Combining SDDP and novel formulations for solving multi-stage capacity expansion problems

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Abstract

This thesis examines how the number of investment stages and problem formulation influence solutions to multi-stage energy capacity expansion problems. Three formulations of an energy capacity expansion problem are given in terms of stochastic dual dynamic programming, each aiming to isolate one type of modeling error. The formulations and the influence of the number of investment stages on their solutions are then compared. It is found that accurate modeling of operating points, as well as the ability to create capacity-building plans, are the most important factors influencing the susceptibility of solutions to the number of stages. The realization of uncertainty is found to have a small influence on the quality of solutions.

Keywords Stochastic Dual Dynamic Programming, Capacity Expansion Planning, Julia

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Tiivistelmä

Tämä opinnäytetyö tutkii, miten päätösvaiheiden määrä ja ongelman muotoilu vaikuttavat ratkaisun laatuun monivaiheisissa energiakapasiteetin laajentamisongelmissa. Kolmea energiakapasiteetin laajentamisongelman muotoilua tarkastellaan, joista jokainen pyrkii eristämään yhdentyyppiset mallinnusvirheet. Päätöskohtien määrään vaikutusta eri muotoilujen ratkaisujen laatuun verrataan. Löydetään että operaatiopisteiden tarkka mallinnus sekä kyky luoda kapasiteetin laajentamissuunnitelmia ovat tärkeimmät tekijät jotka vaikuttavat ratkaisujen laatuun päätösvaiheiden määrän ollessa vakio. Stokastisuuden paljastumisella havaittiin olevan vähäinen vaikutus ratkaisujen laatuun, noin 4% edellämainittunen tekijöiden kokonaisvaikutuksesta.

Avainsanat Mallinnus,kapasiteetin laajennussuunnittelu, Julia

Contents

Abstract 3

Abstract (in Finnish) 4

Contents 5

1 Introduction 6

2 Background 7
- 2.1 Capacity expansion formulation . . . 7
  - 2.1.1 Current formulation . . . 7
  - 2.1.2 Solution methods . . . 9
- 2.2 SDDP . . . 10
  - 2.2.1 SDDP in energy planning . . . 10
  - 2.2.2 Basics of SDDP . . . 10

3 Model formulation 12
- 3.1 Model 1 . . . 12
- 3.2 Model 2 . . . 14
- 3.3 Model 3 . . . 16

4 Case study 18
- 4.1 Input data . . . 18
- 4.2 Uncertainty characterization . . . 19
- 4.3 Hypotheses . . . 23

5 Results 23
- 5.1 Hyper-parameters and convergence . . . 23
- 5.2 Model comparison . . . 23
- 5.3 Limitations . . . 25

6 Conclusions 25

7 Appendix 28
- 7.1 Notation . . . 28

1 Introduction

Capacity expansion problems aim to solve the optimal financial and operational decisions needed for expanding the production capacity of some resource, such as energy grids to meet the demands of the inhabitants or the goods production capacity of factories to meet quotas. As such, tools for solving these problems are of crucial importance both for the public as well as private sector [Luss, 1982]. Energy capacity expansion problems are a subtype of capacity expansion models that often introduce extra complexity due to the limited capacity of transmission lines, multiple nodes with varying needs and production capacities as well as time-variable and uncertain demand and future building costs [Pereira and Pinto, 1991, Baringo and Conejo, 2013].

Various methods have been developed to address this uncertainty and stage-wise nature of investment making, including approximate dynamic programming [Bukenberger and Palmintier, 2018], linear-decision rule approaches [Dominguez et al., 2016] and rolling-window approaches [Dominguez et al., 2015, Domínguez et al., 2021] among many others. One critical modeling decision in all of the approaches is the number and time distribution of the investment stages [Domínguez et al., 2021, Jeon, 2023]. However, the sensitivity of the results on the number of investment stages is not thoroughly examined in the literature with relatively few studies focusing on this critical hyperparameter.

While increasing the number of stages allows for more granular modeling of the underlying problem and, as such, makes results more accurate, adding extra stages comes with computational and numerical challenges. Some formulations see problem size and solution time growing exponentially with respect to the number of stages [Domínguez et al., 2021]. As such, getting the required precision by adding stages might not always be a feasible option. Alternative formulations and models can be examined to alleviate this trade-off between accuracy and computational feasibility and to obtain better performance with revised scenario construction and problem formulation. It is thus important to understand which models are most sensitive to the number of investment stages.

One of the tools that can be used to decrease the computational complexity of such problems is stochastic dual dynamic programming (SDDP). However, capacity expansion problems need to be reformulated to suit the needs and limitations imposed by the framework and are not suitable for SDDP out of the box.

The goal of this thesis is three-fold: first of all, demonstrate how multi-stage energy capacity planning problems can be solved with SDDP, secondly leverage the ability of SDDP to incorporate path-dependency to reduce the computational complexity of multi-stage problems by generating a tree-like scenario structure, and finally evaluate how the number of stages affects solution quality for different formulations of energy capacity expansion problems.

The thesis is structured in the following way: in Section 2 the multi-stage capacity expansion problem definition is introduced and a literature review is presented. Next, core assumptions, strengths, and weaknesses of SDDP are introduced. Section 3 formulates the renewable capacity expansion problem as a stochastic dual dynamic program and introduces three alternate formulations, each aiming to increase the quality of the model. Finally, the sensitivity of the results to the number of investment stages is examined by evaluating model performance in a realistic scenario in Section 4. Both in-group performances of each model with respect to the number of investment stages as well as the between-group performance of the models are compared in Section 5. Finally, directions for further research are proposed in Section 6.

2 Background

2.1 Capacity expansion formulation

2.1.1 Current formulation

While there are many ways to formulate a capacity expansion problem, we will be building on a foundation similar to that described in [Domínguez et al., 2021]

We model a simplified energy system during a fixed planning horizon with the following defining characteristics:

– N nodes, indexed by $n \in 1 : N$

– L transmission lines, indexed by $l \in 1 : L$

– G generators, indexed indexed by $g \in 1 : G$

– O operating points, indexed by $o \in 1 : O$

– T stages, indexed by $t \in 1 : T$

The planning horizon is split into an arbitrary number of stages. At the beginning of each stage, we have the option to invest in power-generating units. At that point, the investment costs for the stage are known but demand during the stage is uncertain. After the investment decisions are made, demand growth uncertainty is revealed and operating decisions are made. The goal of operating decisions is to satisfy time-varying electricity demand at each node. If the demand is not met, an unserved demand penalty cost C_{US} is incurred. The demand can be met by generating power from the generators, each located at some node in the network. The generation capacity of units is limited by the installed capacity P_{gt}^{Ibuilt} as well as the time-dependent availability factor of given power source F_{got}. Electricity can be transferred between the nodes by the aforementioned transmission lines, each of which has its own maximum capacity P_l^{Lmax}. To make this model more tractable and to focus on the goals of this thesis, a few simplifying assumptions are made:

- Since our goal is to model the expansion of power system generation we do not include the possibility to invest in transmission line capacity

- Demand growth and investment costs are stochastic and hence uncertain, other input parameters are deterministic, though they can vary with time

- No unit commitment or ramping constraints are considered, meaning power flow through the system can be changed instantly
- Linear direct current representation of the power system is considered as in Domínguez et al. [2021]
- Instead of modeling the demand and operating decisions for the whole year, yearly variation in demand and power availability is condensed into a smaller group of representative operating points.
- The lifetimes of generating units built are longer than the planning horizon, meaning only units existing during the start of the horizon can be decommissioned. The models can easily be refined to accommodate shorter decommissioning schedules.
- Only one objective is considered - minimizing the expected total costs of our power system over the planning horizon

To accurately describe this system, we need decision variables for operative and investment decisions. For investment decisions, we need $p^I_{gt}(\widetilde{\xi}_{t-1})$, the amount of power generation of generator g built at time t. Here $\widetilde{\xi}_t$ represents the concatenated realizations of all stochastic parameters up to and including stage t. Operating decisions can be characterized by unserved demand $d^{US}_{not}(\widetilde{\xi}_t)$, power generated at each power plant $p^G_{got}(\widetilde{\xi}_t)$, and power flow through each transmission line $p^L_{lot}(\widetilde{\xi}_t)$ at each stage g and operating point o. The details of the stochastic scenarios are further described in Section 4.

Each unit has its own respective stochastic investment cost per megawatt of capacity built at stage t. Thus, the investment costs at stage t are calculated as shown in Equation (1), where $\widetilde{c}^I_{gt}$ refers to the realized annualized investment cost of the generating unit g at stage t and a_t is the years left in the planning horizon and A the annualization factor. Equation (1) further shows the vectorized form of the equation where the indexing on g is dropped, and variables are assumed to be vectors of length G.

$$C^I_t = \sum_{g=1}^{G} A a_t p^I_{gt}(\widetilde{\xi}_{t-1})\widetilde{c}^I_{gt} \tag{1}$$

Operating costs are incurred only from power generation and unserved demand. Operating costs at stage t are given in Equation (2), where H_o refers to the number of hours comprising the operating point o.

$$C^O_t = \sum_{o=1}^{O} H_o(\sum_{n=1}^{N} d^{US}_{not} C_{US} + \sum_{g=1}^{G} p^G_{got} C^G_{gt}) \tag{2}$$

With this description, we arrive at the complete model formulation described below:

$$\underset{d^{US}_{not},p^{G}_{got},p^{L}_{lot},p^{I}_{gt}}{\text{Minimize}} E_{\widetilde{\xi}}\Big\{\sum_{t=1}^{\mathrm{T}}\sum_{g=1}^{G} Aa_t p^I_{gt}\widetilde{c}^I_{gt} + \sum_{o=1}^{O} H_o(\sum_{n=1}^{N} d^{US}_{not}C_{US} + \sum_{g=1}^{G} p^G_{got}C^G_{gt})\Big\} \tag{3a}$$

s.t.

$$\sum_{m=1}^{t} p^I_{gm} \leqslant P^{Imax}_{gt} \ \forall t,g \tag{3b}$$

$$\sum_{m=1}^{t} p^I_{gm} - F_{go}p^G_{got} \geqslant 0 \ \forall t,g,o \tag{3c}$$

$$\sum_{g\in GN(n)} p^G_{got} + \sum_{l\in LN(n)} p^L_{lot} = d^{US}_{not} + \widetilde{d}_{not} \ \forall t,n,o \tag{3d}$$

$$|p^L_{lot}| \leqslant P^{Lmax}_{l} \ \forall t,l,o \tag{3e}$$

$$p^I_{gt}, d^{US}_{not}, p^G_{got} \geqslant 0 \ \forall t,g,o \tag{3f}$$

$$p^I_{gt} \in \sigma\left(\xi_{t-1}\right) \tag{3g}$$

$$d^{US}_{not}, p^G_{got}, p^L_{lot} \in \sigma\left(\xi_t\right), \tag{3h}$$

where (3a) is the expected cost over the planning horizon, (3b) limits capacity construction per each generator node, (3c) limits energy production to the built capacity and the capacity availability, (3d) establishes power balance, with the sum of unserved and served demand being equal to the one transmitted to the node and generated at the node. (3e) and (3f) represent transmission line capacities and non-negativity of investment and generation decisions respectively. Constraints (3g) and (3h) represent non-anticipativity constraints as described in Ruszczynski [1997]. $GN(n)$ and $LN(n)$ represent the index set of generators and transmission lines bringing power to node n,and $\sigma(\xi_t)$ represents the set of functions depending only on realization of stochastic variables up to ξ_t.

2.1.2 Solution methods

Solution methods for multi-stage capacity expansion problems can be roughly split into two approaches: deterministic and stochastic. Deterministic approaches generally rely on exploring all paths of possible scenario realizations. However, with the number of constraints growing exponentially with respect to the number of stages, these methods can become too computationally demanding. [Domínguez et al., 2021, Ruszczynski, 1997]

A theoretical foundation for non-deterministic methods has been laid by Rockafellar and Wets [1991], Ruszczynski [1997] who linked non-anticipativity constraints and scenario structures, creating a new formulation that can be solved by cutting plane methods. They further proposed methods for alleviating the computational complexity with Monte Carlo sampling and regularization.

Domínguez et al. [2021] is one of the more recent papers focusing on comparing various solutions methods against each other. Among the tried solution methods were the following:

- Linear-decision rule approach, where it is assumed that decision variables have a linear relationship with the stochastic variables as described in Kuhn et al. [2011].
- Multi-stage stochastic programming approach, where the multi-stage problem is solved explicitly.
- Rolling-window approach, which approximates the solution to the whole multi-stage problem by solving multiple concurrent two-stage capacity expansion problems.

2.2 SDDP

2.2.1 SDDP in energy planning

Stochastic dual dynamic programming was first applied to energy planning by Pereira and Pinto [1991] and has since received widespread attention and both methodological as well as empirical developments.

Newham and Wood [2007] extended SDDP to allow for investment of integer sizes and restricted each investment to be used only once to allow electricity transmission planning with SDDP. Soares et al. [2017] devised an approach for decreasing solution variability (at the cost of increased expected cost) and Machado et al. [2021] created a way to parallelize the currently single-thread computations of SDDP by decreasing the number of synchronization threads in the algorithm. In this thesis, SDDP.jl [Dowson and Kapelevich, 2021] is used, as it implements these, among many other methodological advances.

2.2.2 Basics of SDDP

Stochastic dual dynamic programming can be used to solve multi-stage problems of the form given in Equation (4) by using the Bellman principle of optimality. Here C_t refers to the cost function, x_t, u_t to the state and control variables, and ω_t, Ω_t to realizations of random variables, and their sample space respectively. The indices corresponding to stages t and x_t may depend on all variables $\omega_i, x_i, u_i \forall i < t$. Similar problem formulations can be found in control theory - utilizing stage-wise costs and decisions (or *controls*) that affect the admissible decision space and costs at the next stages.

$$\begin{aligned} E_{\omega_1 \in \Omega_1} \Bigg[\underset{u_1}{\text{Min}} \Bigg\{ C_1(x_1, u_1, \omega_1) + E_{\omega_2 \in \Omega_2} \Bigg[\underset{u_2}{\text{Min}} \Bigg\{ C_2(x_2, u_2, \omega_2) \\ + E_{\omega_3 \in \Omega_3} \Bigg[\underset{u_3}{\text{Min}} \Big\{ x_3, u_3, \omega_3) + E(...) \Big\} \Bigg] \Bigg\} \Bigg] \Bigg\} \Bigg] \end{aligned} \tag{4}$$

A system solvable by stochastic dual dynamic programming is characterized by the nodes, states, controls, random variables, and the rules controlling them: decision rules, transition functions, and the objective function. In a simple SDDP system without feedback, each node (as represented in Figure 1) can be considered as belonging to a given stage. The node has an incoming state x and an outgoing state x'. Additionally, each node has its own random variables ω, realizations of which are independent of any other events in the model, and are considered as being scenarios. In the node, a decision rule $u = \pi_i(x, \omega)$ is used to map the incoming state to the outgoing state by the transition function $x' = T_i(x, u, \omega)$ while incurring a cost $C_i(x, u, \omega)$, this is the cost appearing in the recursive formula (4). This process is illustrated in Figure 1.

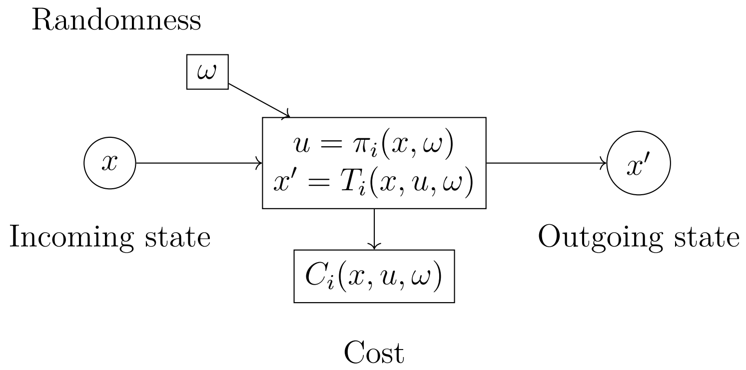

Figure 1: State transition diagram of a single node

Given a network of nodes and the transition probabilities between them, with the transition probabilities being fixed and not depending on realizations of any random variables or decisions made, the SDDP algorithm can minimize the total expected cost of the model by finding optimal decision rules.

The SDDP algorithm iterates over the following phases: A forward pass, where scenario realizations are sampled starting from the first stage until the last one. During the forward pass, approximate subproblems are solved for each of the states, using the approximation of the cost-to-go function derived from the cutting planes of previous iterations.

A backward pass, where the approximations of the cost-to-go functions are refined by adding new cuts to the subproblem as per Kelley's algorithm [Kelley, 1960]. Since cost-to-go is assumed to be convex with respect to the state variables, we receive an under-approximation of the true minimum of the cost-to-go function (and thus the overall optimization function).

This process allows us to refine our lower bound for the solution iteratively. After the iteration is complete, an upper bound for the resulting policy is estimated by sampling realizations of the scenario space and solving related problems with the cost-to-go function refined during the iteration process. This process can then be continued until sufficient convergence or number of iterations is reached.

3 Model formulation

3.1 Model 1

It should now be noted that the capacity expansion model previously described cannot be directly modeled as an SDDP problem and needs to be modified to suit the restrictions that SDDP imposes.

Let us start the process with the objective function described in Equation (3a). The objective contains the sum of expected values over stages. The goal is to split it into a stage-wise component and cost-to-go term while considering the non-anticipativity conditions on each variable. Disregarding constants, stage-wise cost $C_t = C_t^I + C_t^O$ can be described with decision variables p^I_{gt},d^{US}_{not},p^G_{got}, p^L_{lot}, stochastic parameters $\tilde{c}^I_{gt}$,$\tilde{d}_{not}$, and the investment decisions in previous stages p^I_{gm}, as described in (1) (2). Hence, the only decision variables that affect future stage-wise costs are the investment decisions, which do so through the sum $P^{Ibuilt}_{gt} = \sum_{m=1}^{t} p^I_{gm}$, which is equal to the total capacity built until the stage t (that is, the total capacity at the stage t) as described in Equation(5). It follows that the problem can be decomposed as follows: $p^I_{gt}, d^{US}_{not}, p^G_{got}, p^L_{lot} \in u$ are the control variables, $P^{Ibuilt}_{gt} \in x$ are the state variables, with the corresponding transition function shown in Equation (5).

$$P^{Ibuilt}_{g(t+1)} = P^{Ibuilt}_{gt} + p^I_{gt} \tag{5}$$

Finally $C_t \in \sigma(x, u, \omega)$. The stochastic parameters $\tilde{c}^I_{gt}, \tilde{d}_{not}$ are assumed to be independent of state variables and will be modeled by discrete scenarios. The decision process can be formulated as follows:

Stage 0 Investment costs for period 1 become known and capacity is built, no operating decisions are made.

Stage 1 Demand for period 1 becomes known and operating decisions for period 1 are made. Investment costs for period 2 become known and investment decisions for period 2 are made

Stage t Operating costs for period t become known and operating decisions for period t are made. Investment costs for period $t+1$ become known and investment decisions for period t are made.

Stage T Operating costs for period T become known and operating decisions for period T are made

The process is illustrated in Figure 2, where I represents investment stages and O operational stages. Note that the realization of operating costs of period t and investment costs of period $t-1$ are done in the same period. This is possible due to the fact that the operating decisions are not influenced by the investment decisions in the following stages, as operating decisions do not affect the outgoing state, and investment decisions do not affect the constraints or cost of operating decisions of the same stage. We finally obtain the formulation described by (6). Ω_t represents the

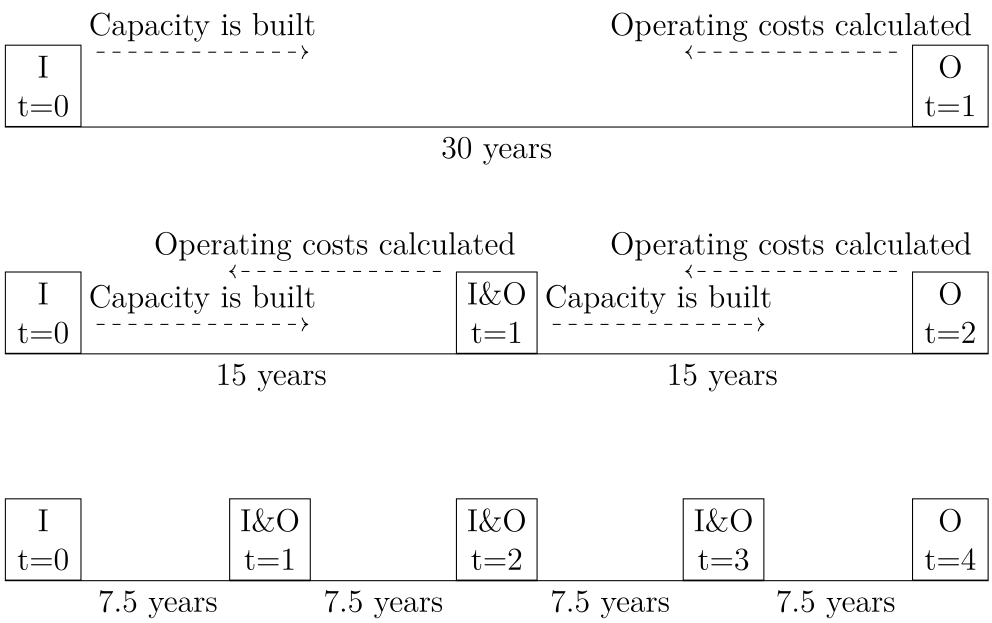

Figure 2: Illustrative figure of Model 1 with 1, 2 and 4 investment stages

discrete and finite set of uncertainty realizations at time t, these will be elaborated on in Section 4

$$\begin{aligned} \underset{u_0}{\text{Min}} \Bigg\{ C_0(P^{Ibuilt}_{g0}, u_0) + E_{\omega_1 \in \Omega_1} \Bigg[\underset{u_1}{\text{Min}} \Bigg\{ C_1(P^{Ibuilt}_{g1}, u_1, \omega_1) \qquad \qquad (6a) \\ + E_{\omega_2 \in \Omega_2} \Bigg[\underset{u_2}{\text{Min}} \Bigg\{ C_2(P^{Ibuilt}_{g2}, u_2, \omega_2) \\ + E_{\omega_3 \in \Omega_3} \Bigg[\underset{u_3}{\text{Min}} \Bigg\{ C_3(P^{Ibuilt}_{g3}, u_3, \omega_3) + E(...) \Bigg\} \Bigg] \Bigg\} \Bigg] \Bigg\} \Bigg] \Bigg\} \end{aligned}$$

Where:

$$C_t(P^{Ibuilt}_t, u_t, \omega_t) = \sum_{g=1}^{G} Aa_t p^I_g \tilde{c}^I_{g\omega_t} + \sum_{o=1}^{O} H_o(\sum_{n=1}^{N} d^{US}_{no} C_{US} + \sum_{g=1}^{G} p^G_{go} C^G_g) \qquad (6b)$$

With decision policy u being restricted by following at each stage t

$$p^{Ibuilt}_{gt} \leqslant P^{Imax}_{gt} \ \forall g \qquad (6c)$$

$$p^{Ibuilt}_{gt} - F_{got} p^G_{got} \geqslant 0 \ \forall g, o \qquad (6d)$$

$$\sum_{g \in GN(n)} p^G_{got} + \sum_{l \in LN(n)} p^L_{lot} = d^{US}_{not} + \tilde{d}_{no\omega_t t} \ \forall n, o \qquad (6e)$$

$$p^I_{gt}, d^{US}_{not}, p^G_{got} \geqslant 0 \ \forall g, o \qquad (6f)$$

$$|p^L_{lot}| \leqslant P^{Lmax}_l \ \forall l, o \qquad (6g)$$

And the transition function between states being

$$P^{Ibuilt}_{g(t+1)} = P^{Ibuilt}_{gt} + p^I_g \qquad (6h)$$

3.2 Model 2

While Model 1 is equivalent to the classical formulation of the capacity expansion problem, if we want to understand how uncertainty affects planning decisions, we must next focus on making decisions and costs compatible across simulations. In the above formulation, as in Domínguez et al. [2021] the last year of each period is used to represent the demand during the whole period. This can overestimate the total demand (in the usual case of positive demand growth), and thus drive higher operating costs and unnecessarily large investments when the amount of stages is low. This leads to the number of investment stages affecting solutions both through increased uncertainty, as well as overestimation of demand, leading to large differences between solutions with different numbers of investment stages.

This effect can be mitigated and the operating costs of simulations with different numbers of investment stages can be made comparable. For example, when comparing models with 1, 2, and 4 investment stages, one can simulate each of these with 4, 2, and 1 operating stages per investment stage respectively. Figure 3 illustrates how the

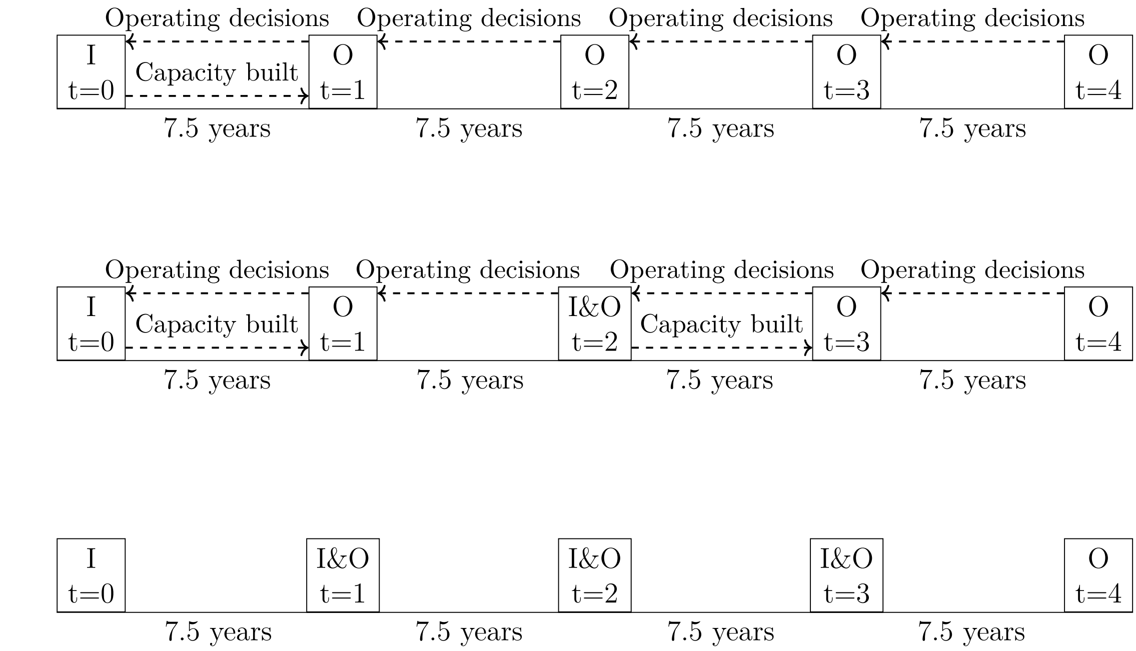

Figure 3: Illustrative figure of Model 2 with 1,2 and 4 investment stages, each having 4 operational stages

new stage structure is built, with each box representing one stage. where for each stage either operating decisions, investment decisions, or both are made. This change makes operating- and investment costs more comparable across the models with different numbers of investment stages. Generally, the process would be as follows:

Stage 0 Investment costs for period 1 become known and capacity is built, no operating decisions are made.

Stage t Operating costs for period t become known and operating decisions for period t are made. If stage t is an investment stage, investment costs for period t+1 become known and investment decisions for period t+1 are made.

Stage T Operating costs for period T become known and operating decisions for period T are made

This model can be implemented simply by increasing the number of stages in (6b) and adding a single new constraint as shown in (7), where $n_{o/d}$ represents the number of operating stages per one investment stage. Effectively the new constraint forces the new capacity built to be 0 for stages where investment decisions are not made.

$$p^I_{gt} = 0 \text{ if } (t-1) \not\equiv 0 \mod n_{o/d} \tag{7}$$

3.3 Model 3

In formulations of Model 1 and Model 2, capacity can only be built at the beginning of each investment stage, and decommissioning of power plants can cause the model to over-invest in generating capacity before necessary. Hence one of the reasons for the decrease in costs with an increase in investment stages would be due to the more granular planning capabilities - not the effect of uncertainty. The aim of the third model is to eliminate the effect of the inability to plan capacity building within an investment stage. This will help us further isolate the effect of uncertainty on the solution quality, as we remove the change in solutions caused by the decommissioning schedule and expected value of demand growth.

This model fixes the issue by creating capacity-building plans. The capacity-building plans are created during the investment stages, and the plan is valid up until the next investment stage. The capacity is then built according to the plan in between investment stages. This finally makes the increase of investment stages affect results only through a better understanding of the uncertainty of the parameters. The process is thus as follows:

Stage 0 Investment costs for period 1 become known and capacity is built, capacity building plans until the next investment stage are made. These are represented by state variable $p_{g0m}^{Iplanned}$, being the capacity of generator g that is going to be built at stage m, as decided at stage 0. No operating decisions are made.

Stage t Operating costs for period t become known and operating decisions for period t are made. If stage t is an investment stage, capacity-building plans until the next investment stage are made. These again are represented by $p_{gtm}^{Iplanned}$, where g and m are as before, and t details during which stage the capacity plans are valid. This plan is then passed onto the next stage as a state variable. If t is not an investment stage, capacity is built according to plan $p_{gtt}^{Iplanned}$, and the plans are passed forward as a state variable.

Stage T Operating costs for a period T become known and operating decisions for period T are made

Model 3 is illustrated in Figure 4, with a similar notation as before. It can be implemented by adding the capacity to be built as a state variable as shown in

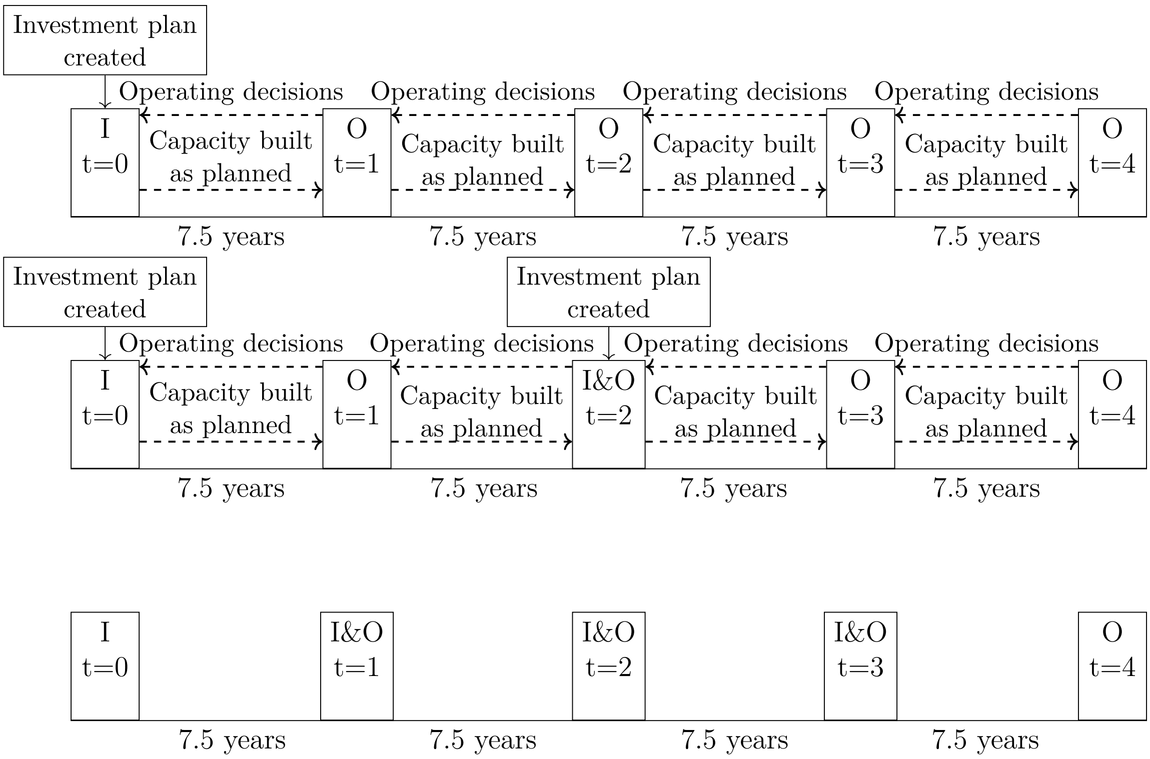

Figure 4: Illustrative figure of Model 3 with 1,2 and 4 investment decision stages, each having 4 capacity building and operational stages

Equation(8a):

$$C_t(P_t^{Ibuilt}, u_t, \omega_t) = \sum_{g=1}^{G} Aa_t p_g^I \tilde{c}_{g\omega_t}^I + \sum_{o=1}^{O} H_o(\sum_{n=1}^{N} d_{no}^{US} C_{US} + \sum_{g=1}^{G} p_{go}^G C_g^G) \tag{8a}$$

With decision policy u being restricted by following at each stage t

$$p_{gt}^{Ibuilt} \leqslant P_{gt}^{Imax} \ \forall g \tag{8b}$$

$$p_{gt}^{Ibuilt} - F_{got} p_{got}^G \geqslant 0 \ \forall g, o \tag{8c}$$

$$\sum_{g \in GN(n)} p_{got}^G + \sum_{l \in LN(n)} p_{lot}^L = d_{not}^{US} + \tilde{d}_{no\omega_t t} \ \forall n, o \tag{8d}$$

$$|p_{lot}^L| \leqslant P_l^{Lmax} \ \forall l, o \tag{8e}$$

$$p_{gt}^I = p_{gtt}^{Iplanned} \text{ if } (t-1) \not\equiv 0 \mod n_{o/d} \tag{8f}$$

$$p_{gt}^I, d_{not}^{US}, p_{got}^G, p_{gtm}^{Iplanned} \geqslant 0 \ \forall g, o, t, m \tag{8g}$$

And the transition function between states being

$$P_{g(t+1)}^{Ibuilt} = P_{gt}^{Ibuilt} + p_g^I \tag{8h}$$

$$p_{g(t+1)m}^{Iplanned} = p_{gtm}^{Iplanned} \text{ if } (t-1) \not\equiv 0 \mod n_{o/d} \forall m \tag{8i}$$

4 Case study

4.1 Input data

This case study is based on the 24-node reliability test system (RTS) network as specified by the IEEE standard [Ordoudis et al., 2016], the location of nodes and transfer lines between them are described in the article. The existing generating units, their locations, and capacities are detailed in Table 4.1. Cost of unserved demand, annualization factor, and initial demand are taken from Domínguez et al. [2021] and are respectively $C_{US} = 10000$, $A = 0.097$, $I_0 = 2508MW$. The decommissioning schedule is also taken from Domínguez et al. [2021].

For modeling investment costs, we split the energy production technologies into two groups that we model separately: mature technologies, which comprise biomass, onshore wind, and solar photovoltaic (PV); and maturing technologies, which comprise concentrated solar power (CSP) and offshore wind. The evolution of costs is characterized by Equations (9) and (10), where $\tilde{c}_{ts}^{ISD}$, $\tilde{c}_{ts}^{ISM}$ are the stochastic coefficients describing the evolution of costs of mature and maturing technologies respectively and $G_{mature}, G_{maturing}$ are the sets of generators representing mature and maturing technologies. Table 2 contains initial investment costs for the new generating units. Note that maximum capacities are 80% larger than in [Domínguez et al., 2021]. This is due to lower capacity availability assumptions, where the original paper could not be replicated due to data availability. If this change is not made the maximum available capacity is not enough for meeting demand.

$$\widetilde{c}^{I}_{g(t+1)} = c^{I}_{gt}\widetilde{c}^{ISD}_{ts} \text{ if } g \in G_{mature} \tag{9}$$
$$\widetilde{c}^{I}_{g(t+1)} = c^{I}_{gt}\widetilde{c}^{ISM}_{ts} \text{ if } g \in G_{maturing} \tag{10}$$

We assume that the demand for each operating point o, for each year is given by Equation(11), such that d^{N}_{n} captures the share of the total demand going into the node and as such $\sum_{n\in N} d^{N}_{n} = 1$ and is constant over operating points and years, as was assumed in [Domínguez et al., 2021]. The variable d^{O}_{o} gives the demand at each operating point o, modeling the seasonal changes in demand. The operating points for wind are taken from [Baringo and Conejo, 2013]. Due to challenges in data availability, solar photovoltaic power (PV) and concentrated solar power (CSP) plants are assumed to have static availability factors of 25%, as per statistics of U.S Energy information association [EIA, 2023] and 75% [Domínguez et al., 2021] respectively. While this decision can influence the absolute numbers of power capacity built, the purpose of this study is to examine new methodologies for finding optimal strategies for multi-stage decision problems, and as such does not require rigorous adherence to real-world scenarios. The term $\widetilde{d^{S}}_{ts}$ is the stochastic demand coefficient at stage t and scenario s. The scenario-generating technique is characterized in the next section.

For each model, we consider 3 versions with varying numbers of investment stages: 1,2, and 4. For Models 2 and 3, the number of operating stages per investment stage will be such that the total amount of operating stages will be 4. This way we ensure that operating points will be similar between all runs and we can better isolate the effect of uncertainty on the capacity planning.

$$d_{no(t+1)} = d^{N}_{n} d^{O}_{o} d_{not} \widetilde{d}^{S}_{ts} \tag{11}$$

Unit	Location node (#)	Variable cost \$/MWh	Capacity (MW)
CCGT1	15	60	600
CCGT2	22	66	600
Nuclear1	21	14	900
Nuclear2	13	16	800
Coal	2	25	500
OnWind1	23	1	750

Table 1: Location, generating capacity and variable cost of existing generating units

4.2 Uncertainty characterization

Now that we have defined most of the model, we are left with the task of characterizing the stochastic variables: the demand $\widetilde{d}^{S}_{ts}$ and the investment cost for mature and maturing technologies $\widetilde{c}^{ISD}_{ts}$, $\widetilde{c}^{ISM}_{ts}$ in each scenario.

Unit	Location node (#)	Investment cost ($/kW)	Variable cost ($/MWh)	Capacity (MW)
Biomass1	21	4114	46	630
Biomass2	23	4920	42	720
Biomass3	16	4885	48	630
OnWind2	16	1953	1	1260
OnWind3	15	1820	1	1260
OnWind4	18	1680	1	1350
OffWind	1	6820	2	1800
PV1	13	2165	1	540
PV2	18	2273	1	540
PV3	1	2085	1	450
CSP1	1	8220	3	450
CSP2	7	8320	3	450
CSP3	2	8667	3	360

Table 2: Locations, costs and maximum capacities of units that can be built

While SDDP can also incorporate other methods for uncertainty characterization, such as objective states [Downward et al., 2020], the scenario method was chosen since objective states cannot accommodate uncertain parameters in the constraints.

The scenario method relies on modeling the uncertainty as a Markov chain of discrete nodes and exploits the fact that SDDP can incorporate path dependency to the nodes by building an approximation of the cost function with respect to the incoming state variables in each node. Being able to model path-dependency here is crucial, as we can construct scenarios to be such that eg. if the number of 'high' and 'low' demand growth realizations is the same, the total demand will be the same irrespective of the order of realizations of these scenarios.

The scenario structure was constructed as follows:

- To align with Domínguez et al. [2021] 3 scenarios for demand growth rate (constant, moderate, and high) and 2 scenarios for investment costs (constant, decreasing) are used for each stage.
- To further maintain consistency with Domínguez et al. [2021] we choose coefficients to be such that the expected value for an increase in annual demand is 1%, and the expected value for the yearly decrease in mature and maturing technologies investment costs to be 0.5% and 1.3% respectively.
- The scenarios are assumed to be equiprobable, independent of past realizations, and influence variables as described in Equations 9,10, and 11
- Lowest demand growth and investment cost scenarios assume that the respective variables do not change $\widetilde{d}^S_{ts}, \widetilde{c}^{ISD}_{ts}, \widetilde{c}^{ISM}_{ts} = 1$
- High and medium demand growth scenarios are chosen to be such that they form a trinomial tree, meaning $\widetilde{d}^S_{t(high)} = (\widetilde{d}^S_{t(medium)})^2$

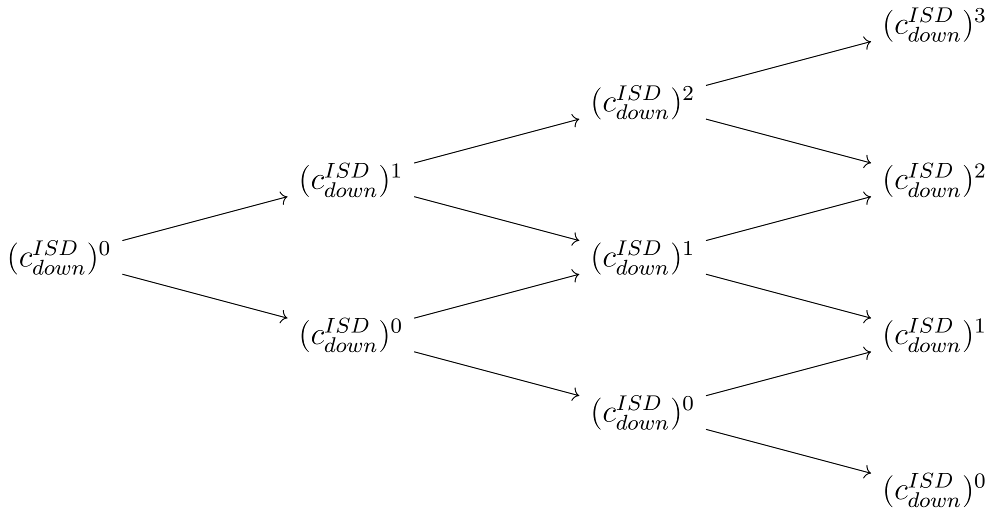

Figure 5: Binomial tree showing possible evolution of investment costs of mature technologies in first 3 stages

Scenario s	$\widetilde{d}_{ts}^{S}$	$\widetilde{c}_{ts}^{ISD}$	$\widetilde{c}_{ts}^{ISM}$
1	1.01^2	1	1
2	1.01	1	1
3	1	1	1
4	1.01^2	0.974	0.99
5	1.01	0.974	0.99
6	1	0.974	0.99

Table 3: Scenarios and respective yearly changes in stochastic coefficients

From the above points, it follows that investment costs either stay constant or decrease by a factor of c_{down}^{ISD} and c_{down}^{ISM} for mature and maturing costs respectively. This is illustrated in the binomial tree shown in Figure 4.2. Demand either stays constant,increases by d_{up}^{S}, or increases by $(d_{up}^{S})^2$ as illustrated in Figure 4.2. Finally, when we take into account the expected values for demand growth and investment growth declines, we can solve for the coefficients, and arrive at the coefficients for scenarios described in Table 4.2. Note that these are yearly changes in the coefficients and thus need to be raised to the power of $30/N_o$ (where N_o stands for the number of operating stages in the model) to get the total change in the stochastic coefficient between two operating stages.

This way of characterizing uncertainty allows us to reduce the scenario space from being exponential with respect to the number of stages to being polynomial, with the number of distinct scenarios per stage being $t(2t-1) = 2t^2 - t$ and the number of distinct nodes in the model being $\sum_{t=1}^{T}(2t^2 - t) = O(T^3)$. This is a considerable reduction in growth rate compared to the original exponential growth presented in [Domínguez et al., 2021].

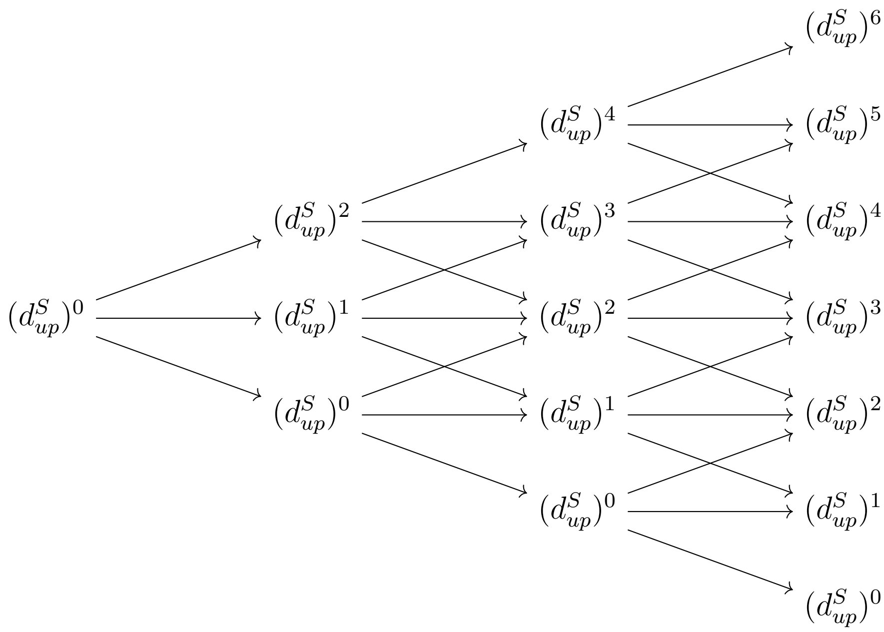

Figure 6: Trinomial tree showing possible evolution of demand in first 3 stages

4.3 Hypotheses

Based on the properties of Models 1, 2, and 3, several hypotheses can be formulated:

H1: Model 1, with its limited ability to build capacity except at the beginning of each period, as well as its tendency to overestimate demand due to only considering last year's operating points into consideration will result in higher investment and operational costs. This model will likely exhibit the largest decline in costs as the number of stages increases.

H2: Model 2 is expected to result in lower investment and operational costs compared to Model 1. The decline in costs with an increasing number of stages is expected to be more moderate compared to Model 1.

H3: Model 3, with its capacity-building plans and ability to build capacity according to those plans, will likely have the lowest investment costs but higher operational costs compared to Model 2. The decline in costs as the number of stages increases is expected to be the smallest among the three models.

H4: As the number of stages increases, the differences in costs between the three models are expected to decrease. This is because a better understanding of uncertainty in parameters can be achieved with a higher number of investment stages. The 4 stage models will have identical solutions, since their problem formulations reduce to each other when there is only 1 operational stage per 1 investment stage.

5 Results

5.1 Hyper-parameters and convergence

The model was solved with SDDP.jl using the default settings [Dowson and Kapelevich, 2021]. The stopping conditions were that the lower bound converges (does not move 0.001 for 20 iterations) or 20,000 iterations are run, whichever comes first. All of the models converged with the bound stalling condition. Solving time was 3 hours on an AMD Ryzen 4000 processor, showing that the scenario reduction technique and SDDP have reduced the computational complexity significantly from the original formulation of Domínguez et al. [2021]. For generating result tables 20 000 realizations of the evolution of demand and investment costs were sampled and solved with the models.

5.2 Model comparison

Table 4 contains the average total costs for each of the models. In line with H1, model 1 sees the greatest decline in costs with an increase in the number of investment stages. With 1 investment stage, the total costs were 230% higher compared to the version with 4 stages.

	Investment stages		
	1	**2**	**4**
Model 1	72643	39542	21971
Model 2	47767	27784	21971
Model 3	23831	22028	21971

Table 4: Average total costs of each model per the number of investment stages

This 230% difference reduces to 117% with 1 investment stage run of model 2. Hence being able to more granularly model operating points managed to reduce our optimality loss by roughly 50%. Model 3 with 1 investment stage had only an 8.5% cost difference to the optimal. The total cost for all of the models with 4 investment stages is the same. This is expected, as the core difference between Model 1 and Models 2 and 3 is the fact that Model 2 can handle multiple operating stages per investment stage, and Model 3 further refines that approach. When there is only one operating stage per investment stage, models are expected to yield the same formulation as Model 1 and thus the same result.

Putting the above numbers in perspective - the optimality loss of model 1 with 1 investment stage was influenced by the following: more granular modeling of operating points caused 49% of the observed effect on total costs, being able to plan capacity building ahead had a total effect of 47%, and the final 4% were caused by being able to change plans once observing the uncertainty realization.

As mentioned above, hypothesis 1 was confirmed by the data, with model 1 seeing by far the greatest absolute costs and greatest decline of costs with an increase in the number of stages. Hypothesis 2 is also validated by the data with Model 2 seeing both lower investment and operational costs than Model 1 as shown in Tables 5 and 7. Due to the ability to create capacity-building plans Model 3 manages to build more capacity than Model 2 while paying less, due to the annualization of costs, as seen in tables 5, and 6. These tables also support hypothesis 3, as we see model 3 having lower investment costs, but higher operating costs than model 2. Hypothesis 4 is also validated clearly, with all of the models converging to the same solution when the number of investment stages was equal to 4. Table 8 shows that Model 3 had the lowest unserved demand cost in most cases - not something discussed in our hypotheses, but an interesting finding for further evaluation.

	Investment stages		
	1	**2**	**4**
Model 1	62890	28235	12385
Model 2	35204	18069	12385
Model 3	13966	12304	12385

Table 5: Average investment costs of each model per the number of investment stages

	Investment stages		
	1	**2**	**4**
Model 1	5373	4198	3884
Model 2	3460	3889	3884
Model 3	4643	4170	3884

Table 6: Average capacity build in each model per the number of investment stages

	Investment stages		
	1	**2**	**4**
Model 1	8726	11307	9269
Model 2	8849	8863	9269
Model 3	9191	9398	9269

Table 7: Average operating costs of each model per the number of investment stages

	Investment stages		
	1	**2**	**4**
Model 1	1027	0	317
Model 2	3714	852	317
Model 3	674	533	317

Table 8: Average unserved demand costs of each model per the number of investment stages

5.3 Limitations

While the initial hypotheses are confirmed, there are caveats, as with any study. First of all, the reference point for optimal costs was a model with 4 stages. The real loss of optimality is likely higher and could be further reduced by increasing the number of stages. Also, while better modeling of operating points (models 2 and 3) allowed for a great reduction in total cost, the final solution was a relatively coarse approximation of real-life capacity planning. For example, all models had the fundamental flaw that the last year's demand for a stage is assumed to be representative of the whole period, which leads to higher total demand than in reality.

Furthermore, the relative magnitudes of the impact of uncertainty, operation point modeling, and capacity building plans might change in a different electric grid configuration or uncertainty characterization. For example, if the uncertainty of demand or investment costs was higher, the effect of uncertainty on total costs could be more than the 4% in this case study.

6 Conclusions

This thesis aimed to shed light on how sensitive are different problem formulations to the number of investment stages in multi-stage capacity expansion problems. Three different models with varying amounts of investment stages were created: one model using the classical formulation, one having enhanced operating point modeling, and the last one introducing capacity-building plans.

These models were combined with a custom scenario-generating technique to reduce computational complexity and solved with SDDP.jl to test four hypotheses: the simple model has the largest loss of optimality when the number of investment stages is decreased; the model with enhanced operating points results in lower investment and operating costs than standard model; model 3 sees lower total costs, with lower investment costs, but higher total costs than model 2. The final hypothesis was that the difference between solutions decreases as the number of stages increases. All of these hypotheses were supported by the results.

The results showed that the ability to accurately model operating points and plan capacity building within an investment stage the most effect on solution accuracy, with the realization of uncertainty being a relatively small factor. This suggests that enhancing operating point modeling could be a potential avenue for further research.

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7 Appendix

7.1 Notation

1. **Indices and sets**

t stages, indexed from 0 to T

l transmission lines, indexed from 1 to L

n nodes, indexed from 1 to N

g generators, indexed from 1 to G

o operating points, indexed from 1 to O

$GN(n)$ set of generators located at node n

$LN(n)$ set of transmission lines having a connection to node n

$G_{mature}, G_{maturing}$ Sets of generators representing mature and maturing technologies respectively

Ω_t Discrete and finite set of scenarios at stage t. A single scenario represented by ω_t

2. **Helping variables**

C_t Total costs at stage$t$

C_t^I Investment costs at stage$t$

C_t^O Operating costs at stage$t$

3. **Unused variables (General formulation of capacity expansion)**

$\tilde{\xi}_t$ Concatenated realizations of all random variables up to stage t

4. **SDDP State variables**

P_{gt}^{IBuilt} Capacity of generator g at stage$t$, can be increased through p_{gt}^{I}

$p_{gtm}^{Iplanned}$ Capacity of generator g to be built at stage m as per plans valid at stage t

5. **SDDP Decision variables**

p_{gt}^{I} Power capacity to be built of generator g at stage t

d_{not}^{US} Amount of unserved demand at node n, operating point o and time t

p_{got}^{G} Power generated at unit g, at operating point o, at time$t$

p_{lot}^{L} Power running through transmission line l at operating point o and stage t

6. **Parameters**

P_{l}^{Lmax} Maximum transmission capacity of line l

P_{gt}^{Imax} Maximum amount of power capacity that can be built for generator g at time$t$

C_{US} Cost per MW of unserved demand

H_o Number of hours in operating point o

a_t Years left in planning horizon at stage t

C_{gt}^{G} Cost to generate one megawatt of capacity from generator g at time t

F_{got} Availability factor of generator g at operating point o and time t

A Annualization factor

$n_{o/d}$ Amount of operating stages per investment stage

7. **Stochastic parameters**

$c_{g\omega t}^{I}$ Cost to build one megawatt of capacity for generator g at stage t in scenario ω

$d_{no\omega t}$ Demand at node n, operating point o, scenario ω and time t