A Decision Model for Electricity Contract Selection Under Uncertainty

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Bachelor's thesis
Espoo 17.1.2024

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Abstract of the bachelor's thesis

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Title A Decision Model for Electricity Contract Selection Under Uncertainty	
Degree programme Bachelor's Programme in Science and Technology	
Major Mathematics and Systems Sciences	**Code of major** SCI3029
Teacher in charge Prof. Ahti Salo	
Advisors MSc (Tech) Leevi Olander,	
Date 17.1.2024 **Number of pages** 22	**Language** English

Abstract

Electricity markets have significantly changed over the recent years. These changes have led to volatile spot markets, which has drawn attention to risk management practices, and how they can be further developed to tackle these new challenges. Consumers have also been heavily influenced by the aforementioned developments, but research on consumers in the energy markets is scarce.

This thesis applies the concept of stochastic dominance to a risk-averse consumer in the Finnish retail electricity markets. The implemented four period decision model considers the uncertainty of the electricity spot price and determines a set of optimal electricity contracts under the assumed risk preferences that minimize the costs associated with these contracts. The uncertainty arising from the electricity spot price is modeled using a binomial lattice. Once the optimal contracts are identified, the results are compared to other decision alternatives.

According to the results, the use of stochastic dominance in choosing electricity contracts proved worthwhile. The model provided clear results that could easily be incorporated into practical decision making. The main result showed that mean-variance optimization did not fully represent the preferences of a risk averse decision maker. Another notable result was that some contract alternatives offered to customers were never preferred by risk averse decision makers.

Keywords Stochastic dominance, decision making, uncertainty, portfolio selection, energy markets

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Tekniikan kandidaatintyön tiivistelmä

Tekijä Ahti Korhonen	
Työn nimi A Decision Model for Electricity Contract Selection Under Uncertainty	
Koulutusohjelma Teknillistieteellinen kandidaattiohjelma	
Pääaine Matematiikka ja systeemitieteet	**Pääaineen koodi** SCI3029
Vastuuopettaja Prof. Ahti Salo	
Työn ohjaaja DI Leevi Olander	
Päivämäärä 17.1.2024 **Sivumäärä** 22	**Kieli** Englanti

Tiivistelmä

Sähkömarkkinoilla tapahtuneiden muutosten takia sähkön spot-hinnat ovat aiempaa huomattavasti epävakaampia, ja siksi riskienhallintakäytäntöihin ja niiden kehittämiseen on suunnattu huomiota. Kuluttajat ovat myös kokeneet nämä muutokset, mutta kuluttajia koskeva tutkimus energiamarkkinoilla on vähäistä.

Tämä tutkimus soveltaa stokastista dominanssia viitekehyksenä riskejä välttävään kuluttajaan Suomen energia markkinoilla. Tutkimuksessa rakennettu neljäperiodinen päätösmalli ottaa huomioon sähkön spot-hinnasta johtuvat epävarmuustekijät ja määrittää päätöksentekijän oletettujen preferenssien mukaan optimaaliset sopimusvaihtoehdot, jotka minimoivat sähkön hankintakustannukset. Sähkön spot-hinnasta johtuvaa epävarmuutta mallinnetaan binomihilan avulla. Kun optimaaliset sopimusvaihtoehdot on identifiotu, tuloksia vertaillaan muihin päätösvaihtoehtoihin.

Tulosten valossa stokastisen dominanssin soveltaminen sähkösopimusten valintaan osoittautui hyödylliseksi, sillä tulokset olivat selkeitä ja niitä pystyttiin hyödyntämään suoraan päätöksentekoon. Päätulos oli sen osoittaminen, että keskiarvo-varianssi optimointi ei pysty täysin kuvaamaan riskejä karttavan päätöksentekijän preferenssejä. Toinen merkittävä tulos oli, että riskejä karttava pätöksentekijä ei koskaan preferoisi tiettyjä kuluttajille tarjottuja sopimustyyppejä.

Avainsanat Stokastinen dominanssi, päätöksenteko, epävarmuus, portfolio valinta, energiamarkkinat

Contents

1 Introduction

Electricity markets are uncertain due to deregulation, renewable energy sources, technological developments, the energy crisis in Europe as well as other factors. These dynamics have led to a volatile spot market in electricity prices, for example, the spot price of electricity increased at most by 150% in 2022 compared to the average price. With these facts in mind, there is a need for risk management tools for hedging these uncertainties (Canelas et al., 2020). Most research in the field of energy markets regarding portfolio optimization has been conducted from the retailer's point of view (for example, see Gökgöz and Atmaca 2012), and the research from the consumer's point of view, especially in the Nordic electricity markets, is scarce. This thesis studies the electricity contract selection problem with the aim of minimizing the costs associated with these contracts by utilizing portfolio optimization and stochastic dominance.

Portfolio optimization is often a multi-objective decision making problem where the trade-offs between risk and return are balanced. Multi-objective decision making can be challenging in that the importance of the objectives can be subjective, meaning that different decision makers or stakeholders may have varying opinions and weightings for the criteria, which can introduce complexities in reaching a consensus. Multi-objective decision making problems often have a set of Pareto optimal solutions instead of one solution. In the set of Pareto optimal solutions, each solution is non-dominated meaning that no objective can be improved without affecting other objectives negatively (Petchrompo and Parlikad, 2019).

Stochastic dominance as a concept in decision theory and risk analysis helps compare and rank random variables based on their probability distributions over outcomes. Stochastic dominance can be used in portfolio selection without knowing the decision maker's risk preferences exactly (Cheong et al., 2007). Incorporating risk preferences in optimization formulations is a viable alternative to using risk-neutral preferences since usually, decision makers are not risk-neutral (Yau et al., 2011).

The main objective of this thesis is to develop a multiperiod decision making model for the selection of optimal electricity contracts from a risk-averse consumers point of view based on the concept of stochastic dominance. We apply this approach to a case study of a consumer in the Finnish energy market, where the decision maker has the possibility to switch contracts every six months. The spot price of electricity is modeled as a binomial tree in which one period is one month and the forecast spans the duration of two years. The decision alternatives are actual electricity contracts offered to consumers in Finland.

2 Background

2.1 Decision making under uncertainty

2.1.1 Portfolio selection

Portfolio selection, or portfolio optimization, is a topic that is well-studied in financial literature but with applications in other domains as well. The background for portfolio optimization originates from Markowitz (1959). In Markowitz's approach to portfolio optimization, also known as mean-variance optimization, investment portfolios are ranked according to two criteria, mean and variance. The variance of a portfolio models the risk whereas the mean of the portfolio returns represents the investment's expected return. Therefore it is rational that the decision maker seeks a portfolio that maximizes their expected return and minimizes risk.

A portfolio is said to be non-dominated when there exists no other portfolio with higher or equal expected returns and less risk. Although variance is a widely accepted risk measure, it has its drawbacks. Variance, which is a symmetric measure, treats upward movements as being equally significant as downward movements. Markowitz's model is also incapable of considering the risk of low probability events as it assumes normally distributed returns (Giorgi, 2005). If portfolio returns, or costs in our case, do not follow a normal distribution, alternative ways of expressing riskiness are needed, for example, stochastic dominance (Leippold, 2015).

Stochastic dominance was introduced in the 1960s by Quirk and Saposnik (1962).

They showed its connection to utility functions and considered first order stochastic dominance for risk-neutral decision makers. Second order stochastic dominance is often considered a relevant choice criterion in portfolio selection since a risk-averse decision maker would prefer a second order stochastically non-dominated solution, which is useful in financial economics (Roman and Mitra, 2009). In practice, stochastic dominance has not been considered relevant in portfolio selection due to its computational complexity until recently (Bawa et al., 1979). To date, some models applying second order stochastic dominance have been developed, for example in R&D portfolio selection (Ringuest et al., 2000) and financial portfolio optimization (Dentcheva and Ruszczynski, 2003).

2.1.2 Stochastic dominance

The idea of stochastic dominance is to split the set of possible decision alternatives into groups of dominated and non-dominated sets (Levy, 2016). A decision alternative $x \in X$ represents a random variable with a known or estimated probability distribution, for example, the returns of a stock portfolio. The cumulative density function $q_x(\theta)$ of decision alternative x is defined as

$$q_x(\theta) \;=\; \mathbb{P}(x \leq \theta),$$

where $\theta \in \Theta$ represents all the possible outcomes of decision alternative x.

The concept of first order stochastic dominance for minimization problems is defined as follows. Alternative x^* is first order stochastically non-dominated, and thus belongs to the non-dominated set X^* if there exists no alternative $x' \in X$ such that

$$q_{x'}(\theta) \;\geq\; q_{x^*}(\theta)$$

for all $\theta \in \Theta$ with at least one strict inequality.

Figure 1 is an example of cumulative density functions $F1$ and $F2$ for two random variables. Using the definition of first order stochastic dominance, it can be seen that neither distribution dominates the other, and thus both are in the non-dominated set X^*. Note that the dominated set may include one distribution that dominates some other, or even all other distributions. No rational decision maker would choose alternatives belonging to the dominated set.

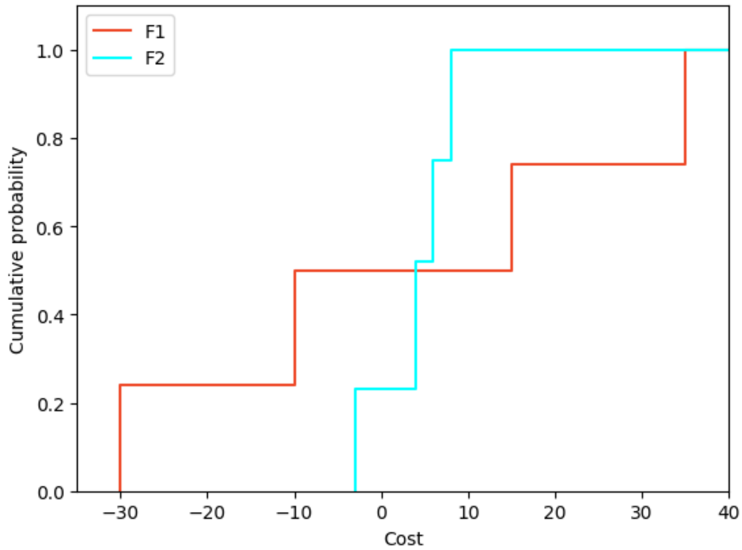

Figure 1: Cumulative density functions of two random variables.

2.1.3 Second order stochastic dominance

The main benefit of second order stochastic dominance is that any risk-averse decision maker would prefer one of the non-dominated alternatives. Risk-aversion means that a decision maker prefers a more certain outcome over a less certain one, even if the expected outcome of the latter is equal to or higher than the more certain outcome (Werner, 2008). The practical difference between first and second order stochastic dominance is that instead of comparing cumulative density functions (CDFs) to each other, cumulative CDFs of the decision alternatives are now compared with each other to determine which alternatives are non-dominated. The definition of second order stochastic dominance is that alternative x^* is second order stochastically non-dominated if there exists no alternative x' such that

$$q^{(2)}_{x'}(\theta) \;\geq\; q^{(2)}_{x^*}(\theta), \tag{1}$$

for all $\theta \in \Theta$ with at least one strict inequality. The definition of $q_x^{(2)}(\theta)$ for decision alternative x is

$$q_x^{(2)}(\theta) \;=\; \int_{-\infty}^{\theta} q_x(t)\, dt.$$

2.2 Scenario modeling

In decision modeling, there is often a need to represent uncertainties so that computational methods can be used to solve them efficiently. Scenario generation is a set of techniques used to create, for example, distributions that represent future developments (Bernaschi et al., 2007). In scenario generation, the idea is to generate a finite set of realistic possible scenarios, for example, in this thesis a collection of realized states for electricity prices will be approximated in the form of a binomial lattice.

2.2.1 Binomial lattice and volatility parameter estimation

To generate scenarios for the electricity price, we apply the model proposed by Cox et al. (1979), which is used, for example, in option pricing (Tian, 1999), pricing of corporate liabilities (Broadie and Kaya, 2007) and valuing IPO's (Kelly, 1998). The model presents a methodology for creating a simple discrete-time model for valuing options. To price options, a binomial lattice for the underlying asset is generated using historical data of the asset's price. From historical data, values for the u and d coefficients, as well as the probability p for an upward movement in the lattice can be approximated. The value of the random variable at each node of the lattice is given by multiplying the previous value with either the u or d parameter.

The parameters of the lattice are chosen to match the expected growth rate ν and the variance of the logarithmic price process σ^2 over a chosen time period. These parameters are defined as

$$\sigma_p^2 = \frac{1}{N-1}\sum_{k=0}^{N-1}[ln\frac{S(k+1)}{S(k)} - \nu_p]^2$$

$$\nu_p = \frac{1}{N} ln \frac{S(N)}{S(0)}$$

Because both expectation and variance are additive in terms of time the annual parameters can be calculated by dividing the periodic parameters by the length of the period. The parameters u, d and p of the lattice are thus given by

$$p = \frac{1}{2} + \frac{1/2}{\sqrt{\sigma^2/(\nu^2 \Delta t) + 1}}$$

$$u = e^{\sigma\sqrt{\Delta t}}$$

$$d = e^{-\sigma\sqrt{\Delta t}}.$$

Figure 2 shows a four-period binomial lattice in which the path denoted with red arrows is one possible scenario. The paths in a binomial lattice are recombining and each node has two possible paths. The starting price represents the current price, which is known. The probability of each outcome is given by the combined probability of upward and downward movements where the movements are assumed to be independent of each other. The probability of a single upward movement is p and the probability of a downward movement is $p - 1$.

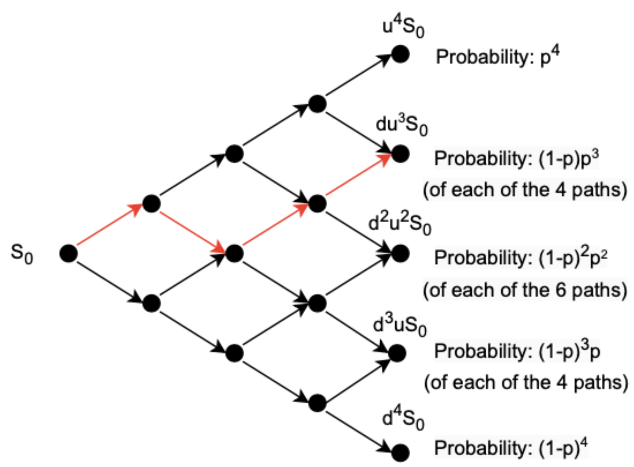

Figure 2: Example of a binomial lattice with four periods.

The binomial lattice may seem relatively simple to model continuous distributions. However, by shortening the period length Δt more accurate distributions can be obtained since there will be a larger number of values after several short steps (Luenberger, 1998).

2.3 Data

The data used in this thesis includes historical spot price data of electricity prices, studio apartments yearly electricity consumption on a monthly level and pricing data of electricity contracts for consumers offered by the largest retailers on the Finnish markets. The Electricity spot price data is from the European network of transmission system operators for electricity (ENTSO-E, 2023). The spot price data has 75215 points. Each point represents the hourly spot price in the Finnish market starting from the 1st of January 2015 until the 31st of July 2023. The spot price data is aggregated into monthly averages. Figure 3 shows the yearly electricity consumption of a studio apartment on a monthly level (Thermopolis, 2023) and Figure 4 illustrates the aggregated monthly electricity spot prices. Note the large spike in the electricity prices in 2022 caused by Russia's invasion of Ukraine and the resulting energy crisis, meaning that the data may not be fully representative of future electricity prices.

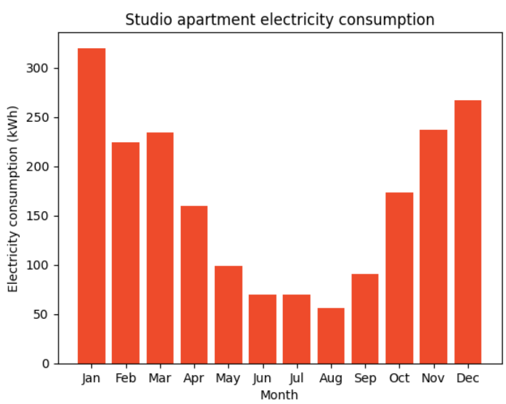

Figure 3: Yearly electricity consumption of a studio apartment.

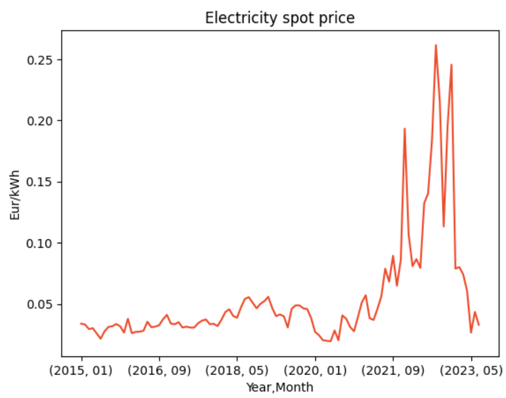

Figure 4: Electricity spot price in Finland between 2015 and 2023.

Table 1 describes the electricity contracts that are the decision alternatives in the model. The contract data is taken from Finnish electricity retailers and thus represents actual decision alternatives.

#	Name	Description	Spot	Margin	Base
1	Fortum Kesto	per consumption indefinite length	no	15,98 c/kWh	4,02 €/month
2	Lumme XS	fixed price indefinite length 1750 kWh/year	no	0 c/kWh	19 €/month
3	Helen Fiksu-sähkö	per consumption 12 month - fixed length	no	9,49 c/kWh	3,99 €/month
4	Helen Pörssisähkö	spot price indefinite length	yes	0,38 c/kWh	3,93 €/month

Table 1: Contracts for the decision model.

3 Case study of electricity contract selection

This section presents a multiperiodic decision model for minimizing the costs of an electricity contract strategy. At the beginning of each six-month time period the decision maker will have an option of selecting a new contract type. The planning horizon length is two years so the model consists of four decision periods. The uncertainty in this model arises from the unknown future electricity spot prices. The decision maker is assumed to be risk averse, so the second order stochastically non-dominated set of all the possible strategies contains cost-minimizing solutions that the decision maker would prefer.

3.1 Electricity price forecast and scenario generation

To model the uncertainty in future electricity spot prices, a binomial lattice with twenty-four periods is approximated from past prices using the model proposed by Cox et al. (1979). The approximated lattice contains prices ranging from 0.556 cents to 44.931 cents per kWh. The prices in the final period are very close to normally distributed as the p parameter, which signals the probability of an upward move, equals 0.4999. The other parameter values are $u = 1.0958$, $d = 0.9126$, $\sigma^2 = 0.1004$ and $\nu = -0.0310$.

Once the binomial lattice has been populated with possible spot prices, all possible scenarios can be obtained by traversing the lattice starting from the initial spot price and exploring all the possible paths that the electricity price might take. The probability of each scenario is given by the binomial formula $\mathbb{P}(s) = p^n(1-p)^{24-n}$, where n is the number of upward movements in scenario s. The number of scenarios obtained is $2^{24} = 16777216$.

3.2 Problem formulation

To model the multiperiodic decision problem, each period has four decision variables $x_{t,i} \in \{0, 1\}$, where $t \in \{1, 2, 3, 4\}$ denotes one of the six month decision periods in the two year time horizon and $i \in \{1, 2, 3, 4\}$ denotes one of the four possible contract alternatives shown in Table 1. The timeline of the decision making model is illustrated in Figure 5.

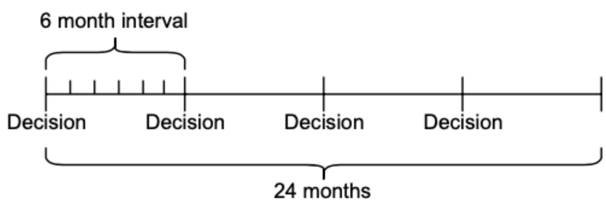

Figure 5: Timeline of the decision making model.

A strategy is denoted by $Z = (x_{1,1}, ..., x_{1,4}, ..., x_{4,1}, ..., x_{4,4})$. In each six-month period only one contract type can be chosen meaning that $\sum_{i=1}^{4} x_{t,i} = 1,\ \forall t$. If the 12-month fixed length contract is chosen, the same contract type must also be chosen in the next six-month period meaning that $x_{t,3} \geq x_{t-1,3} - x_{t-2,3}$, where $x_{t,3}$ for $t < 1$ is zero.

To solve the second order stochastically non-dominated strategies, a cumulative cost distribution is associated with all strategies. The cost of a strategy in one month m depends on the chosen contract type and the scenario s. The cost equations for the contracts are

$$C_1(m) = M_1 k_m + B_1$$

$$C_2(m) = B_2$$

$$C_3(m) = M_3 k_m + B_3$$

$$C_4(m, s) = e_{s,m} k_m + M_4 k_m + B_4,$$

where M is the margin of a given contract, k_m is the consumption for month m, $e_{s,m}$ is the monthly spot price in scenario s and B_i is the base cost of contract i. The contracts are indexed as in Table 1. Using the monthly cost equations, the cost of a strategy in one scenario $s \in S$ is given by

$$C(s) = x_{1,1}[C_1(1, s) + C_1(2, s) + C_1(3, s) + C_1(4, s) + C_1(5, s) + C_1(6, s)] +$$

$$\dots$$

$$x_{1,4}[C_4(1, s) + C_4(2, s) + C_4(3, s) + C_4(4, s) + C_4(5, s) + C_4(6, s)] +$$

$$\dots$$

$$x_{4,1}[C_1(19, s) + C_1(20, s) + C_1(21, s) + C_1(22, s) + C_1(23, s) + C_1(24, s)] +$$

$$\dots$$

$$x_{4,4}[C_4(19, s) + C_4(20, s) + C_4(21, s) + C_4(22, s) + C_4(23, s) + C_4(24, s)],$$

where each row in the sum is the cost of one period for a given contract alternative $Z = (x_{1,1}, x_{1,2}, ..., x_{4,4})$. The sum can be written as

$$C(s) = \sum_{t=1}^{4} \sum_{i=1}^{4} x_{t,i} \sum_{m=(t-1)\cdot 6+1}^{6 \cdot t} C_i(m, s)$$

note the indexing of the m variable which is a translation from 6-month decision periods to 1-month periods. Given the scenario costs, the cumulative cost distribution of one strategy can be obtained from

$$q(\theta) = \sum_{s \in S} \mathbb{I}(C(s) \leq \theta) \cdot \mathbb{P}(s), \ \forall \theta.$$

Here $\mathbb{I} : \mathbb{R} \mapsto \{0, 1\}$ denotes a function indicating whether a given scenario cost is below or equal to the threshold value θ and $\mathbb{P}(s)$ is the probability of scenario s realizing. After calculating cumulative distributions for all possible strategies, the second order stochastically non-dominated set can be found by exhaustively enumerating all possible pairs and comparing them as in (1). In practice, the cumulative CDFs were determined by dividing the costs of a strategy into buckets summing the probabilities in each bucket and finally cumulating the probabilities twice.

3.3 Results

The initial number of possible strategies is 2^{16}, of which 145 are feasible for the given constraints. Since the number of possible strategies was so low, the method of exhaustive enumeration and pairwise second order stochastic dominance comparison could be applied efficiently in 0.6 seconds on a 2020 MacBook Pro with a 2,3 GHz Intel core i7 processor. The Python version used in the modeling was 3.10.13

Table 2 shows all second order stochastically non-dominated strategies. The number in each period denotes the index of the selected contract alternative from Table 1. As can be seen, contract alternative 4 is selected in all decision periods except one. This is caused by the relatively low spot price prediction compared to the costs of the other contract alternatives. Since electricity is the only source of uncertainty, strategies that do not include contract alternative 4 have a standard deviation of 0. Thus among the strategies that do not include contract alternative 4 the one with the lowest cost dominates all others. A future consideration could be adding, for example, electricity consumption as a source of uncertainty. This would make it possible for strategies not containing contract alternative 4 to have a non-zero standard deviation thus providing more interesting results.

The cumulative probability distributions of the second order stochastically non-dominated strategies are in Figure 6. The result is that according to the model, a risk-averse decision maker, regardless of their utility function, would not select contract alternatives 1 or 3 in any decision period

	Contract in period 1	Contract in period 2	Contract in period 3	Contract in period 4	Mean (€)	Std (€)
SSD-Strategy 1	4	2	4	4	414.988	37.202
SSD-Strategy 2	4	4	4	4	427.753	52.598

Table 2: Second order stochastically non-dominated strategies.

For illustration, Figure 7 presents the cumulative probability distributions for one non-dominated strategy and one dominated strategy. In the non-dominated strategy, denoted with red color, contracts 4, 2, 4 and 4 are selected. In the dominated strategy, denoted with light blue, contract alternatives 1, 1, 2 and 4 are chosen. Figure 7 shows that the density function of the non-dominated strategy is greater or equal for all cost values compared to the density function of the dominated strategy, meaning that the costs of the non-dominated strategy are concentrated towards the lower end of the cost axis and thus dominate the other strategy.

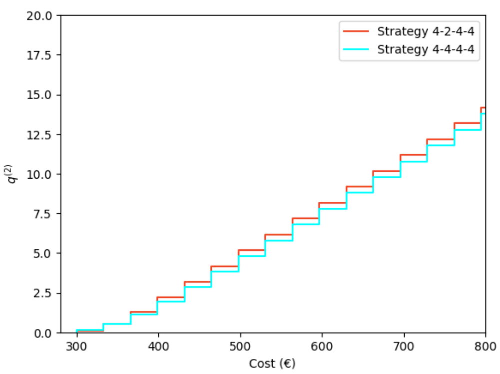

Figure 6: $q^{(2)}$-distributions for both non-dominated strategies.

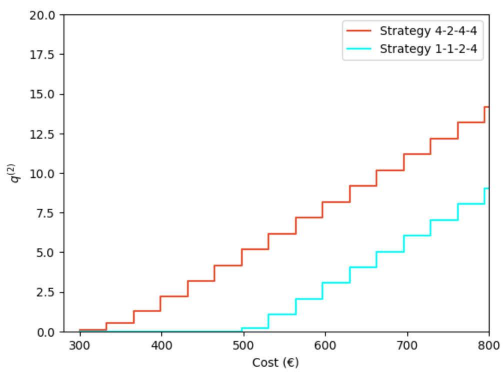

Figure 7: $q^{(2)}$-distributions for one non-dominated strategy and one dominated strategy.

Table 3 shows the strategies that are mean-variance non-dominated with respect to these two criteria

$$min.\ \mu_Z = \sum_{s \in S} \mathbb{P}(s) C(s),$$

$$min.\ \sigma_Z^2 = \sum_{s \in S} \mathbb{P}(s)(C(s) - \mu_Z)^2,$$

where μ_Z is the mean cost of strategy Z and σ_Z^2 is the variance of the costs of strategy Z. There is an overlap between the second order stochastically non-dominated set and mean-variance non-dominated strategies suggesting that there is consistency between the two approaches. However, the second order stochastically non-dominated set also includes strategies that are not included in the mean-variance non-dominated set. This shows that second order stochastic dominance is better at expressing the preferences of a risk-averse decision maker compared to mean-variance optimization. The reason why this comparison is of interest is that mean-variance analysis and expected utility are the two main branches of portfolio selection. The two approaches could also be used jointly by choosing strategies that are both mean-variance and second order stochastically non-dominated.

	Contract in period 1	Contract in period 2	Contract in period 3	Contract in period 4	Mean (€)	Std (€)
MV-Strategy 1	2	2	3	3	435.312	0
MV-Strategy 2	4	2	3	3	420.708	5.428
MV-Strategy 3	4	2	4	3	417.145	19.307
MV-Strategy 4	4	2	4	4	414.988	37.202

Table 3: Mean-variance non-dominated strategies.

4 Conclusions

This thesis studies the selection of electricity contracts from a risk-averse consumer's point of view. The decision making model applies second order stochastic dominance with the aim of minimizing the costs related to the contracts using a multiperiod approach. The uncertainty in the model arises from the future electricity spot prices, which are approximated with a binomial lattice.

Studying stochastic dominance in the context of electricity contract selection proved useful as it indicated results that could easily be used in decision making. The main result was the difference between the non-dominated strategies when using stochastic dominance and the mean-variance approaches. This showed that the mean-variance approach does not fully represent the preferences of risk-averse decision maker. Another notable result was that certain types of contract alternatives were never preferred by a risk-averse decision maker.

The stochastic dominance approach was applied to only a small number of contract choices and the model makes strong assumptions about the behavior of electricity prices such as constant volatility and growth rate. Also discounting of costs was not incorporated into the model, it is however quite relevant in financial settings and could provide more accurate results. Due to these factors, the results should be interpreted with caution.

The energy markets in Europe have changed due to Russia's invasion of Ukraine, so a future research subject could be applying the model to electricity price data collected after February 2022. Other improvements to the model could be made in the scenario modeling part. The binomial lattice does not account for seasonality, which has a strong effect on electricity prices, or changing volatility. In the binomial lattice, the price increases and decreases are constant compared to the sharp spikes and changing volatility in real electricity spot prices. Customer behavior is also a topic that is relevant in the context of energy markets. Sharp spikes in electricity prices tend to lead to noticeable demand responses by consumers.

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