecast model for LGD was presented in Miu and Ozdemir (2017) and more discussion is provided in Section 3.2.4.

Consider an example where the LGD is estimated with a linear regression model with two risk drivers, the loan-to-value (LTV) and the collateral location. The LTV is a numerical risk driver, which is calculated as the ratio between the current loan amount and the residential collateral value allocated to the loan contract. The LTV changes over time depending on the loan amount and the collateral value. The loan amount can be approximated according to a re-payment schedule and the collateral value can be modelled using house price indexes. The collateral location is a binary variable which indicates if the collateral is placed in the a capital region or not. Thus, the location risk driver can be fairly well assumed to be constant for the remaining lifetime of the asset.

The example estimation equation is $\widehat{\mathrm{LGD}}_{i,t_R+\tau} = \widehat{\beta}_0 + \widehat{\beta}_1 \cdot \mathrm{LTV}_{i,t_R+\tau-1} + \widehat{\beta}_2 \cdot \mathrm{LOC}_{i,t_R+\tau-1}$, where the estimated coefficients are $\widehat{\beta}_0 = 0.002$, $\widehat{\beta}_1 = 0.43$ and $\widehat{\beta}_2 = -0.02$. An example for calculating the LGD for stage 1 and stage 2 is shown in Table 2. It can be seen that due to the increasing collateral value and the decaying loan amount, the LGD estimates change over time.

**Table 2:** Example of applying the model $\widehat{\mathrm{LGD}}_{i,t_R+\tau} = 0.002 + 0.43 \cdot \mathrm{LTV}_{i,t_R+\tau-1} - 0.02 \cdot \mathrm{LOC}_{i,t_R+\tau-1}$ to an asset with five years of remaining lifetime both for the case of stage 1 and 2 allocations.

| Date | Stage | Loan | Collateral | LTV | Location | LGD |
|---|---|---|---|---|---|---|
| $t_R$ | - | 1000 | 1300 | 0.769 | 1 | - |
| $t_R+1$ | 1, 2 | 800 | 1350 | 0.593 | 1 | 0.313 |
| $t_R+2$ | 2 | 600 | 1400 | 0.429 | 1 | 0.237 |
| $t_R+3$ | 2 | 400 | 1450 | 0.276 | 1 | 0.166 |
| $t_R+4$ | 2 | 200 | 1500 | 0.133 | 1 | 0.101 |
| $t_R+5$ | 2 | 0 | 1550 | 0 | 1 | 0.039 |

The next subsections describe methodology on how the IRB LGD model can be calibrated for IFRS 9 purposes and how point-in-time adjustments are introduced to the through-the-cycle or hybrid estimates. In other words, the LGD estimates in Table 2 need to be either adjusted using and output module, which scales the estimates up or down according the macroeconomic scenarios. Alternatively, the macroeconomic model is based on modelling the risk drivers for different scenarios such that the changes is risk drivers yield varying LGD estimates depending on the macroeconomic scenarios.

### 3.2.2 Calibration

The LGD model must be calibrated to the IFRS 9 realized loss levels, because the realized losses in IRB are generally higher compared to IFRS 9. Relevant adjustments to realized LGD calculations are applied such as removal of indirect costs, applying the effective interest rate in the cash flow discounting, and to reconsider the maximum length of recovery process again if needed in order to minimize conservatism. Furthermore, adjusted incomplete defaults may be included for the calibration similarly as for the long-run average calibration of the IRB model.

The main assumption for re-calibrating the LGD estimates is that the rank ordering model estimated for IRB purposes allows for approximately same risk differentiation for IFRS 9 purposes. The intuition is straight forward. If there is a high risk asset in IRB, it will be a high risk asset also in IFRS 9 (RiskQuest, 2020). Same applies for low risk assets. This approach allows to keep synergy between the IRB and IFRS 9 models.

In the case of continuous LGD estimates for a particular portfolio, the calibration may be done by scaling the estimates with a scaling factor or by applying linear regression as a mapping function. Let $\mathrm{LGD}_i^*$ be the realized LGD for asset $i$ calculated on the basis of the IFRS 9 standard such that incomplete defaults are also considered in an appropriate manner. Let $\widehat{\mathrm{LGD}}_i$ be the non-calibrated LGD estimate for asset $i$ given by the IRB LGD model. The mapping equation is

$$\mathrm{LGD}_i^* = \gamma_1 \cdot \widehat{\mathrm{LGD}}_i + \gamma_0 + \xi_i, \tag{5}$$

where $\gamma_1$ and $\gamma_0$ are unknown regression parameters and $\xi_i$ is a random error. The coefficient $\gamma_1$ can be seen as a scaling factor and $\gamma_0$ can be seen as a correction after the scaling, i.e., the mean value of the IFRS 9 LGD estimate given that the IRB LGD estimate is zero. The parameters are estimated according to (4). Furthermore, the calibrated estimates $\widehat{\mathrm{LGD}}_i^* = \widehat{\gamma}_1 \cdot \widehat{\mathrm{LGD}}_i + \widehat{\gamma}_0$ can be bounded between zero and one by $\min[0, \max[\widehat{\mathrm{LGD}}_i^*, 1]]$ (Yashkir and Yashkir, 2013).

In the case of discrete LGD grades the calibration may be done by re-calculating the LRA LGD values for each grade using the IFRS 9 realized loss data. This is, however, left for future research as the thesis focuses on the continuous calibration approach.

### 3.2.3 Loss Rate Model

In this section, the theory on loss rate modelling is presented. The loss rate model is used to incorporate PIT adjustments to the LGD estimates for different scenarios in the ECL calculations. The PIT adjustments to the LGD estimates can be seen as an "output adjustment". The macroeconomic model is based on finding correlation and causal relationships between the loss rates over time and the macroeconomic factors. This section includes theory on how the observed loss rate is defined and how the loss rate forecasts can be used to adjust the LGD estimates. Moreover, time series modelling with ordinary least squares regression theory and the model development process is presented.

Let the time horizon for the loss time series be $t = 1, \ldots, T$, where each $t$ denotes the time of default with a specified frequency, e.g., quarterly. Note that the specified time period may include incomplete defaults which are adjusted for future cash flows with appropriate methodology. The portfolio level average LGD, or observed loss rate, can be calculated for all $t$ such that

$$\overline{\mathrm{LGD}}_t = \frac{1}{N_t} \sum_{i=1}^{N_t} \mathrm{LGD}_{i,t}, \tag{6}$$

where $N_t$ is the number of defaults at time $t$ and $\mathrm{LGD}_{i,t}$ is the realized LGD for asset $i$ which defaulted at $t$. As the average value is not a robust metric, it might get influenced by extreme values of LGD realizations. Thus, these observations, when present, need to be investigated and handled properly.

Equation (6) can be seen as a PIT LGD time series, and it is similar to, e.g., the EAD weighted average LGD estimates over time in Joubert et al. (2021). The question is, how can the forecasts for this series be used to adjust LGD estimates for different scenarios? Assuming that the LGD model already exhibits PIT properties via collateral risk drivers for example, the loss rate time series can be forecasted for the different scenarios and the estimates are adjusted according to the deviations with respect to the baseline scenario as was done in Joubert et al. (2021).

In Joubert et al. (2021) an error correction model was built for the LGD estimate time series, which was used to forecast LGD for the baseline, weak and strong scenarios for $T_f$ forecasting time points. The adjustment scalars for the weak and strong scenarios were then calculated as the ratio of the average weak or strong forecasts and the average baseline forecasts. The scalars for the weak and strong scenarios were then used to adjust the LGD estimates for the scenarios accordingly.

This thesis uses the same approach, but rather than modelling the time series for the estimated LGD time series, the modelling is conducted to the observed loss rate (6), and the scenario scalars are calculated on the basis of the stage allocation of the asset and its remaining maturity.

Let $\overline{\mathrm{LGD}}_{\tau,base}$, $\overline{\mathrm{LGD}}_{\tau,weak}$, $\overline{\mathrm{LGD}}_{\tau,strong}$ be the forecasts for the average LGD at time $\tau = 1, \ldots, T_f$ for the baseline, weak and strong scenarios, respectively. The final forecast time $T_f$ will be determined by the stage allocation and maturity of the asset. For example, in stage 1 the forecast window is one year, which on quarterly frequency would require a window of four quarters. Modifications for the forecasts need to be incorporated in the case of different time frequencies between the LGD model and the loss rate model. Suppose the LGD model is defined on a yearly frequency and the loss rate model is on quarterly frequency. Hence, the quarterly loss rate forecasts should be aggregated to obtain a yearly scenario forecast such that the aggregation is aligned with the maturity of the asset at each time point during the maturity. Various aggregation methods for this task can be applied such as averaging the quarterly values for the particular forecasting year together. If the loss rate model is built on the same frequency as the LGD model is specified, then no adjustments are required for the forecasts.

Assuming that the loss rate forecasts are aggregated on the correct frequency with respect to the LGD model, for each $\tau = 1, \ldots, T_f$, the scalars can be calculated by

$$\delta_{\tau,s} = \frac{\overline{\text{LGD}}_{\tau,s}}{\overline{\text{LGD}}_{\tau,base}}, \quad s \in \{weak, strong\}, \tag{7}$$

as in Joubert et al. (2021). Furthermore, as in Joubert et al. (2021) the scenario scalars are used to adjust the individual LGD estimates for the weak and strong scenarios for all $\tau = 1, \ldots, T_f$ such that

$$\widehat{\text{LGD}}_{i,t_R+\tau,s} = \delta_{\tau,s} \cdot \widehat{\text{LGD}}_{i,t_R+\tau}, \quad s \in \{weak, strong\}. \tag{8}$$

The problem in this type of scalar calculation is that the baseline scenario estimates will not be affected by any macroeconomic factors via the loss rate model. In some situations this is perfectly reasonable if the LGD estimates implicitly contain information about a baseline scenario via risk drivers, e.g., collateral values. However, a simple method can be introduced to overcome this issue. Assume that the observed loss rate series fluctuates around a LRA LGD value for a particular portfolio. The LRA LGD is calculated by

$$\overline{\text{LGD}} = \frac{1}{N} \sum_{i=1}^{N} \text{LGD}_i. \tag{9}$$

Thus, the LRA LGD is just the default weighted average of the all realized losses in the portfolio over time. Hence, rather than comparing the loss rate forecasts in the weak and strong scenarios to the baseline forecast as in (7), the comparison can be done such that all scenarios are compared to the long-run average value (9). Specifically,

$$\delta_{\tau,s} = \frac{\overline{\text{LGD}}_{\tau,s}}{\overline{\text{LGD}}}, \quad s \in \{base, weak, strong\}. \tag{10}$$

This approach makes it possible to calculate how much the forecasts deviate from the long-run average and apply the scalars $\delta_{\tau,s}$ to individual forecasts as in (8), but now for the set of scenarios $s \in \{base, weak, strong\}$.

When forecasting the loss or default rates it is common for the institutions to define a forecasting period. According to the IFRS 9 Implementation by EU institutions monitoring report EBA (2023) par. 82 institutions usually use forecasts up to three years. According to EBA (2023) par. 83 institutions also consider gradual reversion of macroeconomic factors to the long-term macroeconomic conditions at the end of the forecasting period. This is applied to avoid the use of excessively long forecasts in the ECL calculations. Hence, for stage 2 assets where maturities go beyond, e.g., the three years forecasting period, it is required to ensure that the LGD estimate adjustments also follow the mean reversion.

#### 3.2.3.1 Regression

The observed loss rate series can be modelled using ordinary least squares (OLS) regression method which was described in Section 3.1. The OLS time series model is an intuitive approach for modelling the loss rates over time. The main assumptions is that the loss rates over time fluctuate around a long-run average value, and the deviations from the average can be captured by the changes in the macroeconomic environment. The time series OLS model is specified as

$$z_t = \alpha_0 + \sum_{j=1}^{J} \alpha_j M_{j,t} + u_t, \tag{11}$$

where $z_t$ is the dependent time series, $M_{j,t}$ is the $j$:th macroeconomic factor, $\alpha_j$ is the coefficient for the factor $j$, $\alpha_0$ is the intercept and $u_t$ is a random error. The model parameters can be estimated using the Equation (4) to obtain $\widehat{\alpha}_0, \widehat{\alpha}_1, \ldots, \widehat{\alpha}_J$.

For OLS time series modelling, it is important that both the dependent and independent time series are stationary to correctly model the dynamics and correlations. This further relates to the topic of "spurious regression", which should be avoided. Spurious regression is a phenomenon where two series $a_t$ and $b_t$ appear related, for example via similar trend, but in reality they have nothing to do with each other (Hyndman and Athanasopoulos, 2021). Spurious regression is often associated with non-stationarity of $a_t$ and $b_t$. However, Granger et al. (2001) showed that spurious relationship can be present even if the variables are stationary. Therefore, the results of the estimated models should always be analyzed carefully.

Stationarity of time series is usually defined in terms or weak stationarity, as strict stationarity is often not feasible in practice because it requires a common distribution function that does not change over time (Kirchgässner et al., 2012). Let $\{w_t\}_{t=1}^{T}$ be a real valued sample from a stochastic time series process. According to Kirchgässner et al. (2012) the process $\{w_t\}_{t=1}^{T}$ is weak stationary when the following conditions are met: the mean does not change over time, i.e., $E[w_t] = \mu_t = \mu$ for all $t = 1, \ldots, T$, the variance of the process is constant and finite over time, i.e., $Var[w_t] = E[(w_t - \mu_t)^2] = \sigma^2$, and the covariance function $Cov[w_t, w_s] = E[(w_t - \mu_t)(w_s - \mu_s)]$ depends only on the distance between two random variables and not the time point $t$.

The stationarity of time series can be determined with statistical tests. Two common tests are the Dickey-Fuller test, which tests the unit root null hypothesis, and the Kwiatkowski–Phillips–Schmidt–Shin test, which tests the stationarity hypothesis, see, e.g., Kirchgässner et al. (2012). However, these tests should be interpreted with caution, see, e.g., Cochrane (1991). If the stationarity tests give unintuitive results, then one can assume stationarity of a time series by making assumptions about the order of integration. The order of integration $I(d)$ means that the series has to be differenced $d$ times in order to make the series stationary (Kirchgässner et al., 2012). However, in some modelling data sets it might be that differencing has to be applied due to non-stationary behaviour in a particular subset, even if the series can be argued to be $I(0)$, for example. Differencing can be done by the backward difference operator $\nabla = (1 - B)$, where $B$ is a backward shift operator $Bw_t = w_{t-1}$ (Box and Jenkins, 1976). Hence, a first order differenced series is obtained by $\nabla w_t = (1 - B)w_t = w_t - w_{t-1}$. Other related transformations are the growth rate transform $(w_t - w_{t-1})/w_{t-1}$ and the log-difference transform $\log(w_t) - \log(w_{t-1})$ (Kirchgässner et al., 2012). The differencing can also be done with respect to lags larger than one, i.e., $\nabla^l w_t = (1 - B^l)w_t = w_t - w_{t-l}$ (Box and Jenkins, 1976).

The error term $u_t$ in the regression Equation (11) is assumed to be a white noise process. A white noise process is a stationary time series process that has the properties $E[u_t] = 0$, $Var[u_t] = \sigma^2$ for all $t = 1, \ldots, T$ and $Cov[u_t, u_s] = 0$ for all $t \neq s$ (Kirchgässner et al., 2012). This requires that the following OLS assumptions need to be checked: the error terms have zero mean and constant variance, and the errors terms are not autocorrelated. It is also convenient to check if the residuals are normally distributed and unrelated to the independent variables (Hyndman and Athanasopoulos, 2021).

#### 3.2.3.2 Model Development

In developing a macroeconomic regression model, the dependent time series must be first defined as in Equation (6). Second, the macroeconomic factors are specified and stationarity is assessed for all series. Required variable transformations are applied to reach stationarity. Third, analysis of appropriate lags for the stationary macroeconomic factors is conducted. Finally, the model is to be estimated using a proposed set of stationary, potentially lagged, macroeconomic factors. The model estimation includes model selection, residual diagnostic checking, and analysis of the model structure and statistical properties of the estimated parameters.

Appropriate lags for the macroeconomic factors can be assessed with the cross-correlation function as proposed by Box and Jenkins (1976). In Box and Jenkins the estimate for the cross-correlation function for the input sequence $x_t$ and the output sequence $y_t$ is defined as

$$r_{xy}(k) = \frac{c_{xy}(k)}{s_x s_y}, \ k = 0, \pm 1, \pm, 2, \cdots,$$

where $k$ is a lag, $c_{xy}(k)$ is the estimate for the cross-covariance and $s_x$ and $s_y$ are the estimated standard deviations for the sequences $x_t$ and $y_t$, respectively. The estimate for cross-covariance $c_{xy}(k)$ is defined as

$$c_{xy}(k) = \begin{cases} \frac{1}{T}\sum_{t=1}^{T-k}(x_t - \overline{x})(y_{t+k} - \overline{y}) & k = 0, 1, 2, \ldots \\ \frac{1}{T}\sum_{t=1}^{T+k}(y_t - \overline{y})(x_{t-k} - \overline{x}) & k = 0, -1, -2, \ldots, \end{cases}$$

where $T$ is the number of time points, and $\bar{x}$ and $\bar{y}$ are the average values of sequences $x_t$ and $y_t$, respectively (Box and Jenkins, 1976). The largest absolute lag of the cross-correlation function can indicate towards a suitable lag to be used for the macroeconomic factors in the OLS model. Box and Jenkins (1976) note that in practice at least 50 observations are required to get useful estimates for the cross-correlation. Furthermore, the selected lags should be in a reasonable range. Too large lags can affect the economical sensibility of the model, reduce the number of data for model estimation, and introduce application related issues to ECL calculation in a way that scenarios might not influence the results as the macroeconomic information is used from history to forecast future loss rates. Negative lags $k < 0$ are also known as "leads" and they can also be present in the model as was described in Bellotti and Crook (2012).

Once suitable macroeconomic factors have been identified a variable selection procedure can take place. If the total number of factors $J$ (including lags or leads) is not too large, it is easy to estimate all possible model configurations. If $J$ is large, then more advanced forward or backward selection algorithms can be incorporated. However, depending on the amount of data it might be reasonable to restrict the maximum number of variables to be included in the model. Moreover, macroeconomic factors have often high correlation, which can cause issues related to multicollinearity. This can be tackled by including only few different factors that cover different macroeconomic aspects.

The chosen model selection criteria is the corrected Akaike Information Criterion ($\mathrm{AIC}_c$). The model configuration that minimizes the $\mathrm{AIC}_c$ is selected for further analysis. The $\mathrm{AIC}_c$ is defined as

$$\mathrm{AIC}_c = -2\log(\hat{L}) + \frac{2T}{T-n-1}n,$$

where $n$ is the number of estimated parameters in the model, $T$ is the number of data points in the time series and $\hat{L} = p(\widehat{\Theta} \mid \mathcal{D})$ is the maximized likelihood function value obtained via $\widehat{\Theta} = \arg\max_{\Theta} \log p(\Theta \mid \mathcal{D})$ (Stoica and Selen, 2004). Here, $p(\Theta \mid \mathcal{D})$ is the likelihood function of the model, $\Theta$ is the parameter vector and $\mathcal{D}$ is a data vector. For details, see, Stoica and Selen (2004). With small sample size $T$ the $\mathrm{AIC}_c$ is more suitable than the standard $\mathrm{AIC} = -2\log(\hat{L}) + 2n$, because the penalty term is larger, and hence, $\mathrm{AIC}_c$ has smaller risk of overfitting (Stoica and Selen, 2004). For OLS, the likelihood function is obtained from the normal distribution as the error term is assumed to be normally distributed with zero mean and constant variance. Thus, the likelihood is defined as

$$L = \prod_{i=1}^{T} \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{1}{2}\frac{u_i^2}{\sigma^2}),$$

where $u_i^2$ is the squared error term in the regression equation and $\sigma^2$ is the variance of the error terms $u_i$.

The corrected AIC is not the only criterion that can be used in the model evaluation and selection. To accompany the $\mathrm{AIC}_c$ criterion one can analyze the coefficient of determination ($R^2$) or its more suitable variant, the adjusted coefficient of determination ($R_a^2$), which takes into account the number of variables in the regression equation. According to Chatterjee and Simonoff (2013) the coefficient of determination is

$$R^2 = 1 - \frac{\sum_{i=1}^{T}(z_i - \widehat{z}_i)^2}{\sum_{i=1}^{T}(z_i - \overline{z})^2}$$

where $\overline{z}$ is the mean value of $z_i$, and further, the adjusted $R^2$ is

$$R_a^2 = R^2 - \frac{T}{T-n-1}(1 - R^2).$$

Once the model with the minimum $\mathrm{AIC}_c$ is found, then the model assumptions and the model structure are analyzed. The OLS assumptions are verified by performing diagnostic checks for the residuals $\widehat{u}_t = z_t - \widehat{z}_t$, where $\widehat{z}_t$ is the fitted value. The zero mean assumption of the residuals is trivial in the case of OLS estimates, as the OLS minimizes the sum of squared residuals, which implies that the sum of the residuals is zero. The normality of residuals is tested by calculating the Shapiro-Wilk test, which tests the null hypothesis that the data comes from a normal distribution, see, e.g., Shapiro and Wilk (1965). Additionally, normality is verified by plotting the histogram of residuals or the quantile-quantile plot (Chatterjee and Simonoff, 2013). According to Hyndman and Athanasopoulos (2021) the constant variance of the residuals, i.e., homoscedasticity, can be tested by plotting the residuals against time, against fitted values $\widehat{z}_t$ and against macroeconomic factors $M_{j,t}$. Non-linearities in the scatter plots may suggest that there are non-linear effects present in the data which are not captured by the linear model. To test autocorrelation of the residuals the estimated autocorrelation function (ACF) is visualized. The estimate for the ACF is defined as

$$r_k = \frac{c_k}{c_0},$$

where $c_k$ is the estimated autocovariance for lags $k = 0, 1, \ldots, k^*$ defined as

$$c_k = \frac{1}{T}\sum_{t=1}^{T-k}(w_t - \overline{w})(w_{t+k} - \overline{w}),$$

where $k^* < T$, and $\overline{w}$ is the mean of the time series $w_t$ (Box and Jenkins, 1976). The approximate 95% confidence interval for the estimated ACF is given by $\pm 2/\sqrt{T}$ (Kirchgässner et al., 2012). Additionally, the Breusch-Godfrey test (BG-test) is applied for multiple lags to test the null hypothesis that the residuals are not autocorrelated up to a specified lag (Uyanto, 2020).

After performing the diagnostics tests, two hypothesis tests regarding the OLS regression are conducted. The F-test is conducted to test overall significance of the model and $t$-tests are conducted for each estimated regression coefficient to check if the particular macroeconomic factor gives additional predictive power (Chatterjee and Simonoff, 2013). The confidence level of the hypothesis tests is set at 5%. For the reliability of the t-test and calculation of confidence intervals it is important that the residuals are normally distributed as otherwise the results can be misleading (Chatterjee and Simonoff, 2013).

If the OLS model includes multiple macroeconomic factors, it is required to test if there is multicollinearity present. Chatterjee and Simonoff (2013) explain that multicollinearity is a phenomena where some explanatory variables are highly correlated with each other, and it can result to instability of regression coefficients. Multicollinearity can be detected with the variance inflation factor (VIF), which is defined as $\text{VIF}_j = 1/(1 - R_j^2)$, where $R_j^2$ is the coefficient of determination of the factor $j$ which is estimated by OLS using the other factors (Chatterjee and Simonoff, 2013). A cut-off value for VIF is usually set to 5 or 10 (Craney and Surles, 2002). Other model dependent cut-off values have also been studied, see, e.g., Craney and Surles (2002). Moreover, the correlation between the macroeconomic factors can be calculated to complement the VIF analysis.

The loss rate model should have the following properties: 1) it is economically plausible, 2) it satisfies statistical properties, and 3) it has good predictive power. Hence, the development might require iterative steps. For example, some model configuration might optimize a selected performance criterion and satisfy OLS assumptions, but the economical intuition of the model is not good, e.g., due to wrong signs of the estimated coefficient which imply contradicting behaviour to the loss rates. Thus, in the end, it is better to select a model that is economically plausible but with lower predictive power rather than a model that is not economically plausible but has good predictive power. This logic relates to stress testing applications, where according to Malone and Wurm (2017), a level of "realism" needs to be present, which not only considers the accuracy of the macroeconomic model, but also the objective of replicating the behaviour of a variable conditional to the stressed macroeconomic scenarios.

### 3.2.4 Risk Driver Model

A method for adjusting the LGD estimates to be dependent on the macroeconomic conditions is to use a risk driver based model that aims to exploit the influence of macroeconomic to risk drivers, which impact the risk associated to the losses. Assuming that the LGD model includes risk drivers which are sensitive to the macroeconomic conditions, such as collateral values, this model is applicable regardless of how macroeconomic factors seem to impact the observed losses.

This type of method was presented in Miu and Ozdemir (2017), where the IFRS 9 LGD was modelled by estimating the recovery cash flows based on projections of collateral values according to a macroeconomic model for annualized growth rates of the collateral. The forecasted cash flow recoveries were compared against the exposure at default (EAD) to obtain the expected LGD. Specifically, the LGD estimation equation presented by Miu and Ozdemir (2017) is

$$E_{t_R}[\text{LGD}_{t_R+\tau}] = 1 - \frac{\phi \cdot V_{t_R} \cdot \exp(\tau \cdot E_{t_R}[r^V_{t_R,t_R+\tau}])}{E_{t_R}[\text{EAD}_{t_R+\tau}]}, \tag{12}$$

where $t_R$ is the reporting date, $\tau$ is the time point after the reporting date (e.g., in years), $E_{t_R}[\text{EAD}_{t_R+\tau}]$ is the expected EAD if the asset defaults at $t_R + \tau$, $\phi$ is a fraction that describes the present value of net recoveries obtained from the collateral value at the time of default, $V_{t_R}$ is the collateral value at reporting date and $E_{t_R}[r^V_{t_R,t_R+\tau}]$ is the expected value of the annualized growth rate for the collateral from $t_R$ to $t_R + \tau$. The $r^V_{t_R,t_R+\tau}$ is modelled with a linear regression model using macroeconomic factors (Miu and Ozdemir, 2017). The described approach is intuitive. However, the Equation (12) does not follow the regression-like structure for LGD in Section 3.1. Hence, an adaptation of the model is required.

As in Section 3.2.1, the LGD for each time point in the remaining maturity of the asset is estimated by calculating the LGD estimate based on risk driver information one-year prior to the assumed default moment. However, just updating the risk driver values for the future years will result into a problem: for the stage 1 there will be no differences in the LGD estimates for different scenarios, because the risk driver information is taken at reporting date $t_R$ to obtain the LGD estimate for $t_R + 1$. Thus, the risk driver information should be taken from the time of default such that the risk driver value at reporting date is forecasted to the time of default with respect to the macroeconomic scenarios as in Miu and Ozdemir (2017).

Consider the loan-to-value (LTV) risk driver. At reporting date the LTV is

$$\mathrm{LTV}_{t_R} = \frac{L_{t_R}}{V_{t_R}},$$

where $L_{t_R}$ is the loan amount at $t_R$ and $V_{t_R}$ is the collateral value at $t_R$. In accordance with Miu and Ozdemir (2017), the collateral value and the loan amount should be estimated for the time of default. Thus, the forward-looking LTV for $t_R+\tau$ is expressed by

$$\mathrm{LTV}_{t_R+\tau} = \frac{\mathrm{EAD}_{t_R+\tau}}{V_{t_R+\tau}}.$$

The EAD at $t_R+\tau$ is estimated with a separate EAD model or according to the payment plan of the loan, see, e.g., Miu and Ozdemir (2017). The collateral value $V_{t_R+\tau}$ is estimated by updating the collateral value $V_{t_R}$ with a model using macroeconomic factors such as house price indexes (Miu and Ozdemir, 2017).

Another problem arises if the risk driver information is forecasted to the default date and used in the ECL calculations. Suppose that the ECL is calculated for stage 1 assets, and the LTV explains the loss risk in a way that a larger LTV is riskier than a low LTV. Let the LTV at reporting date be $\mathrm{LTV}_{t_R} = L_{t_R}/V_{t_R} = 1000/1300 = 0.769$, the loan amount estimate at default date be $\mathrm{EAD}_{t_R+1} = 800$ (loan is paid 200 units back during one year), and the estimated collateral value be $V_{t_R+1} = 1350$ (the collateral value increases 50 units due to increasing house prices). The LTV at the time of default is $\mathrm{LTV}_{t_R+1} = L_{t_R+1}/V_{t_R+1} = 800/1350 = 0.593$, which is considerably lower than the value at reporting date due to decreasing nominator and increasing denominator. Hence, this potentially results in a lower LGD estimate compared to the reporting date LTV if no adjustments are made, which can cause a calibration related issue compared to historically observed losses because the LTV distribution shifts.

To overcome this problem for the continuous LGD estimates, there are several options. For example, the LGD model presented in Section 3.1 uses standardized continuous risk drivers. Hence, if the LTV distribution at default times is known, then the standardization parameters can be calculated again to standardize the default date LTV to zero mean and unit variance. Thus, the model can be then applied as is. Another approach is that the calibration mapping function (5) is estimated again such that the default date risk driver information is used in the model input. This will impact the mapping function parameters $\gamma_1$ and $\gamma_0$ in Equation (5). If there is no default date LTV data available or the distribution is identical to the reporting date distribution, then it is possible to simulate the default date LTV values by applying the risk driver model to historical data.

Collateral values can be modelled with different methods and granularity can be introduced with respect to the collateral types and locations, for example. The easiest way to incorporate collateral value forecasts is to use collateral valuation models used by the institution. Other methods can be to approximate collateral values using macroeconomic factors or by developing entirely new models. Miu and Ozdemir (2017) model the annualized growth rates of collateral values with annualized growth rates of macroeconomic factors. A similar approach can be employed, which aims to model only the time frequency specific growth rates, e.g., quarterly or yearly.

Let $V_{i,t_R}$ and $V_{i,t_D}$ be the collateral value at reporting date and default date of asset $i$, respectively. Let the time frequency be set to quarterly. The complete growth rate from reporting date to default date is calculated by $\widetilde{V}_{i,t_D} = (V_{i,t_D} - V_{i,t_R})/V_{i,t_R}$. The complete growth rates can be converted to quarterly or yearly rates by assuming equal growth in each quarter from $t_R$ to $t_D$. Note that the growth rate is not defined if the quarterly difference from $t_R$ to $t_D$ is zero. Let the number of quarters between $t_R$ and $t_D$ be $q_i \in [1, 2, 3, 4]$. The quarterly growth rate is given by $v^Q_{i,t_D} = (1 + \widetilde{V}_{i,t_D})^{1/q_i} - 1$, and the yearly growth rate is given by $v^Y_{i,t_D} = (1 + v^Q_{i,t_D})^{q_i} - 1$.

Using the loss rate model theory in Section 3.2.3, an OLS model for the quarterly or yearly growth rates can be defined as

$$v_t^{freq} = \alpha_0 + \sum_{j=1}^{J} \alpha_j M_{j,t} + u_t,$$

where $freq \in \{Q, Y\}$ is the applied growth rate frequency, $v_t^{freq}$ is the average growth rate at $t$ and $M_{j,t}$ is a macroeconomic factor that aims to explain the collateral value growth. Similar OLS model diagnostics and tests are applied in this type of model as well.

After developing the model with, e.g., yearly growth rates using yearly growths in macroeconomic factors to be in line with the yearly ECL calculations, the collateral values are updated for individual assets by

$$V_{i,t_R+\tau} = V_{i,t_R} \cdot \prod_{k=1}^{\tau} (1 + v_k^Y).$$

Depending on the quality or availability of data, as well as the collateral types in the portfolio, this type of model is challenging to be developed. In some situations a particular macroeconomic factor can be deemed as a realistic approximation of the collateral value behaviour by expert judgement, and thus, it is possible to directly approximate the collateral value growth rates using some macroeconomic factor. For example, residential collateral values can be directly updated by the growth rate of the house prices in according locations.

### 3.2.5 Testing

In the context of IFRS 9 and PIT estimates there is no common way in literature to explicitly assess whether the PIT LGD estimates are suitable or sensitive enough to the macroeconomic conditions. Thus, in this thesis the following aspects are evaluated:

1. *Predictive power*: the PIT LGD estimates, in theory, should have better predictive power compared to the TTC LGD estimates.

2. *Backscoring*: the PIT LGD estimates are backscored to historical data to empirically determine if they react more to the macroeconomic environment compared to the TTC or hybrid LGD esimates.

3. *Economic impact*: the impact of macroeconomic scenarios is intuitive in terms of the PIT LGD estimates.

The idea behind these testing aspects come from the general credit risk modelling and stress testing perspectives. The developed models should be as accurate as possible and at the same time they should behave in an intuitive manner such that they can be used for decision making.

The tests are done both for the calibration data set (used for calibration) and the OOS and OOT samples extracted from the calibration data set (used only for testing). For predictive power, the used metrics aim to evaluate the "calibration" and "discriminatory power" performance of the model (Loterman et al., 2012). As in Loterman et al. the calibration performance is tested with the coefficient of determination $R^2$, the root mean squared error (RMSE) and the mean absolute error (MAE). These RMSE and MAE are are defined as

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\text{LGD}_i - \widehat{\text{LGD}}_i)^2},$$

$$\text{MAE} = \frac{1}{N}\sum_{i=1}^{N} \mid \text{LGD}_i - \widehat{\text{LGD}}_i \mid,$$

where $\text{LGD}_i$ are the realized values, $\widehat{\text{LGD}}_i$ the estimated values. The discriminatory power is tested with the Pearson's correlation, the Spearman's rank correlation and the Kendall's tau as in Loterman et al. (2012). These performance metrics are used to understand how the model performs more broadly instead of focusing on just one or two metrics.

## 3.3 Data Simulation

### 3.3.1 Descriptions

This section presents the data used for LGD data simulation and model development. The data is composed of macroeconomic factors gathered from Statistics Finland and risk drivers that are assumed to impact realized LGD values in a residential mortgage portfolio. Four risk drivers are selected for the data simulation and model development purposes, and they are defined in the following list:

1. *Loan-to-Value*: The ratio of loan amount and the residential collateral value at reporting date. The impact for LGD is assumed to be direct. Larger LTV implies larger losses as a large LTV implies low collateralization of the contract.

2. *Location*: A binary variable that describes if the location of the residential collateral. Two locations are specified according to Statistics Finland. The locations are 1. greater Helsinki (value = 1), and 2. whole country excluding greater Helsinki (value = 0). The impact for LGD is assumed to be direct. The residential collaterals located in greater Helsinki are assumed to be more valuable, implying lower losses.

3. *Arrears*: The average number of months in arrears in the past 12-months. Assumed to have a direct impact on LGD. More arrears can imply worse payment capability, which implies larger risk of collateral liquidation and losses, although Matuszyk et al. (2010) found an counter-intuitive opposite impact to LGD using this risk driver.

4. *Income*: The monthly income. Assumed to have opposite impact on LGD. Larger salaries can imply better payment capabilities, and therefore, lower losses.

These are not the only risk drivers or even the best risk drivers to explain LGD in a residential mortgage portfolio. These are chosen mainly for the illustrative purposes of the thesis and the intuitive relation in terms of LGD. The macroeconomic factors are shown in the following list:

1. *Consumer price index* (CPI) (StatFin, 2023a). Assumed to have direct impact on LGD as high inflation can lower payment capabilities.

2. *12-month Euribor interest rate* (EUR12) (SuomenPankki, 2023). Used in cash flow discounting. Assumed to have direct impact on LGD as high interest rates can lower payment capabilities.

3. *Gross domestic product* (GDP) (StatFin, 2023b). Assumed to have indirect impact on LGD as a good economy can imply lower losses.

4. *House price index* (HPI) of old dwellings (whole country, greater Helsinki, whole country excluding greater Helsinki) (StatFin, 2023c). Assumed to have direct impact to the residential collateral values in these regions and indirect impact on LGD.

The sourced macroeconomic factors are in quarterly frequency, and they are illustrated in Figure 7. The time horizon for the macroeconomic factors and the simulation study is set from 2005 Q1 to 2019 Q4, which gives 15-years of data, i.e., 60 quarters. This is a decent amount of data for modelling purposes, although ideally there would be even more data available.

For the sake of simplicity, the CPI, GDP and HPI series are assumed to be $I(1)$ series, i.e., they are required to be differenced once to reach stationarity. This is assumed due to their linearly growing trend over time and stochastic nature. By the same reasoning, the series for EUR12 and LGD are assumed to be $I(0)$ series, as their values can be assumed to be bounded and to fluctuate around some long-run average over time. However, this is not always true in a subset of a series. For example, the 12-month Euribor trends downwards in the specified time period, which forces the series to be differenced once to remove the trend. Thus, the quarter-to-quarter growth rate transformation $(M_t - M_{t-1})/M_{t-1}$ is applied for CPI, GDP and HPI, and the first difference $M_t - M_{t-1}$ is applied to EUR12. The transformed series are in Figure 8.

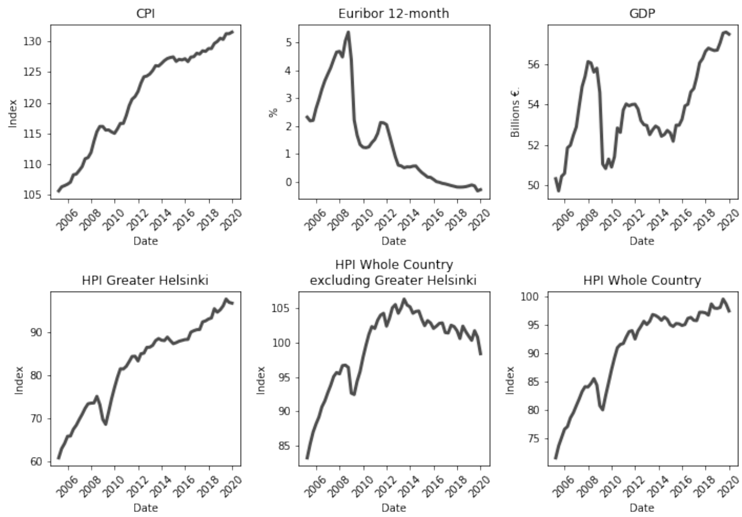


**Figure 7:** Visualizations of sourced quarterly macroeconomic factors from 2005 Q1 to 2019 Q4.

### 3.3.2 Algorithm

The data is simulated by generating random variables according to a pre-defined correlation structure using Gaussian copulas as described, e.g., in McNeil et al. (2015) and Li et al. (2013). The over time distribution for the realized LGD and risk drivers is guided via linear relationships to macroeconomic factors. The distribution choices for the LGD and risk drivers are presented in Table 3.

The Gaussian copula simulation algorithm according to McNeil et al. (2015) is briefly described. The idea is to first simulate random variables $\boldsymbol{Y}$ from the multivariate normal distribution $N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where $p$ is the number of variables. To simulate $\boldsymbol{Y}$, variables $\boldsymbol{Z}$ are first generated independently from a standard normal distribution,

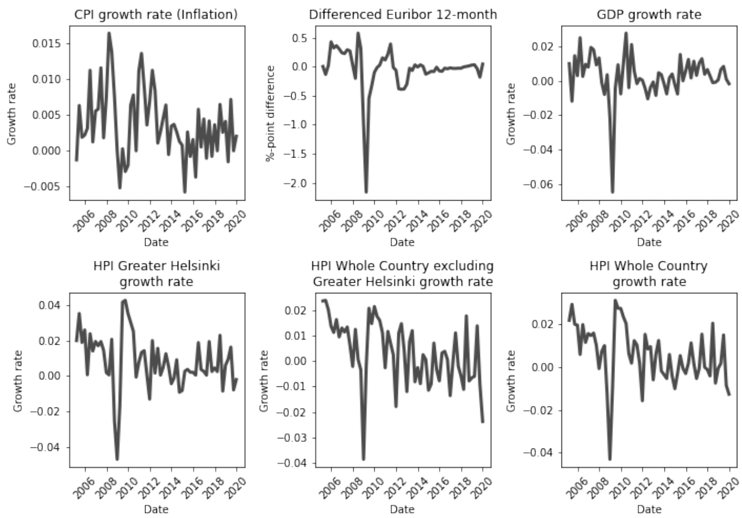


**Figure 8:** Visualizations of transformed quarterly macroeconomic factors from 2005 Q1 to 2019 Q4.

**Table 3:** Loss given default and risk driver distributions.

| Variable name | Distribution |
|---|---|
| Loss given default (LGD) | $\mathrm{LGD}_t \sim Beta(\alpha_t, \beta = 0.5)$ |
| Collateral location (LOC) | $\mathrm{LOC} \sim Bernoulli(p = 0.3)$ |
| Loan-to-value (LTV) | $\mathrm{LTV}_t^{\mathrm{LOC}} \sim Gamma(k_t, \theta = 0.1)$ |
| Arrears (ARR) | $\mathrm{ARR}_t \sim Lognormal(\mu = 0.01, \sigma = 0.7)$ |
| Income (INC) | $\mathrm{INC}_t \sim Gamma(k = 10, \theta = 350)$ |
| Months in default (MID) | $\mathrm{MID}_t \sim 10 \cdot Lognormal(\mu = 0, \sigma = 0.6)$ |
| Exposure at default (EAD) | $\mathrm{EAD}_t \sim Gamma(k = 1, \theta = 70000)$ |

and the Cholesky decomposition is applied for the matrix $\boldsymbol{\Sigma}$ to obtain a Cholesky factor $\boldsymbol{L}$ (McNeil et al., 2015). Details for the Cholesky decomposition can be found, e.g., in Higham (1990). The normally distributed variables are obtained by setting $\boldsymbol{Y} = \boldsymbol{\mu} + \boldsymbol{L}\boldsymbol{Z}$. Here, $\boldsymbol{\mu}$ is set as a zero vector and a positive definite correlation matrix is used a a covariance matrix by assuming unit variance in each variable. Hence, the variable $\boldsymbol{Y}$ will have a distribution of $\boldsymbol{Y} \sim N_p(\boldsymbol{0}, \boldsymbol{\Sigma})$. The Gaussian copula $\boldsymbol{U}$ is applied to the variables $\boldsymbol{Y}$, i.e., the cumulative distribution function $\Phi(\cdot)$ of a standard normal distribution is applied for each variable in $\boldsymbol{Y}$. The copula $\boldsymbol{U}$ will have uniform marginal distributions $\boldsymbol{U} \sim U(0, 1)$.

After the copula simulation is completed it is possible to generate random variables $\boldsymbol{X}$ with any desired distribution according to the propositions for quantile and probability transformations presented in McNeil et al. (2015). The propositions state that: Let $F$ be a cumulative distribution function (CDF) and let $F^{-1}(y) = \inf\{x : F(x) \geq y\}$ be the generalized inverse function. For quantile transformation, if $U \sim U(0, 1)$ has a standard uniform distribution, then $P(F^{-1}(U) \leq x) = F(x)$. For probability transformation, if $Y$ has a continuous univariate CDF, $F$, then $F(Y) \sim U(0, 1)$.

(McNeil et al., 2015)

The correlation matrix $\boldsymbol{\Sigma}$ that will be applied for the copula simulation is presented in Figure 9. Only moderate correlations are considered in order to simulate the generally low predictive power of the LGD models. Moreover, it is assumed that the risk drivers are not correlated with each other.

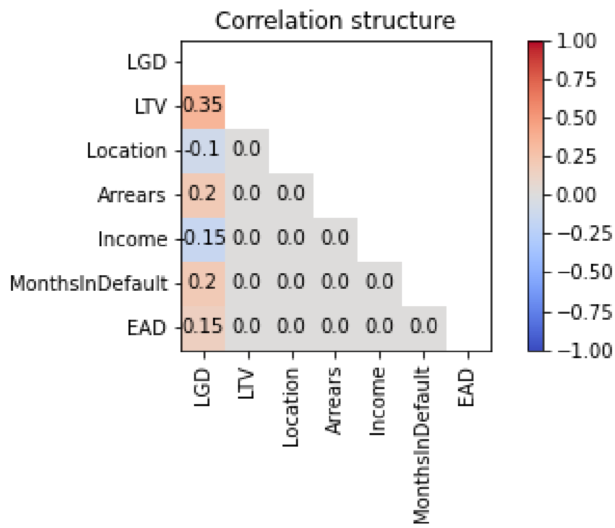


**Figure 9:** Input correlation matrix for the simulation algorithm.

In addition to the risk driver and LGD simulations, few other variables are simulated. Random default dates are generated for each sample by generating uniformly random monthly end dates between a specified time horizon $[T_{start}, T_{end}]$. The default dates are used to join macroeconomic data. Additionally, monthly end reporting dates are generated by randomly selecting one to twelve months prior to the default date.

The distributions for LGD and LTV have included macro-dependency. The dependency is seen via the mean values of the variables $E_{\text{LOC}}[\text{LGD}_t]$ and $E_{\text{LOC}}[\text{LTV}_t]$ at time $t$ and location variable LOC. The LGD is assumed to be Beta-distributed and the mean value is simulated with a linear model of form

$$E_{\text{LOC}}[\text{LGD}_t] = \beta_1 \cdot \text{GDP}_t + \beta_2 \cdot \text{CPI}_t + \beta_3 \cdot \text{HPI}_{t,\text{LOC}} + \beta_4 \cdot \text{EUR12}_t + \beta_5 \cdot t + \beta_0 + \epsilon_t.$$

Note that here each macroeconomic factor stands for the transformed factor, e.g., $\text{GDP}_t$ is the quarterly growth rate of GDP. Once the expected value of the LGD distribution at time $t$ is simulated, then the distribution for LGD at $t$ is obtained by updating the parameter $\alpha_{t,\text{LOC}}$ for the Beta-distribution such that

$$E_{\text{LOC}}[\text{LGD}_t] = \frac{\alpha_{t,\text{LOC}}}{\alpha_{t,\text{LOC}} + \beta} \implies \alpha_{t,\text{LOC}} = \frac{E_{\text{LOC}}[\text{LGD}_t] \cdot \beta}{1 - E_{\text{LOC}}[\text{LGD}_t]}.$$

For LTV, the simulation works similarly. The expected value of the distribution is simulated via

$$E_{\text{LOC}}[\text{LTV}_t] = \frac{E_{\text{LOC}}[\text{LTV}_{t-1}]}{\beta_1 \cdot \text{HPI}_{t,\text{LOC}} + \beta_0} + \epsilon_t.$$

The parameter update for the Gamma-distribution is done by

$$E_{\text{LOC}}[\text{LTV}_t] = k_{t,\text{LOC}} \cdot \theta \implies k_{t,\text{LOC}} = E_{\text{LOC}}[\text{LTV}_t]/\theta.$$

After generating random samples, the variable distributions are treated for non-realistic values and outliers. The months in default (MID) values are converted to integers by rounding to the closest integer. All distributions are winsorized to the 0.999 quantile. The collateral value risk driver at default date is also calculated from the simulated EAD and default date LTV risk drivers by $\text{Collateral}_{default} = \text{EAD}/\text{LTV}_{default}$. For reporting date values, the exposure amount risk driver (Exposure) at reporting date is simulated by increasing the EAD amount uniformly randomly by 0 to 10%. The collateral value at reporting date is simulated by multiplying the values with $\text{HPI}^{\text{LOC}}_{reporting}/\text{HPI}^{\text{LOC}}_{default}$, which are calculated on quarterly frequency between the reporting and default date. Here the HPI is the index value instead of the growth rates. Moreover, a random error is added to the reporting date collateral values by generating from $N(0, \sigma = 0.3)$ for the cases where the quarterly difference between the reporting date and default date is not zero, and the growth rates are restricted between ±5%. For zero quarterly difference the collateral value is assumed to be the same. Winsorization is applied for the newly calculated variables. The location, income and arrear risk drivers are assumed to be the same on reporting date and default date.

The second phase of the simulation generates the recovery process cash flows to calculate the IFRS 9 LGD realizations from the simulated IRB LGD realizations. First, the algorithm calculates the IRB recovery amount by $\text{EAD} \cdot (1 - \text{LGD})$. Second, the cash flows are simulated as a single write-off amount from default date to the months in default date. As the calculated recovery amount is already assumed to be discounted, then the non-discounted cash flow is obtained by multiplying the recovery amount by $(1 + \text{EUR12} + 5\%)^{\text{MID}}$, where EUR12 is the 12-month Euribor at the default date and the 5% is the IRB add-on. Third, the non-discounted cash flow still includes the indirect costs which are removed by decreasing the non-discounted cash flow uniformly 0 to 5%. The effective interest rate (EIR) is simulated by generating a uniformly random interest rate margin between 0.1% and 5% which is added to the 12-month Euribor at default date. The IFRS 9 recovery rate can be now calculated by dividing the non-discounted cash flow without indirect costs with $(1 + \text{EIR})^{\text{MID}}$. To avoid excessive recovery amounts going above the original EAD, the IFRS 9 recovery amounts are set to be at most the EAD amount. The IFRS 9 LGD realization can be now computed.

The third and final phase of the simulation is to generate the model development samples (IS, OOS, OOT) and calibration samples (CAL). From all simulated observations 50% is randomly sampled for the calibration data sets and the other 50% is used for the IS, OOS and OOT. All incomplete cases are removed from the development data set sampling. The OOT sample is created by taking all observations that get resolved after a specified $T_{OOT}$ time point (Tanoue et al., 2017), which is set to 31-12-2018 in this simulation. The IS and OOS samples are simply created by randomly dividing the data to 80% IS and 20% OOS from the remaining observations. The calibration samples are treated in almost similar way. Now the incomplete defaults are not removed, and the simulated realized LGD values simulate the estimation of the realized LGD for incomplete cases. The calibration data is randomly sampled with 80%-20% ratio, where the 80% is used for model calibration sample (CAL), which is assumed to entail the requirements explained in EBA (2017). The 20% is split to OOS and OOT samples such that the OOT sample includes incomplete defaults and resolved defaults after 31-12-2018. The OOS calibration set are the remaining observations. Note that for model calibration the OOT sample works more like a “recent time” OOS sample, because the CAL sample used for the model calibration includes also the most recent information. This can give insights to the model performance on most recent data rather than just using one OOS sample from the entire calibration time horizon. To avoid confusion with the development samples, the out-of-sample and out-of-time samples for calibration purposes are abbreviated as OOSC and OOTC, respectively.

# 4 Results

## 4.1 Data

Two data sets are simulated. In the first data set, no trend is introduced to the LGD time series, which illustrates a situation where the observed loss rates of a financial institution fluctuate around the LRA LGD value and the deviations from this LRA LGD are mainly caused by the macroeconomic conditions. In the second data set, a trend component is introduced to the LGD time series which causes the observed losses to get smaller over time. This illustrates a situation where the credit portfolio quality gets better over time due to, e.g., business related reasons, but the trend is not clearly caused by macroeconomic conditions. Moreover, given that the modelling period is restricted, the trend imposes uncertainty for the future, i.e., it is not known whether the observed losses stabilize or start to increase, which relates to the context of economic cycles.

**Table 4:** The parameters for time series simulations.

| | GDP | CPI | $HPI_{LOC=0,1}$ | EUR12 | const. | time | error |
|---|---|---|---|---|---|---|---|
| LGD 1 | -0.9 | 0.5 | -1, -1 | 0.0005 | 0.2 | - | N(0,0.01) |
| LGD 2 | -0.9 | 0.5 | -1, -1 | 0.0005 | 0.3 | -0.003 | N(0,0.01) |
| LTV 1 | - | - | 0.6, 0.6 | - | - | - | N(0,0.001) |
| LTV 2 | - | - | 0.6, 0.6 | - | - | - | N(0,0.001) |

For both data sets 100000 samples are simulated such that the time interval for the macroeconomic factors is considered on between quarter one of 2005 and fourth quarter of 2019. The parameters used in the LGD and LTV time series simulations for both data sets are shown in Table 4. For LTV the initial expected value is set to 0.6, which gives the parameter value $k_0 = 0.6/0.1 = 6$.

The simulated distributions for risk drivers in the first data set are displayed in Figure 10, while the simulated LGD distributions for both IRB and IFRS 9 cases are displayed in Figure 11. The simulated correlation structure of the first data set is in Figure 12. The same visualizations for the second data are almost identical with only very minor differences, and therefore, they are not displayed here.

The calibration sample is anlyzed. The LGD time series both for the IRB and IFRS 9 realizations levels for the first data set is shown in Figure 13. The IFRS 9 realization level is lower compared to the IRB realizations due to the use of a different cash flow discounting factor and removal of indirect costs, and the LGD series does not have down or up going trend. Figure 14 shows the LGD time series of the second data set. The figure is otherwise similar, but a downward trend is seen, which is obtained by simulating a trend component. In reality, the trend can be due to various reasons related to business or data, for example. Furthermore, the risk driver time series in the calibration sample of the first data set are visualized in Figure 15, where the key takeaway is to see the downward trend of LTV risk driver due to increasing house prices. Same behaviour is present in the second data sets risk drivers.

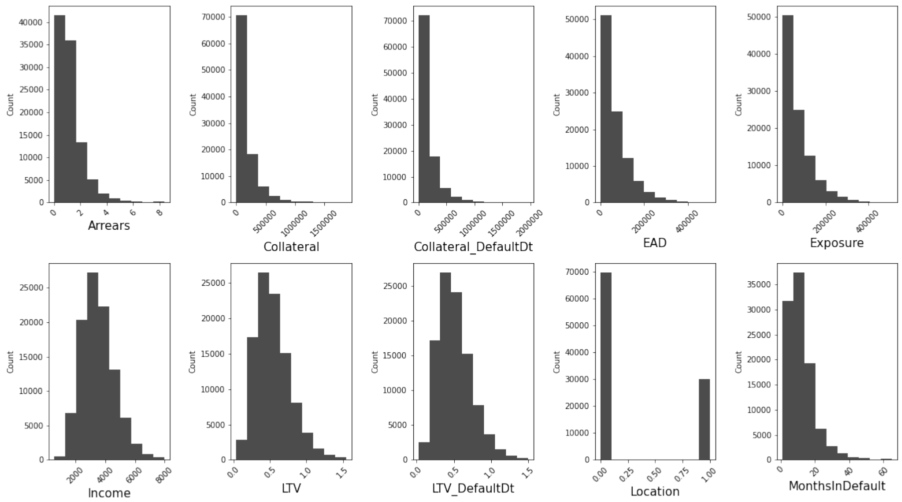


**Figure 10:** Simulated risk driver distributions for all samples in the first data set.

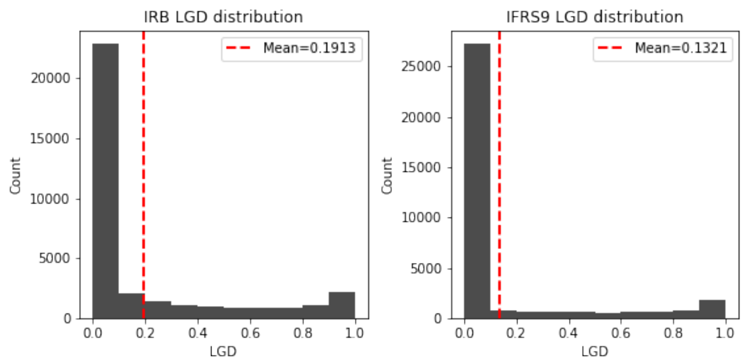


**Figure 11:** The IRB and IFRS 9 LGD distributions for all samples in the first data set.

The number of observations in the development and calibration samples are displayed in Table 5. The numbers are same for both data sets. The number of observations is generally large compared to studies with only 4000 observations (Witzany et al., 2010). The simulation illustrates the data amounts for large portfolios with data available for multiple years.

**Table 5:** The observation sizes for the different samples.

| Sample | Number of observations |
|---|---|
| Development IS | 34620 |
| Development OOS | 8654 |
| Development OOT | 3284 |
| Calibration CAL | 40000 |
| Calibration OOS | 8690 |
| Calibration OOT | 1310 |

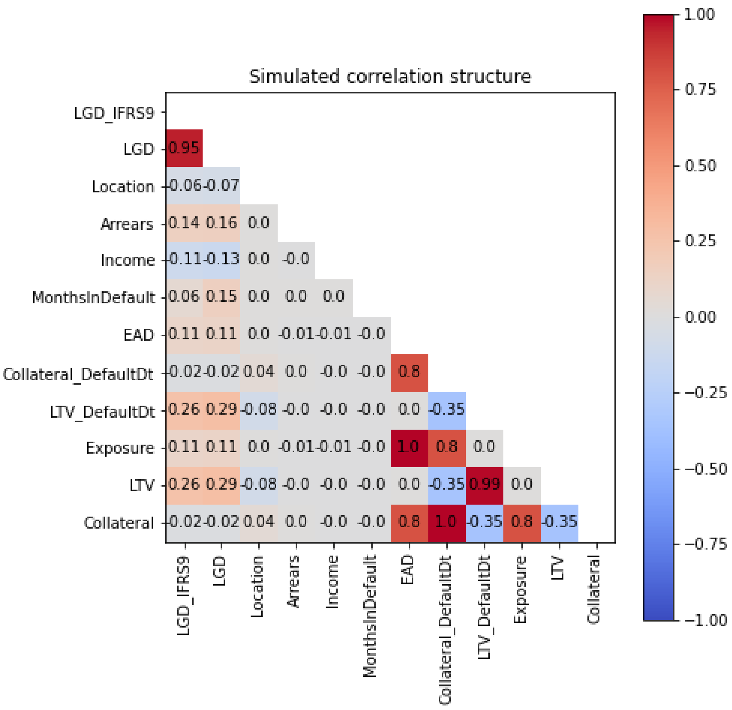


**Figure 12:** Output correlation structure for all samples in the first data set.

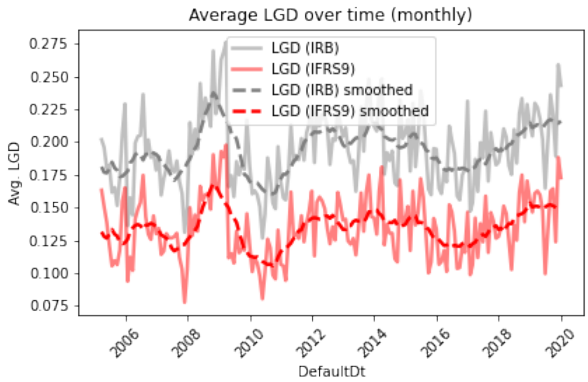


**Figure 13:** Data set 1. Monthly observed loss rate over time in the simulated calibration sample. Both IRB (top) and IFRS 9 (bottom) realizations are displayed with a 12-month centered moving average smoothed line.

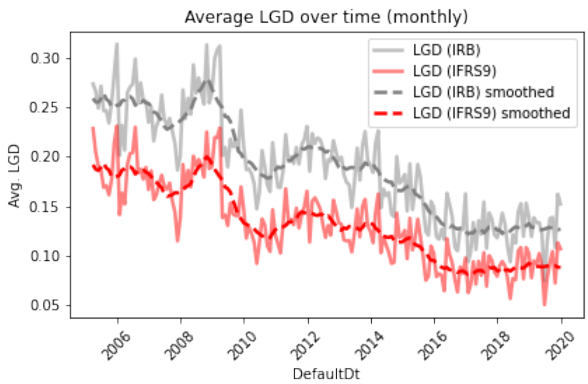


**Figure 14:** Data set 2. Monthly observed loss rate over time in the simulated calibration sample. Both IRB (top) and IFRS 9 (bottom) realizations are displayed with a 12-month centered moving average smoothed line.

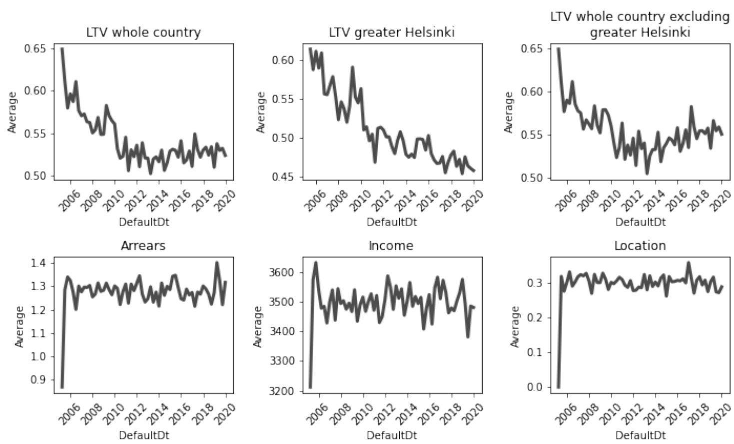


**Figure 15:** Data set 1. The risk driver time series for the calibration sample.

## 4.2 Model Estimation and Calibration

The IRB model parameters are estimated using the development IS sample using the four risk drivers and the IRB LGD realization as a target variable. The parameter estimation results for the linear regression model for both data sets are in Table 6. The results indicate that the estimated parameters are intuitive with respect to the coefficient signs. Specifically, an increase in LTV or ARR will increase LGD, while an increase in INC or having the collateral location in greater Helsinki (LOC=1) decrease LGD.

**Table 6:** The estimated IRB LGD model parameters for data sets 1 and 2.

| Risk driver | Data set 1 estimates $\widehat{\beta}$ | Data set 2 estimates $\widehat{\beta}$ |
|---|---|---|
| constant | $\widehat{\beta}_0 = 0.2016$ | $\widehat{\beta}_0 = 0.2086$ |
| LTV | $\widehat{\beta}_1 = 0.0846$ | $\widehat{\beta}_1 = 0.0886$ |
| ARR | $\widehat{\beta}_2 = 0.0483$ | $\widehat{\beta}_2 = 0.0492$ |
| INC | $\widehat{\beta}_3 = -0.0395$ | $\widehat{\beta}_3 = -0.0403$ |
| LOC | $\widehat{\beta}_4 = -0.0344$ | $\widehat{\beta}_4 = -0.0328$ |

The IRB LGD estimates for these models are restricted between zero and one. Table 7 shows the performance metrics of the IRB LGD model in both data sets. The results show that the models perform well also in the OOS and OOT samples which were not used for estimating the parameters. Moreover, the coefficient of determination ($R^2$) values are in line with the findings in literature. However, the $R^2$ in the second data set OOT sample is really low, which is due to the LGD distribution skewing towards zero. These models are used as a basis for the IFRS 9 model calibration with the assumption that the models have been tested and validated for IRB or other internal modelling purposes.

**Table 7:** The IRB LGD model performances for data sets 1 and 2.

| Sample | $R^2$ | RMSE | MAE | Pearson | Spearman | Kendall |
|---|---|---|---|---|---|---|
| 1. IS | 0.1226 | 0.2881 | 0.2189 | 0.3502 | 0.401 | 0.2781 |
| 1. OOS | 0.1379 | 0.2931 | 0.2218 | 0.3729 | 0.421 | 0.2929 |
| 1. OOT | 0.1275 | 0.2866 | 0.2131 | 0.3577 | 0.42 | 0.2923 |
| 2. IS | 0.1271 | 0.2919 | 0.2235 | 0.3565 | 0.4041 | 0.2804 |
| 2. OOS | 0.1415 | 0.2971 | 0.2275 | 0.3777 | 0.4238 | 0.2949 |
| 2. OOT | 0.0108 | 0.2571 | 0.2017 | 0.3176 | 0.3985 | 0.2863 |

The models are calibrated to the IFRS 9 realization level using the mapping function (5). To recap, the mapping function is estimated by using the IFRS 9 LGD realizations as target variable and the non-calibrated IRB LGD estimate as an explanatory variable. The estimated mapping function parameters for both data sets are in Table 8. From the estimated parameters $\widehat{\gamma}_1$ it can be seen that they are below one, i.e., the IRB LGD estimates are scaled on a lower level. This also implies that

**Table 8:** The IFRS 9 LGD mapping function parameters for data sets 1 and 2.

| Variable | Data set 1 estimate $\widehat{\gamma}$ | Data set 2 estimate $\widehat{\gamma}$ |
|---|---|---|
| constant | $\widehat{\gamma}_0 = -0.0318$ | $\widehat{\gamma}_0 = -0.0359$ |
| $\widehat{\text{LGD}}$ | $\widehat{\gamma}_1 = 0.873$ | $\widehat{\gamma}_1 = 0.844$ |

the IFRS 9 LGD estimates increase by $\widehat{\gamma}_1$ if the IRB LGD estimates increase by one unit. The constant term in the mapping function $\widehat{\gamma}_0$ ensures proper linear calibration, i.e., it functions as the mean value of IFRS 9 estimate if the IRB LGD estimate is zero. The final mapping function estimates are restricted between zero and one. This calibration step can be seen as a non-PIT calibration as the PIT adjustments are not yet applied. The performance metrics for the calibration samples for both data sets are in Table 9. The results are good, as the coefficient of determination values remain around 10% and the other performance metrics have similar magnitudes compared to the IRB LGD model performances in Table 7. According to the findings in Loterman et al. (2012), the correlation metrics (Pearson, Spearman, and Kendall) are decent as they are approximately in the range of 0.25 to 0.3.

**Table 9:** The IFRS 9 calibrated LGD model performance for data sets 1 and 2.

| Sample | $R^2$ | RMSE | MAE | Pearson | Spearman | Kendall |
|---|---|---|---|---|---|---|
| 1. CAL | 0.1047 | 0.2725 | 0.1892 | 0.3237 | 0.3215 | 0.249 |
| 1. OOSC | 0.1014 | 0.2623 | 0.1833 | 0.3209 | 0.3019 | 0.2346 |
| 1. OOTC | 0.1146 | 0.2867 | 0.2009 | 0.344 | 0.3363 | 0.2596 |
| 2. CAL | 0.1070 | 0.2697 | 0.1853 | 0.3272 | 0.3229 | 0.2506 |
| 2. OOSC | 0.1051 | 0.2672 | 0.1843 | 0.3243 | 0.3113 | 0.2413 |
| 2. OOTC | 0.0617 | 0.2364 | 0.168 | 0.3152 | 0.2922 | 0.2315 |

The portfolio level LRA LGD values are calculated for both data sets using all available observations in the calibration sample by calculating the average of all IFRS 9 realized losses. Here it is assumed that the incomplete defaults are already properly addressed in the calibration sample. The LRA LGD results are in Table 10. The LRA LGD values can be used in calculating the adjustment scalars for individual LGD estimated in the case of the loss rate model. The results show that for IFRS 9 the average losses are around 13% while for IRB the average losses are approximately 19%. The impact of the discounting factor in realized loss calculations is notable.

**Table 10:** The LRA LGD values for both simulated data sets calculated using the calibration sample.

| Data set | IRB LRA LGD | IFRS 9 LRA LGD |
|---|---|---|
| 1 | 0.1947 | 0.1346 |
| 2 | 0.1894 | 0.1313 |

## 4.3 Point-in-Time Adjustments

This section discusses the point-in-time adjustments for the LGD estimates. The models for the two data sets are considered separately. First putting emphasis on the data set without trending LGD and then focusing on the second data set with trending LGD over time. The loss rate modelling is discussed first and the application of "output adjustments" to adjust the LGD estimates for different scenarios are presented with examples. Lastly, the risk driver model is discussed and the application of "input adjustments" is presented with and example.

### 4.3.1 Loss Rate Model for Data Without Trend

The first data set is analyzed. The observed loss rate time series is calculated on quarterly frequency according to Equation (6), where the default dates are converted to quarterly frequency. Similar time series aggregation is performed for the non-PIT LGD estimates to see how the average estimates behave over time. Figure 16 shows the observed and estimated loss rates. The observed loss rate tends to fluctuate around the LRA LGD value without trends. Thus, the observed loss rate is stationary and no differencing or other treatments are required to be applied. The estimated series is rather stable over time which implies that the LGD estimates do not capture the macroeconomic fluctuations as much as seen in the observed loss rates. However, the effect of changing LTV distribution is seen as the estimate series slightly trends down. This suggests that the estimates react to the changes in the collateral values, and hence, they are more hybrid than through-the-cycle.

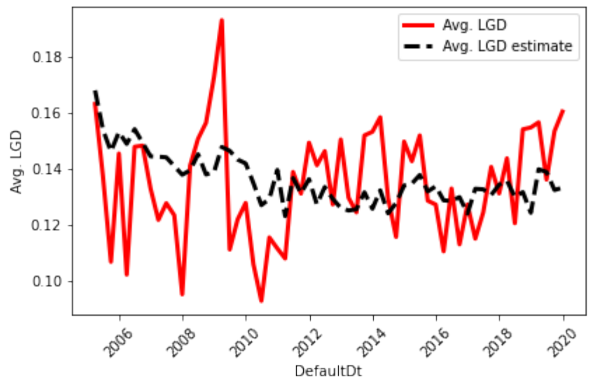


**Figure 16:** Data set 1 modelling. The quarterly aggregated observed loss rate (Avg. LGD) and the LGD estimates.

The next step is to make the initial selections and stationary treatments for the macroeconomic factors. In this thesis only four factors are considered and the stationary transformations are the quarterly growth rates for GDP, CPI and HPI, while the EUR12 is transformed with a quarterly difference. The cross-correlation coefficients between the input sequences (stationary macroeconomic factors) and the output sequence (stationary observed loss rate) are calculated and the results are in Figure 17. The first row in the figure shows the estimated cross-correlation coefficients for all lags and the second row shows the coefficients for -4 to 4 quarter lags in order to restrict the lag numbers to a one-year horizon with respect to the default date.

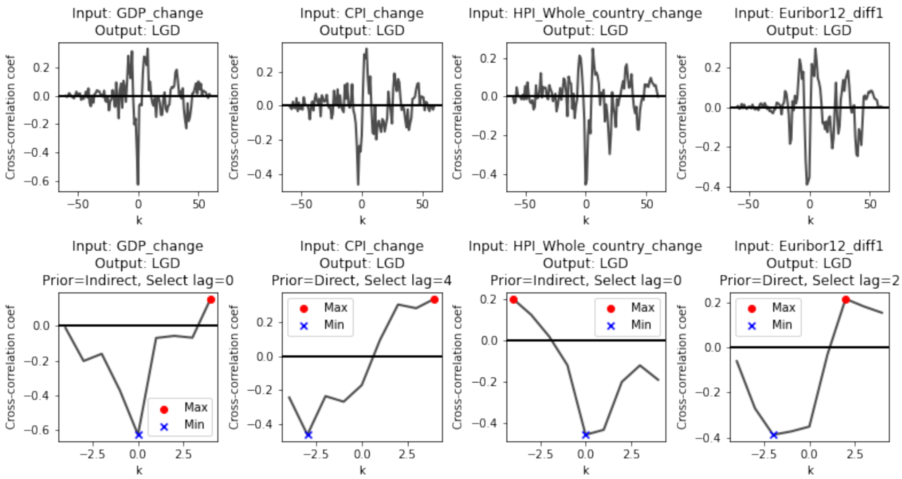


**Figure 17:** Data set 1 modelling. The cross-correlation results.

The lags are selected such that the cross-correlation is maximized or minimized with respect to the prior economical intuition of how the macroeconomic factor impacts LGD. For the GPD and the whole country HPI the impact to LGD is assumed to be indirect, and hence, the lag with minimal cross-correlation selected. The lag zero minimized the cross-correlation for both GDP and HPI. Thus, no operations are required for GDP and HPI. The result indicates that the changes the macroeconomic factors impact immediately the loss rates at default dates. For CPI and EUR12 the impact to LGD is assumed to be direct, and hence, cross-correlation is maximized. The lags $k = 4$ and $k = 2$ maximize the cross-correlation for the factors CPI and EUR12, respectively. The lags indicate that the changes in previous quarters with respect to the default date impact the LGD at default date.

Once the lags (or leads $k < 0$) have been selected and applied for each macroeconomic factor, the model selection can be performed. It is possible to calculate all possible model configurations as there are only four macroeconomic factors, which gives a total of 15 model configurations. The model configuration with minimal corrected AIC is selected. Two other conditions are also used. All coefficients (except the intercept) are required to be statistically significant and the F-test should reject the null hypothesis. The confidence level for these test is set to 5%. The candidate model structure is in Table 11, which includes the parameter estimates, the standard errors, t-test statistics, t-test p-values and the 95% confidence intervals. The performance results and other statistical tests are in Table 12.

From the model structure it can be seen that the coefficient signs are correct, i.e., they are negative for GDP and HPI, meaning that an increase in those factors indicates a decrease in the losses. All coefficient estimates $\widehat{\alpha}$ are significant according to the t-tests. However, as there is not much data available (only 60 observations) it is seen

**Table 11:** Data set 1 modelling. Loss rate OLS model structure and statistics.

| Variable | Estimate $\widehat{\alpha}_j$ | Std. error | t-stat | p-value | 95% conf. int. |
|---|---|---|---|---|---|
| constant | 0.1395 | 0.002 | 69.202 | 0.000 | $[0.135, 0.143]$ |
| GDP | -0.8511 | 0.154 | -5.531 | 0.000 | $[-1.159, -0.543]$ |
| HPI | -0.4473 | 0.149 | -3.007 | 0.004 | $[-0.745, -0.149]$ |

**Table 12:** Data set 1 modelling. Statistical tests and performances.

| Criterion | Value | Test | Statistic | p-value |
|---|---|---|---|---|
| $AIC_c$ | -336.0439 | F-test | 27.2242 | 0.0000 |
| $R_a^2$ | 0.4706 | Shapiro-Wilk | 0.9892 | 0.8734 |

that the confidence intervals for the coefficients are wide for GDP and HPI. The constant parameter is in line with the LRA LGD value of $\overline{\text{LGD}} = 0.1346$. Furthermore, the adjusted $R^2$ value is approximately 47%, which can be considered decently good.

The diagnostic plots are in Figure 18. The residuals are normally distributed according to the histogram, quantile-quantile-plot and the Shapiro-Wilk test that gives a p-value of 0.8734, not rejecting the null hypothesis of normality. The residuals are homoscedastic, as the residuals over time have only slight changes in variance. The scatter plot of fitted values and residuals does not indicate any notable non-linear patterns, and the scatter plots between the macroeconomic factors and residuals in Figure 19 are also scattered equally and non-linearities are not seen. The residuals are not autocorrelated as the ACF plot shows only small autocorrelation values for all lags that are under the 95% confidence, and the Breusch-Godfrey (BG) test for 1 to 12 qaurterly lags also indicates that no autocorrelation is present as none of the lags reject the null hypothesis (p-values are larger than 5%). Multicollinearity is analyzed as there are more than one factors in the regression equation. The VIF values for both GDP and HPI are 1.1355 (lower than threshold 5) and the Pearson correlation is 0.299. Hence, no multicollinearity is present is statistical terms. However, economically the GDP and HPI exhibit correlated patterns. In summary, all diagnostic tests are satisfied.

The predicted loss rates against the observed loss rates are displayed in Figure 20. The historical predictions capture the fluctuations in the loss rate. However, the predictions are only fitted values with the data used in the model training.

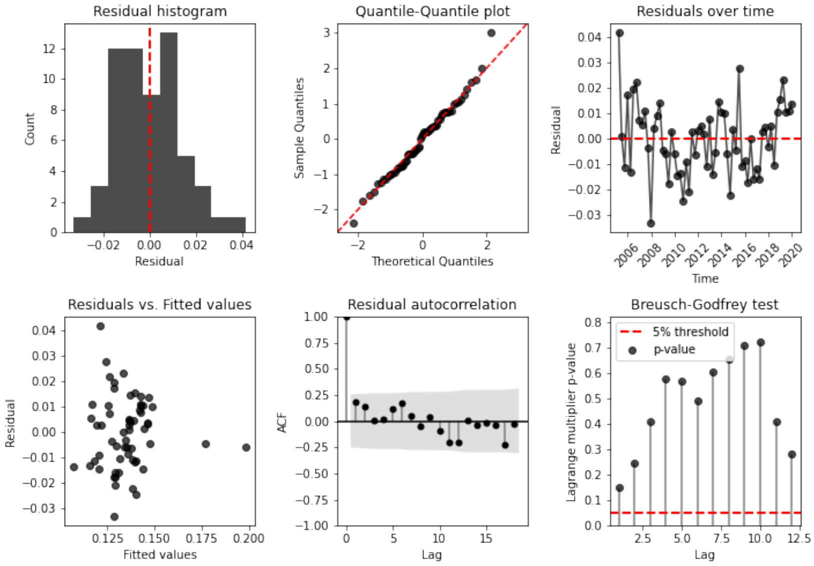


**Figure 18:** Data set 1 modelling. Diagnostic plots for the observed loss rate OLS model.

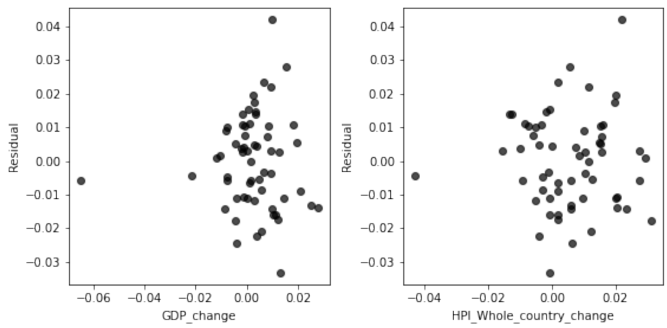


**Figure 19:** Data set 1 modelling. Scatter plots of residuals and macroeconomic factors.

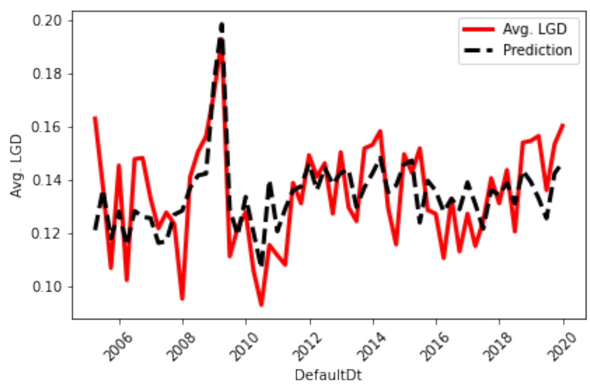


**Figure 20:** Data set 1 modelling. The observed and predicted loss rates.

The PIT adjustment scalars are calculated according to Equation (10) by comparing the predictions against the LRA LGD value of the portfolio $\overline{\text{LGD}} = 0.1346$. The scalars are used to adjust the LGD estimates by multiplying the IFRS 9 calibrated LGD estimate with the scalar associated with the default date. The comparison between the non-PIT and PIT IFRS 9 LGD estimate time series is in Figure 21. The PIT estimates are much more volatile compared to the non-PIT estimates and they react to the economic conditions. Table 13 shows the performance metrics for PIT adjusted estimates for the CAL, OOSC and OOTC samples. The results show that the PIT adjusted estimates are slightly better in all metrics and samples compared to the non-PIT adjusted results in Table 9. Only in the OOTC sample the MAE is better for the non-PIT estimates compared to the PIT estimates. As the PIT adjustments did not weaken the overall LGD model performance and the loss rate model is reasonable in both statistical and economical sense, the LGD model with the PIT adjustments can be used in ECL calculations.

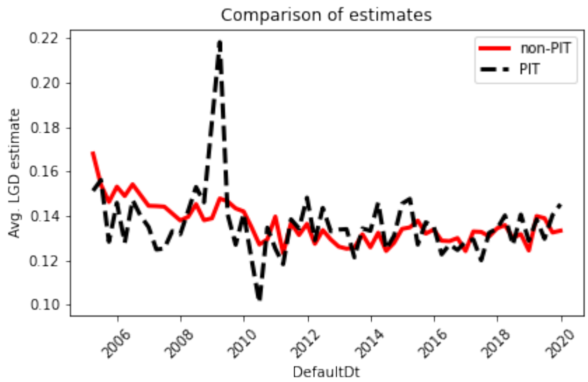


**Figure 21:** Comparison between the non-PIT and PIT average LGD estimates over time.

**Table 13:** The IFRS 9 calibrated and PIT adjusted LGD model performance for data set 1.

| Sample | $R^2$ | RMSE | MAE | Pearson | Spearman | Kendall |
|---|---|---|---|---|---|---|
| CAL | 0.1078 | 0.2721 | 0.1888 | 0.3284 | 0.3264 | 0.2528 |
| OOSC | 0.104 | 0.2619 | 0.1829 | 0.3252 | 0.307 | 0.2385 |
| OOTC | 0.1181 | 0.2861 | 0.2017 | 0.3479 | 0.3379 | 0.2611 |

An example of using this model in application is presented. Suppose that the reporting date is at the beginning of the year. Let the baseline forecasts for GDP and HPI growth rates for the end of the current year be $\text{GDP}_{base} = 0.01$ and $\text{HPI}_{base} = 0.02$. Moreover, let the weak and strong scenarios be $\text{GDP}_{weak} = -0.02$, $\text{GDP}_{strong} = 0.04$, $\text{HPI}_{weak} = -0.03$ and $\text{HPI}_{strong} = 0.05$. Assume that the quarterly growths are equal in each quarter. The quarterly rates are given by $\widetilde{M}^Q = (1 + \widetilde{M}^Y)^{1/4} - 1$, where $\widetilde{M}^Y$ is the yearly growth rate of the macroeconomic factor. Note that if the factors are lagged or the reporting date is at, e.g., the second quarter of the year, then the forecasts of next years or previous realizations need to be used as well. The quarterly forecasts are given by $\overline{\text{LGD}}_t = \widehat{\alpha}_1 \cdot \text{GDP}_t + \widehat{\alpha}_2 \cdot \text{HPI}_t + \widehat{\alpha}_0$. For all $\tau = 1, 2, 3, 4$ quarters the forecasts are

$$\begin{aligned}
\overline{\text{LGD}}_{t_R+\tau,base} &= -0.8511 \cdot 0.0025 - 0.4473 \cdot 0.005 + 0.1395 = 0.1351 \\
\overline{\text{LGD}}_{t_R+\tau,weak} &= -0.8511 \cdot (-0.005) - 0.4473 \cdot (-0.0076) + 0.1395 = 0.1471 \\
\overline{\text{LGD}}_{t_R+\tau,strong} &= -0.8511 \cdot 0.0099 - 0.4473 \cdot 0.0123 + 0.1395 = 0.1256.
\end{aligned}$$

Averaging the forecasts for all $\tau = 1, 2, 3, 4$ in each scenario gives the yearly forecast. Diving these forecasts with the LRA LGD value presented in Table 10 the adjustment scalars become $\delta_{t_R+1,base} = 0.1351/0.1346 = 1.0037$, $\delta_{t_R+1,weak} = 0.1471/0.1346 = 1.093$ and $\delta_{t_R+1,strong} = 0.1256/0.1346 = 0.9329$. If the non-PIT LGD estimate for a stage 1 asset is $\widehat{\text{LGD}}^*_{i,t_R+1} = 0.1$, then the different scenario adjustments give $\widehat{\text{LGD}}^*_{i,t_R+1,base} = \delta_{t_R+1,base} \cdot 0.1 = 0.1004$, $\widehat{\text{LGD}}^*_{i,t_R+1,weak} = \delta_{t_R+1,weak} \cdot 0.1 = 0.1093$ and $\widehat{\text{LGD}}^*_{i,t_R+1,strong} = \delta_{t_R+1,strong} \cdot 0.1 = 0.0933$. If the LGD estimates are already assumed to be PIT or hybrid, then the method in Joubert et al. (2021) can be applied. Here, the baseline forecast stays as $\widehat{\text{LGD}}^*_{i,t_R+1,base} = 0.1$, but the adjustment scalars become $\delta_{t_R+1,weak} = 0.1471/0.1351 = 1.089$ and $\delta_{t_R+1,strong} = 0.1256/0.1351 = 0.9294$. This same idea can be extended for stage 2 assets for yearly $t_R + \tau$ time points.

### 4.3.2 Loss Rate Model for Data With Trend

Similar analysis is carried out for the second data set where a trend is seen in the observed loss rate. The original observed loss rate for the second data set is in Figure 22. The observed loss rate has down going trend, and the estimated series has similar behaviour as with the previous data set, i.e., it is stable but moderate fluctuations are present due to the changing LTV risk driver.

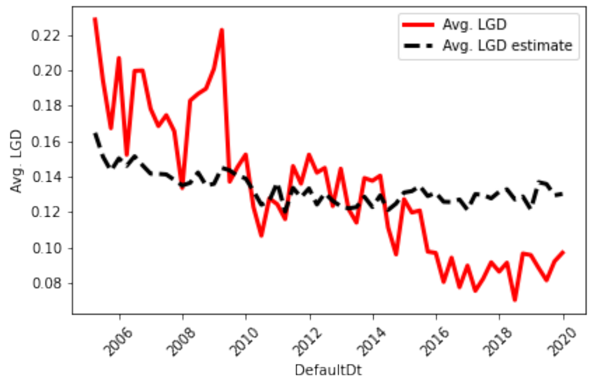


**Figure 22:** Data set 2 modelling. The quarterly aggregated observed loss rate (Avg. LGD) and the LGD estimates.

Different modelling options can be conducted. The first option is to conduct modelling for the loss rate series without any transformations, as LGD is assumed to be an $I(0)$ series. However, depending on the amount of data and the macroeconomic factors, the results can become spurious. A second option is to remove the trend from the time series by estimating the linear or polynomial trend. This requires the assumption that the loss rates are stationary around a trend, which is not a very reasonable assumption. Moreover, it can cause challenges in application as the time component should also be included. A third approach is to conduct the modelling on the differenced time series data, i.e., the target variable becomes $\nabla \text{LGD}_t = \text{LGD}_t - \text{LGD}_{t-1}$. Log-differences and four-quarter (yearly) differences can also be applied. This step ensures stationarity for the model estimation phase, given that the LGD time series is not stationary in a particular time period. The differenced loss rate is in Figure 23. No trend is now observed and the series is stationary.

The cross-correlation analysis is performed as previously. The GDP and HPI are assumed to have indirect impact on LGD, i.e., cross-correlation is minimized. For CPI and EUR12 direct impact is assumed, and thus, cross-correlation is maximized. The results are in Figure 24. For GDP and HPI lags $k = -4$ and $k = 0$ are selected, respectively, as they minimize the cross-correlation. For CPI and EUR12 the lags $k = -2$ and $k = 1$ are selected, respectively, as they maximize the cross-correlation. Negative lags are called are leads, which indicate that the future values of these factors with respect to the default date impact the LGD at default date.

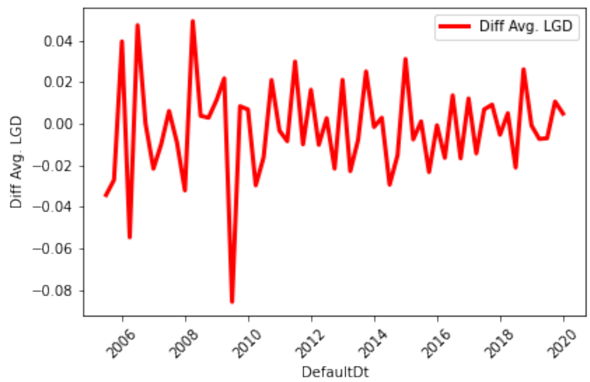


**Figure 23:** Data set 2 modelling. The differenced observed loss rate.

The model selection is performed by minimizing the corrected AIC and selecting configurations with only significant coefficients for the macroeconomic factors. Only configurations that pass the F-test are considered. The best model is the uni-variate OLS model with the HPI factor. Before going into further details, the model diagnostics are shown in Figure 25. The diagnostic plots for the ACF and the BG-test clearly indicate that significant residual autocorrelation is present in the model, which violates the OLS assumptions. Hence, the model is not applicable.

There are multiple options to overcome the residual autocorrelation problem in OLS modelling. The easiest option is to try out alternative variable transformations for the dependent time series, for example. If autocorrelation is still present, then other modelling methods should be investigated. If autocorrelation is seen to be significant only at the first lag of residuals, i.e., first order autocorrelation, then the

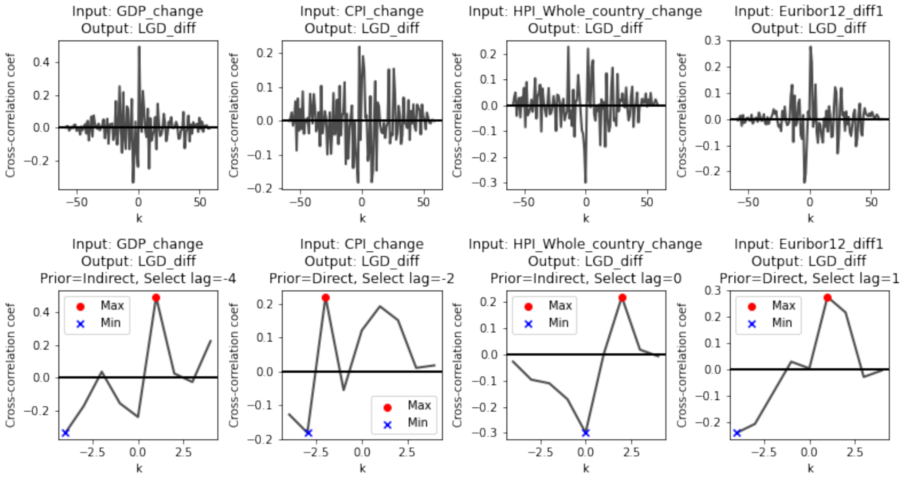


**Figure 24:** Data set 2 modelling. The cross-correlation results for the differenced loss rate output.

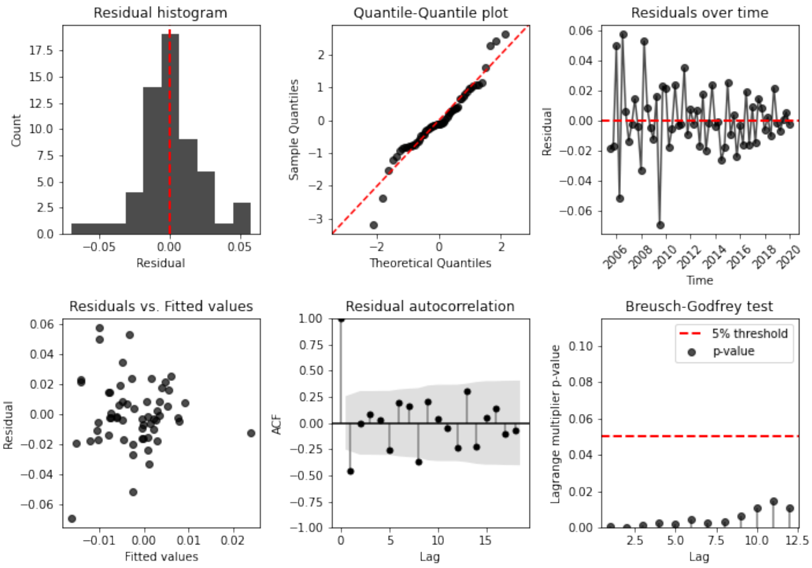


**Figure 25:** Data set 2 modelling. Diagnostic plots for the differenced loss rate OLS model.

Cochrane-Orcutt or Prais-Winsten procedure can be applied (Chatterjee and Simonoff, 2013). These procedures adjust the estimated OLS parameters in a way that the first order autocorrelation is addressed. Another approach is to estimate a regression with time series error model as is presented in Tsay (1984). Here, the OLS model entails its straightforward interpretation but the residual series is considered as an autoregressive and moving average process. In this model the parameters can be estimated simultaneously and multiple autoregressive orders can be considered. A third alternative is to extend the OLS model such that the regression equation includes lagged observations of the dependent variable. This type of model is also known as an “autoregressive distributed lag” (ADL) model, see, e.g., Hassler and Wolters (2006). The downside of this model, however, is that the regression coefficients for the macroeconomic factors change meaning, while this is not the case in the regression with time series errors model.

Testing of alternative modelling methods is left for further research, although they can be easily incorporated by following the framework and methodology presented in this thesis. Instead, an alternative variable transformation for the observed loss rate is tested. The yearly difference $\nabla^4 \overline{\mathrm{LGD}}_t = \overline{\mathrm{LGD}}_t - \overline{\mathrm{LGD}}_{t-4}$ is applied. The new transformation is in Figure 26. The trend is again removed and stationarity is assumed.

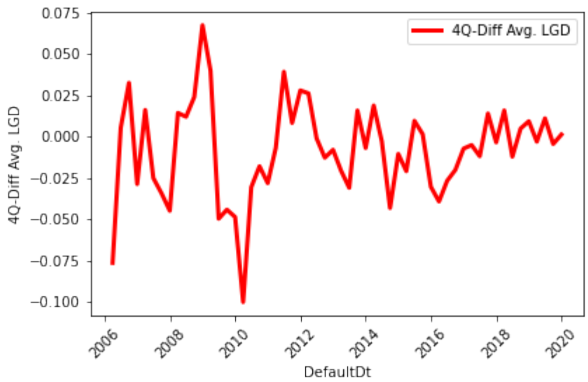


**Figure 26:** Data set 2 modelling. The yearly differenced observed loss rate.

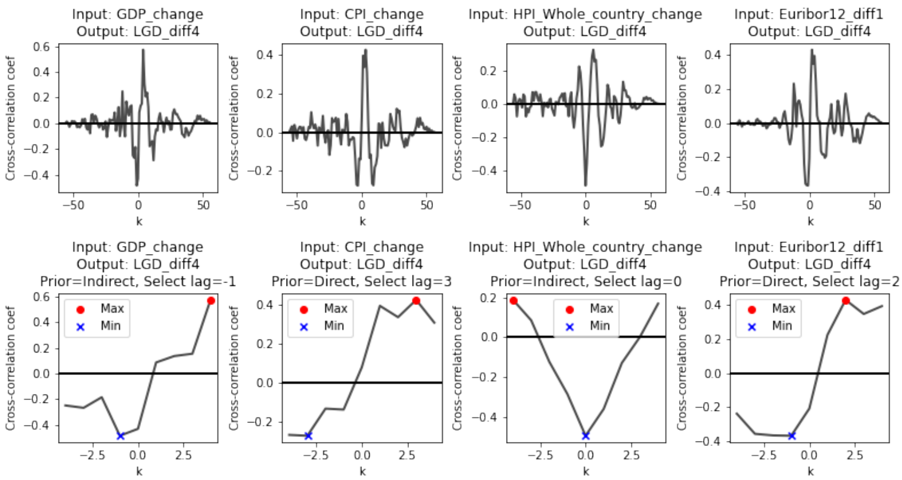


**Figure 27:** Data set 2 modelling. The cross-correlation results for the yearly differenced loss rate output.

The cross-correlation results are in Figure 27. The results change slightly compared to the previously used quarter to quarter differenced loss rate. The minimal cross-correlations for GDP and HPI are found at lags $k = -1$ and $k = 0$, respectively. The maximal cross-correlations for CPI and EUR12 are found at lags $k = 3$ and $k = 2$, respectively.

The model selection follows similar structure as previously. Table 14 shows the model candidate that minimizes the corrected AIC and has satisfactory statistical properties. The model structure is plausible as the parameter for GDP is negative and the parameter for EUR12 is positive. The coefficients are statistically significant, except for the intercept which is expected to be close to zero. The negative sign of the intercept is due to the decreasing losses over time. The 95% confidence intervals are also quite wide for the coefficients which is due to the low amount of data. Note that here the number of data points is 53 which is smaller compared to the number of data points in data set one which had 60. The reduction of data points is due to taking the yearly difference for the loss rate which removes four data points, and also lagging the GDP and EUR12 by -1 and 2, respectively, which removes another 3 data points. Thus one must be cautious when applying these types of variable transformations because it reduces the amount of data to be used for model estimation.

The model performance results and other statistical tests are displayed in Table 15. The results indicate that this model is also decently good as the adjusted $R^2$ values is approximately 45%.

**Table 14:** Data set 2 modelling. Yearly differenced loss rate model structure and statistics.

| Variable | Estimate $\widehat{\alpha}_j$ | Std. error | t-stat | p-value | 95% conf. int. |
|---|---|---|---|---|---|
| constant | -0.0034 | 0.003 | -1.143 | 0.259 | $[-0.009, 0.003]$ |
| GDP lead 1 | -1.1206 | 0.230 | -4.880 | 0.000 | $[-1.582, -0.659]$ |
| EUR12 lag 2 | 0.0304 | 0.008 | 4.023 | 0.000 | $[0.015, 0.046]$ |

**Table 15:** Data set 2 modelling. Statistical tests and performances.

| Criterion | Value | Test | Statistic | p-value |
|---|---|---|---|---|
| $AIC_c$ | -255.2902 | F-test | 22.0436 | 0.0000 |
| $R_a^2$ | 0.4473 | Shapiro-Wilk | 0.9724 | 0.2553 |

The diagnostic plots are in Figure 28. The residuals are normally distributed according to the histogram and Shapiro-Wilk test p-value of 0.2553. The quantile-quantile-plot implies normality as well, but the left tail of the residual distribution seems to be skewed. The residuals are homoscedastic as the over time plot is stable. The fitted values and residuals are uniformly scattered and do not show non-linear patterns. The scatter plots between the macroeconomic factors and the residuals in Figure 29 indicate that there are no non-linear patterns present. The residual autocorrelation is also removed according to the ACF plot and BG-test for 1 to 12 lags. Thus, this model configuration can be assumed to satisfy the OLS assumptions. Additionally, the VIF values for both macroeconomic factors are 1.0125 and Pearson correlation is -0.0942. Hence, no multicollinearity is assumed to be present.

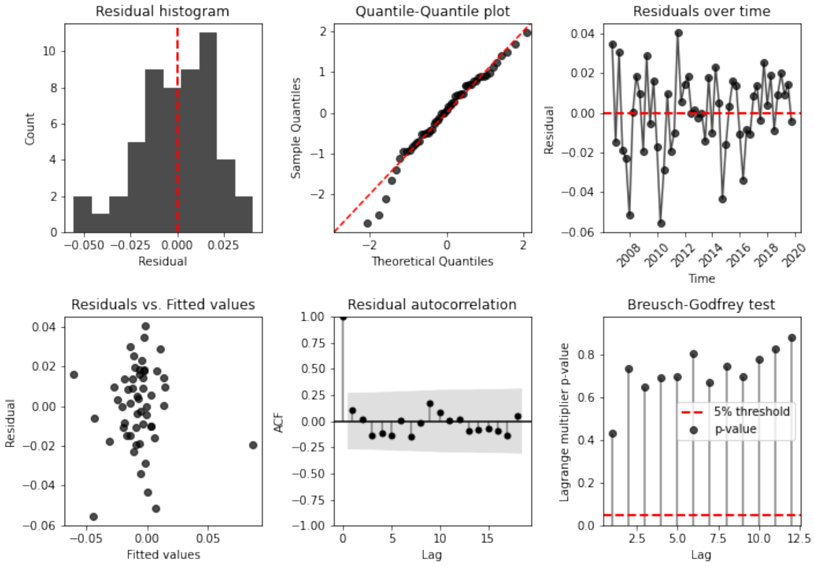


**Figure 28:** Data set 2 modelling. Diagnostic plots for the yearly differenced loss rate OLS model.

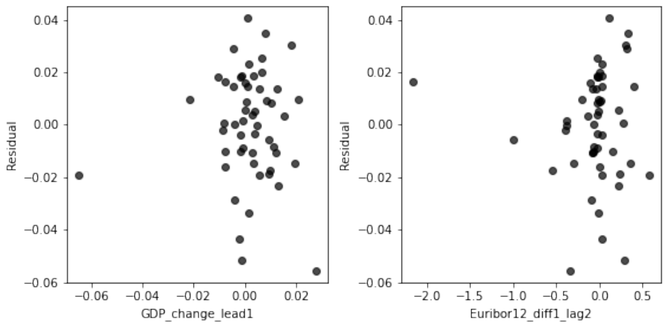


**Figure 29:** Data set 2 modelling. The scatter plots between the residuals and macroeconomic factors.

The predictions for the yearly differenced and original LGD series are in Figures 30 and 31, respectively. The predictions fit well to the data and the trends and macroeconomic fluctuations in the loss rates are captured. As the predictions are done as in one-step, i.e., the loss rate value four quarters previously is known when the next one is predicted. When forecasting, the results can be based only to the latest known observations.

As done previously, the PIT adjustment scalars are calculated by dividing the predictions for the loss rates with the LRA LGD value of $\overline{\text{LGD}} = 0.1313$. The scalars are applied adjust the non-PIT LGD estimates. Comparison between the PIT and non-PIT estimate time series is in Figure 32. The performance metrics Table 16 are calculated to compare the PIT and non-PIT estimates. The metrics for the non-PIT estimates are displayed again as they are different from the ones in Table 9 due to a slightly different number of data points in the calibration data, as the loss rate model

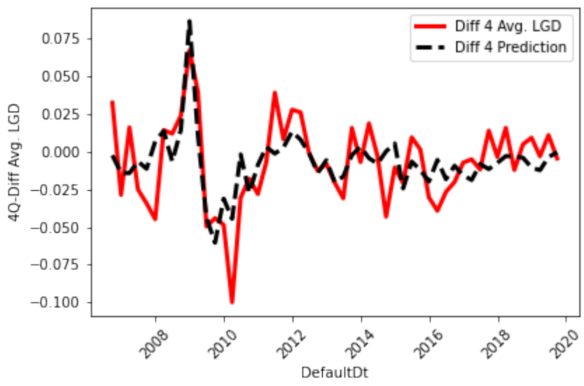


**Figure 30:** Data set 2 modelling. The observed and predicted yearly difference of loss rates.

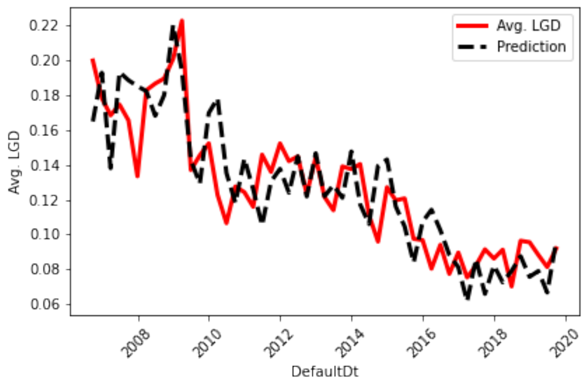


**Figure 31:** Data set 2 modelling. The observed and predicted loss rates.

included differencing and lags which remove data points. The results show that for all samples and all performance metrics the PIT adjusted estimates are better compared to the non-PIT estimates.

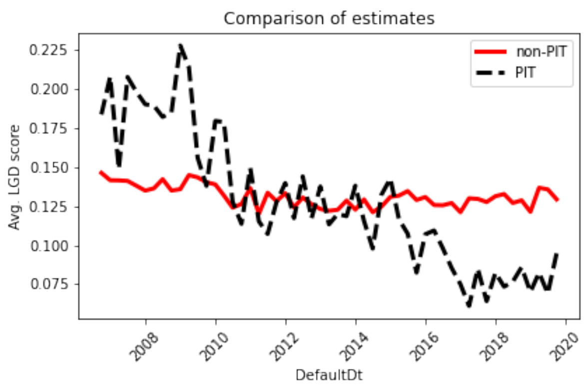


**Figure 32:** Data set 2 modelling. Comparison between the non-PIT and PIT average LGD estimates over time.

**Table 16:** Data set 2 modelling. The IFRS 9 calibrated, both non-PIT and PIT adjusted, LGD model performance.

| Sample | $R^2$ | RMSE | MAE | Pearson | Spearman | Kendall |
|---|---|---|---|---|---|---|
| CAL | 0.1044 | 0.2663 | 0.1823 | 0.3234 | 0.3185 | 0.2475 |
| CAL (PIT) | 0.1128 | 0.265 | 0.177 | 0.3366 | 0.3361 | 0.2609 |
| OOSC | 0.1023 | 0.2638 | 0.1812 | 0.3203 | 0.307 | 0.2384 |
| OOSC (PIT) | 0.1076 | 0.2631 | 0.1806 | 0.333 | 0.3235 | 0.2517 |
| OOTC | 0.0583 | 0.2377 | 0.1687 | 0.3108 | 0.2837 | 0.2249 |
| OOTC (PIT) | 0.0962 | 0.2329 | 0.1368 | 0.325 | 0.2871 | 0.2277 |

In conclusion, the model for the yearly differenced loss rate can be used for ECL applications to adjust the LGD estimates for different macroeconomic scenarios as the macroeconomic model satisfies statistical properties, it is economically intuitive, and the PIT adjusted estimates are better compared to the non-PIT estimates.

In comparison to the model developed for the first data set some aspects need to be addressed when the yearly differenced loss rate model is applied. Let $\tau$ be a discrete step forward in time in quarterly frequency. The loss rate forecast for $t_R + \tau$, where $t_R$ is the reporting date quarter, is calculated by $\overline{\text{LGD}}_{t_R+\tau} = \nabla^4 \overline{\text{LGD}}_{t_R+\tau} + \overline{\text{LGD}}_{t_R+\tau-4}$. Hence, forecasting the next four quarters for stage 1 assets, for example, requires that the loss rates $\overline{\text{LGD}}_{t_R-3}$, $\overline{\text{LGD}}_{t_R-2}$, $\overline{\text{LGD}}_{t_R-1}$ and $\overline{\text{LGD}}_{t_R}$ are known. This is a challenge, because the latest observed loss rates can be biased. The bias is usually due to most recent defaults that have not yet been resolved as the recovery processes for defaults can take several years. Thus, the most recent realized losses are typically addressed with a method that estimates the losses for the incomplete defaults as was discussed in Section 3.1. However, it can also be possible that the most recent loss rates are representative and unbiased. If it is not possible to use most recent loss rates for forecasting, then a pragmatic approach can be incorporated. For example, it can be assumed that the best approximation for recent loss rates are equal to the LRA LGD, which implies that the forecasts for the differenced losses indicate how much deviation will be seen from the LRA LGD value in a particular macroeconomic scenario.

For example, consider the same yearly forecasts for GDP as presented in the example for the first data set, i.e., $\text{GDP}_{1,base} = 0.01$. Additionally, it is required that there is a forecast for the second year for GDP due to the lead of 1 quarter. Let the second year GDP forecast be $\text{GDP}_{2,base} = 0.02$. The quarterly growths are $\text{GDP}_{1,base} = 0.0025$ and $\text{GDP}_{2,base} = 0.005$. Let the quarterly differences for EUR12 for the first year forecast be $\text{EUR12}_{1,base} = 0.02$ and the quarterly differences in the previous year (due to lagged factor) be $\text{EUR12}_{0,base} = 0.05$. The percentage point differences are assumed to be equal in each quarter. Assume that the loss rate values four quarters previously are equal to the LRA LGD value in Table 10, i.e., $\overline{\text{LGD}} = 0.1313$. The differenced loss rate forecasts are given by the equation

$\nabla^4\overline{\text{LGD}}_t = \widehat{\alpha}_1 \cdot \text{GDP}_{t+1} + \widehat{\alpha}_2 \cdot \text{EUR12}_{t-2} + \widehat{\alpha}_0$. The next four quarters forecasts are

$$\begin{aligned}
\nabla^4\overline{\text{LGD}}_{t_R+1} &= -1.1206 \cdot 0.0025 + 0.0304 \cdot 0.05 - 0.0034 = -0.0046\\
\nabla^4\overline{\text{LGD}}_{t_R+2} &= -1.1206 \cdot 0.0025 + 0.0304 \cdot 0.05 - 0.0034 = -0.0046\\
\nabla^4\overline{\text{LGD}}_{t_R+3} &= -1.1206 \cdot 0.0025 + 0.0304 \cdot 0.02 - 0.0034 = -0.0055\\
\nabla^4\overline{\text{LGD}}_{t_R+4} &= -1.1206 \cdot 0.005 + 0.0304 \cdot 0.02 - 0.0034 = -0.0083.
\end{aligned}$$

The loss rate forecasts for the next four quarters are

$$\begin{aligned}
\overline{\text{LGD}}_{t_R+1} &= \nabla^4\overline{\text{LGD}}_{t_R+1} + \overline{\text{LGD}}_{t_R-3} = 0.1267\\
\overline{\text{LGD}}_{t_R+2} &= \nabla^4\overline{\text{LGD}}_{t_R+2} + \overline{\text{LGD}}_{t_R-2} = 0.1267\\
\overline{\text{LGD}}_{t_R+3} &= \nabla^4\overline{\text{LGD}}_{t_R+3} + \overline{\text{LGD}}_{t_R-1} = 0.1257\\
\overline{\text{LGD}}_{t_R+4} &= \nabla^4\overline{\text{LGD}}_{t_R+4} + \overline{\text{LGD}}_{t_R} = 0.123,
\end{aligned}$$

where $\overline{\text{LGD}}_{t_R-3} = \overline{\text{LGD}}_{t_R-2} = \overline{\text{LGD}}_{t_R-1} = \overline{\text{LGD}}_{t_R} = \overline{\text{LGD}} = 0.1313$ are set to the LRA LGD value. The average of there forecasts is 0.1255, which can be used as the yearly forecast. Comparing this number to the LRA LGD value gives the one-year adjustment scalar $\delta_{t_R+1,base} = 0.1255/0.1313 = 0.9559$, which can be used to adjust the individual LGD estimates for the baseline scenario. Similar logic can be used for the weak and strong scenarios as well. Moreover, for stage 2 assets, the quarterly forecasts $t_R + \tau$ where $\tau > 4$, the forecasts are dynamically updated by using the produced forecasts $\overline{\text{LGD}}_{t_R+\tau-4}$.

Another challenge is the incorporation of synergy between the mean reversion of macroeconomic factors and through-the-cycle reversion of the risk parameters as described in EBA (2023) par. 82 and 83. Specifically, after the forecasting period it might be that the macroeconomic factors converge to their long-term values in each scenario, but it still needs to be ensured that the loss rate forecast also converge to the long-run averages. The investigation for the optimal methods regarding these challenges is left for future research.

### 4.3.3 Risk Driver Model for Data With Trend

This section covers the input adjustment results using the risk driver model in Section 3.2.4. The risk driver model can be used to forecast risk driver values such that the macroeconomic scenarios are incorporated into the ECL calculations for both stage 1 and 2 assets. Alternatively, if a loss rate model is used as an output adjustment, the risk driver model can be used for simply updating the risk driver values in stage 2 calculations. In situations with no clear correlation with respect to the loss rates and macroeconomic factors, the risk driver model can be used instead. The justification for the use of the risk driver model should be based on the macro-sensitivity of the risk drivers, which consequently make the LGD model macro-sensitive.

To illustrate the risk driver model development process and application, only the second simulated data set with trend is used. The reason for this is that the development process and application is exactly similar as it would be for the first data set without trend, and the effect of the risk driver model is seen for a slightly more complex data set.

The first step of the process is to understand the LGD model structure, i.e., what are the risk drivers, how the risk drivers are transformed, what is the underlying LGD model, and what are the outputs of the model. The LGD model has four risk drivers: loan-to-value (LTV), location, income and arrears. The collateral location can be assumed to be stable over time, unless the collateral is changed in the loan contract to another collateral. In terms of the income risk driver it could be possible to develop a model which follows, e.g., an earnings level index and unemployment rates. However, the model can easily become complex as income can depend on multiple factors such as the job, job location, and industry. The arrear risk driver is also fairly complex to model as the number of arrears can be a cause of, e.g., income or payment behaviour related characteristics. Hence, the modelling will focus on the LTV risk driver as the collateral values can be modelled with macroeconomic factors and the exposure amount can be modelled with, e.g., a separate exposure at default (EAD) model (Miu and Ozdemir, 2017).

The collateral value growth rates are first calculated by comparing the default date and reporting date values with each other. The dates are converted to quarterly frequency. Only observations where the quarterly difference between the reporting date and default date is greater or equal to one are considered. In this calibration data set, there are 34359 observations to be used for calculating the collateral growth rates. The other 5641 observations defaulted in the same quarter as the reporting date, and thus, there is no difference in the collateral values. According to Section 3.2.4, the calculated collateral growth rates are first converted to quarterly rates by assuming equal growth in each quarter, and further, the growth rates are converted to yearly growth rates. To add granularity into the model the yearly collateral growth rates can be modelled with respect to the collateral location if there is such macroeconomic information available.

The yearly collateral and house price index (HPI) growth rates are visualized for the two different locations in Figure 33, where it can be seen that the growth rates seem to behave similarly, but with a small lag. The cross-correlation analysis reveals that for both locations a lag $k = -1$ maximizes the cross-correlation, i.e., the increase in HPI increases collateral values as expected. Hence, a lead of one quarter is applied to the yearly HPI growth rates.

Two OLS models are estimated, where the location based yearly collateral value growth rates are modelled with the location based yearly HPI growth rates with a one quarter lead. The estimated models for both locations are in Table 17. The models indicate reasonable economical behaviour as the coefficients are positive, i.e., the collateral value growth rates increase when the HPI increases. The intercepts are also positive, which indicate that the collateral values increase even if the HPI growths are zero. A negative intercept would indicate depreciation of the collateral values (Miu and Ozdemir, 2017). The performance metrics and statistics are in Table 18. The collateral value model for the whole country excluding greater Helsinki (LOC=0) performs reasonably well with an adjusted $R^2$ of approximately 32%, while the collateral value model for greater Helsinki (LOC=1) has an adjusted $R^2$ of approximately 19%.

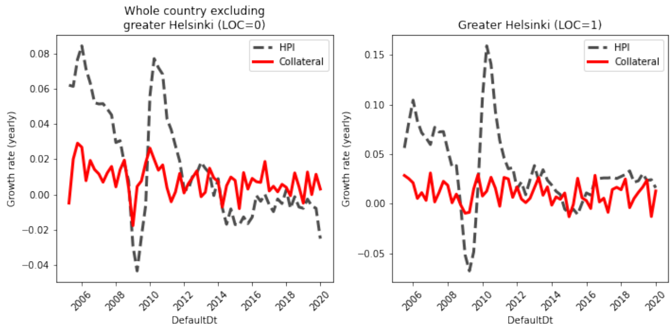


**Figure 33:** The yearly collateral and HPI growth rates for two locations on quarterly frequency.

**Table 17:** Risk driver model structure and statistics for both locations LOC=0 and LOC=1.

| Variable | Estimate $\widehat{\alpha}_j$ | Std. error | t-stat | p-value | 95% conf. int. |
|---|---|---|---|---|---|
| constant | 0.0058 | 0.001 | 5.498 | 0.000 | $[0.004, 0.008]$ |
| $\mathrm{HPI}_0$ lead 1 | 0.1610 | 0.03 | 5.289 | 0.000 | $[0.100, 0.222]$ |
| constant | 0.0061 | 0.002 | 3.404 | 0.001 | $[0.003, 0.010]$ |
| $\mathrm{HPI}_1$ lead 1 | 0.1293 | 0.034 | 3.775 | 0.000 | $[0.061, 0.198]$ |

**Table 18:** Statistical tests and performances for risk driver models.

| Model | Criterion | Value | Test | Statistic | p-value |
|---|---|---|---|---|---|
| LOC=0 | $\mathrm{AIC}_c$ | -410.5313 | F-test | 27.9740 | 0.0000 |
| | $R_a^2$ | 0.3174 | Shapiro-Wilk | 0.9755 | 0.2785 |
| LOC=1 | $\mathrm{AIC}_c$ | -361.5324 | F-test | 14.2532 | 0.0004 |
| | $R_a^2$ | 0.1886 | Shapiro-Wilk | 0.9781 | 0.3775 |

The diagnostic plots for both whole country excluding greater Helsinki model and greater Helsinki models are displayed in Figures 34 and 35, respectively. In the diagnostic plots for the LOC=0 model the OLS assumptions are mostly satisfied, i.e., the residuals are not autocorrelated and the residuals are homoscedastic. According to the Shapiro-Wilk test the residuals are normally distributed, but the histogram and quantile-quantile-plot show that residuals are skewed. This can make the statistical tests of the model parameters unstable. Moreover, slight non-linearity is present in the scatter plot of fitted values and residuals.

For the LOC=1 model all other assumptions are satisfied, but according to the BG-test there is autocorrelation present for lags 2 and 3. This is not necessarily a problem as there is no first order autocorrelation present according to the BG-test and the ACF plots. Moreover, all lags above 3 do not indicate autocorrelation. Thus, it could be argued that this model can still be used in application.

The exposure at default has to be also forecasted to properly forecast the LTV

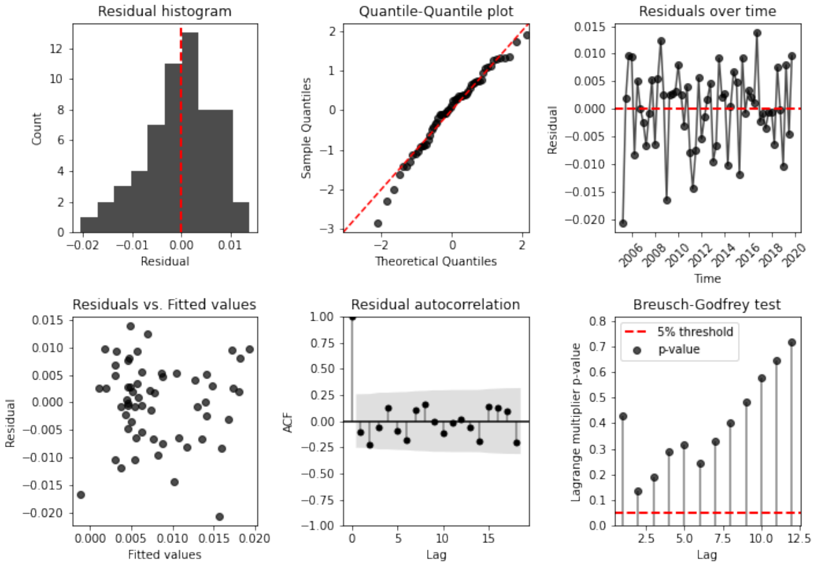


**Figure 34:** Diagnostic plots for the whole country excluding greater Helsinki collateral value model (LOC = 0).

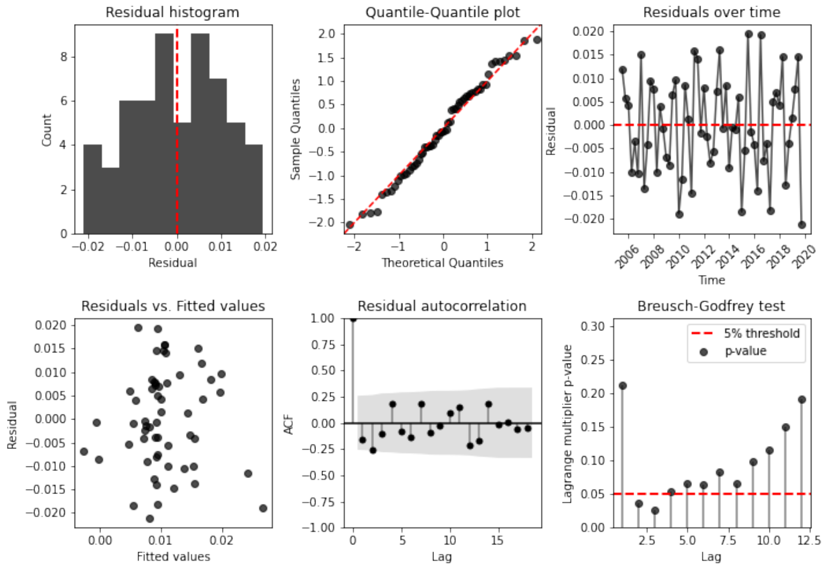


**Figure 35:** Diagnostic plots for the greater Helsinki collateral value model (LOC = 1).

values at default date. A very simple model to forecast EAD is employed. The model calculates an average conversion factor $C$ from exposure at reporting date to EAD by first calculating the ratio between EAD and exposure at reporting date for all calibration samples and then taking the average. Thus, the EAD for asset $i$ is forecasted by $\widehat{\mathrm{EAD}}_i = \widehat{C} \cdot \mathrm{Exposure}_i$, where $\mathrm{Exposure}_i$ is the exposure at reporting date and $\widehat{C} = (1/N) \cdot \sum_{i=1}^{N} \mathrm{EAD}_i / \mathrm{Exposure}_i$ is the estimated conversion factor. The conversion factor is quantified to be $\widehat{C} = 0.9579$. Hence, the EAD is on average approximately

4.2% smaller compared to the exposure at reporting date.

The next topic to consider is the calibration correction of the IFRS 9 LGD model. As discussed in Section 3.2.4, a risk of consistently under- or overestimating losses can occur if the default date LTV values are used and no corrections are made to the model calibration. The risk is caused from the LTV distribution shift, as the general trend in collateral values and exposures leads to decreasing LTV values. In the model setting used in this thesis there are two options how the risk of LTV distribution shift can be addressed. The first option is to fit a new calibration mapping function (5) such that the scores from the IRB LGD model are calculated by applying the default date LTV values and the existing standardization parameters. The second approach is to re-fit the standardization parameters $\mu$ and $\sigma$ for the standardization function $X^* = (X - \mu)/\sigma$ using the default date LTV values. The second approach scales the default date LTV values to zero mean and unit variance, which allows use the calibrated IFRS 9 LGD model as it is. Thus, the second approach will be used as it only changes the parameters associated with the LTV risk driver.

The mean and variance of the LTV at default date in the calibration data are calculated. The default date mean is $\mu_{default} = 0.5164$ and the default date variance is $\sigma^2_{default} = 0.0518$. Both of these are lower compared to the LTV at reporting date in the development IS sample, which gave $\mu_{reporting} = 0.5418$ and $\sigma^2_{reporting} = 0.0572$. The default date LTV is standardized by $\text{LTV}_i^* = (\text{LTV}_i - \mu_{default})/\sigma_{default}$. The over time LGD estimates are in Figure 36, which shows that the non-PIT estimates and the corrected PIT estimates using the newly standardized LTV values are overlapping. The PIT estimates where the default date LTV has been applied using the old standardization parameters shows that the time series mean is consistently under the non-PIT adjusted estimates. The performance metrics for the corrected PIT estimates are in Table 19, which indicate that they are better compared to the non-PIT estimates in the CAL and OOSC samples for all metrics expect the MAE. For the OOTC sample only the Spearman and Kendall metrics are better. Note that the performances are calculated by applying the known default date LTV values and not the forecasted default date LTV values.

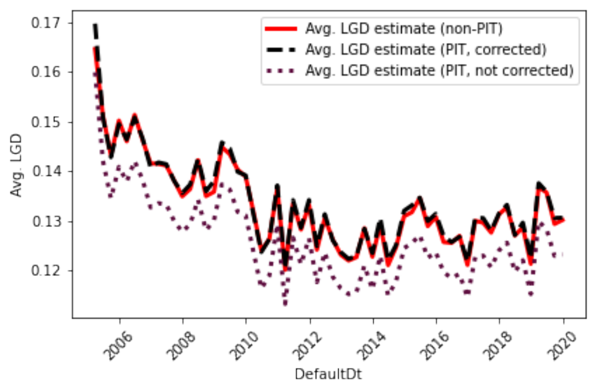


**Figure 36:** Comparison of the IFRS 9 calibrated LGD estimates for the risk driver forecast model.

**Table 19:** The IFRS 9 calibrated LGD model performance with both non-PIT and PIT adjusted using default date LTV.

| Sample | $R^2$ | RMSE | MAE | Pearson | Spearman | Kendall |
|---|---|---|---|---|---|---|
| CAL | 0.107 | 0.2697 | 0.1853 | 0.3272 | 0.3229 | 0.2506 |
| CAL (PIT) | 0.1082 | 0.2695 | 0.1855 | 0.3292 | 0.324 | 0.2514 |
| OOSC | 0.1051 | 0.2672 | 0.1843 | 0.3243 | 0.3113 | 0.2413 |
| OOSC (PIT) | 0.1069 | 0.2669 | 0.1843 | 0.3271 | 0.3144 | 0.2438 |
| OOTC | 0.0617 | 0.2364 | 0.168 | 0.3152 | 0.2922 | 0.2315 |
| OOTC (PIT) | 0.0598 | 0.2367 | 0.1687 | 0.3141 | 0.2924 | 0.2317 |

The collateral value and EAD models are now developed and the calibration corrections are in place. Thus, it is possible to backscore the exact model to historical observations, which simulates stage 1 yearly LGD estimates using the known location based yearly growth rate HPI information one year after the reporting date. Specifically, the one-year forward-looking LTV values from reporting date $t_R$ are

$$\widehat{\mathrm{LTV}}_{i,t_R+1,\mathrm{LOC}} = \frac{\widehat{\mathrm{EAD}}_{i,t_R+1}}{\widehat{V}_{i,t_R+1,\mathrm{LOC}}} = \frac{\widehat{C} \cdot \mathrm{Exposure}_{i,t_R}}{(1+\widehat{v}^Y_{t_R+1,\mathrm{LOC}}) \cdot V_{i,t_R,\mathrm{LOC}}},$$

where $\widehat{v}^Y_{t,\mathrm{LOC}} = \widehat{\alpha}_{1,\mathrm{LOC}} \cdot \mathrm{HPI}^Y_{t+1,\mathrm{LOC}} + \widehat{\alpha}_{0,\mathrm{LOC}}$ is the yearly growth rate of the collateral in location LOC $\in \{0,1\}$, $\mathrm{HPI}^Y_{t,\mathrm{LOC}}$ is the yearly growth of HPI in location LOC, and $V_{i,t_R,\mathrm{LOC}}$ is the residential collateral value at reporting date.

To get a broader sense of the developed collateral value models, it is also tested to directly update the collateral values according to the yearly growth of HPI in the different locations by $v^Y_{t,\mathrm{LOC}} = \mathrm{HPI}^Y_{t,\mathrm{LOC}}$. The direct updating of collateral values based on the HPI illustrates a situation where it is deemed by expert judgement that the HPI sufficiently enough represents the behaviour of the residential collaterals in the portfolio.

The backscores and the observed loss rate are in Figure 37. It can be seen that the PIT adjusted estimates using the developed collateral value and EAD modules are almost identical to the non-PIT adjusted estimates. The PIT adjusted estimates using directly the HPI growths are slightly more volatile, but they seem to also overlap with the non-PIT estimates in the latest years. Thus, the estimates are not as volatile compared to the loss rate model adjustments, but the collateral value forecasting approach is still applicable.

For example, consider the collateral value model developed for the greater Helsinki area. Table 20 shows calculations for an asset for the next four years of maturity. The other risk driver values are set to ARR = 1, LOC = 1 and INC = 3000 for all time points. The exposure, or EAD, is calculated by $\mathrm{Exposure}_{t_R+\tau} = C^\tau \cdot \mathrm{Exposure}_{t_R}$. The standardization parameters for the ARR and INC risk drivers are $\mu_{\mathrm{ARR}} = 1.289$, $\mu_{\mathrm{INC}} = 3497.881$, $\sigma^2_{\mathrm{ARR}} = 0.976$ and $\sigma^2_{\mathrm{INC}} = 1225581.221$. The IRB LGD model and the IFRS 9 mapping function parameters for the second data set are in Tables 6 and 8, respectively.

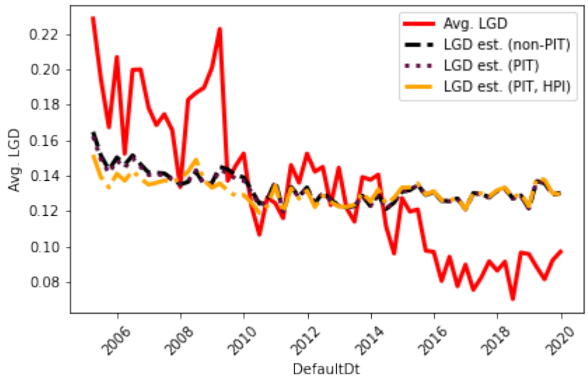


**Figure 37:** The risk driver model backscores to historical data.

**Table 20:** Example IFRS 9 LGD estimate calculations for the next four years using example yearly HPI scenario forecasts, the developed collateral value model (1) and the expert based HPI update model (2).

| | $t_R$ | $t_R+1$ | $t_R+2$ | $t_R+3$ | $t_R+4$ |
|---|---|---|---|---|---|
| $\text{HPI}^Y_{t,base}$ | - | 0.02 | 0.02 | 0.01 | 0.01 |
| $\text{HPI}^Y_{t,weak}$ | - | -0.04 | -0.03 | 0.0 | 0.0 |
| $\text{HPI}^Y_{t,strong}$ | - | 0.04 | 0.03 | 0.02 | 0.02 |
| $v^Y_{t,base}$ | - | 0.0087 | 0.0087 | 0.0074 | 0.0074 |
| $v^Y_{t,weak}$ | - | 0.0010 | 0.0022 | 0.0061 | 0.0061 |
| $v^Y_{t,strong}$ | - | 0.0113 | 0.0100 | 0.0087 | 0.0087 |
| Exposure | 60000 | 57477 | 55060 | 52744 | 50526 |
| (1) $V_{t,base}$ | 80000 | 80697 | 81400 | 82004 | 82612 |
| (1) $V_{t,weak}$ | 80000 | 80076 | 80256 | 80748 | 81242 |
| (1) $V_{t,strong}$ | 80000 | 80904 | 81713 | 82425 | 83143 |
| (2) $V_{t,base}$ | 80000 | 81600 | 83232 | 84064 | 84905 |
| (2) $V_{t,weak}$ | 80000 | 76800 | 74496 | 74496 | 74496 |
| (2) $V_{t,strong}$ | 80000 | 83200 | 85696 | 87410 | 89158 |
| (1) $\text{LTV}_{t,base}$ | 0.75 | 0.7123 | 0.6764 | 0.6432 | 0.6116 |
| (1) $\text{LTV}_{t,weak}$ | 0.75 | 0.7178 | 0.686 | 0.6532 | 0.6219 |
| (1) $\text{LTV}_{t,strong}$ | 0.75 | 0.7104 | 0.6738 | 0.6399 | 0.6077 |
| (2) $\text{LTV}_{t,base}$ | 0.75 | 0.7044 | 0.6615 | 0.6274 | 0.5951 |
| (2) $\text{LTV}_{t,weak}$ | 0.75 | 0.7484 | 0.7391 | 0.708 | 0.6782 |
| (2) $\text{LTV}_{t,strong}$ | 0.75 | 0.6908 | 0.6425 | 0.6034 | 0.5667 |
| (1) $\widehat{\text{LGD}}^*_{i,t,base}$ | - | 0.18 | 0.1682 | 0.1573 | 0.1469 |
| (1) $\widehat{\text{LGD}}^*_{i,t,weak}$ | - | 0.1818 | 0.1714 | 0.1606 | 0.1503 |
| (1) $\widehat{\text{LGD}}^*_{i,t,strong}$ | - | 0.1794 | 0.1674 | 0.1562 | 0.1456 |
| (2) $\widehat{\text{LGD}}^*_{i,t,base}$ | - | 0.1774 | 0.1633 | 0.1521 | 0.1415 |
| (2) $\widehat{\text{LGD}}^*_{i,t,weak}$ | - | 0.1919 | 0.1888 | 0.1786 | 0.1688 |
| (2) $\widehat{\text{LGD}}^*_{i,t,strong}$ | - | 0.173 | 0.1571 | 0.1442 | 0.1322 |

The example shows that using this method it is possible to obtain LGD estimates for the different scenarios without explicitly adjusting the outputs of the LGD model. The direct use of yearly HPI growth forecasts yields larger differences between the scenarios compared to the developed collateral value model. Hence, it terms of estimate volatility the expert based model is better. However, it must be noted that in terms of both input and output PIT adjustments, the resulting estimates are always dependent on the used scenarios in the ECL calculations. Arguing that the differences between the LGD estimates in each scenario are “too low” or “too high” is not reasonable. It is more convenient to ask first what is a proper range for LGD estimates in terms of ECL and how the model reacts to stressed scenarios. Scenarios should be then assessed whether they are specified correctly. If no changes in scenarios are made, then alternative modelling methods can be investigated. Ultimately, expert judgement can be incorporated as overlays to address deficiencies in the IFRS 9 models.

To summarize, the risk driver model based on collateral values does not exceed as well as the loss rate models presented previously. The performance metrics using known default date LTV data are only slightly better compared to the non-PIT estimates and in some cases slightly worse. Additionally, the backscoring of the developed model revealed that over time the estimates are not so responsive to the macroeconomic fluctuations. This is due to the fact that only the HPI is used, and the effects are seen in the LTV value directly, but only partially in the LGD estimates. Moreover, the developed linear models had a positive intercept, which diminishes the influence of decreasing HPI growth rate scenarios.

The results should not be interpreted to mean that the presented risk driver model is not a convenient choice, because the model is still able to exploit the macroeconomic environment in terms of collateral values, which quantify the risk related to LGD. The risk driver model is a reasonable choice in the case when it is not possible to apply a loss rate model for a particular portfolio. Furthermore, even if this risk driver model is not used for forecasting LGD estimates for different scenarios for stage 1 and 2 assets, it can still be used for risk driver lifetime updates purely in stage 2. Combinations of the risk driver and loss rate models can be used as well, i.e., the risk driver forecast model is first applied to get scenario forecast via risk drivers and then an output adjustment is applied according to the loss rate model. However, depending how the models perform this can result in “double counting” of scenario effects.

## 4.4 Simulation and Model Risk

This section discusses the scope of using simulated data and the risks related to the simulation algorithm, which can have impact on the modelling results. Moreover, the PIT adjustments are analyzed and improvement suggestions are proposed.

Data simulation was conducted due to the sensitive nature of financial data regarding losses. The scope was to simulate approximately realistic LGD and risk driver data that could be seen in a residential mortgage portfolio. Due to the complex nature of default recovery processes the simulation algorithm is not able to provide perfectly realistic data, but the data is realistic enough to obtain the similar model performances as with real data and to test the PIT adjustment framework. Hence, making the algorithm even more realistic by considering more granular cash flow simulations, the effect of default rates, varying lengths of recovery processes, and non-linear effects in data will mostly impact the modelling results rather than the developed framework and the PIT adjustment methods.

The simulation algorithm is based on generating random samples from a multivariate normal distribution using a predefined correlation structure and transforming the generated samples to proper distributions using Gaussian copulas. Hence, the choice of the copula and the correlation structure can affect the results by making the LGD model better or worse, e.g. in terms of $R^2$. Missing values are not added to the random samples and the distributions were designed and treated in a way that no significant outliers are present. In practice, LGD data typically contains missing values and outlying observations due to various reasons. Negative values and values much larger than one can be also present in LGD data. These data quality aspects mostly impact the choices of risk driver transformations and outlier treatments, which can be relevant when developing risk driver models for input adjustments.

The expected values of LGD and LTV over time are simulated using linear models and macroeconomic factors. Using a linear model is a strong assumption, because in reality the effects can be non-linear. Moreover, the effects are simulated for the entire modelling time horizon, but in reality it may be possible that the macroeconomic shocks in terms of losses are seen only in very bad economic conditions. Losses can also have high correlation with the observed default rates, and thus, default rates could be also used in modelling the loss rates. Furthermore, the effects of macroeconomic factors were designed in a way that they directly impact the loss rates at a specified default date, and thus, the potential effect of lags or leads was not introduced in the simulation. In terms of LGD the lags or leads can be very important, because the realized LGD is calculated from recovered cash flows obtained, e.g., from payments or liquidating collaterals. Hence, in an economical perspective the cash flows obtained from liquidating residential collateral can be affected by the house price indexes at the liquidation date. This is the same idea as the current-LTV calculation in Qi and Yang (2009).

The handling of maximum recovery length defaults was left out from the simulation by assuming that there are no excessively long defaults in the data, and to be in line with the unbiased view of IFRS 9. In addition, the handling of incomplete defaults was left outside the scope of this thesis as there is not much, if any, public literature available on the methods and they can be very institution specific. Hence, these maximum recovery length and handling of incomplete cases could be studied more as those can have impact to the observed loss rate time series, which can then affect the OLS time series modelling results. Moreover, simulating the interest rates could be investigated more closely, which relates to the discounting factor differences between IRB and IFRS 9 that affect the observed loss levels.

In terms of the developed IRB LGD model there could be room for improvement such that different model structures could be tested as they can have impact on, e.g., how the input adjustments work or how the output adjustments are developed. For example, output adjustments could be also developed for write-off ratios and zero-loss rates using the methods provided in this thesis. In terms of calibration methodology this thesis focused on continuous scale calibration. Thus, discrete LGD grade calibration could be also studied. For LGD grades the loss rate time series modelling methods can be applicable with minor modifications, because the updating of LGD estimates is based on how much over or under the LRA LGD the losses will go based in the scenarios. In terms of the risk driver model one must ensure that the distributions of the LGD grades is in line with the historically observed loss rates when forward-looking information is incorporated.

The parameters of the loss rate models are heavily dependent of the simulation algorithm. Tuning the linear model simulation parameters can lead to substantially different results, which is something that one needs to be aware. Moreover, as described before, the treatment of incomplete defaults and long recovery processes can also impact the results. Therefore, it is important that the model development process, final results and decisions can be justified in statistical, economical and business terms. For example, the loss rate model for the data set with trending loss rates, the GDP and Euribor were selected to be the final macroeconomic factors. This was surprising as the simulation algorithm gave very small weight on the differenced Euribor factor. Hence, the OLS results may be spurious, and it could have been driven by the large drop around 2009-2010 in the yearly differenced loss rate as a similar large drop was seen also in the differenced Euribor. However, the results were intuitive in economical sense and the statistical properties were also satisfied. Thus, the model could be used in application. Additionally, the developed time series models should be monitored and constantly re-calibrated when time goes by.

The testing of the predictive performance of the OLS time series models was performed only for the calibration sample which was used for fitting the model. The reason for this is that the number of quarterly data points in a time period of 15-years is very small, and hence, removing data points for testing purposes is not ideal. If more time series data would be available, then predictive performance testing could be done for an out-of-time time series sample. In practice the amount of data can be even smaller that 15-years, as the IRB model development requires at least five years of data to be used for retail exposures (Temim, 2019). IFRS 9 does not have this requirement, but usually the amount of data is the same as was used in the IRB model development.

The risk driver model considered the modelling of the collateral values as was done in Miu and Ozdemir (2017) in order to update the LTV values. The LTV was the most impacting risk driver in the model according to the regression coefficients and simulated correlation structures, and the collateral values are macro-sensitive. Hence, applying the LTV forecasts as an input adjustment is justified. However, similar risk driver models can be applied for other macro-sensitive risk drivers as well to introduce additional macroeconomic conditionality to the LGD estimates. Modelling alternative risk drivers can be, however, more challenging depending on the definition of the risk driver.

Simulating reporting date collateral values and reporting date exposures have direct impact on the differences between the reporting date and default date LTV values, which have large impact on the resulting risk driver models presented in Section 3.2.4. The simulation algorithm was designed to restrict the growth rates of reporting and default date collateral values to a specific range in order to not be too consistent with the HPI growths. As the coefficients in the collateral value models were estimated to be much smaller than one, it could be argued that it would be better to use the expert based model that directly updates collateral values according to the HPI growth rates. The expert based model gave also slightly more volatile LGD estimates over time. The use of the expert based model, however, should be explicitly justified. For example, as the collateral values in this study were assumed to be residential collateral values, then the use of HPI can be appropriate. One needs to still be careful about how the particular HPI is defined, i.e., which types of houses it considers. If the collateral values used in the LTV calculations include also other types of securities such as guarantees or valuable items, then one needs to investigate if there are macroeconomic factors that are able to model these types of collaterals.

The continuous risk drivers were normalized to zero mean and unit variance. This had a major advantage in terms of incorporating forward-looking information of risk drivers as model input. If the forward-looking distribution is known or can be approximated by utilizing known macroeconomic factors, then new standardization parameters can be estimated. This allows to use the IFRS 9 calibrated LGD model as because the shift in the risk driver distribution is corrected. However, if very strong shifts in the variance of the risk driver occur, for example, then the approach might become infeasible. Using other variable transformation techniques such as binning with weights of evidence values as in Matuszyk et al. (2010), the same calibration correction may not work, unless the risk drivers were standardized prior to the binning. Instead, one could possibly try re-estimating the mapping function used for IFRS 9 calibration by applying the forward-looking information and the binning structure of the risk driver.

# 5 Conclusions

This thesis has presented a framework for calibrating loss given default (LGD) models used in regulatory capital calculations under the internal ratings-based (IRB) approach to be compliant with the IFRS 9 accounting standard in the calculation of expected credit losses (ECL). The scope was restricted to non-defaulted secured residential mortgages, but the methods can be extended to other portfolios and defaulted assets as well. For testing the framework, a simulation algorithm was developed for generating LGD data with macroeconomic dependencies. The IRB LGD was modelled using linear regression, which served as a basis for the methods presented in the framework.

The IFRS 9 accounting standard calls for point-in-time (PIT) and forward-looking estimates, meaning that the LGD estimates a required to be conditioned on the expected macroeconomic environment, which is an opposite philosophy compared to IRB, where estimates aim to be through-the-cycle (TTC), i.e., not dependent on the macroeconomic environment. The PIT estimates are used in scenario probability weighted ECL calculations. Hence, the estimates are calculated for different macroeconomic scenarios. The framework presented two methods for applying PIT adjustments to the LGD estimates. The PIT adjustments were divided into "output" and "input" adjustments. Output adjustments aim to directly adjust the LGD estimates. A similar method was presented in Joubert et al. (2021), where an error correction model was used to adjust the LGD estimates. Input adjustments aim to adjust the risk drivers (explanatory variables) used in the LGD model, which are then reflected in the LGD estimates. The method was built upon the study of Miu and Ozdemir (2017), where the IFRS 9 LGD was estimated by forecasting collateral values.

An ordinary least squares (OLS) model for the observed loss rate time series was used as an output adjustment method. The calculation of the observed loss rate time series, using the loss rate forecasts to adjust the LGD estimates for different macroeconomic scenarios, and the OLS model development process were considered in detail. The OLS time series model is a simple and flexible approach to model dependencies between macroeconomic factors and loss rates over time. Various variable transformation techniques can be employed to address non-stationarity of time series and also non-linearities. For example, quarterly or yearly differencing can be incorporated remove trends and seasonality. Moreover, to address non-linearities one could apply log-transformations or include polynomial macroeconomic factors in the regression equation (Hyndman and Athanasopoulos, 2021). Overall, the OLS model for loss rates was reasonable for both trending and non-trending loss rates, and examples were provided how those models could be incorporated for ECL calculations. However, the OLS does not address situations were the residuals are autocorrelated, but the methodology can be easily extended to, e.g., regression with time series errors as in Tsay (1984).

Methods were provided for using the forecasts of the loss rate model to adjust individual LGD estimates for macroeconomic scenarios. The methods were built upon the study of Joubert et al. (2021), where adjustments for weak and strong scenarios were calculated by comparing the weak and strong scenarios forecasts to the baseline scenario forecasts. This way of calculating the adjustment scalars, however, assumes that the LGD estimates are already PIT or hybrid, and it does not introduce adjustments for the baseline scenario. A variation of calculating the adjustment scalars was presented, which essentially compares the loss rate forecasts in all scenarios to the portfolio specific long-run average (LRA) LGD value, which aims to describe how much in each scenario the loss rates go below or above the LRA LGD. The variation allows to incorporate adjustments also for the LGD estimates in the baseline scenario. Furthermore, the adjustment scalars were calculated such they correspond to the particular time point in the maturity of the asset. For stage 1 assets a one year forecasts is used. For stage 2 assets with, e.g., two years of remaining maturity, the forecasts for the first and second year are used to adjust the LGD estimates in the two time points of the assets maturity, respectively. The aim is to keep synergy between the LGD estimates and the loss rates which were aggregated to the default dates. In Joubert et al. (2021) only one fixed forecasting period was specified and the average of the scenario forecasts over that period was used to compute the adjustment scalars.

The second PIT adjustment method was a risk driver model, which focused on forecasting the collateral values used in the most relevant risk driver of the LGD model that is the loan-to-value (LTV). LTV has been identified in literature to be one of the most important risk driver for modelling LGD, see, e.g., Qi and Yang (2009). The risk driver model was built on the study of Miu and Ozdemir (2017), where a similar model was presented, which relied on the forecasting of collateral values using a macroeconomic linear regression model. However, the LGD estimation technique was different compared to traditional regression models. Hence, this thesis presented an adaptation of the approach used in Miu and Ozdemir (2017) to be used in a linear regression model for LGD. Aspects to make additional corrections to the calibration of the LGD model were discussed to use the forecasted risk driver values consistently in the LGD model. The main idea in the input adjustment method is to incorporate scenario based forward-looking information of risk drivers to the LGD model such that the LGD estimates do not have to be explicitly adjusted. Moreover, this model can be used for updating risk driver values for stage 2 assets in the case it is not needed to be used for stage 1 assets.

The risk driver model did not perform as well as the loss rate model according to many performance metrics on testing samples, and the resulting LGD estimates over time were not as volatile. However, it should be noted that the performance metrics were only marginally better or worse in the loss rate and risk driver model based PIT adjustments, respectively. The risk driver model can be used as a valid alternative to the loss rate model in the case that a proper macroeconomic model is not found for the loss rate. Not finding a proper loss rate model is plausible due to the complex behaviour of realized losses that depend on the recovered cash flows from the default recovery processes. Additional research can be done for the risk driver model, where the focus could be in incorporating adjustments to alternative risk drivers and using different variable transformation techniques. Moreover, in terms of discrete LGD grades, the possible grade distribution changes caused by the adjustments in the risk driver values could be studied in more detail.

Before applying the PIT adjustments, the calibration of the IRB LGD rank ordering model to correspond the realized losses calculated on the basis of IFRS 9 was applied to keep synergy between the IRB and IFRS 9 such that the risk differentiation stays the same, but the risk quantification is different. The calibration of continuous LGD estimates was performed using linear regression as a mapping function to set the IRB LGD scores to a lower level. In IFRS 9, the realized losses are generally lower than those for IRB, due to a lighter cash flow discounting factor, not using indirect costs associated with collection procedures, and taking into account all cash flows for an unbiased view. The calibration of discrete LGD grades was not studied in detail, but a possible method was mentioned such as estimating the LRA LGD values for each grade using the IFRS 9 realized loss data.

Future research can focus on various topics which can be incorporated into the presented framework. For example, the LGD model structure can be analyzed more closely, taking into account different variable transformations techniques, multi-stage models and complex non-linear models. Closer attention can also be provided for different secured or unsecured portfolios with various types of collaterals. The loss rate time series modelling can focus on more advanced methods to address autocorrelation issues, see, e.g., Tsay (1984). Synergy between the scenarios and the developed PIT adjustment models can be investigated more closely, because the used scenarios drive the results of the ECL calculations. Macroeconomic scenarios can exhibit mean reversion of the macroeconomic factors, but it still needs to be ensured that the loss rate and LGD forecasts also converge to the TTC values after the specified forecasting period.

The focus of this thesis was on non-defaulted assets that fall into the stage 1 and 2 allocations in ECL calculations. Hence, the defaulted assets (stage 3) can be considered further research. The main difference in stage 3 compared to the stages 1 and 2 is that the assets are defaulted, i.e., the probability of default is one. This does not allow for similar calculation of ECL, where the marginal probability of default is used at each time point of the assets remaining maturity. Moreover, for defaulted observations the exposure at default date is not relevant, as the focus is in the current exposure amount. The methods presented in this thesis can be used for stage 3 modelling as well, e.g., in terms of the loss level calibration. However, additional modifications may need to be introduced such as using the reference dates instead of default dates and taking into account the time in default of the asset and the possible transitions from the stage 3 to stages 1 or 2.

Overall, the topics of modelling LGD for regulatory capital and IFRS 9 ECL purposes are very broad and various aspects need to be taken into account, not just selecting the best modelling methods. The concept of IFRS 9 LGD has not been discussed as extensively in literature compared to IRB modelling. Thus, the framework presented in this thesis opens up various methodological aspects in modelling IFRS 9 LGD with the IRB LGD models used as a basis. The framework can be applied in real ECL calculations for provisioning, used for stress testing exercises, and it can be enhanced by adding granularity to the modelling methods and modifying the method to be applicable for defaulted assets as well.

# References


BCBS. Basel II: International convergence of capital measurement and capital standards: A revised framework - comprehensive version, 2006. URL https://www.bis.org/publ/bcbs128.htm. (accessed: 05.05.2023).

BCBS. Basel III: A global regulatory framework for more resilient banks and banking systems, 2011. URL https://www.bis.org/publ/bcbs189.htm. (accessed: 04.05.2023).

BCBS. Guidance on accounting for expected credit losses, 2015. URL https://www.bis.org/bcbs/publ/d311.htm. (accessed: 05.06.2023).

D. O. Beerbaum and S. Ahmad. Credit risk according to IFRS 9: Significant increase in credit risk and implications for financial institutions. *Available at SSRN 2654120*, 2015.

T. Bellotti and J. Crook. Loss given default models incorporating macroeconomic variables for credit cards. *International Journal of Forecasting*, 28(1):171–182, 2012.

G. E. Box and G. M. Jenkins. *Time series analysis: Forecasting and control, Revised Edition*. Holden Day, San Francisco., 1976.

S. Chatterjee and J. S. Simonoff. *Handbook of regression analysis*, volume 5. John Wiley & Sons, 2013.

J. H. Cochrane. A critique of the application of unit root tests. *Journal of Economic Dynamics and Control*, 15(2):275–284, 1991.

T. A. Craney and J. G. Surles. Model-dependent variance inflation factor cutoff values. *Quality Engineering*, 14(3):391–403, 2002.

EBA. Consultation paper: Guidelines on PD estimation, LGD estimation and the treatment of defaulted exposures, 2016. URL https://www.eba.europa.eu/publications-and-media/events/consultation-guidelines-pd-estimation-lgd-estimation-and-treatment. (accessed: 21.06.2023).

EBA. Guidelines on PD estimation, LGD estimation and the treatment of defaulted exposures, 2017. URL https://www.eba.europa.eu/regulation-and-policy/model-validation/guidelines-on-pd-lgd-estimation-and-treatment-of-defaulted-assets#pane-new-3cd4feaf-807f-4122-9d50-97dc4859d840. (accessed: 19.05.2023).

EBA. IFRS 9 implementation by EU institutions: 2023 monitoring report, 2023. URL https://www.eba.europa.eu/sites/default/files/2023-11/25b12d35-9c28-4335-a589-166c77198920/Final%20Report%20on%20IFRS9%20implementation%20by%20EU%20institutions.pdf. (accessed: 18.12.2023).

J. S. Fons. Using default rates to model the term structure of credit risk. *Financial Analysts Journal*, 50(5):25–32, 1994.

M. R. González, A. Partal Ureña, and P. G. Fernández-Aguado. *Proposal on ELBE and LGD in-default: Tackling capital requirements after the financial crisis*. Number 2165. ECB Working Paper, 2018.

C. W. Granger, N. Hyung, and Y. Jeon. Spurious regressions with stationary series. *Applied Economics*, 33(7):899–904, 2001.

M. Gubareva. How to estimate expected credit losses–ECL–for provisioning under IFRS 9. *The Journal of Risk Finance*, 22(2):169–190, 2021.

U. Hassler and J. Wolters. Autoregressive distributed lag models and cointegration. *Allgemeines Statistisches Archiv*, 90:59–74, 2006.

N. J. Higham. Analysis of the Cholesky decomposition of a semi-definite matrix. *Oxford University Press*, 1990.

R. J. Hyndman and G. Athanasopoulos. *Forecasting: Principles and practice, 3rd edition*. OTexts: Melbourne, Australia, 2021. URL OTexts.com/fpp3. (accessed: 04.11.2023).

IASB. IFRS 9 Financial Instruments, 2014. URL https://www.ifrs.org/. (accessed: 04.05.2023).

M. Joubert, T. Verster, and H. Raubenheimer. Default weighted survival analysis to directly model loss given default. *South African Statistical Journal*, 52(2):173–202, 2018.

M. Joubert, T. Verster, H. Raubenheimer, and W. D. Schutte. Adapting the default weighted survival analysis modelling approach to model IFRS 9 LGD. *Risks*, 9(6): 103, 2021.

R. Kellner, M. Nagl, and D. Rösch. Opening the black box–quantile neural networks for loss given default prediction. *Journal of Banking & Finance*, 134:106334, 2022.

P. King and H. Tarbert. Basel III: An overview. *Banking & financial services policy report*, 30(5):1–18, 2011.

G. Kirchgässner, J. Wolters, and U. Hassler. *Introduction to modern time series analysis*. Springer Science & Business Media, 2012.

D.-Q. Li, X.-S. Tang, K.-K. Phoon, Y.-F. Chen, and C.-B. Zhou. Bivariate simulation using copula and its application to probabilistic pile settlement analysis. *International Journal for Numerical and Analytical Methods in Geomechanics*, 37 (6):597–617, 2013.

G. Loterman, I. Brown, D. Martens, C. Mues, and B. Baesens. Benchmarking regression algorithms for loss given default modeling. *International Journal of Forecasting*, 28(1):161–170, 2012.

S. W. Malone and M. A. Wurm. Modeling stressed LGDs for macroeconomic scenarios. *Moody's Analytics Risk Perpectives: Managing Disruption*, 9, 2017.

A. Matuszyk, C. Mues, and L. C. Thomas. Modelling LGD for unsecured personal loans: Decision tree approach. *Journal of the Operational Research Society*, 61(3): 393–398, 2010.

A. J. McNeil, R. Frey, and P. Embrechts. *Quantitative risk management: Concepts, techniques and tools-revised edition*. Princeton university press, 2015.

P. Miu and B. Ozdemir. Adapting the Basel II advanced internal-ratings-based models for international financial reporting standard 9. *Journal of Credit Risk*, 13(2), 2017.

Z. Novotny-Farkas. The interaction of the IFRS 9 expected loss approach with supervisory rules and implications for financial stability. *Accounting in Europe*, 13 (2):197–227, 2016.

M. Qi and X. Yang. Loss given default of high loan-to-value residential mortgages. *Journal of Banking & Finance*, 33(5):788–799, 2009.

M. Qi and X. Zhao. Comparison of modeling methods for loss given default. *Journal of Banking & Finance*, 35(11):2842–2855, 2011.

RiskQuest. Inconsistency between AIRB and IFRS 9 PD modelling requirements, 2020. URL https://www.riskquest.com/inspiration/news-events/inconsistency-between-airb-and-ifrs-9-pd-modelling-requirements. (accessed: 9.12.2023).

W. D. Schutte, T. Verster, D. Doody, H. Raubenheimer, and P. J. Coetzee. A proposed benchmark model using a modularised approach to calculate IFRS 9 expected credit loss. *Cogent Economics & Finance*, 8(1):1735681, 2020.

S. S. Shapiro and M. B. Wilk. An analysis of variance test for normality (complete samples). *Biometrika*, 52(3/4):591–611, 1965.

StatFin. Official Statistics of Finland (OSF): Consumer price index [online publication], 2023a. ISSN 1799-0254. URL https://stat.fi/en/statistics/khi. (accessed: 5.10.2023).

StatFin. Official Statistics of Finland (OSF): Quarterly national accounts [online publication], 2023b. ISSN 1797-9765. URL https://stat.fi/en/statistics/ntp. (accessed: 5.10.2023).

StatFin. Official Statistics of Finland (OSF): Prices of dwellings in housing companies [online publication], 2023c. ISSN 2323-8801. URL https://stat.fi/en/statistics/ashi. (accessed: 5.10.2023).

C. Stephanou and J. C. Mendoza. Credit risk measurement under Basel II: An overview and implementation issues for developing countries. *World Bank Policy Research Working Paper*, (3556), 2005.

P. Stoica and Y. Selen. Model-order selection: A review of information criterion rules. *IEEE Signal Processing Magazine*, 21(4):36–47, 2004.

SuomenPankki. Suomen Pankki - Tilastot - Korot , 2023. URL https://www.suomenpankki.fi/fi/Tilastot/korot/taulukot2/. (accessed: 5.10.2023).

Y. Tanoue, A. Kawada, and S. Yamashita. Forecasting loss given default of bank loans with multi-stage model. *International Journal of Forecasting*, 33(2):513–522, 2017.

J. Temim. The IFRS 9 impairment model and its interaction with the basel framework. *Moody's analytics, Electronic Journal*, 8, 2019.

E. Tobback, D. Martens, T. Van Gestel, and B. Baesens. Forecasting loss given default models: Impact of account characteristics and the macroeconomic state. *Journal of the Operational Research Society*, 65:376–392, 2014.

R. S. Tsay. Regression models with time series errors. *Journal of the American Statistical Association*, 79(385):118–124, 1984.

S. S. Uyanto. Power comparisons of five most commonly used autocorrelation tests. *Pakistan Journal of Statistics and Operation Research*, pages 119–130, 2020.

J. Witzany, M. Rychnovsky, and P. Charamza. Survival analysis in LGD modeling. *Available at SSRN 1574452*, 2010.

X. Yao, J. Crook, and G. Andreeva. Enhancing two-stage modelling methodology for loss given default with support vector machines. *European Journal of Operational Research*, 263(2):679–689, 2017.

O. Yashkir and Y. Yashkir. Loss given default modelling: Comparative analysis. *Journal of Risk Model Validation*, 7(1):25–59, 2013.