Cost-efficient portfolios of reinforcement actions to secure the performance of transportation networks

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Abstract

Transportation networks like public transport and highway systems are a critical part of modern society's infrastructure. They enable the fast and reliable transportation of goods, people and information. Due to their critical nature, it is extremely important to secure their performance against disruptions caused by deterioration of the network's components or external hazards like natural disasters or terrorist attacks for example. This motivates the decision makers to invest in reinforcement actions, which secure the network's performance.

The decision makers need some help in identifying those reinforcement actions, which have the biggest positive impact on the performance of the network while also having minimal cost. A collection of such reinforcement actions is called a cost-efficient portfolio. This thesis presents an algorithm for identifying these portfolios. Transportation networks are modeled as undirected and unweighted graphs consisting of nodes and edges connecting them.

Two types of reinforcement actions are considered: Type I, which reinforce existing edges, and Type II, which add new edges to the network. We illustrate the algorithm with an example network with 10 nodes and 12 edges, where Type II reinforcement actions have a cost double the cost of Type I actions. There are 12 Type I actions and 13 Type II actions to consider. The results show us that it is far more beneficial to favor Type II reinforcement actions. However, there was one exception to this which was the reinforcement of a particular edge connecting one weakly connected node to the rest of the network, which was reinforced in most cost-efficient portfolios. The results infer that decision makers should favor Type II reinforcement actions when they are not unreasonably expensive.

One could support the decision makers with the cost-efficient portfolios given by the algorithm by recommending those portfolios or by recommending just some reinforcement actions, which have a high relative share amongst the cost-efficient portfolios. Additionally suggesting to avoid those reinforcement actions, which are not amongst any of the cost-efficient portfolios is possible.

Keywords Critical infrastructure, transportation network, decision analysis

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Tiivistelmä

Kuljetusverkot, kuten joukko- ja ilmaliikenne, ovat äärimmäisen tärkeä osa modernin yhteiskunnan infrastruktuuria. Ne mahdollistavat tavaroiden, ihmisten ja informaation nopean ja luotettavan liikkumisen paikasta toiseen. Niiden tärkeyden takia on ratkaisevaa turvata niiden toimintakyky olosuhteista riippumatta. Äärimmäiset sääilmiöt, komponenttien rappeutuminen ja muut ulkoiset uhat vahingoittavat verkon toimintakykyä, mikä motivoi päätöksentekijöitä investoimaan verkon vahvistamiseen.

Päätöksentekijät kaipaavat apua niiden vahvistustoimien identifioimiseen, joilla on isoin positiivinen vaikutus verkon toimintakykyyn, jotka ovat samalla mahdollisimman edullisia. Kokoelmaa tällaisia vahvistustoimia kutsutaan kustannustehokkaiksi portfolioiksi. Tässä kandidaatintyössä esitellään algoritmi, joka identifioi juuri tällaiset portfoliot. Kuljetusverkkoja mallinnetaan suuntaamattomina ja painottamattomina verkkoina, jotka koostuvat solmuista ja niitä yhdistävistä kaarista.

Vahvistustoimia on kahdenlaisia: ensimmäisen tyypin vahvistustoimessa vahvistetaan jo olemassa olevia kaaria ja toisen tyypin taas lisäävät uusia kaaria verkkoon. Kuvaamme algoritmin toimintaa esimerkkiverkolla, jossa on 10 solmua ja 12 kaarta, kun toisen tyypin vahvistustoimet olivat kustannukseltaan kaksinkertaisia ensimmäisen tyypin vahvistustoimiin verrattuna. Tulokset osoittavat, että on paljon kannattavampaa rakentaa uusia kaaria kuin vahvistaa jo olemassa olevia. Tästä poikkeuksena oli yksi kriittiseksi osoittautunut kaari, jota vahvistettiin lähes jokaisessa kustannustehokkaassa portfoliossa. Tästä päätellen uusien kaarien rakentaminen on lähes aina vanhojen vahvistamista kannattavampaa, kun niiden rakentaminen ei ole kohtuuttoman kallista.

Päätöksentekijöitä voi tukea algoritmin palauttamien kustannustehokkaiden portfolioiden avulla sekä suosittelemalla kokonaisia portfolioita että antamalla suosituksia yksittäisistä vahvistustoimista, joilla on suuri esiintyvyys kustannustehokkaissa portfolioissa. Lisäksi päätöksentekijöitä voi kehottaa välttämään niitä vahvistustoimia, jotka eivät esiinny lainkaan kustannustehokkaissa portfolioissa.

Avainsanat Kriittinen infrastruktuuri, kuljetusverkko, päätösanalyysi

Contents

1 Introduction

Transportation networks like highway systems, public transport networks of a metropolitan area or non-physical networks like content delivery networks are a part of modern society's critical infrastructure. Networks like these are critical in enabling modern society to function. We use them for food and goods distribution and also for the transportation of people and information across short and vast distances alike. The criticality of these transportation networks motivates decision makers (DM) like network operators or a government representative, to secure their performance against disruptions caused by natural disasters and deterioration of the network components. Additionally, their critical nature makes them prone to external hazards like terrorist attacks. This brings up the question of how to reinforce these transportation networks to be more resilient against these threats and minimize the effect of these disruptions.

The next question that one might ponder is what the performance of a network means. This can be measured in many ways, which may include looking at distances in the network or how connected the network is. The measurement is dependent on the preferences of the DM or a possible regulatory context, which is often the case with transportation networks like public transport and other government-mandated networks. We will be discussing performance measuring more in-depth in Section 3.

In this thesis, we introduce, discuss and explore how to improve and secure the performance of transportation networks. Reinforcement actions can be for example: upgrading the components of the network to be more reliable, constructing new connections to the network to provide alternate routes or placement of emergency supplies for rebuilding or fixing destroyed components quickly after a disaster. The DM is interested in implementing these reinforcement actions in such a way, that has the most positive impact on the performance of the network while also being inexpensive to implement.

The DM seeks to identify, which reinforcement actions they should implement to the network depending on their budget. We introduce a framework for distinguishing those portfolios of reinforcement actions, which have the best positive impact on the transportation network in question while also accomplishing this as inexpensively as possible. This framework could then assist DM by recommending reinforcement actions to them.

This thesis is structured as follows, in Section 2 we go through some background on the problem by exploring earlier work in the area. In Section 3 we discuss the methods required for improving the performance of transportation networks more in-depth by walking through every step of the process in detail. At the end of the section, the entire procedure is captured in Algorithm 2. In Section 4 we go through an illustrative example, which clarifies the procedure presented in the previous section better. We also introduce a method to conduct sensitivity analysis on the problem at hand to make some more robust conclusions about the results themselves. Lastly Section 5 summarizes the key points of the thesis neatly.

2 Background

Kangaspunta and Salo (2014) presents a general framework for securing the performance of transportation networks and how to approach computing the cost-efficient portfolios of node and edge reinforcement actions. The problem is modeled as a multi-objective optimization problem: maximize the network's expected performance and minimize the investment cost of the applied portfolio of reinforcement actions. We adapt this general framework for our application in this thesis.

An alternative approach to improving the performance of transportation networks is taken in Ip and Wang (2011). They modeled the problem as a multi-objective optimization problem, where they wanted to maximize the network's resilience and minimize its friability. The paper uses the concept of independent passageway sets to quantify the resilience of a network. We adapt the concept of independent passageway sets for building one performance metric.

In Latora and Marchiori (2001) the concepts of *global efficiency* and *local efficiency* are introduced for measuring the performance of networks like communication and transportation systems. They analyze these two performance metrics for unweighted and weighted graphs representing real-world networks. From their paper, we use *global efficiency* as a performance metric in this thesis.

Another approach is taken in Cappanera and Scaparra (2011) for identifying how to allocate protective resources to transportation networks. They introduce a multilevel optimization model, which identifies optimal strategies for protecting a transportation network. Their model proves to be effective even with large networks having over 200 nodes and 1000 edges.

Haritha and Anjaneyulu (2024) compare different measures of resilience for networks. They conduct a thorough analysis of varying measures on abstract networks with different topologies. They adapt the concept of independent passageway sets from Ip and Wang (2011) and use them to build the measure *number of independent paths*. This is similar to our performance metric, which is built with independent passageway sets.

3 Methods

3.1 Transportation Networks

Transportation networks can be modeled as unweighted and undirected graphs. Denote by $G(V, E)$ a transportation network, where $V = \{1, 2, 3, ..., N\}$ is the set of nodes in the network and $E \subseteq \{(i, j) \mid i, j \in V,\ i \neq j\}$ is the set of edges represented by pairs of nodes. For readability purposes, we label the edges with an arbitrary function $f : E \to \{1, 2, 3, ..., M\}$, where M = |E|. An edge being disrupted means that the edge is non-operational and we can model this by removing the disrupted edge from the graph. Without loss of generality, we consider edge disruptions. Then a node disruption can be modeled by removing all edges connected to the node. Additionally, we only consider two possible states for each edge: either fully operational or disrupted. Also, we assume here that there are no ripple effects in the disruptions of edges, that is that all of the disruptions are uncorrelated from each other. From now on edge k refers to the edge $(i, j) \in E$, which is labeled with k, that is $f(i, j) = k \in \{1, 2, 3, ..., M\}$. Denote by x_k the state of edge k, which indicates whether or not the edge is operational or disrupted, more precisely

$$x_k = \begin{cases} 1, & \text{if edge } k \text{ is operational} \\ 0, & \text{otherwise} \end{cases}, \quad k = 1, 2, 3, ..., M$$

The state of a network is defined to be a binary vector $x = [x_1, x_2, x_3, ..., x_M] \in \{0, 1\}^M = \mathcal{X}$, where $\mathcal{X}$ is called the state space. The collection of disrupted edges of the network in state x is denoted by $D^x = \{(i, j) \in E \mid f(i, j) = k \ \wedge \ x_k = 0\} \subseteq E$. The disrupted network is modeled without the disrupted edges. Therefore let the network, which is in state x, be $G(V, E^x)$, where $E^x = E \setminus D^x$.

The probability of edge k being operational is $\mathbb{P}(x_k = 1) = 1 - p_k$ and the probability of it being disrupted is $\mathbb{P}(x_k = 0) = p_k$. These probabilities are collected to a probability vector $p = [p_1, p_2, p_3, ..., p_M] \in [0, 1]^M$. Since we consider all edge disruptions to be independent of each other and therefore uncorrelated we can compute the probability of the network being in a particular state x with (1).

$$\mathbb{P}(x \mid p) = \prod_{k=1}^{M} x_k \cdot (1 - p_k) + (1 - x_k) \cdot p_k \tag{1}$$

3.2 Measuring the Performance

A passageway between two nodes $v_1, v_m \in V$ is a sequence of edges denoted by $P = [(v_1, v_2), ..., (v_{m-1}, v_m)]$ one could use to get from node v_1 to node v_m. This brings us to one of our performance metrics, where we consider the number of different independent passageways between nodes. These can be characterized by the concept of an independent passageway set, which is introduced in Ip and Wang (2011) and is defined in Definition 3.1.

Definition 3.1 (Independent passageway set)**.** If a set of passageways between two nodes $i, j \in V$ contain no common edges with other passageways in the set, the set is an independent passageway set for the nodes i and j denoted by $L(i, j)$.

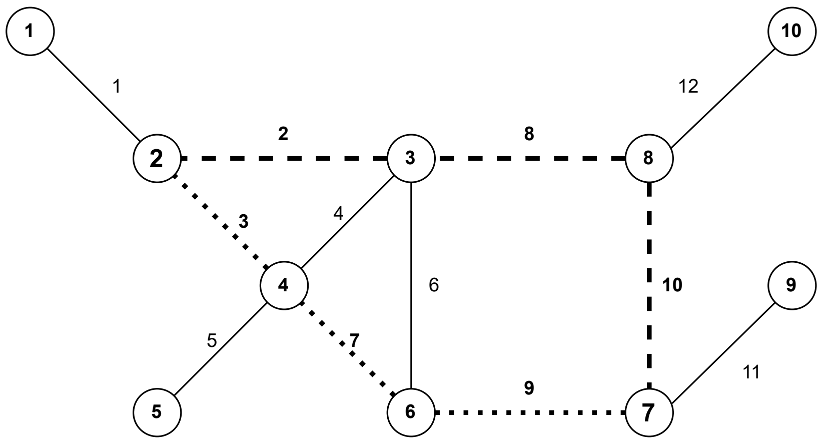

Figure 1: An example of an independent passageway set.

Since the users of transportation networks often seek to use the shortest possible passageway to get from node $i \in V$ to node $j \in V$, we consider only those independent passageway sets, which consist of the shortest independent passageways possible. From now on this kind of set for nodes i, j is denoted by $L(i, j)$. The cardinality of this set is denoted as $k(i, j) = |L(i, j)|$. From Figure 1 we can see that there are two different independent passageways between nodes 2 and 7 since all other paths between these two nodes would have common edges with these two paths. Therefore $k(2, 7) = 2$.

In addition to introducing the concept of independent passageway sets, an algorithm for finding these independent passageway sets is presented in Ip and Wang (2011). Because in this thesis we are only interested in the cardinality of the set $k(i, j)$ for nodes $i, j \in V$ we can further simplify their algorithm. To find this cardinality we need the concepts of distance between nodes and degree of a node in a graph. Let $d(i, j)$ be the minimum distance between two nodes $i, j \in V$ in the graph. This distance can be computed with Dijkstra's algorithm for example. If no passageway between nodes i, j exists, we define $d(i, j) = \infty$. The degree of a node is defined as the number of edges connected to the node denoted by $d_i, i \in V$. With these concepts, we can use Algorithm 1, which is adapted from Ip and Wang (2011), for finding $k(i, j),\ i, j \in V$.

Algorithm 1 Procedure for computing $k(i, j)$

$N(i, j) \leftarrow \min\{d_i, d_j\}$
$k(i, j) \leftarrow 0$
while $d(i, j) \neq \infty \wedge k < N(i, j)$ **do**
    $k(i, j) \leftarrow k(i, j) + 1$
    Delete all edges in the found shortest passageway
end while
Recover all deleted edges
Output the result $k(i, j)$

Now we can use the procedure described in Algorithm 1 to calculate $k(i,j)$ for all nodes $i, j \in V$. The sum of these is called *number of independent paths* like introduced in Haritha and Anjaneyulu (2024). We use the average of these as one of our performance metrics. It can be computed for a network G in a given state x like in (2).

$$v_1(x) = \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=i+1}^{N} k(i,j) \tag{2}$$

In addition to looking at the number of independent passageways between nodes, we consider the distances between these nodes, which can be interpreted in a transportation network for example as the number of exchanges between different vehicles in a public transportation network or the number of different flights needed to take to go from point A to point B in a network of airports and flight routes. To capture this aspect we use *global efficiency* introduced in Latora and Marchiori (2001). It can be computed for a network G in a given state x as in (3).

$$v_2(x) = \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=i+1}^{N} \frac{1}{d(i,j)} \tag{3}$$

Let us assume that the two attributes of the network the two performance metrics are measuring are mutually preferentially independent and also that a few other technical details hold for them as described in Dyer and Sarin (1979). With these assumptions, we can combine them with a linear combination to get the additive multi-attribute value function defined in (4) for the performance of the network G in some state x. For this, we need some weights for the performance metrics. Let $w = (w_1, w_2) \in S$, where $S \subseteq S^0 = \{w \in \mathbb{R}^2 \mid w_1 + w_2 = 1,\ w_1, w_2 \geq 0\}$ be the vector consisting of the two weights. It belongs to the set of feasible weights S, which is a subset of all possible weights S^0.

$$V(w,x) = w_1 v_1(x) + w_2 v_2(x),\ w \in S \tag{4}$$

Now with the help of (1) and (4) we can define the expected utility for the network G given some weight vector w and the disruption probabilities p. This is done in (5).

$$\mathbb{E}[V(w,x) \mid p] = \sum_{x \in \mathcal{X}} \mathbb{P}(x \mid p) \cdot V(w,x) \tag{5}$$

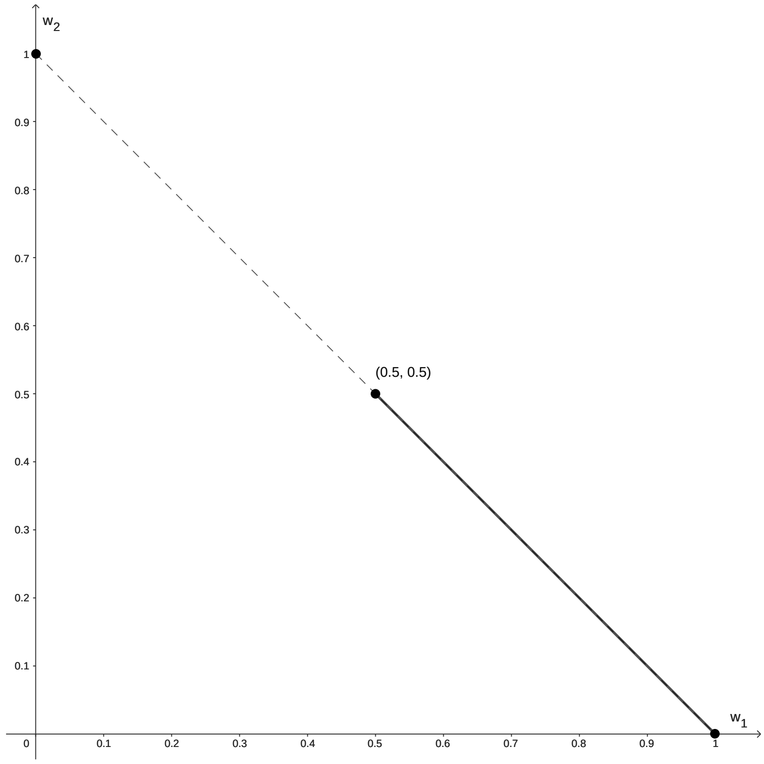

Figure 2: An example of a possible set of feasible weights.

The set of feasible weights S depends entirely on the preferences of the DM between the two performance metrics. These preferences can be captured in many ways as described in Morton (2018). An example of a possible set of feasible weights S, which resulted from the preference $w_1 \geq w_2$ is visualized in Figure 2 as the bold line segment. The dotted line represents those possible weights, which are no longer feasible given this preference.

Due to the fact the DM themself do not always know their preferences regarding the exact weights for the performance metrics, evaluating the performance seems difficult due to the uncountably infinite size of the set of feasible weights. To help with this problem we define $S_{ext} \subseteq S$ to be the set of extreme points of S. Luckily it is sufficient to just examine the performance for the weights in this set according to Liesiö et al. (2008). The performance for the weights in S_{ext} gives us enough information about the performance for all weights in S to compare the effects of reinforcement actions on the performance of the network.

3.3 Securing the Performance

We consider two types of reinforcement actions. Type I action is reinforcing existing edges in the graph. Reinforcing an existing edge reduces its disruption probability. Type II actions then add new edges to the network with some disruption probability. Consider that there are r alternate reinforcement actions in the set of reinforcement actions $R = \{1, 2, 3, ..., r\}$, which can be combined in all possible ways. This leads to 2^r different combinations of reinforcement actions. Let a portfolio of reinforcement actions $q = [q_1, q_2, q_3, ..., q_r] \in \{0,1\}^r$ be such a combination, where $q_m = 1$ if reinforcement action $m \in R$ is implemented in the portfolio and $q_m = 0$ otherwise. Denote by $Q = \{0,1\}^r$ the set of all portfolios of reinforcement actions.

Implementing a portfolio of reinforcement actions $q^j \in Q$ to the network $G(V,E)$ modifies it, which results in a network $G(V,E_j)$. In the case that q^j includes no Type II actions $E = E_j$ and when it does $E \subset E_j$. Applying a portfolio might also modify the original disruption probabilities as discussed earlier. If edge $k \in \{1,2,3,...,M_j\}$, where $M_j = |E_j|$, is reinforced in portfolio q^j it lowers its disruption probability to some $p'_k < p_k$. Therefore when portfolio q^j is applied to the network $G(V,E)$ the disruption probability of edge k becomes

$$p_k(q^j) = \begin{cases} p'_k, & \text{if edge k was reinforced in portfolio } q^j \\ p_k, & \text{otherwise} \end{cases}$$

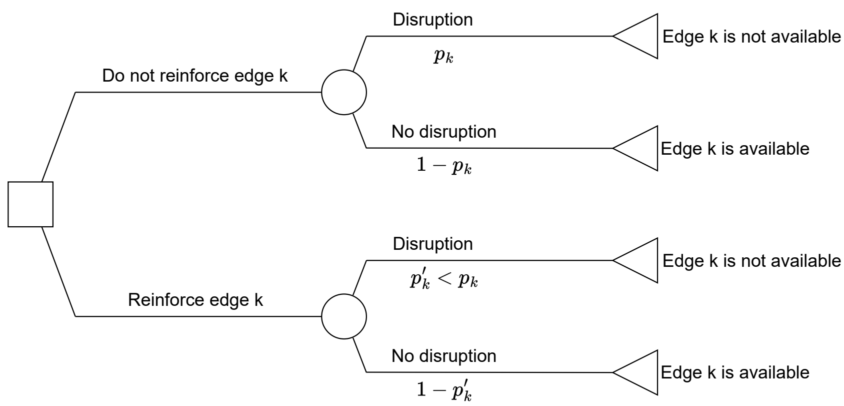

Figure 3: Decision tree for reinforcing edge k.

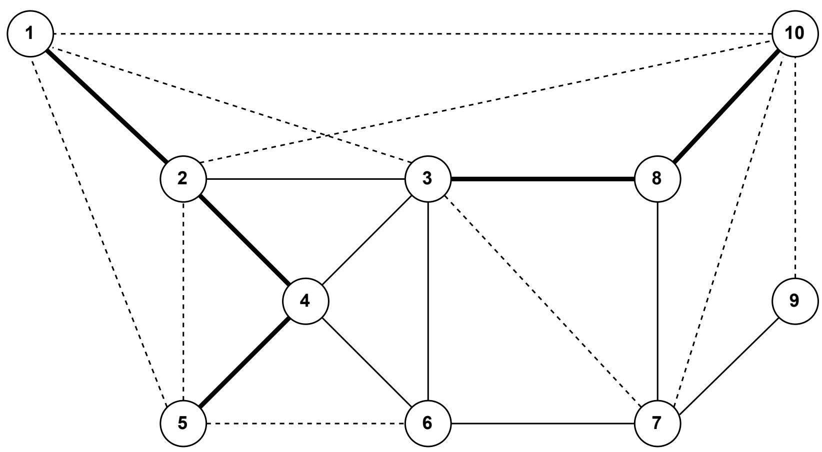

Figure 4: An example of possible reinforcement actions.

Both types of reinforcement actions will always increase the expected utility for the performance of the network because it monotonically increases when disruption probabilities decrease and when the number of edges in the network increases. There is a decision tree to capture the process of choosing to reinforce a particular edge in the network in Figure 3. Figure 4 represents an example of possible edges to add depicted with dotted lines and possible edges to reinforce in bold to an example network with 10 nodes and 12 edges.

In addition to considering the two performance metrics, the DM seeks to minimize investment costs. All reinforcement actions come with some cost, which can all be collected into a cost vector $c = [c_1, c_2, c_3, ..., c_r] \in \mathbb{R}^r_+$, where the elements are the costs of the reinforcement actions associated by index. Let us assume that there are no cost synergies between the reinforcement actions and therefore the cost of a reinforcement action does not depend on whether or not any other reinforcement actions are implemented. With the cost vector c and this assumption we can define the cost of a portfolio of reinforcement actions as a dot product of the portfolio and the cost vector as in (6).

$$C(q) = \sum_{k=1}^{r} c_k q_k, \;\; q \in Q \tag{6}$$

Now with the cost of a portfolio, we can prune out those portfolios, which are too expensive. Given some budget b the set of feasible portfolios of reinforcement actions is defined as $Q_F = \{q \in Q \mid C(q) \leq b\} \subseteq Q$. Since the DM seeks to maximize performance, while minimizing investment costs, we need to identify those portfolios, which outperform other ones while also having a lower or equal cost. We start by defining what it means to outperform another portfolio. This is captured with the concept of dominance, which is defined in Definition 3.2. Additionally, we are also interested in identifying portfolios, which perform equally well so we can prune the one, which is more expensive. Two portfolios $q^k, q^j \in Q_F$ are considered to be equal with regard to efficiency denoted by $q^k \sim q^j$ if and only if $\forall w \in S_{ext}$: $\mathbb{E}[V(w,x) \mid q^k] = \mathbb{E}[V(w,x) \mid q^j]$.

Definition 3.2 (Dominance between portfolios)**.** A portfolio $q^k \in Q_F$ dominates portfolio $q^j \in Q_F$ denoted by $q^k \succ q^j$ if and only if

$$\begin{cases} \forall w \in S_{ext} : \mathbb{E}[V(w,x) \mid q^k] \geq \mathbb{E}[V(w,x) \mid q^j] \\ \exists w \in S_{ext} : \mathbb{E}[V(w,x) \mid q^k] > \mathbb{E}[V(w,x) \mid q^j] \end{cases}$$

The dominance between portfolios is not sufficient for pruning inefficient portfolios from the set of feasible portfolios, since it does not take into account the cost of portfolios. We define that a feasible portfolio $q^k \in Q_F$ dominates with cost another feasible portfolio $q^j \in Q_F$ if it dominates it and has at most the same cost associated with it or in the other case if they are equal with regard to efficiency and the cost of portfolio q^k is strictly less than the cost of portfolio q^j.

Definition 3.3 (Dominance with cost between portfolios)**.** A portfolio $q^k \in Q_F$ dominates with cost another portfolio $q^j \in Q_F$ denoted by $q^k \succ_C q^j$ if and only if

$$(q^k \succ q^j \wedge C(q^k) \leq C(q^j)) \vee (q^k \sim q^j \wedge C(q^k) < C(q^j)$$

We are especially interested in those portfolios, which are non-dominated with taking cost into account otherwise known as cost-efficient portfolios. With the help of Definition 3.3 we can now prune inefficient portfolios from the set of feasible portfolios Q_F to get the set of cost-efficient portfolios Q_{CE}, that is those portfolios which are not dominated with cost by any other portfolios in the set of feasible portfolios. In Definition 3.4 the formal definition of a cost-efficient portfolio is presented. The procedure for finding this set is presented and explained in Section 3.4.

Definition 3.4 (Cost-efficient portfolio)**.** A portfolio $q^k \in Q_F$ is cost-efficient and therefore belongs to the set of cost-efficient portfolios $Q_{CE} \subseteq Q_F$ if and only if

$$\nexists q^j \in Q_F : q^j \succ_C q^k$$

Table 1: Example of a set of feasible portfolios.

Portfolio	Cost	$\mathbb{E}[V(x, w^1) \mid q^k]$	$\mathbb{E}[V(x, w^2) \mid q^k]$
q^1	0.0	0.90	0.40
q^2	1.0	0.95	0.45
q^3	1.0	0.95	0.44
q^4	1.0	1.00	0.43
q^5	1.9	1.15	0.47
q^6	2.0	1.15	0.47
q^7	2.0	1.16	0.46

Let us go through a small example of cost-efficiency. Consider that there are seven feasible portfolios and two extreme points w^1 and w^2 as in Table 1. Starting from the top: we consider portfolio q^1, which has a cost of 0. We see that there are no other feasible portfolios with equal or lower cost and therefore according to Definition 3.4 it is cost-efficient. Now consider portfolios q^2, q^3, and q^4 which all have a cost of 1. We can see that portfolio q^2 dominates portfolio q^3 since their costs are equal and we see that

$$\begin{cases} \mathbb{E}[V(x, w^1) \mid q^2] = \mathbb{E}[V(x, w^1) \mid q^3] \\ \mathbb{E}[V(x, w^2) \mid q^2] > \mathbb{E}[V(x, w^2) \mid q^3] \end{cases}$$

There is no dominance between portfolios q^2 and q^4. Additionally, we notice that portfolio q^5 dominates q^6, since the latter is more expensive $C(q^5) < C(q^6)$, while they have the same expected performance for both extreme points

$$\begin{cases} \mathbb{E}[V(x, w^1) \mid q^5] = \mathbb{E}[V(x, w^1) \mid q^6] \\ \mathbb{E}[V(x, w^2) \mid q^5] = \mathbb{E}[V(x, w^2) \mid q^6] \end{cases}$$

Now that we prune out portfolios q^3 and q^6 there is no longer dominance between any of the other feasible portfolios. Therefore $Q_{CE} = \{q^1, q^2, q^4, q^5, q^7\}$.

3.4 Finding the Cost-Efficient Portfolios

We have now gone over how to model transportation networks as graphs, how to calculate the expected utility for the performance of such networks and how to compare portfolios of reinforcement actions with our notion of expected utility. With these, we can determine if a portfolio of reinforcement actions is cost-efficient according to Definition 3.4. Next, we are going to go over the procedure for finding the cost-efficient portfolios of reinforcement actions when given a transportation network $G(V, E)$, the set of extreme points S_{ext} of the set of feasible weights S, the possible r alternative reinforcement actions and their costs and effects. This procedure is presented in Algorithm 2, which is adapted from Kangaspunta and Salo (2014).

Algorithm 2 Procedure for finding the cost-efficient portfolios

1: $Q^0 \leftarrow [0, 0, 0, ..., 0] \in Q_F$
2: **for** l = 1, 2, 3, ..., r **do**
3: $\quad Q^l \leftarrow \{q^k \in Q_F \mid q^j \in Q^{l-1} : q^k_l = 1 \wedge \forall i \neq l : q^k_i = q^j_i\}$
4: $\quad$ **for** $q^k \in Q^l$ **do**
5: $\quad\quad$ Compute $\mathbb{E}[V(w, x) \mid q^k]\ \forall w \in S_{ext}$
6: $\quad$ **end for**
7: $\quad Q^l \leftarrow Q^l \setminus \{q^k \in Q^l \mid \exists q^j \in Q^{l-1} : q^j \succ_C q^k\}$
8: $\quad Q^{l-1} \leftarrow Q^{l-1} \setminus \{q^j \in Q^{l-1} \mid \exists q^k \in Q^l : q^k \succ_C q^j\}$
9: $\quad Q^l \leftarrow Q^l \cup Q^{l-1}$
10: **end for**
11: $Q_{CE} \leftarrow Q^r$
12: Output the result Q_{CE}

In Step 1 we initialize Q^0 with the trivial cost-efficient portfolio portfolio containing no reinforcement actions at all. In Step 3 we consider only those feasible portfolios, which add the reinforcement action l to a portfolio of Q^{l-1} for iteration l. We compute the expected utility of the performance for all weights w in the set of extreme points S_{ext} given portfolio q^k is applied to the network in Step 5. After we have done that for all portfolios in Q^l, in Step 7 we prune out those portfolios from Q^l which are dominated with cost by at least one portfolio in Q^{l-1} and vice versa in Step 8. Then, in Step 9 of iteration l we can combine these two sets to be Q^l. Lastly, after the final iteration r we can save Q^r as the set of cost-efficient portfolios Q_{CE} and output it in Steps 11 and 12.

4 Results

4.1 Example Network

We illustrate the procedure depicted in Algorithm 2 with a small example. First let us consider a transportation network, which has 10 nodes and 12 edges connecting them as depicted in Figure 5. Estimating the disruption probabilities for each edge in the network is difficult due to the complexity of estimating the probabilities of different events like earthquakes or terrorist attacks causing disruptions. For the sake of the example let us arbitrarily choose that each edge has a disruption probability of 0.2, $p_k = 0.2,\ \forall k \in \{1, ..., 12\}$.

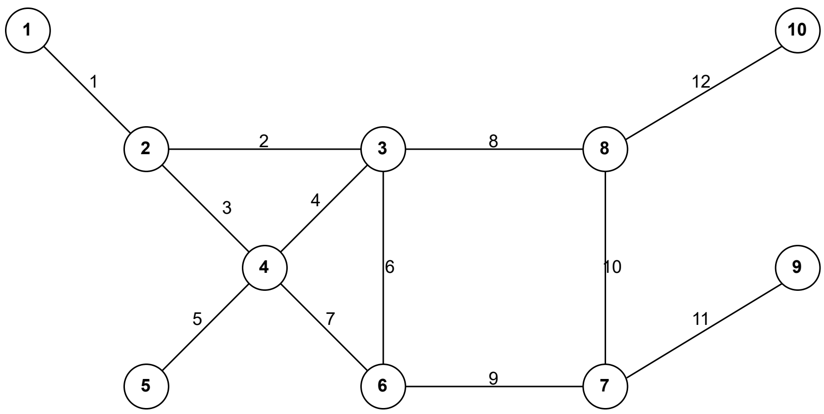

Figure 5: Example network with 10 nodes and 12 edges.

Let us consider a total of 25 alternative reinforcement actions to choose from. The first 12 of them are Type I actions each corresponding to reinforcing its corresponding edge. These reinforcement actions lower the disruption probability of the edge it corresponds to from 0.2 to 0.1 and all have a cost of 1 unit. Additionally, there are 13 Type II actions as depicted in Figure 6, which all have an estimated disruption probability of 0.3 and come with a cost of 2 units. With these 25 reinforcement actions, one could construct 2^{25} different portfolios. With the restriction of having a budget $b = 9$, this number then reduces to around 250 000 different feasible portfolios.

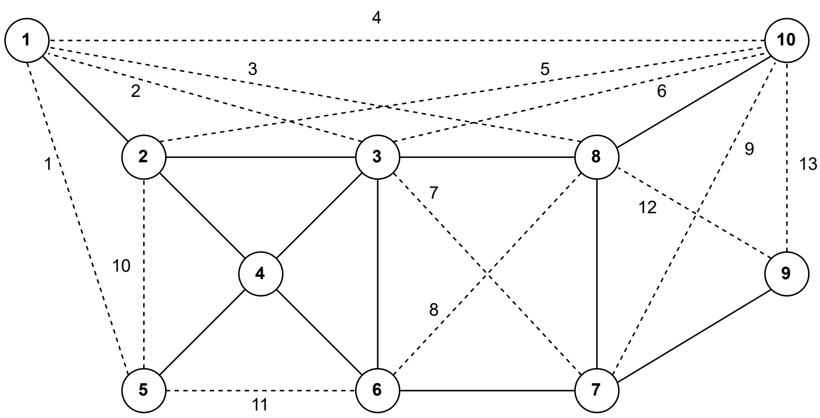

Figure 6: Possible Type II actions.

Let us also assume that the DM has not stated any preference regarding the two performance metrics defined in (2) and (3). That means that our set of feasible weights is just $S = S^0 = \{w \in \mathbb{R}^2 \mid w_1 + w_2 = 1,\ w_1, w_2 \geq 0\}$ and its corresponding set of extreme points is $S_{ext} = \{(1,0),(0,1)\}$. With the help of the methods presented in Section 3 and Algorithm 2, computing the set of cost-efficient portfolios yields 28 cost-efficient portfolios. They are numbered from 1 to 28 in ascending order about the cost of the portfolio (see Appendix A). Computing these took around 110 minutes on a modern CPU (6 cores, 12 threads, 3.7 GHz).

To get a better understanding of the results, in Figure 7 all 28 cost-efficient portfolios with varying costs are plotted. On the x-axis is the expected average number of independent passageways of the portfolio and on the y-axis the expected *global efficiency*. From it, we can see that there seems to be a positive correlation between the two expectations of the performance metrics, which was expected since both of them measure the connectivity of the network.

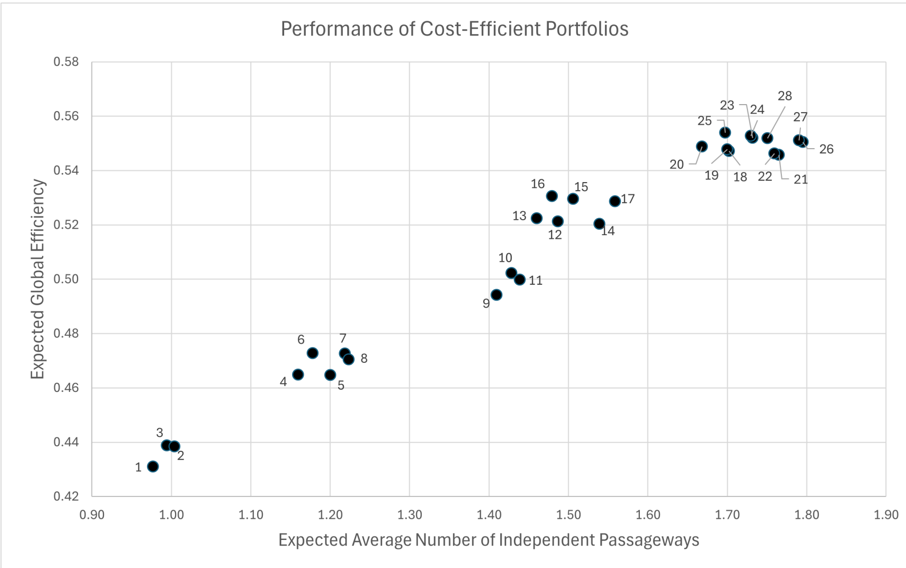

Figure 7: Performance of the cost-efficient portfolios.

Additionally in Figures 8 and 9 the expected values of the two performance metrics are plotted against the cost of a portfolio for all 28 cost-efficient portfolios. From these two figures, we can see that there is approximately a 79% increase in the expected average number of independent passageways and a 28% increase in expected *global efficiency* when comparing the expected effects of those portfolios having a cost of 9 units to the expected performances of the original network.

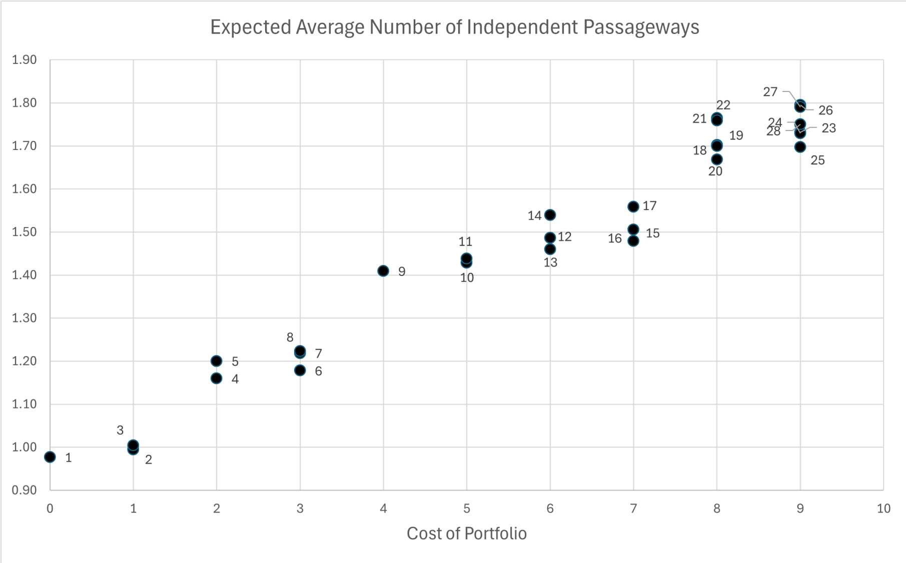

Figure 8: Expected average number of independent passageways as a function of cost.

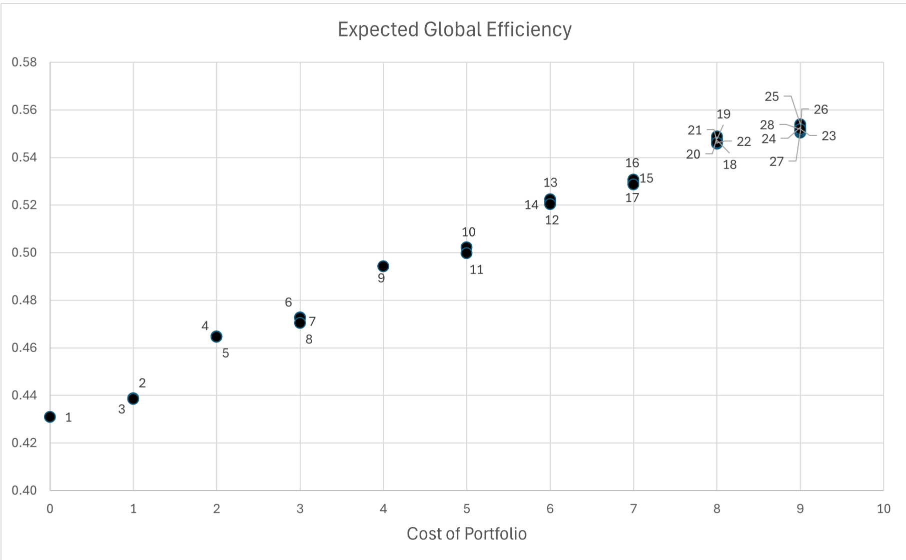

Figure 9: Expected *global efficiency* as a function of cost.

Since the budget in this example problem was 9 units, let us take a closer look at those portfolios utilizing the whole budget. In Figure 10 those six portfolios are plotted with the x-axis having the expected average number of independent passageways and on the y-axis is the expected *global efficiency*. Comparing the two extremes here portfolio 25 and 26 we see that the resulting expected *global efficiency* from implementing portfolio 25 is about 0.6% better than that of portfolio 26, but on the other hand portfolio 26's resulting expected average number of independent passageways is approximately 5.7% larger than that of portfolio 25. In the case where the DM does not value *global efficiency* as much as the average number of independent passageways of the network, they should probably favor portfolios 26 and 27 over the other portfolios, when they are seeking to spend their whole budget.

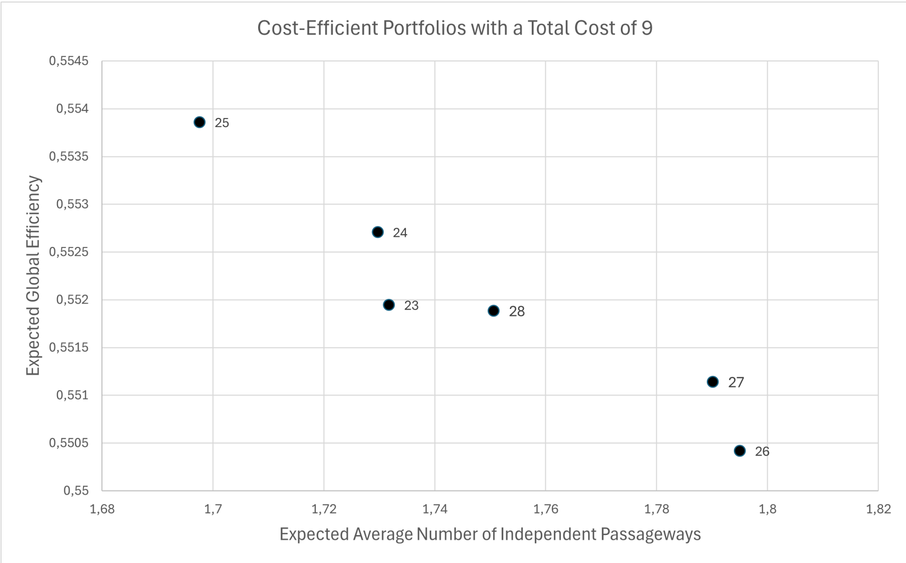

Figure 10: Cost-efficient portfolios with a total cost of 9 units.

4.2 Sensitivity Analysis

The results of Algorithm 2 are highly dependent on parameters like the disruption probabilities of the edges, the costs and effects of the reinforcement actions and the set of feasible weights representing the preferences of the DM. It is instructive to conduct some sensitivity analysis about those parameters where there is uncertainty to get a better understanding of the results and identify robust reinforcement actions. For this we can use the core index of a reinforcement action, which was introduced in Liesiö et al. (2008) and defined in (7). First, let us limit ourselves to only those cost-efficient portfolios that have a total cost equal to some level $\beta \in \mathbb{R}_+$ the DM is seeking to invest to secure the network performance $Q_{CE}^{\beta} = \{q \in Q_{CE} \mid C(q) = \beta\} \subset Q_{CE}$. Then we compute the relative share of each reinforcement action in this set of cost-efficient portfolios to get the core indexes of all the reinforcement actions.

$$CI(m) = \frac{|\{q \in Q_{CE}^{\beta} \mid q_m = 1\}|}{|Q_{CE}^{\beta}|}, \; m = 1, 2, 3, ..., r \tag{7}$$

A reinforcement action that has a core index of one indicates to us that it is in all portfolios of Q_{CE}^{β}. Based on this we can safely recommend this reinforcement action to the DM when they are looking to invest β units in reinforcing the transportation network. A reinforcement action with a core index of zero on the other hand indicates to us that that reinforcement action is not in any of the portfolios in Q_{CE}^{β} and thus we can safely discard the reinforcement action from the selection. For those reinforcement actions for which $0 < CI(m) < 1$ we can not draw such conclusions without more information about the parameters. In the case, that the DM trusts all of the parameters and we have no further information, we can recommend reinforcement action for which $0 < CI(m) < 1$, but even in this case the resulting recommendation is not guaranteed to be optimal.

One way how to make more robust recommendations is to use decision rules like *minimax regret* or *maximin*, see Greco et al. (2016) for more information. When where there is uncertainty in the parameters, we can still draw some meaningful conclusions with sensitivity analysis. Suppose that we have some real-valued uncertain parameters $(y_1, y_2, y_3, ..., y_n) \in \mathbb{R}^n$ and some confidence intervals $(I_1, I_2, I_3, ..., I_n)$ for each of them. With these, we can define the uncertainty set to be $\mathcal{D} = I_1 \times I_2 \times I_3 \times ... \times I_n$. We can then take a similar approach as we took with the weights of the two performance metrics and we can just consider the set extreme points $\mathcal{D}_{ext}$ of the uncertainty set $\mathcal{D}$ for similar reasons as with the weights in Section 3.2. Then one could conduct sensitivity analysis using this set of extreme points and the concept of core indexes to make more robust recommendations for reinforcement actions.

Let us consider a scenario, where we have uncertainty in the disruption probabilities of new edges and in the costs of them in our example problem from Section 4.1. For simplicity let us consider the two cases separately. In the first case, suppose all we know is that each new edge has a disruption probability $p_k \in [0.2, 0.4],\ k = 13, 14, 15, ..., 25$. This would result in an uncertainty set with 13 dimensions, which has 2^{13} extreme points. Computing the cost-efficient portfolios for all of these points would be very costly time-wise because computing the cost-efficient portfolios just once in Section 4.1 took almost 2 hours. To simplify the computations we assume that all new edges still have the same disruption probability. Additional simplification is to compute the cost-efficient portfolios just for four values on this interval $p_k \in \{0.2, 0.25, 0.35, 0.4\}$ to illustrate this process of sensitivity analysis. The resulting core indexes of the reinforcement actions, when $\beta = 9$ was used, are plotted together with the core indexes of the reinforcement actions from the default scenario in Figure 11. Those reinforcement actions, which have $CI(m) = 0$ are omitted from the plot.

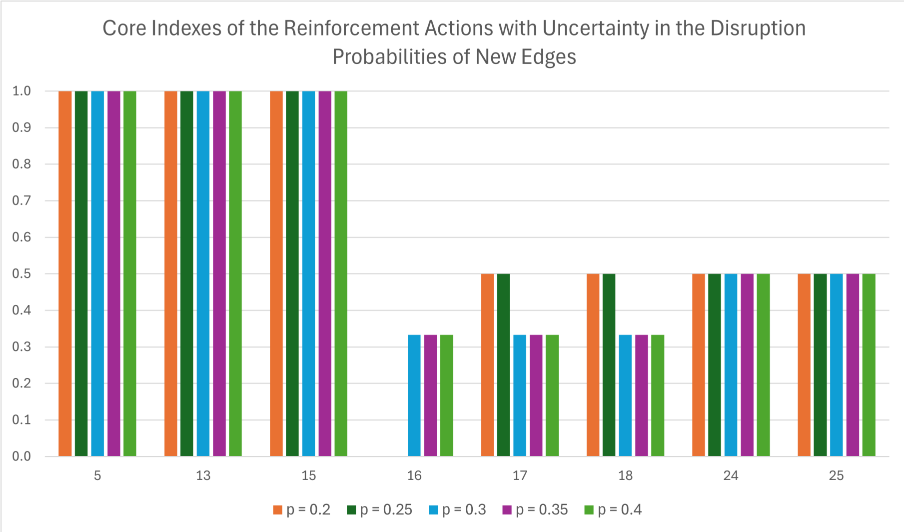

Figure 11: Core indexes with uncertainty in the disruption probabilities of new edges.

From Figure 11 we can see that there are three reinforcement actions 5, 13 and 15, which have a $CI(m) = 1$. This tells us that these three reinforcement actions are selected in every cost-efficient portfolio, which has a total cost of 9 units, in this case with varying disruption probabilities of new edges. Based on this and with the assumptions we made we can recommend these three reinforcement actions to the DM when they seek to use the full budget of 9 units. If one wishes to be even more confident about a recommendation like this, one would have to conduct more sensitivity analysis for example using the uncertainty sets discussed earlier.

We can safely discard the reinforcement actions, which have a $CI(m) = 0$ because they are not present in any of the cost-efficient portfolios with a total cost of 9. That leaves us to consider the reinforcement actions 16, 17, 18, 24 and 25. We can not draw such conclusions about these as we can for the three reinforcement actions with $CI(m) = 1$. The choice for the reinforcement actions selected to use the rest of the budget depends on the DMs preferences, but we can for example recommend reinforcement actions 24 and 25 over the other ones in the case there is no further information or preferences from the DM. This is due to the case that they are more reliable when we know that there is uncertainty in the disruption probabilities of new edges.

For the second case, we consider uncertainty in the costs of adding new edges. Again for simplicity, we assume that all new edges have the same cost. In this case, we only know that adding a new edge has a cost $c_k \in [1.5, 2.5]$, $k = 13, 14, 15, ..., 25$. Computing the cost-efficient portfolios with just four different values $c_k \in \{1.5, 1.75, 2.25, 2.5\}$ yields us the results presented in Figure 12. The results are quite similar to the ones in Figure 11 and we can draw similar conclusions from it. What we can say is that these results do not seem to vary as much as in the first case, when there was uncertainty in the disruption probabilities of new edges. The core indexes are constant across all five choices of costs for the new edges.

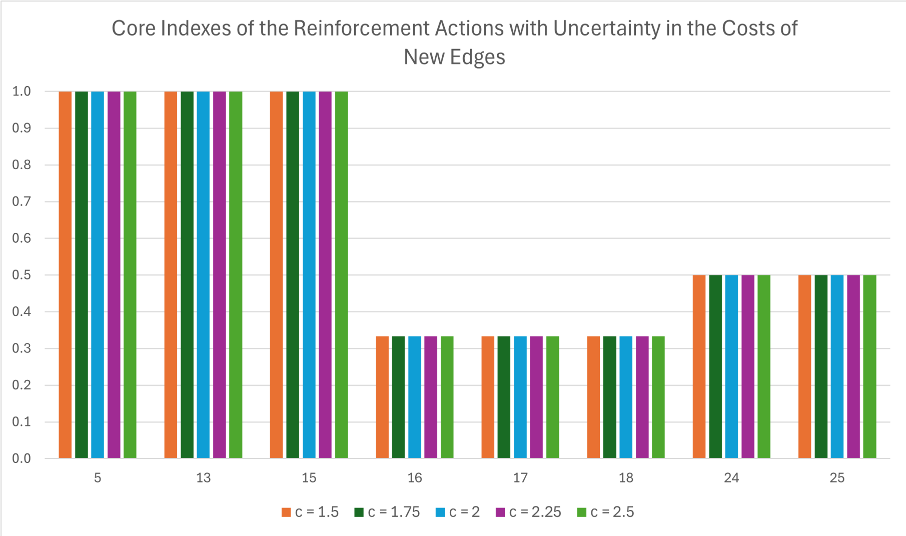

Figure 12: Core indexes with uncertainty in the costs of Type II actions.

Based on the sensitivity analysis done on the two uncertain parameters, in this example, we would recommend the DM to choose the reinforcement actions 5, 13 and 15 since they have a $CI(m) = 1$ in both cases. The rest of the budget should be spent on 16, 17, 18, 24 or 25 depending on the preferences of the DM. They should not consider other reinforcement actions not listed here since they have a $CI(m) = 0$ and are therefore not selected in any of the cost-efficient portfolios with a total cost of 9 units.

This kind of sensitivity analysis should be conducted more extensively when facing uncertainty in the parameters and wanting to make more robust recommendations for reinforcement actions. In this example, it should have been done with the help of the resulting uncertainty set $\mathcal{D}$ constructed on both the uncertain disruption probabilities and costs of new edges.

5 Summary

This thesis introduces an application of the framework from Kangaspunta and Salo (2014) by considering two types of reinforcement actions: reinforcing existing edges and adding new edges to the network. The framework allows the identification of cost-efficient portfolios of reinforcement actions on transportation networks, where the transportation network's edges may be disrupted due to deterioration of components, extreme weather conditions, natural disasters or other external hazards like terrorist attacks.

Additionally, we explore a method for sensitivity analysis to make robust recommendations of reinforcement actions when faced with uncertainty in the parameters. This method is useful for identifying robust reinforcement actions with uncertainty in the parameters, but it is computationally expensive because it requires the main algorithm used in this thesis to be run multiple times. This is because the algorithm itself already proved to be computationally expensive even for relatively small networks having around 10 nodes and 12 edges.

To further improve this framework one could explore alternate ways for determining the expected utility for the performance of a network in a faster way. The expected utility used in this thesis proved to be computationally expensive as its complexity was exponential in the number of edges in the network. What could be done with a completely alternative way of evaluating performance, for example adapting the resilience of a transportation network defined in Ip and Wang (2011). Another approach would be to speed up the computation of the expected utility by reducing the size of the state space by rare-event approximation or by fixing some edges to be always operational.

The second aspect one could explore is the possibility of getting rid of the assumption of uncorrelated edge disruptions. This could be done for example with the aid of Bayesian networks to model the correlated disruption probabilities. A Bayesian network could then be used to compute the probabilities of the possible states the network can be in and would replace the presented method in (1) for computing the probability of a state. This would allow us to model more accurately threats, which affect multiple edges at the same time like large-scale natural disasters and terrorist attacks. Another problem one could dive deeper into is the elicitation of the weights given to the performance metrics. This could be done with many well-known methods discussed in Morton (2018).

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A Cost-Efficient Portfolios for the Example Network

In Table A1 the resulting cost-efficient portfolios of the example problem from Section 4.1 are presented. They are numbered in ascending order about their cost from 1 to 28. Also, the portfolios themselves, their costs and their respective effects are presented in the table. Their effects being their expected average number of independent passageways $\mathbb{E}[v_1(x)]$ and their expected *global efficiency* $\mathbb{E}[v_2(x)]$ of the resulting networks when the corresponding portfolio has been applied.

Table A1: Cost-efficient portfolios.

Index	Portfolio	Cost	$\mathbb{E}[v_1(x)]$	$\mathbb{E}[v_2(x)]$
1	[0000000000000000000000000]	0	0.9771	0.4310
2	[0000100000000000000000000]	1	0.9946	0.4388
3	[0000000100000000000000000]	1	1.0041	0.4384
4	[0000000000000010000000000]	2	1.1598	0.4648
5	[0000000000000001000000000]	2	1.2004	0.4647
6	[0000100000000010000000000]	3	1.1781	0.4727
7	[0000100000000001000000000]	3	1.2188	0.4726
8	[0000000100000001000000000]	3	1.2236	0.4705
9	[0000000000000001000000001]	4	1.4094	0.4942
10	[0000100000000001000000001]	5	1.4284	0.5022
11	[0000000100000001000000001]	5	1.4387	0.4997
12	[0000000000000010100000010]	6	1.4868	0.5212
13	[0000000000000010010000010]	6	1.4601	0.5224
14	[0000000000000010100000001]	6	1.5393	0.5203
15	[0000100000000010100000010]	7	1.5060	0.5295
16	[0000100000000010010000010]	7	1.4792	0.5306
17	[0000100000000010100000001]	7	1.5586	0.5286
18	[0000000000001011000000010]	8	1.7021	0.5472
19	[0000000000001010100000010]	8	1.7000	0.5478
20	[0000000000001010010000010]	8	1.6684	0.5489
21	[0000000000001011000000001]	8	1.7649	0.5457
22	[0000000000001010100000001]	8	1.7596	0.5462
23	[0000100000001011000000010]	9	1.7318	0.5519
24	[0000100000001010100000010]	9	1.7297	0.5527
25	[0000100000001010010000010]	9	1.6976	0.5539
26	[0000100000001011000000001]	9	1.7951	0.5504
27	[0000100000001010100000001]	9	1.7902	0.5511
28	[0000100000001010010000001]	9	1.7507	0.5519